PHYSICAL REVIEW A **86**, 031602(R) (2012)

# Controlled manipulation of light by cooperative response of atoms in an optical lattice

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(Received 23 December 2011; published 11 September 2012)

We show that atoms in an optical lattice can respond cooperatively to resonant incident light and that such a response can be employed for precise control and manipulation of light on a subwavelength scale. Specific collective excitation modes of the system that result from strong light-mediated dipole-dipole interactions can be addressed by tailoring the spatial phase profile of the incident light. We demonstrate how the collective response can be used to produce optical excitations at well-isolated sites on the lattice.

DOI: 10.1103/PhysRevA.86.031602 PACS number(s): 03.75.Lm, 32.80.Qk

Accurate control of ultracold atomic gases in periodic optical lattices, in which interactions are well understood, opens the door to unique and intriguing opportunities to study many-particle phenomena and their applications. Experimental progress has lead to observations of novel strongly interacting states, e.g., in quantum phase transitions [1–5] and fermionic pair condensation [6]. Many-body quantum entanglement has been generated via controlled atom collisions [7], lattice systems have been used for preparation of spin-squeezed states for sub-shot-noise interferometry [8], and the atoms can now even be manipulated in a single-spin level at a specific lattice site [9]. On the other hand, recent developments in nanofabrication of arrays of circuit elements acting as plasmonic resonators has stimulated interest in photonic metamaterials. A metamaterial is an artificially tailored crystal consisting of subwavelength-scale structures that can manipulate light on a nanoscale. Here we show that a basic Mott-insulator state of a neutral gas of ultracold atoms confined in an optical lattice, or artificial light crystal, exhibits strongly interacting electric-dipole transitions leading to a cooperative response. Such collective behavior can influence resonant imaging and may also be employed to form a metamaterial for precise control and manipulation of optical fields on a subwavelength scale, providing an interesting nanophotonic tool.

Here we consider an ultracold gas of atoms confined in a two-dimensional (2D) optical lattice with precisely one atom per site [9]. Resonant, coherent light whose spatial phase-profile is adjusted, e.g., by a hologram or spacial light modulator, illuminates the lattice. The scattered light mediates strong many-particle dipole-dipole (DD) interactions between atoms, leading to a cooperative atom response. The optical excitations of the atoms exhibit collective modes with resonance frequencies and radiative linewidths that dramatically differ from those of an isolated atom. We demonstrate the idea of subwavelength-scale light manipulation by engineering the spatial phase profile of an incident monochromatic plane wave. The tailored incident field produces localized dipolar subwavelength-scale excitations of the atoms in desired locations in the lattice even though the atoms are not addressed individually. By dynamically adjusting the phase pattern of the field, the excitations can be controlled and moved around in the lattice.

The particular example of subwavelength-scale localization of optical excitations we study here has attracted considerable interest in nanophotonics with possibilities for microscopy and data storage applications. Spatial and temporal modulation of ultrashort laser pulses leads to excitation of energy hot spots in nanostructures [10,11]. Also interactions between induced currents and plasmonic waves on nanostructures permit the excitation of subwavelength hot spots by amplitude- or phase-modulated monochromatic fields [12–14].

Nontrivial collective optical properties result from a cooperative response of the strongly interacting, closely spaced atoms: Recurrent scattering events, in which a photon is repeatedly scattered by the same atom, lead to collective modes with strongly modified spatial configurations and radiation rates [15–19]. Such scattering processes can result in light localization that is analogous to Anderson localization of electrons [20–22]. The resonant response is very different from the studies of off-resonant optical diagnostics of atomic correlations in optical lattices [23–29]. Photonic band gaps for atomic lattices have previously been calculated in Ref. [30].

For our lattice system we numerically calculate the optical response by stochastically sampling the atomic positions according to their spatial distributions, then solving the recurrent scattering events to all orders for each stochastic realization. We find a strong resonant response in the case of closely spaced atoms with the near-field emission pattern from the atoms forming sharp, narrow spatially localized amplitude peaks. The response is sensitive to detuning of the incident light from the atomic resonance and to the spatial separation between the atoms. Tuning light off resonant or increasing the lattice spacing rapidly leads to suppressed interactions. If the atoms are not confined strongly enough to the individual lattice sites, the resulting increased disorder in the atomic positions due to quantum fluctuations also suppresses the strong collective effects in the ensemble-averaged response.

We take the atoms to occupy the lowest-energy band in a 2D square optical lattice of periodicity $a$ in the $xy$ plane [31]. We assume that the atoms are tightly confined to the lowest vibrational state in the $z$ direction of an oblate external potential and that they reside in the Mott-insulator state with precisely one atom per site. In a combined harmonic trap and the lattice the single-occupancy state of bosonic atoms can exist in a weak harmonic trap or can be engineered from the typical "wedding-cake" Mott-insulator ground state by manipulating the multioccupancy states, e.g., by single-site addressing or by inducing atom parity-dependent losses [9]. In a deep lattice, the

 

vibrational ground-state (Wannier) wave function in each site is approximately that of a harmonic oscillator with frequency $\omega = 2\sqrt{s}E_R/\hbar$ [resulting in a Gaussian density profile $\rho_j(\mathbf{r})$ with the $1/e$ width $\ell = as^{-1/4}/\pi$ in the $xy$ plane], where $s$ denotes the lattice depth in the units of the lattice-photon recoil energy $E_R = \pi^2\hbar^2/(2ma^2)$ [32].

We illuminate the lattice with a monochromatic incident field $\mathbf{E}_{\text{in}}(\mathbf{r},t)$ whose frequency $\Omega$ is nearly resonant on an electric-dipole transition. This impinging field excites the dipole transition of the atoms, producing scattered light that, in turn, impact the driving of neighboring atoms and alter their scattered light. The scattered photons can mediate strong interactions between closely spaced atoms, so that the atomic system responds to light cooperatively, exhibiting *collective* excitation eigenmodes. Here, we show how to exploit these interactions for controlling and manipulating light on a subwavelength scale. As a specific example, we prepare subwavelength-scale spatially localized collective excitations of the atoms in isolated regions of the lattice by considering an incident plane-wave illumination of the atoms with an approximately sinusoidal phase profile. Such a response is distinct from that which would be seen if the atoms did not interact.

In order to model the cooperative atom response to light, we assume the incident field is sufficiently weak that saturation of the excited state can be neglected. For simplicity, we consider the atomic internal states as an effective two-level system consisting of a single electronic ground and excited state. A desired two-level configuration could be realized with a cycling transition by shifting all other transitions out of resonance or with a $J = 0 \to J = 1$ transition with all except one excited state shifted out of resonance [31]. On impact, light drives atomic transitions inducing a polarization density $\mathbf{P}^+(\mathbf{r},t) = \sum_j \mathbf{P}_j^+(\mathbf{r},t)$, where the polarization within each site $j$, $\mathbf{P}_j^+(\mathbf{r},t) = \mathbf{d}_j\delta(\mathbf{r}-\mathbf{r}_j)$, and $\mathbf{d}_j$ is the electric-dipole moment of an atom at site $j$ with fixed orientation $\hat{\mathbf{d}}$. To facilitate numerical evaluation of the lattice response, we express the polarization in terms of stochastic amplitudes $e_j$, representing the coherence of atoms $j$ realized for a stochastic sampling of atomic positions from the atomic density distributions $\rho_j(\mathbf{r}_j)$, such that $\mathbf{P}_j^+(\mathbf{r},t) = e^{-i\Omega t}\wp\hat{\mathbf{d}}\,\delta(\mathbf{r}-\mathbf{r}_j)e_j(t)$, where $\wp$ is the atomic dipole matrix element. When the atomic dynamics evolve on time scales much longer than the light propagation time across the optical lattice [17], the induced polarization produces the scattered electric field $\mathbf{E}_{\text{S},j}^+(\mathbf{r}) = k^3/(4\pi)\int d^3r'\mathbf{G}(\mathbf{r}-\mathbf{r}')\cdot\mathbf{P}_j^+(\mathbf{r}')$, where $\mathbf{G}(\mathbf{r}-\mathbf{r}')$ is the monochromatic dipole radiation kernel representing the radiated field at $\mathbf{r}$ from a dipole residing at $\mathbf{r}'$ [33]. Thus, the atom at site $j$ experiences driving by the sum of fields $\mathbf{E}_{\text{S},j'}$ scattered from all other atoms in the lattice and the incident field $\mathbf{E}_{\text{in}}$; these scattered fields are proportional to the amplitudes $e_{j'}$ of their atoms of origin. These multiple-scattering processes therefore produce collective dynamics described by

$$\dot{e}_j = (i\delta - \Gamma/2)e_j + \sum_{j'\neq j}\mathcal{C}_{j,j'}e_{j'} + F_j, \tag{1}$$

$$\mathcal{C}_{j,j'} \equiv \frac{3\Gamma}{2i}\hat{\mathbf{d}}^*\cdot\mathbf{G}(\mathbf{r}_j-\mathbf{r}_{j'})\cdot\hat{\mathbf{d}}, \tag{2}$$

where $F_j \equiv e^{i\Omega t}\wp\hat{\mathbf{d}}^*\cdot\mathbf{E}_{\text{in}}^+(\mathbf{r}_j)/(i\hbar)$ is the direct driving of the atom in site $j$ by the incident field, $\delta \equiv \Omega - \omega_{e,g}$ is the detuning of the field from resonance, and $\Gamma$ is the spontaneous emission rate. For each stochastic realization of atomic positions, interactions between $N$ atoms in an optical lattice lead to the formation of $N$ *collective* atomic excitations, each with its own resonance frequency and spontaneous emission rate, which could have either superradiant or subradiant characteristics.

By exploiting the strong DD interactions, one can tailor the incident field so that it excites specific linear combinations of collective modes, providing a desired response. Here we consider a linearly polarized ($\hat{\mathbf{e}}_y$ chosen to be in the lattice plane), phase-modulated field whose positive frequency component reads

$$\mathbf{E}_{\text{in}}^+(\mathbf{r},t) \simeq \hat{\mathbf{e}}_y E_0 e^{i\varphi(x,y)}e^{i(kz-\Omega t)}, \tag{3a}$$

$$\varphi(x,y) = \frac{\pi}{2}\sin(\kappa x)\sin(\kappa y). \tag{3b}$$

A field profile of this form, however, contains evanescent plane-wave components whose transverse wave vectors exceed the carrier wave number $k = 2\pi/\lambda = \Omega/c$. We therefore approximate it through the truncated Fourier expansion [31]. One can produce such a field, e.g., by a hologram or a spatial light modulator. Figure 1 illustrates how the field Eq. (3) can excite a checkerboard pattern of localized excitations of atoms in a lattice. In these calculations, we neglect the width of the atomic wave functions in the $z$ direction, as a width on the order of $\ell$ in the $z$ direction has a negligible effect on the calculated response. We fully incorporate, however, quantum fluctuations of the atomic positions on the lattice plane in the vibrational ground state of each site. The many-atom correlations of the specific one-atom Mott state are also included. As we discuss below, interactions between the atoms are vital to the realization of this pattern. The atoms are arranged in an $18\times 18$ square lattice with the spacing $a = 0.55\lambda$, and a site residing at $(x_0,y_0) = (a/2,a/2)$. The incident field [Eq. (3)] has a modulation period $2\pi/\kappa = 6a$, indicating the periodicity of $\mathbf{E}_{\text{in}}(\mathbf{r},t)$ of six sites. We choose the dipole orientation $\hat{\mathbf{d}} \approx \hat{\mathbf{e}}_z + 0.1\hat{\mathbf{e}}_y$ to be slightly rotated from the normal to the lattice plane so that the atoms scatter fields largely within the plane while allowing them to be driven by the incident field. The collective mode spontaneous emission rates range from a very subradiant $3\times 10^{-3}\Gamma$ to the superradiant $5\Gamma$, while their frequency shifts from the single-atom resonance range from $-2\Gamma$ to $0.9\Gamma$.

We first consider an infinitely deep lattice in which the atoms are perfectly confined at the center in their respective sites, i.e., with $\ell = 0$. In this case, the atomic positions are deterministic and Eq. (1) reduces to a coupled set of linear equations whose steady-state solutions for $|e_j|^2$ are shown in Fig. 1(a). A subset of atoms residing at the local minima of $\varphi(x,y)$ are more strongly excited than those in the surrounding lattice sites, while those at the local maxima of $\varphi(x,y)$ are roughly as weakly excited as their surroundings. The peaks sit on a background with excitations roughly 0.2 times those of the most excited atoms. We find a subwavelength excitation full width at half maximum (FWHM) width of the peak to be less than $0.9\lambda$. This results in a checkerboard pattern with

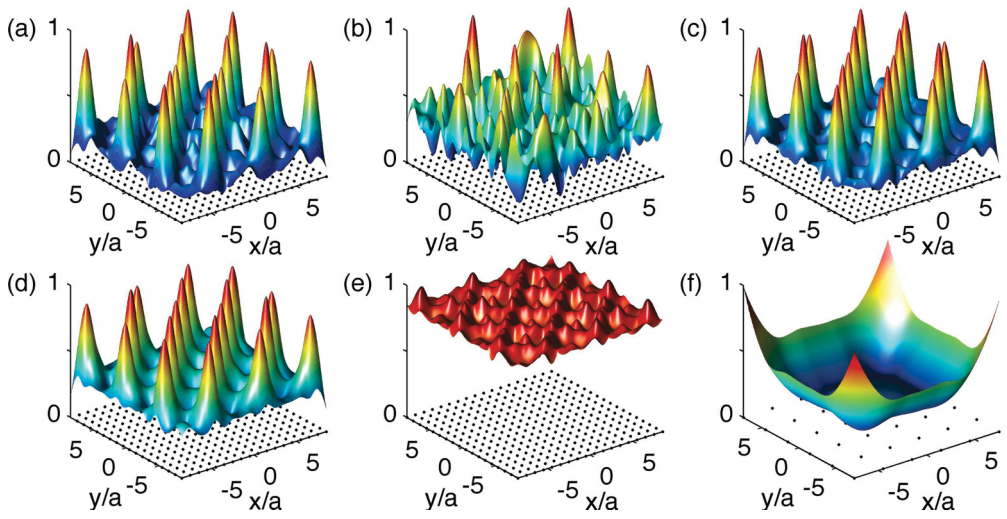


FIG. 1. (Color online) The atomic excitation intensities $\overline{|e_j|^2}/\max_{j'}\overline{|e_{j'}|^2}$ resulting from the cooperative response of an optical lattice system to a phase-modulated incident field [Eq. (3)] with modulation period of six sites. The black dots indicate the positions of the populated sites. In (a), the atoms are perfectly confined at the center of each site and their wave functions have width $\ell = 0$. (b) represents the response of a single stochastic realization of atomic positions sampled from a 2D Gaussian variables of width $0.12a$ centered on the lattice sites. Ensemble average responses over several thousand realizations of atomic positions are shown for (c), (e), (f) $\ell = 0.12a$ and (d) $\ell = 0.21a$. In (e), the incident field is detuned from the resonance of an isolated atom by $10\Gamma$, while the detuning is zero in all other cases. In (f), atoms were removed from 8/9 of the sites, resulting in an effective spacing of $a' = 3a$, thus suppressing the role of the other sites from the response and destroying the excitation pattern.

the localized excitations separated by $3\sqrt{2}$ sites (three sites in the both directions) sitting on a background of weakly excited atoms. The regularity of the response can be broken by slightly altering the period or the orientation of the phase modulation. Note that the periodicity of the incident field is significantly larger than the width of the localized excitations.

In a more realistic scenario, the width of the lattice site atomic wave function due to zero-point fluctuations cannot be neglected and the scattered light sources are essentially distributed over the atomic densities. To obtain the average atomic excitations $\overline{|e|_j^2}$ that dominate the near-field emission, we solve Eq. (1) through Monte Carlo integration [34]. We obtain a large number of realizations of atomic positions $\mathbf{r}_j$ in each site $j$ sampled from a probability distribution matching the single-atom density function. Then for each realization, we solve Eq. (1) as if the atoms were localized at the sample points. We then compute $|e_j|^2$ for each sample, and perform an ensemble average over all realizations of atomic positions. Each stochastic realization represents a possible outcome of a single experimental run in which atomic positions are localized due to the measurement of scattered photons.

Quantum fluctuations of atomic positions in individual sites can dramatically affect the response. We demonstrate this in Fig. 1(b) which shows the excitation intensities for a single stochastic realization of atomic positions in which a single atom in each site is independently sampled from the Gaussian density distribution of width $\ell = 0.12a$ (corresponding to the lattice height $s \simeq 50$). The nonregularity of the lattice alters the collective modes for the sample realization. Although the atoms are driven by the same incident field that gives rise to the pattern in Fig. 1(a), the displacement drastically alters the collective interaction, and yields an optical response with a significant stochastic noise and a less regular array of localized peaks. Such effects can be washed out when one calculates the ensemble average of the response that corresponds to expectation values obtained over many experimental realizations. The excitation intensity $\overline{|e_j|^2}$ averaged over 6400 position realizations for Wannier functions of width $\ell = 0.12a$ is shown in Fig. 1(c) for the same parameters as those in Fig. 1(b). With atomic wave function of this width, the collective interactions producing the pattern of Fig. 1(a) survive the averaging process, providing an excitation with subwavelength FWHM and a background excitation comparable to that calculated for perfectly localized atoms in an infinitely deep lattice. Weaker confinement only moderately diminishes the cooperative interactions. For $\ell = 0.21a$, corresponding to $s \simeq 5$, Fig. 1(d) shows a weakening contrast of the pattern with a background excitation approximately 0.3 times that of the peaks, which themselves have slightly broader FWHM of $1.2\lambda$.

We can illustrate the essential nature of cooperative interactions in the formation of this excitation pattern by suppressing the DD interactions with an increased detuning of incident light from the single-atom resonance [see Fig. 1(e)]; in this case the probability of a multiple-scattering process for a two-level atom reduces geometrically with the number of single-scattering events involved in that process [18]. While the incident field has the same phase modulation, the excitation pattern of Figs. 1(a) and 1(c) is not preserved.

The pattern formation in Figs. 1(a) and 1(c) truly represents a cooperative response where the interactions between all the atoms in the lattice, including also the weakly excited ones, are essential. We illustrate this in Fig. 1(f), where all of the atoms not residing at the maxima or minima of the incident field phase modulation $\varphi$ have been removed from the lattice. In effect, the lattice spacing was tripled to $a' = 3a = 1.65\lambda$, with an atom

residing at $(x_0, y_0) = (a'/2, a'/2)$. If the removed atoms had not played an essential role, the response of the lattice would show a checkerboard pattern of strongly excited and weakly excited atoms. However, the system response displayed in Fig. 1(f) shows that the atoms in the interior of the lattice are excited roughly evenly even though each atom is driven with an opposite phase to that of its nearest neighbor. Atoms at the edge of the sample are more strongly excited due to finite-size effects.

Sharp localized excitations may be broadened by heating and losses that can inhibit the cooperative atom response. Raman transitions to other vibrational center-of-mass states heat up the atoms, broadening the atomic density distributions in individual sites and increasing the hopping amplitude of the atoms between the adjacent sites. Such processes could be reduced, e.g., due to the orthogonality of the eigenfunctions in each site, if the electronic ground and excited state atoms approximately experience the same lattice potential even if the system is not in the Lamb-Dicke regime. Alternatively, if the atomic linewidth is much larger than the trapping frequency, the collective response may reach a steady state before the heating becomes deleterious.

In conclusion, we have shown that resonant DD interactions between atoms in an optical lattice lead to a collective response that can be exploited in manipulation of light on a subwavelength scale. To illustrate this, we studied an example of engineering a checkerboard pattern of isolated atomic excitations. Unlike in nanofabricated metamaterial samples [10–13], here the effect is not based on interactions between plasmonic and current excitations but purely electric-DD interactions between neutral atoms without magnetic contributions. Moreover, the positions of the excitations can be dynamically altered simply by translating the phase-modulation pattern, so that the collective excitation pattern adiabatically follows the change in the phase pattern. By understanding these interactions, the characteristics of an incident field could be engineered to produce more complex excitations. Our example also demonstrates how a cooperative response can have implications on the resonant absorption imaging of 2D atomic samples in which case deviations from the column density results have been experimentally observed [35]. Moreover, the nontrivial relationship between incident field modes and the collective excitations to which they couple could be of importance, e.g., to imaging and to the implementation of quantum memories in which collective optical excitations facilitate storage and retrieval.

We acknowledge financial support from the Leverhulme Trust and the EPSRC.

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