Dynamically stabilized decoherence-free states in non-Markovian open fermionic systems

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Decoherence-free subspaces (DFSs) provide a strategy for protecting the dynamics of an open system from decoherence induced by the system-environment interaction. So far, DFSs have been primarily studied in the framework of Markovian master equations. In this work, we study decoherence-free (DF) states in the general setting of a non-Markovian fermionic environment. We identify the DF states by diagonalizing the nonunitary evolution operator for a two-level fermionic system attached to an electron reservoir. By solving the exact master equation, we show that DF states can be stabilized dynamically.

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I. INTRODUCTION

Quantum decoherence is a fundamental issue in open quantum systems and is also the most important obstacle to the realization of a large-scale quantum computer. Unavoidable interactions with noisy environments typically render initially prepared pure states mixed very rapidly. Nonetheless, a subspace of Hilbert space can exist where a system undergoes a unitary evolution irrespective of the interaction with its environment. Such a decoherence-free subspace (DFS) can in principle provide a theoretically perfect strategy to protect a system against quantum decoherence. The possibility of DFSs was already pointed out by Zurek [1], who observed that if the interaction Hamiltonian of the system with the environment has a degenerate eigenvalue, then superpositions of the corresponding eigenstates remain coherent. Generally, degenerate eigenvalues can notably arise if the coupling has a certain symmetry, and the resulting protection against decoherence has been observed early in the context of rotational tunneling [2–9]. In the late 1990s, DFSs were rediscovered independently by several groups [10–15]. The theory of DFSs has already been extensively discussed in the context of the symmetries of the Hamiltonian [12,13], semigroup dynamics in the language of a quantum master equation [14–19], and the operator sum representation based on Kraus operators [20]. Meanwhile, the existence of DFSs has also been verified experimentally with polarization-entangled photons [21–23], trapped ions [24,25], nuclear spins using nuclear magnetic resonance techniques [26–29], and neutron interferometry [30]. These experiments show that encoding quantum information in a DFS can significantly prolong the storage time. Therefore, DFSs have attracted wide interest for applications in fault-tolerant quantum computation [17,31,33–38], long distance quantum communication [39], quantum key distribution [40], quantum teleportation [41], quantum metrology [42], robust quantum repeaters [43,44], and coherent quantum control [45–47]. The requirement of symmetry in the coupling to the environment is not always necessary [48]. Recently, a new Heisenberg-limited metrology protocol was proposed which exploits the evolution of a DFS due to a collective change of the couplings to the environment [49].

According to Refs. [14,16], a DFS in a system ruled by Markovian dynamics is defined formally by the vanishing of the nonunitary Lindblad (decohering) part in the Markovian master equation [50]; see Eq. (3). This defines a Hilbert subspace in which the dynamics is locally (in time) unitary. Even so, decoherence can still arise if the Hamiltonian of the system drives a state out of the DFS. This happens on the time scale of the system Hamiltonian, which is typically much longer than the microscopic decoherence times [32]. However, leaking out of the DFS can be suppressed by coupling the system relatively strongly to the environment [33,51]. An alternative definition of a DFS in the Markovian context has been given in Ref. [18]. These authors not only derived necessary and sufficient conditions for the vanishing of the nonunitary part of the master equation [see Eq. (4)], but also found criteria for globally DF states, for which the dynamics resulting from the full master equation (including the Hamiltonian part) remains unitary.

The extension of such DFS criterion to non-Markovian dynamics is not straightforward. The general non-Markovian master equation of the Nakajima-Zwanzig form [52] involves a complicated time-nonlocal memory integrand in the nonunitary terms. However, the exact master equations that describe the general non-Markovian dynamics have been recently developed for some classes of open quantum systems, including quantum Brownian motion [53–55], entangled cavities with vacuum fluctuations [56], coupled harmonic oscillators [57,58], quantum dot electronic systems in nanosturctures [59,60], various nanodevices with time-dependent external control fields [61], nanocavity systems including initial system-reservoir correlations [62], and photonic networks imbedded in photonic crystals [63]. These exact master equations can all have the nonunitary Lindblad form, but the decoherence rates are time dependent and may become negative during the time evolution. This is different from the Markovian case, where the decoherence rates are always positive. Since the fact that all rates have the same sign plays an essential role in showing the necessity of the DFS criterion, other possibilities of creating a DF state may arise in non-Markovian cases.

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In this work, we study the dynamics of an open fermionic system by solving the exact non-Markovian fermionic master equation [59–61]. We find pure DF states which arise from the fact that certain time-dependent decoherence rates in the master equation switch themselves off after the system reaches a stable state. We call these DF states dynamically stabilized DF states. They are generated in particular for non-Markovian environments. The mechanism of how the DF states arise is therefore very different from the known mechanism obtained from the Markovian master equation. For a two-level fermionic system coupled to an electron reservoir, we find two dynamically stabilized DF states that possess full quantum coherence between the singly occupied states of the original two levels. Practical applications of the dynamically stabilized DF states in electron spin and charge qubits for quantum information processing are expected.

The paper is organized as follows. In Sec. II, we briefly review the general criterion of DFSs based on the Markovian master equation formalism. In Sec. III, we discuss the dynamics of electron systems in nanostructures via the exact master equation. In particular, we consider a two-level electron system whose two levels are coupled identically to an electron reservoir. We find that the system Hilbert space can be split into two closed subspaces. Then in Sec. IV, in terms of the full Lindblad generator, we discuss all the possibilities of how DF states can arise in the system. We find that the vanishing of one of the two decoherence rates in the problem can give rise to dynamically stabilized DF states. With only one decoherence term in the master equation remaining, the well-known criterion of DFSs for Markovian decoherence still provides a necessary and sufficient condition for DF states. In Sec. V, we investigate under what conditions the dynamically stabilized DF states can be reached by the same dissipative process that allows their existence. We find that the initial state and the details of the dynamics given by time-dependent nonequilibrium Green's functions determine which DF state can be reached. In Sec. VI, we discuss the generation of DF states in the Born-Markovian (BM) dynamics within our exact framework. We show that the concept of DF states in the BM dynamics is a special case of the exact solution. Finally, a conclusion is given in Sec. VI.

II. CRITERION FOR DFS IN MARKOVIAN CASE

In this section, we briefly review the general criterion of DFS based on the Markovian master equation following Ref. [18]. Consider an open system S coupled to an external noisy environment E; we can write the total Hamiltonian as

$$H = H_S \otimes I_E + I_S \otimes H_E + H_I, \tag{1}$$

where I is the identity operator and H_I denotes the interaction Hamiltonian between the system and its environment. If the dynamics of the system is Markovian and the system and the environment are initially decoupled, the master equation for the density matrix of the system takes the Lindbladian form [64–66]

$$\dot{\rho}(t) = -i[\widetilde{H}_S, \rho(t)] + L[\rho(t)], \tag{2a}$$

$$L[\rho] = \frac{1}{2}\sum_{\alpha=1}^{N} a_\alpha (2F_\alpha \rho F_\alpha^\dagger - F_\alpha^\dagger F_\alpha \rho - \rho F_\alpha^\dagger F_\alpha), \tag{2b}$$

where $\widetilde{H}_S = H_S + \Delta$ is the renormalized system Hamiltonian with Δ a possible Hermitian contribution from the environment ("Lamb shift"), F_α are orthogonal operators on the system Hilbert space $\mathcal{H}_S$, and a_α denote real positive coefficients. Thus, the commutator involving $\widetilde{H}_S$ in Eq. (2a) determines an effectively unitary evolution of the system, while the decohering effect induced by the environment is totally accounted for by the nonunitary term $L[\rho]$.

For the Markovian master equation (2), the condition of an instantaneous DFS at time t amounts to the vanishing of $L[\rho]$, that is [18],

$$L[\rho_{\mathrm{DF}}(t)] = 0. \tag{3}$$

This ensures that $\rho_{\mathrm{DF}}(t)$ obeys a unitary evolution $\dot{\rho}_{\mathrm{DF}}(t) = -i[\widetilde{H}_S, \rho_{\mathrm{DF}}(t)]$ at time t. This does not imply necessarily unitary evolution at all times, as the evolution due to $\widetilde{H}_S$ can drive the system out of the DFS. In [14] the DFS found from Eq. (3) was therefore called DFS to the first order.

A sufficient and necessary condition for a state $|k_{\mathrm{DF}}\rangle$ to be locally DF is given in terms of the operators $\{F_\alpha\}$ [18] by

$$\begin{aligned} F_\alpha |k_{\mathrm{DF}}\rangle &= c_\alpha |k_{\mathrm{DF}}\rangle, \quad \forall \alpha, k_{\mathrm{DF}} \quad \text{and} \\ \sum_{\alpha=1}^{N} a_\alpha F_\alpha^\dagger F_\alpha |k_{\mathrm{DF}}\rangle &= \sum_{\alpha=1}^{N} a_\alpha |c_\alpha|^2 |k_{\mathrm{DF}}\rangle. \end{aligned} \tag{4}$$

Equation (4) means that the DF states are degenerate eigenstates of all the operators $\{F_\alpha\}$ and degenerate eigenstates of $\sum_{\alpha=1}^{N} a_\alpha F_\alpha F_\alpha^\dagger$. A special case is given by $c_\alpha = 0\ \forall \alpha$, and the second condition in Eq. (4) is then automatically fulfilled. In Ref. [18] it was shown that the space spanned by the $|k_{\mathrm{DF}}\rangle$ is a DFS at all times, if and only if in addition to satisfying (4) it is also invariant under $\widetilde{H}_S$.

The generalization of Eq. (4) to non-Markovian master equations is not straightforward. However, it was recently shown [67] that under certain conditions the solution of a non-Markovian master equation [52] with a finite-time memory kernel can be at the same time a solution of a local-in-time non-Markovian master equation. In principle, local-in-time generalizations of the Markovian master equation (2b) may be obtained by making both the rates a_α and the operators F_α time dependent. Interestingly, the exact master equations describing the general non-Markovian dynamics for a large class of bosonic and fermionic systems [56,59–63] have been developed recently. They all have the Lindbladian form of Eq. (2), but the decoherence rates a_α in the nonunitary term (2b) are time dependent and local in time, whereas the operators F_α are time independent. The time-dependent decoherence rates are determined microscopically and nonperturbatively by the retarded and correlation Green's functions in nonequilibrium Green's function theory [68], where the backactions from reservoirs are fully taken into account. As a

result, the non-Markovian dynamics is fully characterized by the time-nonlocal retarded integrand in the Dyson equation, which governs the nonequilibrium Green's functions [69,70]. In addition to being time dependent, the decoherence rates can become negative for short times, representing in a certain sense the backflow of information from the environment to the system [59,71–74]. Since the fact that all a_α have the same sign is an important requirement for showing that Eq. (4) is necessary for a Markovian DF state (see the argument after Eq. (3.15) in Ref. [18]), additional DF states may arise in the non-Markovian case. In the following, we provide a new mechanism for generating dynamically stabilized DF states in fermionic systems, based on the exact fermionic master equation developed recently [59–61].

III. EXACT MASTER EQUATION

We consider a general nanoelectronic system with N energy levels coupled to an electron reservoir. The Hamiltonian for the system, the electron reservoir, and the interaction between them read

$$\begin{aligned} H_S &= \sum_{i=1}^{N} \epsilon_i a_i^\dagger a_i, \quad H_B = \sum_k \varepsilon_k c_k^\dagger c_k, \\ H_I &= \sum_{i=1}^{N} \sum_k (V_{ik} e^{i\phi_i} a_i^\dagger c_k + V_{ik} e^{-i\phi_i} c_k^\dagger a_i). \end{aligned} \tag{5}$$

Here $a_i^\dagger$ and a_i are electron creation and annihilation operators for ith level with energy ϵ_i. The operators $c_k^\dagger$ and c_k denote electron creation and annihilation operators for the energy level ε_k of the electron reservoir. The coupling strength between the system and the reservoir is described by the $V_{ik} \in \mathbb{R}$, with an explicit phase ϕ_i. The total Hamiltonian is $H = H_S + H_B + H_I$.

The exact master equation for the system was obtained in [59–61],

$$\dot\rho(t) = -i[\widetilde H_S(t), \rho(t)] + L[\rho(t)], \tag{6a}$$

where the renormalized system Hamiltonian $\widetilde H_S(t)$ and the decoherence term $L[\rho(t)]$ take the form

$$\widetilde H_S(t) = \sum_{i,j=1}^{N} \widetilde\epsilon_{ij}(t) a_i^\dagger a_j, \tag{6b}$$

$$\begin{aligned} L[\rho(t)] = \sum_{i,j=1}^{N} \{ &\kappa_{ij}(t)[2a_j\rho(t)a_i^\dagger - a_i^\dagger a_j \rho(t) - \rho(t) a_i^\dagger a_j] \\ &+ \widetilde\kappa_{ij}(t)[2a_i^\dagger \rho(t) a_j - a_j a_i^\dagger \rho(t) - \rho(t) a_j a_i^\dagger]\}. \end{aligned} \tag{6c}$$

Here the shifted energy $\widetilde\epsilon_{ij}(t)$ and the decoherence rates $\kappa_{ij}(t)$ and $\widetilde\kappa_{ij}(t)$ are all time dependent (but local in time). They are determined microscopically and nonperturbatively in terms of the retarded and correlation Green's functions by eliminating completely all the reservoir degrees of freedom (i.e., tracing over all the states of the environment). Their explicit forms are shown in Eq. (9) of Ref. [59].

To be more specific, let us consider the case of a nanoelectronic system with $N = 2$. Physically, such a system may be realized by a double quantum dot system in which each dot has a single active energy (on-site) level, coupled to electrodes with all spins polarized in both the dots and the electrodes. Another example for $N = 2$ is given by a single-level quantum dot coupled to electrodes with allowed spin flips between two antiparallel directions.

Furthermore, we assume that the two energy levels of the system are degenerate: $\epsilon_1 = \epsilon_2 = \epsilon_0$. Practically, the energy degeneracy is easier to be realized in the second setting than in the first. We also assume that both levels have the same coupling strength to the electron reservoir, that is, $V_{1k} = V_{2k} = V_k/\sqrt{2}$. Then we introduce two effective fermion operators

$$A_+ = \frac{1}{\sqrt2}(a_1 + e^{i\phi} a_2), \quad A_- = \frac{1}{\sqrt2}(-e^{-i\phi} a_1 + a_2),$$

with $\phi = \phi_1 - \phi_2$. The Hamiltonian (5) can be rewritten in terms of $A_\pm$ as follows

$$\begin{aligned} &H_S = \epsilon_+ A_+^\dagger A_+ + \epsilon_- A_-^\dagger A_-, H_B = \sum_k \varepsilon_k c_k^\dagger c_k, \\ &H_I = \sum_k V_k [e^{i\phi_1} A_+^\dagger c_k + e^{-i\phi_1} c_k^\dagger A_+]. \end{aligned} \tag{7}$$

where the effective energy levels $\epsilon_\pm = \epsilon_0$ are still degenerate.

The system Hamiltonian is diagonalized in terms of $A_\pm$, such that the original system is equivalent to an effective system which has two decoupled energy levels $\epsilon_\pm$, out of which only one energy level (ϵ_+) couples to the electron reservoir. As a result, the corresponding exact master equation becomes

$$\dot\rho(t) = -i[\widetilde H_S(t), \rho(t)] + L[\rho(t)], \tag{8a}$$

with the new expressions of $\widetilde H_S(t)$ and $L[\rho(t)]$

$$\widetilde H_S(t) = \widetilde\epsilon_+(t) A_+^\dagger A_+ + \epsilon_- A_-^\dagger A_-, \tag{8b}$$

$$\begin{aligned} L[\rho(t)] = &\kappa(t)[2A_+\rho(t)A_+^\dagger - A_+^\dagger A_+ \rho(t) - \rho(t) A_+^\dagger A_+] \\ &+ \widetilde\kappa(t)[2A_+^\dagger \rho(t) A_+ - A_+ A_+^\dagger \rho(t) - \rho(t) A_+ A_+^\dagger]. \end{aligned} \tag{8c}$$

The decoherence rates are $\kappa(t) = \gamma(t) - \frac{\widetilde\gamma(t)}{2}$ and $\widetilde\kappa(t) = \frac{\widetilde\gamma(t)}{2}$. The renormalized energy level $\widetilde\epsilon_+(t)$ and the rates $\gamma(t)$ and $\widetilde\gamma(t)$ are determined exactly by

$$\widetilde\epsilon_+(t) = -\mathrm{Im}[\dot u(t) u^{-1}(t)], \tag{9a}$$

$$\gamma(t) = -\mathrm{Re}[\dot u(t) u^{-1}(t)], \tag{9b}$$

$$\widetilde\gamma(t) = \dot v(t) - 2v(t)\mathrm{Re}[\dot u(t) u^{-1}(t)], \tag{9c}$$

and $u(t)$ and $v(t)$ are the retarded and correlation Green's functions in the Schwinger-Keldysh nonequilibrium Green's function theory [61]. They obey the integrodifferential equations

$$\frac{d}{dt}u(t) + i\epsilon_+ u(t) + \int_{t_0}^{t} g(t-\tau) u(\tau) d\tau = 0, \tag{10a}$$

$$v(t) = \int_{t_0}^{t} d\tau_1 \int_{t_0}^{t} d\tau_2 u(\tau_1) \widetilde g(\tau_2 - \tau_1) u^*(\tau_2), \tag{10b}$$

subjected to the boundary condition $u(t_0)=1$. The integration kernels read $g(\tau)=\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}J(\omega)e^{-i\omega\tau}$ and $\widetilde{g}(\tau)=\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}J(\omega)f(\omega)e^{-i\omega\tau}$, where the spectral density of the reservoir is given by $J(\omega)=2\pi\sum_k|V_k|^2\delta(\omega-\omega_k)$, and $f(\omega)=1/(e^{\beta(\omega-\mu)}+1)$ is the initial electron distribution of the reservoir.

Since the effective energy level ϵ_- is apparently decoupled from the electron reservoir, one may naively think that the state generated by the operator $A_-^\dagger$, namely $A_-^\dagger|0\rangle$, becomes naturally a DF state. This is actually not true. The term with $\widetilde{\kappa}(t)$ in Eq. (8c) will drive this state into another state since $A_+^\dagger A_-^\dagger|0\rangle\neq 0$. However, there is an occupation constant of motion in this system, $[A_-^\dagger A_-,H]=0$. This symmetry separates the system Hilbert space into two closed subspaces $\mathcal{H}_+=\{|\mathsf{v}\rangle,|+\rangle\}$ and $\mathcal{H}_-=\{|-\rangle,|\mathsf{d}\rangle\}$, corresponding to the occupation $N_-\equiv\langle A_-^\dagger A_-\rangle=0$ and 1, respectively. Here $|\mathsf{v}\rangle$ ($|\mathsf{d}\rangle$) is the vacuum (doubly occupied) electron state, while $|\pm\rangle=A_\pm^\dagger|\mathsf{v}\rangle$ are superpositions of singly occupied states of the original two levels,

$$\begin{aligned}|+\rangle&=\left(|1\rangle+e^{-i\phi}|2\rangle\right)/\sqrt{2},\\ |-\rangle&=\left(-e^{i\phi}|1\rangle+|2\rangle\right)/\sqrt{2},\end{aligned}\tag{11}$$

and $|i\rangle=a_i^\dagger|\mathsf{v}\rangle$ $(i=1,2)$. The two relative phases are ϕ and $\phi+\pi$, where ϕ is arbitrary. As a consequence, starting from any initial state in the closed subspace $\mathcal{H}_+$ ($\mathcal{H}_-$), the system will be kept in this subspace throughout the evolution process.

Equation (8) has the standard form of Lindblad master equation, except that the decoherence rates, $\kappa(t)$ and $\widetilde{\kappa}(t)$, can depend on time and even become negative. They are local in time and determined microscopically and nonperturbatively from Eq. (10). The DFS criterion of Eq. (4) is then still sufficient but may not be necessary. The operators $A_+^\dagger$ and A_+ act on the four basis states $\{|\mathsf{v}\rangle,|-\rangle,|+\rangle,|\mathsf{d}\rangle\}$ according to the following relations:

$$\begin{aligned}A_+^\dagger|\mathsf{v}\rangle&=|+\rangle,\quad A_+|\mathsf{v}\rangle=0,\\ A_+^\dagger|+\rangle&=0,\quad A_+|+\rangle=|\mathsf{v}\rangle,\\ A_+^\dagger|-\rangle&=|\mathsf{d}\rangle,\quad A_+|-\rangle=0,\\ A_+^\dagger|\mathsf{d}\rangle&=0,\quad A_+|\mathsf{d}\rangle=|-\rangle.\end{aligned}$$

One sees that both $A_+^\dagger$ and A_+ have a fourfold degenerate eigenvalue $c_\alpha=0$, but the corresponding eigenspaces are only two dimensional, given by $\{|+\rangle,|\mathsf{d}\rangle\}$ for $A_+^\dagger$ and $\{|-\rangle,|\mathsf{v}\rangle\}$ for A_+. Since these two spaces do not overlap, condition (4) is not satisfied, and as long as both $\kappa(t)$ and $\widetilde{\kappa}(t)$ are positive, there is no DFS. In the next section, we explore whether additional DF states can exist if $\kappa(t)$ and $\widetilde{\kappa}(t)$ are not both positive and show that a DFS can arise dynamically, when at least one of the two rates $\kappa(t)$ and $\widetilde{\kappa}(t)$ vanishes.

IV. DF STATES

A. Eigenstates of the Lindblad operator

Since, as discussed above, the Markovian DFS criterion (4) may not be necessary for a non-Markovian system, we define a local (in time) DFS by the vanishing of the nonunitary term $L[\rho(t)]$. Clearly, this leads to local unitary time evolution. In the following, we discuss all possibilities of how $L[\rho(t)]=0$ can arise by calculating the eigenvalues of the full Lindblad generator L in Liouville space. If L has a zero eigenvalue, the corresponding eigenstate is DF. Conversely, a local DF state is by definition an eigenstate of L with zero eigenvalue. However, as the eigenstates of L do not necessarily have all the properties of a density matrix, such states may not be physical. One must therefore examine for each eigenstate whether it is a physical state or can be combined with other eigenstates corresponding to the same (degenerate) eigenvalue to form a physical state. In this way one can find all physically possible DF states.

In the basis of $\{|\mathsf{v}\rangle,|+\rangle,|-\rangle,|\mathsf{d}\rangle\}$, the density matrix takes the general form

$$\begin{aligned}\rho(t)=&\rho_{\mathsf{vv}}(t)|\mathsf{v}\rangle\langle\mathsf{v}|+\rho_{++}(t)|+\rangle\langle+|+\rho_{+-}(t)|+\rangle\langle-|\\ &+\rho_{+-}^*(t)|-\rangle\langle+|+\rho_{--}(t)|-\rangle\langle-|+\rho_{\mathsf{dd}}(t)|\mathsf{d}\rangle\langle\mathsf{d}|.\end{aligned}\tag{12}$$

Coherences between states with different particle numbers are not permitted due to the particle number superselection rule. Since there are only six nonzero elements of $\rho(t)$, we can define the basis $\{|1\rangle\equiv|\mathsf{v}\rangle\langle\mathsf{v}|,\ |2\rangle\equiv|+\rangle\langle+|,\ |3\rangle\equiv|+\rangle\langle-|,\ |4\rangle\equiv|-\rangle\langle+|,\ |5\rangle\equiv|-\rangle\langle-|,\ |6\rangle\equiv|\mathsf{d}\rangle\langle\mathsf{d}|\}$ in Liouville space, which is orthogonal with respect to the scalar product $\langle A|B\rangle=\mathrm{tr}A^\dagger B$. In this basis, the density matrix can be rewritten as a column vector, $|\rho(t)\rangle=[\rho_{\mathsf{vv}}(t),\rho_{++}(t),\rho_{+-}(t),\rho_{-+}(t),\rho_{--}(t),\rho_{\mathsf{dd}}(t)]^T$. The Lindblad operator L is represented by a matrix L_t, and its action on a density matrix reduces to a simple matrix multiplication of L_t with $|\rho(t)\rangle$, where L_t reads

$$L_t=\begin{bmatrix}-2\widetilde{\kappa}(t) & 2\kappa(t) & 0 & 0 & 0 & 0\\ 2\widetilde{\kappa}(t) & -2\kappa(t) & 0 & 0 & 0 & 0\\ 0 & 0 & -(\kappa(t)+\widetilde{\kappa}(t)) & 0 & 0 & 0\\ 0 & 0 & 0 & -(\kappa(t)+\widetilde{\kappa}(t)) & 0 & 0\\ 0 & 0 & 0 & 0 & -2\widetilde{\kappa}(t) & 2\kappa(t)\\ 0 & 0 & 0 & 0 & 2\widetilde{\kappa}(t) & -2\kappa(t)\end{bmatrix}.\tag{13}$$

The eigenvalues and the corresponding eigenstates of L_t are

$$\begin{aligned} l_1 &= 0, \\ l_2 &= -2[\kappa(t)+\widetilde{\kappa}(t)], \\ l_3 &= -[\kappa(t)+\widetilde{\kappa}(t)], \\ l_4 &= -[\kappa(t)+\widetilde{\kappa}(t)], \\ l_5 &= -2[\kappa(t)+\widetilde{\kappa}(t)], \\ l_6 &= 0, \end{aligned} \tag{14}$$

and

$$\begin{aligned} |l_1\rangle &= \frac{1}{\kappa(t)+\tilde{\kappa}(t)}[\kappa(t)|1\rangle+\widetilde{\kappa}(t)|2\rangle], \\ |l_2\rangle &= \frac{1}{2}(-|1\rangle+|2\rangle), \\ |l_3\rangle &= |3\rangle, \\ |l_4\rangle &= |4\rangle, \\ |l_5\rangle &= \frac{1}{2}(-|5\rangle+|6\rangle), \\ |l_6\rangle &= \frac{1}{\kappa(t)+\tilde{\kappa}(t)}[\kappa(t)|5\rangle+\widetilde{\kappa}(t)|6\rangle]. \end{aligned} \tag{15}$$

We see that all eigenvalues come in pairs, and there are always at least two eigenvalues equal to zero. In writing Eq. (15) we have assumed that $\kappa(t)+\widetilde{\kappa}(t)\neq 0$. The case of $\kappa(t)+\widetilde{\kappa}(t)=0$ is discussed below. The normalization used for $|l_1\rangle$ and $l_6\rangle$ is convenient, as in this way these two states can be interpreted directly as density matrices if both $\kappa(t)$ and $\widetilde{\kappa}(t)$ are positive. If one of the rates is negative (and the other nonzero), both states become nonpositive and therefore cease to be physical states. Moreover, since they have orthogonal support, no linear combination of them can bring about a positive state. Similarly, states $|l_2\rangle$ and $|l_5\rangle$, as well as any linear combination of them, are clearly nonpositive. Finally, states $|l_3\rangle$ and $|l_4\rangle$ as well as any linear combination of them are traceless and are therefore not physical states either. Since $|l_2\rangle,\ldots,|l_5\rangle$ are independent of $\kappa(t)$ and $\widetilde{\kappa}(t)$ [and are therefore *never* physical states, regardless of the values of $\kappa(t)$ and $\widetilde{\kappa}(t)$], the only possibility of having a physical DF state is through $|l_1\rangle$ and $|l_6\rangle$ with both $\kappa(t)$ and $\widetilde{\kappa}(t)$ non-negative, or one of them vanishing (in the latter case one may always choose the eigenvector with positive global sign).

If $\kappa(t)=0$ and $\widetilde{\kappa}(t)\neq 0$, $|l_1\rangle$ and $|l_6\rangle$ are two pure DF states $|2\rangle$ and $|6\rangle$; that is, the subset of $\{|+\rangle,|\mathrm{d}\rangle\}$ contains all the possible DF states in this case. Likewise, if $\widetilde{\kappa}(t)=0$ and $\kappa(t)\neq 0$, $|l_1\rangle$ and $|l_6\rangle$ are the two pure DF states $|1\rangle$ and $|5\rangle$, that is, $\{|\mathrm{v}\rangle,|+\rangle\}$. Below, by examining an explicit example, we show that the vanishing of one of the decoherence rates is physically feasible after some time t_s, when the system reaches its steady state; see Fig. 1.

If $\kappa(t)>0$ and $\widetilde{\kappa}(t)>0$, the two eigenstates $|l_1\rangle$ and $|l_6\rangle$ are time-dependent mixed states. They are decoherence free as much as they are locally stationary states due to local detailed balance. From a perspective of application for quantum information processing, these states are less interesting. They are the analogs of thermal equilibrium states that are stationary under a Markovian relaxation process.

It remains to consider the case $\kappa(t)+\widetilde{\kappa}(t)=0$. All eigenvalues vanish, but as long as $\kappa(t)\neq 0$, the eigenvectors are still given by Eq. (15) with the only difference that the normalization of $|l_1\rangle$ and $|l_6\rangle$ through the prefactor $1/[\kappa(t)+\widetilde{\kappa}(t)]$ has to be removed. $|l_1\rangle$ becomes colinear with $|l_2\rangle$, and $|l_6\rangle$ colinear with $|l_5\rangle$. The dimension of the space of eigenvectors of L_t is reduced to four, and L_t can therefore not be fully diagonalized. As the linearly independent eigenvectors $|l_1\rangle,\ldots,|l_4\rangle$ are never physical states, this means that L_t has no eigenstates that are physically possible, and therefore no DF states exist.

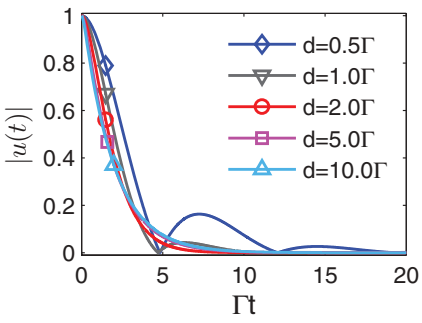

FIG. 1. (Color online) The exact solutions of decoherence rates $\kappa(t)$ and $\widetilde{\kappa}(t)$ for different external bias voltage $\mu=eV$: (a), (b) for the weakly non-Markovian dynamics (with $d=10\Gamma$); (c), (d) for the strong non-Markovian dynamics (with $d=0.5\Gamma$). Here we set $\epsilon_0=0.2\Gamma$, $k_BT=0.3\Gamma$. We note that generally $\kappa(t)$ and $\widetilde{\kappa}(t)$ are nonzero during the time evolution. However, for a large positive (or negative) bias voltage, one of them is switched off after some specific time.

If $\kappa(t)=\widetilde{\kappa}(t)=0$ at some time (for example, if the retarded Green's function $u(t)$ took a nonzero steady value, it would be possible that both rates vanish for $t>t_s$), we have $L_t=0$, and the whole Hilbert space becomes a dynamically stabilized DFS. From a quantum information perspective this would be of course an ideal situation. Unfortunately, in the fermionic system interacting with an electron reservoir considered here, it appears that this situation does not arise, as shown in Fig. 1.

In summary, as long as we restrict ourselves to pure states for our non-Markovian master equation and discard the "trivial" case of $L_t=0$, the DFS is still given entirely by the Markovian criterion (4). This is because the pure DF states only exist if exactly one of the two terms [proportional to either $\kappa(t)$ or $\widetilde{\kappa}(t)$] in the Lindblad superoperator remains, and the logic of the proof of necessity of condition (4) remains intact in such a situation. However, the time dependence of $\kappa(t)$ and $\widetilde{\kappa}(t)$ brings about a new freedom and allows for the dynamical stabilization of DF states through the switching off of one of the decoherence rates.

B. Physical realization of DF states

In the following, we discuss to what extent the DF states just discussed can be reached through the same non-Markovian dynamics described by L_t. We consider the double quantum dot system from above coupled to an electron reservoir with a spectral density of the Lorentz form [59,75,76],

$$J(\omega)=\frac{\Gamma d^2}{(\omega-\epsilon_+)^2+d^2}, \tag{16}$$

where Γ is the system-reservoir coupling strength and d is the bandwidth of the effective reservoir spectrum. In

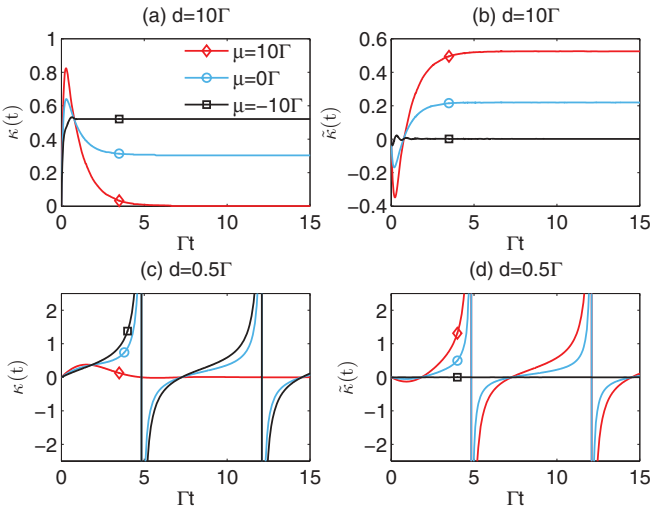

FIG. 2. (Color online) The exact solution of $|u(t)|$ for an electron reservoir with the Lorentz spectral density. Here we take $\epsilon_0 = 0.2\Gamma$. Note that $|u(t)|$ decays exponentially for large bandwidth d, corresponding to the weakly non-Markovian dynamics. For a small bandwidth $d < 2\Gamma$, the strong non-Markovian memory effect brings the short-time oscillations for $|u(t)|$.

the well-known wide-band limit (i.e., $d \to \infty$), the spectral density approximately becomes a constant one, $J(\omega) \to \Gamma$. This corresponds to the Markovian limit.

With the above spectral density, the solution of $u(t)$ obeying Eq. (10a) can be obtained analytically

$$u(t) = \begin{cases} \frac{e^{-i\epsilon_+ t}}{2d_\Gamma}\left[d_\Gamma^+ e^{-\frac{d_\Gamma^- t}{2}} - d_\Gamma^- e^{-\frac{d_\Gamma^+ t}{2}}\right], & d \neq 2\Gamma, \\ \left[1 + \frac{dt}{2}\right] e^{-(i\epsilon_+ + \frac{d}{2})t}, & d = 2\Gamma, \end{cases} \tag{17}$$

where $d_\Gamma = \sqrt{d^2 - 2\Gamma d}$ and $d_\Gamma^\pm = d \pm d_\Gamma$. Obviously, after some time t_s, $u(t)$ always decays to zero, as shown in Fig. 2 where the different behaviors of the amplitude of $u(t)$ correspond to different bandwidths d. For the bandwidth $d \gtrsim 2\Gamma$ (weakly non-Markovian case), $u(t)$ exponentially decays to zero, which is a result similar to the Markovian dynamics; see the discussion in Sec. VI. When the bandwidth $d < 2\Gamma$, the non-Markovian memory effect of the reservoir becomes significant, which induces a short-time oscillation for $|u(t)|$.

Using the solution of Eq. (17), combined with Eqs. (10b) and (9), we can easily calculate the decoherence rates $\kappa(t)$ and $\tilde{\kappa}(t)$ in Eq. (8). The result is plotted in Fig. 1 for the cases of weakly ($d = 10\Gamma$) and strongly ($d = 0.5\Gamma$) non-Markovian dynamics. In experiments, one can adjust the external bias (μ) to raise or lower the Fermi surface of the electron reservoir. Here we display three cases where the bias is much higher ($\mu = 10\Gamma$), relatively small ($\mu = 0$), and much lower ($\mu = -10\Gamma$) than the quantum dot energy level ϵ_0. First, we see that when $d = 0.5\Gamma$, as shown in Figs. 2(c) and 2(d), $\kappa(t)$ and $\tilde{\kappa}(t)$ can jump from a positive value to a negative value repeatedly during the evolution process, which corresponds to the forth and back flows of the information between the system and the environment, in evidence of the strong non-Markovian dynamics.

We observe that no matter what the values of the width d and the bias μ are, the two decoherence rates $\kappa(t)$ and $\tilde{\kappa}(t)$ never satisfy $\kappa(t) + \tilde{\kappa}(t) = 0$. However, if we apply a large bias to raise (or lower) the Fermi surface of the electron reservoir much higher (or much lower) than the dot level, one of the two decoherence rates is switched off after a time scale of a few $1/\Gamma$, which implies the existence of a DFS.

As shown in Figs. 2(a) and 2(b), in the weakly non-Markovian case, when applying a positive bias $\mu = 10\Gamma$, the decoherence rate $\kappa(t)$ shows a positive peak in the beginning and then decays rapidly to a zero steady value on a time scale of a few $1/\Gamma$, while $\tilde{\kappa}(t)$ turns negative first and then climbs up to a nonzero steady value. For the strongly non-Markovian regime ($d = 0.5\Gamma$), the same situation happens; see the red curves in Figs. 2(c) and 2(d). The decoherence rate reaches $\kappa(t) = 0$ on a time scale t_s of a few $1/\Gamma$, while $\tilde{\kappa}(t)$ keeps jumping from positive values to negative values repetitively. In this case, the dynamics for $t > t_s$ is described by the following master equation:

$$\dot{\rho}(t) = -i[\widetilde{H}_S(t), \rho(t)] + L[\rho(t)], \tag{18a}$$

$$\widetilde{H}_S(t) = \widetilde{\epsilon}_+(t) A_+^\dagger A_+ + \epsilon_- A_-^\dagger A_-, \tag{18b}$$

$$L[\rho(t)] = \widetilde{\kappa}(t)[2A_+^\dagger \rho(t) A_+ - A_+ A_+^\dagger \rho(t) - \rho(t) A_+ A_+^\dagger]. \tag{18c}$$

Then the states $|+\rangle$ and $|\mathsf{d}\rangle$ become possible dynamically stabilized DF states, since $A_+^\dagger|+\rangle = 0$ and also $A_+^\dagger|\mathsf{d}\rangle = 0$ leads to the vanishing of the nonunitary part $L[\rho(t)]$ after $t > t_s$. Therefore, the Markovian DFS criterion of Eq. (4) still is both necessary and sufficient for a time-local DFS when only one decoherence rate remains.

On the other hand, if we apply a negative bias to the electron reservoir (e.g., $\mu = -10\Gamma$), as shown by the black curves in Fig. 2, we find that for both weakly and strongly non-Markovian dynamics, the decoherence rate $\tilde{\kappa}(t)$ goes to zero very quickly, while $\kappa(t)$ either reaches a nonzero steady value (in the weakly non-Markovian regime) or keeps jumping from a positive value to a negative value for all times (in the strongly non-Markovian regime). Then the master equation for $t > t_s$ is effectively given by

$$\dot{\rho}(t) = -i[\widetilde{H}_S(t), \rho(t)] + L[\rho(t)], \tag{19a}$$

$$\widetilde{H}_S(t) = \widetilde{\epsilon}_+(t) A_+^\dagger A_+ + \epsilon_- A_-^\dagger A_-, \tag{19b}$$

$$L[\rho(t)] = \kappa(t)[2A_+ \rho(t) A_+^\dagger - A_+^\dagger A_+ \rho(t) - \rho(t) A_+^\dagger A_+]. \tag{19c}$$

Again, the DFS criterion Eq. (4) is then both necessary and sufficient for DF states. Since $A_+|-\rangle = 0$ and $A_+|\mathsf{v}\rangle = 0$, then we obtain that $|-\rangle$ and $|\mathsf{v}\rangle$ are possible dynamically stabilized DF states.

In summary, these results show that the vanishing of one of the decoherence rates implies that $\{|+\rangle, |\mathsf{d}\rangle\}$ or $\{|-\rangle, |\mathsf{v}\rangle\}$ are dynamically stabilized DF states.

V. PHYSICAL REALIZATION FOR DYNAMICALLY STABILIZED DF STATES

In the following, by examining the general exact solution of master equation (8), we prove that all pure dynamically stabilized DF states $\{|+\rangle,|\mathsf{d}\rangle\}$ and $\{|-\rangle,|\mathsf{v}\rangle\}$ predicted above are physically realizable through the decoherence process given by the same master equation. We also give the exact conditions for generating these DF states in terms of the initial state, and a condition on the Green's functions $u(t)$ and $v(t)$.

First, by solving Eq. (8), we can exactly give the elements of $\rho(t)$ in Eq. (12) in terms of the initial $\rho(t_0)$ and the functions of $u(t)$ and $v(t)$ as

$$\begin{aligned}
\rho_{\mathsf{vv}}(t) &= [1-v(t)]\rho_{\mathsf{vv}}(t_0)+[1-v(t)-|u(t)|^2]\rho_{++}(t_0),\\
\rho_{++}(t) &= v(t)\rho_{\mathsf{vv}}(t_0)+[v(t)+|u(t)|^2]\rho_{++}(t_0),\\
\rho_{+-}(t) &= u(t)e^{i\epsilon_- t}\rho_{+-}(t_0),\\
\rho_{--}(t) &= [1-v(t)]\rho_{--}(t_0)+[1-v(t)-|u(t)|^2]\rho_{\mathsf{dd}}(t_0),\\
\rho_{\mathsf{dd}}(t) &= v(t)\rho_{--}(t_0)+[v(t)+|u(t)|^2]\rho_{\mathsf{dd}}(t_0).
\end{aligned} \tag{20}$$

This expression for $\rho(t)$ is valid for an arbitrary spectral density of the reservoir. This solution confirms the fact that $\mathcal{H}_+ = \{|\mathsf{v}\rangle,|+\rangle\}$ and $\mathcal{H}_- = \{|-\rangle,|\mathsf{d}\rangle\}$ are two independent closed subspaces. For any initial state in the subspace $\mathcal{H}_+$ ($\mathcal{H}_-$), the system will be dynamically stabilized in this subspace.

The general solution (20) shows that if $v(t)=1$ is satisfied when $t>t_s$, the initial vacuum state $|\mathsf{v}\rangle$ converges to the stabilized state $|+\rangle$, while if $v(t)+|u(t)|^2=1$ is reached when $t>t_s$, the initial singly occupied state $|+\rangle$ converges to the dynamically stabilized DF state $|+\rangle$. That is, the same dynamically stabilized DF state $|+\rangle$ can be generated from different initial states in the same subspace of $\mathcal{H}_+$ under different stabilization conditions for $u(t)$ and $v(t)$. Similarly, one can obtain the remaining dynamically stabilized DF states $|\mathsf{v}\rangle$, $|-\rangle$, and $|\mathsf{d}\rangle$ from different initial states under different stabilization condition, as shown in Table I.

Besides the initial states listed in Table I, another more general initial pure state is a superposition of $|+\rangle$ and $|-\rangle$, namely $|\Phi\rangle=\alpha|+\rangle+\beta|-\rangle$ with $|\alpha|^2+|\beta|^2=1$. In this case, as one can easily check from Eq. (20), the resulting state becomes

$$\begin{aligned}
\rho_{\mathsf{vv}}(t) &= [1-v(t)-|u(t)|^2]|\alpha|^2,\\
\rho_{++}(t) &= [v(t)+|u(t)|^2]|\alpha|^2,\\
\rho_{+-}(t) &= u(t)e^{i\epsilon_- t}\,\alpha\beta^*,\\
\rho_{--}(t) &= [1-v(t)]|\beta|^2,\\
\rho_{\mathsf{dd}}(t) &= v(t)|\beta|^2.
\end{aligned} \tag{21}$$

TABLE I. The dynamically stabilized DF states for different choices of the initial state of the system, plus the different stabilization conditions.

Initial state	Stabilization condition	DF state
$\|\mathsf{v}\rangle$	$v(t)=0$ (or 1)	$\|\mathsf{v}\rangle$ (or $\|+\rangle$)
$\|+\rangle$	$v(t)+\|u(t)\|^2=0$ (or 1)	$\|\mathsf{v}\rangle$ (or $\|+\rangle$)
$\|-\rangle$	$v(t)=0$ (or 1)	$\|-\rangle$ (or $\|\mathsf{d}\rangle$)
$\|\mathsf{d}\rangle$	$v(t)+\|u(t)\|^2=0$ (or 1)	$\|-\rangle$ (or $\|\mathsf{d}\rangle$)

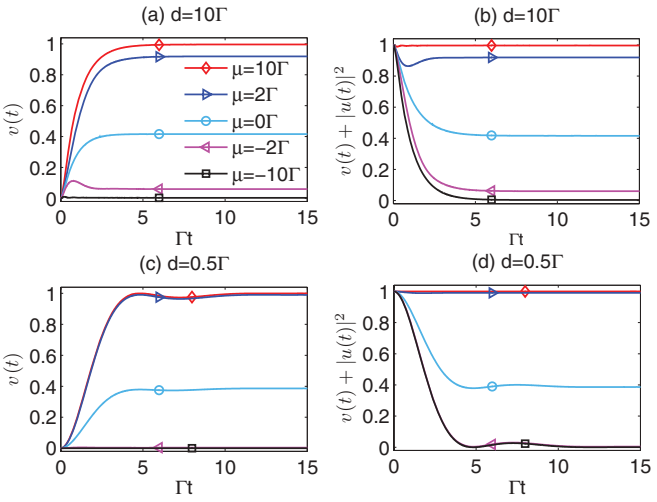

FIG. 3. (Color online) The exact solution of $v(t)$ and $v(t)+|u(t)|^2$ for an electron reservoir with varying the external bias voltage $\mu=eV$: (a), (b) for the weakly non-Markovian dynamics (with $d=10\Gamma$); (c), (d) for the strongly non-Markovian dynamics (with $d=0.5\Gamma$). Here we set $\epsilon_0=0.2\Gamma$, $K_BT=0.3\Gamma$. By applying a large positive (or negative) bias voltage, the steady value of $v(t)$ and $v(t)+|u(t)|^2$ will approach 1 (or 0).

In the case that both α and β are nonzero, one has to have $v(t)=0$ and $|u(t)|^2=1$ to generate a stabilized (pure) DF state $|\Phi\rangle$. This is impossible unless the system is totally decoupled from the environment from the beginning. In other words, except for a stabilized mixed state, no stabilized DF state can be obtained. As a conclusion, Table I lists all the possible pure stabilized DF states in this system. The present results confirm the statement in the last section that the only possible pure dynamically stabilized DF states are $|\mathsf{v}\rangle,|\pm\rangle$, and $|d\rangle$.

For the Lorentz spectral density, we display the functions of $v(t)$ and $v(t)+|u(t)|^2$ in Fig. 3. Interestingly, by comparing Figs. 3(a) with 3(b) and Figs. 3(c) with 3(d), we find that in the strongly non-Markovian case ($d=0.5\Gamma$), applying a relatively small bias voltage (for example, $|\mu|=2\Gamma$) makes the steady values of $v(t)$ and $v(t)+|u(t)|^2$ quickly approach 1 or 0, whereas this is not the case in the weakly non-Markovian regime ($d=10\Gamma$), where $v(t)\to 0.9$ for $\mu=2\Gamma$ [and $v(t)\to 0.1$ for $\mu=-2\Gamma$]. This indicates that the backflow of information from the reservoir, due to a strongly non-Markovian memory of the reservoir, helps the stabilization of the system in the states of $\{|+\rangle,|\mathsf{d}\rangle\}$ or $\{|\mathsf{v}\rangle,|-\rangle\}$. This facilitates the physical realization of the dynamically stabilized DF states.

Furthermore, we can now prove that under the stabilization conditions listed in Table I, one of the decoherence rates vanishes when $t>t_s$. In fact, $\kappa(t)$ and $\widetilde{\kappa}(t)$ in Eq. (9) can be further simplified as

$$\kappa(t)=\frac{|u(t)|^2}{2}\frac{d}{dt}\frac{1-v(t)}{|u(t)|^2}, \tag{22a}$$

$$\widetilde{\kappa}(t)=\frac{|u(t)|^2}{2}\frac{d}{dt}\frac{v(t)}{|u(t)|^2}. \tag{22b}$$

It clearly shows that the condition $v(t)=1$ [or $(v(t)+|u(t)|^2=1]$ implies $\kappa(t)=0$, and the condition $v(t)=0$ [or $v(t)+|u(t)|^2=0$] indicates $\widetilde{\kappa}(t)=0$, namely, one

decoherence rate is turned off for $t > t_s$. In this case, the Markovian DFS criterion still provides a necessary and sufficient condition for generating dynamically stabilized DF states. However, for the non-Markovian dynamics, we have shown in Table I that which final state is realized depends both on the initial state and the details of the dynamics given by the Green's functions $u(t)$ and $v(t)$.

VI. COMPARISON TO THE BORN-MARKOV DYNAMICS

In the previous sections, using the exact master equation, we have shown the generation of DF states for a system coupled to a non-Markovian reservoir. For comparison, in the following, we discuss the corresponding results in terms of the BM master equation.

The BM dynamics usually corresponds to the case where the coupling strength between the system and the electron reservoir is very weak, and the characteristic correlation time of the electron reservoir is sufficiently shorter than that of the system. In such a case, the electron reservoir has no memory effect on the evolution of the system. Then the solutions of $u(t)$ and $v(t)$ are reduced to [59–61]

$$\begin{aligned} u_{\rm BM}(t) &= e^{-i\widetilde{\epsilon}_+ t-\frac{1}{2}J(\epsilon_+)t}, \\ v_{\rm BM}(t) &= [1-e^{-J(\epsilon_+)t}]f(\epsilon_+), \end{aligned} \tag{23}$$

where $\widetilde{\epsilon}_+ = \epsilon_+ + (\delta\epsilon_+)_{\rm BM}$ with the energy shift $(\delta\epsilon_+)_{\rm BM} = \mathcal{P}\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\frac{\Gamma(\omega)}{\omega-\epsilon_+}$. Note that for the wide-band limit, we simply have $\widetilde{\epsilon}_+ = \epsilon_+$ and $J(\epsilon_+) = \Gamma$. Substituting Eq. (23) into Eq. (9), we obtain the constant rates

$$\begin{aligned} \kappa(t) &= \tfrac{1}{2}J(\epsilon_+)[1-f(\epsilon_+)], \\ \widetilde{\kappa}(t) &= \tfrac{1}{2}J(\epsilon_+)f(\epsilon_+). \end{aligned} \tag{24}$$

Here $f(\epsilon_+)$ is the fermion distribution function of the electron reservoir at the frequency ϵ_+, that is,

$$f(\epsilon_+) = \frac{1}{e^{(\epsilon_+-\mu)/K_BT}+1}. \tag{25}$$

Thus the exact master equation is reduced to the BM master equation, where the decoherence rates are time independent. This gives the standard Lindblad form for the Markovian dynamics. Based on the general DFS criterion of Eq. (4), we see that there is in general no DFS for this system in the BM limit.

However, if we apply a large positive bias $\mu = eV$ such that $(\mu-\epsilon_+)/k_BT \gg 1$, then $f(\epsilon_+) \to 1$, which leads to $\kappa \to 0$. $\widetilde{H}_S$ leaves the states $|+\rangle$ and $|{\rm d}\rangle$ invariant during the time evolution. The relation $A_+^\dagger|+\rangle = 0$ (or $A_+^\dagger|{\rm d}\rangle = 0$) guarantees $L[\rho(t)] = 0$ for these states. The states $\{|+\rangle, |{\rm d}\rangle\}$ are therefore DF states in the BM limit under large positive bias. Likewise, applying a large negative bias to the electron reservoir such that $(\epsilon_+-\mu)/K_BT \gg 1$, then $f(\epsilon_+) \to 0$ and $\widetilde{\kappa}(t) \to 0$. In this case, the states $\{|{\rm v}\rangle, |-\rangle\}$ are the DF states in the BM limit.

In conclusion, the states $\{|+\rangle, |{\rm d}\rangle\}$ and $\{|{\rm v}\rangle, |-\rangle\}$ are also DF in the BM limit if one of the decoherence rates is switched off by properly tuning the bias voltage on the electron reservoir. This result is consistent with the result in the non-Markovian case discussed above. The apparent difference is that the DF states in the BM limit seem to exist without the dynamical stabilization processes. However, this difference is not crucial in reality. It is well known [64,65] that the BM master equation with the constant decoherence rates, Eq. (24), is derived under the condition $t \gg \tau_r$, where τ_r is the characteristic time of the reservoir [64]. In other words, a stabilization time scale t_s has implicitly been used in deriving the BM master equation, such that the decoherence rates become time independent for $t > t_s \gg \tau_r$. Therefore, the concept of the dynamically stabilized DF states, based on the exact non-Markovian master equation, gives the generalized picture of DFSs. It contains the Markovian DFSs as a special case.

VII. CONCLUSION

In summary, we have investigated the DFS of a non-Markovian fermionic open system, based on an exact non-Markovian master equation developed recently. The master equation has a nonunitary term of the standard Lindblad form, but the corresponding decoherence rates are time dependent and local in time. They are determined microscopically and nonperturbatively from the Schwinger-Keldysh nonequilibrium Green's functions and fully account for the non-Markovian memory effect.

As a concrete example, we have studied a fermionic system with two degenerate energy levels coupled identically to a fermionic reservoir. We find that the whole Hilbert space is split into two closed subspaces. For any initial state in one of the subspaces, the system will remain in this subspace forever. By diagonalizing the full Lindblad operator, we found that physical DF states exist if and only if one of the two relevant decoherence rates switches itself off dynamically. Such a situation can be achieved under practical conditions. It should be pointed out that two of the DF states are coherent superpositions with an arbitrary relative phase between the two original energy levels, which may be of physical interest for quantum computation.

In addition, which DF state is reached as a result of the dissipative dynamics depends both on the initial state and the details of the dynamics, as expressed by the time-dependent nonequilibrium Green's function. We show this explicitly by solving exactly the non-Markovian master equation. Interestingly, we find that the strongly non-Markovian memory can help to stabilize the DF states compared to the Markovian case.

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