Localization length of nearly periodic layered metamaterials

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We have analyzed numerically the localization length of light ξ for nearly periodic arrangements of homogeneous stacks (formed exclusively by right-handed materials) and mixed stacks (with alternating right- and left-handed metamaterials). Layers with index of refraction n_1 and thickness L_1 alternate with layers of index of refraction n_2 and thickness L_2. Positional disorder has been considered by shifting randomly the positions of the layer boundaries with respect to periodic values. For homogeneous stacks, we have shown that the localization length is modulated by the corresponding bands and that ξ is enhanced at the center of each allowed band. In the limit of long wavelengths λ, the parabolic behavior previously found in purely disordered systems is recovered, whereas for $\lambda \ll L_1 + L_2$ a saturation is reached. In the case of nearly periodic mixed stacks with the condition $|n_1 L_1| = |n_2 L_2|$, instead of bands there is a periodic arrangement of Lorenztian resonances, which again is reflected in the behavior of the localization length. For wavelengths of several orders of magnitude greater than $L_1 + L_2$, the localization length ξ depends linearly on λ with a slope inversely proportional to the modulus of the reflection amplitude between alternating layers. When the condition $|n_1 L_1| = |n_2 L_2|$ is no longer satisfied, the transmission spectrum is very irregular and this considerably affects the localization length.

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I. INTRODUCTION

During recent decades, a new type of artificial material, so-called left-handed (LH) metamaterials, have attracted a great deal of attention. They present negative indices of refraction for some wavelengths [1], with considerable applications in modern optics and microelectronics [2–5]. Metamaterials can resolve images beyond the diffraction limit [6,7], act as an electromagnetic cloak [8–10], enhance quantum interference [11], or yield to slow light propagation [12].

Regarding the localization length in disordered systems, the presence of negative refraction in one-dimensional (1D) disordered metamaterials strongly suppresses Anderson localization [13]. As a consequence, an unusual behavior of the localization length ξ at long wavelengths λ has been observed. Asatryan *et al.* reported a sixth power dependence of ξ with λ under refractive-index disorder [14,15] instead of the well-known quadratic asymptotic behavior $\xi \sim \lambda^2$ [16–19]. Recently, Mogilevtsev *et al.* [20] also found a suppression of Anderson localization of light in 1D disordered metamaterials combining oblique incidence and dispersion, while Torres-Herrera *et al.* [21] have developed a fourth-order perturbation theory to resolve the problem of nonconventional Anderson localization in bilayered periodic-on-average structures. The effects of polarization and oblique incidence on light propagation in disordered metamaterials were also studied in Ref. [22].

In this paper, we calculate numerically the localization length of light ξ for a 1D arrangement of layers with index of refraction n_1 and thickness L_1 alternating with layers of index of refraction n_2 and thickness L_2. In order to introduce disorder in our system, we change the position of the layer boundaries with respect to the periodic values, maintaining the same values of the refraction indices n_1 and n_2. This is the case of positional disorder, in contrast to the compositional disorder, where there exist fluctuations of the index of refraction [23].

Two structures are analyzed in detail: homogeneous (H) stacks, composed entirely of the traditional right-handed (RH) materials with positive indices of refraction, and mixed (M) stacks, with alternating layers of left- and right-handed materials. For the sake of simplicity, the optical path in both layers is the same; that is, the condition $|n_1 L_1| = |n_2 L_2|$ is satisfied in most of the work. These periodic-on-average bilayered photonic systems have already been studied analytically by Izrailev *et al.* [24,25]. These authors have developed a perturbative theory up to second order in the disorder to derive an analytical expression for the localization length for both H and M stacks. In our case, we have obtained two equations for the localization length ξ as a function of the wavelength λ from our numerical results. For H stacks, a quadratic dependence of ξ for long wavelengths is found, as previously reported in the literature. On the other hand, the localization length saturates for lower values of λ. An exhaustive study of ξ in the allowed and forbidden bands (gaps) of weakly disordered systems is carried out. We show that the localization length is modulated by the corresponding bands and that this modulation decreases as the disorder increases. For low-disordered M stacks and wavelengths of several orders of magnitude greater than the grating period $\Lambda = L_1 + L_2$, the localization length ξ depends linearly on λ with a slope inversely proportional to the modulus of the reflection amplitude between alternating layers.

The plan of the work is as follows. In Sec. II we carry out an exhaustive description of our one-dimensional disordered system and the numerical method used in our localization length calculations. A detailed analysis of ξ in the allowed bands and gaps of homogeneous stacks is performed in Sec. III, where a practical expression for the localization length as a function of λ and the disorder is derived. In Sec. IV we calculate ξ for mixed stacks of alternating LH and RH layers. A linear dependence of the localization length at long wavelengths is found for low-disordered M stacks. Finally, we summarize our results in Sec. V.

II. SYSTEM DESCRIPTION AND NUMERICAL MODEL

Let us consider a one-dimensional arrangement of layers with index of refraction n_1 alternating with layers of index of

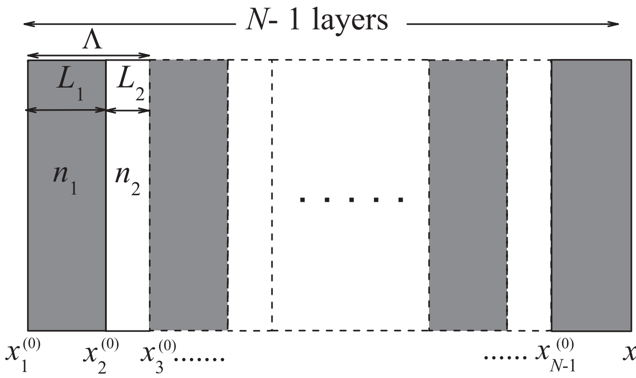

FIG. 1. A periodic arrangement of layers with index of refraction n_1 and thickness L_1 alternating with layers of index of refraction n_2 and thickness L_2. The grating period is $\Lambda = L_1 + L_2$.

refraction n_2. The width of each one is the sum of a fixed length L_i for $i = 1,2$ and a random contribution of zero mean and a given amplitude. The wave numbers in layers of both types are $k_i = \omega n_i/c$, where ω is the frequency and c is the vacuum speed of light. As previously mentioned, the grating period of our system Λ is defined as the sum of the average thicknesses L_1 and L_2 of the two types of layers, that is, $\Lambda = L_1 + L_2$. We have introduced the optical path condition $|n_1L_1| = |n_2L_2|$ for simplicity (in the case of left-handed layers $n_i < 0$, so the absolute value has been written to consider these type of materials). Without disorder, each layer would be limited by two boundaries $x_j^{(0)}$ and $x_{j+1}^{(0)}$, where N is the total number of boundaries. The periodic part of the system considered is schematically represented in Fig. 1.

In the presence of disorder, the position of the corresponding boundaries are

$$x_j = x_j^{(0)} + \xi_j\delta, \quad j = 2,\dots,N-1, \tag{1}$$

except for the first and the last boundary, so as to maintain the same total length L. The parameters ξ_j are zero-mean independent random numbers within the interval $[-0.5,0.5]$. Throughout all our calculations, we have chosen values of the disorder parameter δ less than L_1 and L_2.

For each L, we calculate the transmission coefficient of our structure T and average its logarithm, $\ln T$, over 800 disorder configurations. Then, we obtain numerically the localization length ξ via a linear regression of $\ln T$ [23]:

$$\lim_{L\to\infty} -\frac{\langle \ln T\rangle}{2L} = \frac{1}{\xi}. \tag{2}$$

Here, the angular brackets $\langle\cdots\rangle$ stand for averaging over the disorder. We choose six values of the total length L to perform the linear regression of Eq. (2). The localization length ξ is evaluated as a function of the disorder parameter δ and the frequency of the incident photon ω.

We calculate the transmission coefficient of our system via the characteristic determinant method, first introduced by Aronov *et al.* [26]. This is an exact and nonperturbative method that provides the information contained in the Green's function of the whole system. In our case, the characteristic determinant D_j can be written as [26]

$$D_j = A_jD_{j-1} - B_jD_{j-2}, \tag{3}$$

where the index j runs from 1 to N and the coefficients A_j and B_j can be written as

$$A_j = 1 + \lambda_{j-1,j}\,\frac{r_{j-1,j}}{r_{j-2,j-1}} \tag{4}$$

and

$$B_j = \lambda_{j-1,j}\,\frac{r_{j-1,j}}{r_{j-2,j-1}}\left(1 - r_{j-2,j-1}^2\right). \tag{5}$$

The parameters $r_{j-1,j}$, which are the reflection amplitudes between media $j-1$ and j, are given by

$$r_{j-1,j} = -r_{j,j-1} = \frac{Z_{j-1} - Z_j}{Z_{j-1} + Z_j}, \tag{6}$$

where Z_j corresponds to the impedance of layer j and can be be expressed for normal incidence in terms of its dielectric permittivity ϵ_j and magnetic permeability μ_j as

$$Z_j = \sqrt{\frac{\mu_j}{\epsilon_j}}. \tag{7}$$

The quantity $\lambda_{j-1,j}$ entering Eqs. (4) and (5) is a phase term [26],

$$\lambda_{j-1,j} = \lambda_{j,j-1} = \exp[2ik_{j-1}|x_j - x_{j+1}|]. \tag{8}$$

Here k_{j-1} is the wave number in a layer with boundaries x_j and x_{j+1}. This recurrence relation facilitates the numerical computation of the determinant. The initial conditions are the following:

$$A_1 = 1; \quad D_0 = 1; \quad D_{-1} = 0. \tag{9}$$

The transmission coefficient of our structure T is given in terms of the determinant D_N by

$$T = |D_N|^{-2}. \tag{10}$$

III. LOCALIZATION LENGTH FOR HOMOGENEOUS STACKS

Before dealing with mixed stacks, we present results for low-disordered homogeneous systems with underlying periodicity, which has not been previously studied. In this section we perform a detailed analysis of the localization length ξ in the allowed bands and in the forbidden gaps of disordered H stacks as a function of the disorder δ, the incident wavelength λ, and the reflection coefficient between alternating layers $|r_{j-1,j}|^2$.

As is well known, in the absence of disorder the transmission spectrum of right-handed systems presents allowed and forbidden bands whose position can be easily determined via the following dispersion relation obtained from the Bloch-Floquet theorem [27]:

$$\cos(\beta\Lambda) = \cos(k_1L_1)\cos(k_2L_2) - \frac{1}{2}\left(\frac{Z_2}{Z_1} + \frac{Z_1}{Z_2}\right)\sin(k_1L_1)\sin(k_2L_2), \tag{11}$$

where β is the Block wave vector. When the modulus of the right-hand side of Eq. (11) is greater than 1, β has to be taken as imaginary. This situation corresponds to a forbidden band.

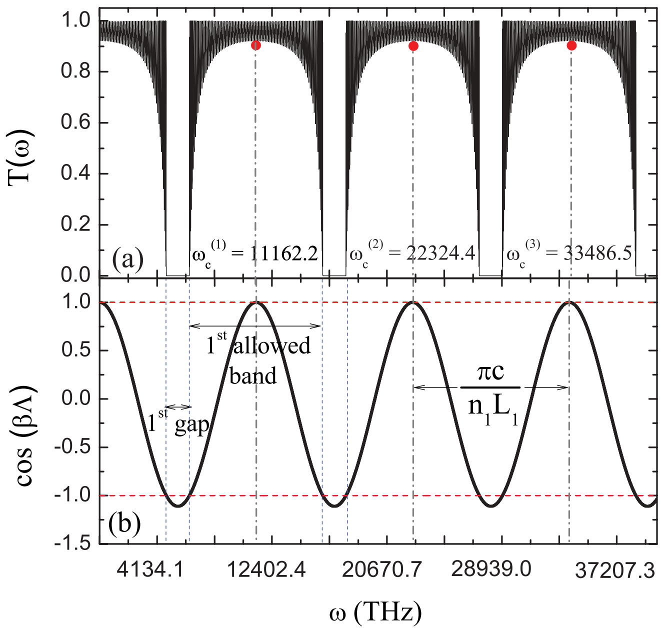

FIG. 2. (Color online) (a) The transmission coefficient T and (b) the parameter $\cos(\beta\Lambda)$ versus the frequency ω for the homogeneous periodic system described in the text (99 layers).

Taking into account the condition $|n_1L_1| = |n_2L_2|$, Eq. (11) reduces to

$$\cos(\beta\Lambda) = \cos^2(k_1L_1) - \frac{1}{2}\left(\frac{Z_2}{Z_1} + \frac{Z_1}{Z_2}\right)\sin^2(k_1L_1). \quad (12)$$

On the other hand, when $\cos(\beta\Lambda)$ is equal to unity, the incident frequency ω is located at the center of the mth allowed band, $\omega_c^{(m)}$. After some algebra, we obtain from Eq. (12)

$$\omega_c^{(m)} = m\pi\left(\frac{c}{n_1L_1}\right) = m\pi\left(\frac{c}{n_2L_2}\right). \quad (13)$$

Let us first consider a periodic H stack formed by 50 layers of length $L_1 = 52.92$ nm and index of refraction $n_1 = 1.58$ alternating with 49 layers of length $L_2 = 39.38$ nm and $n_2 = 2.12$. The total size of our structure is 4.57 μm and the reflection coefficient between alternating layers is 0.05259. Figure 2(a) represents the transmission coefficient T as a function of the frequency ω to illustrate its behavior. Also shown are the centers of each allowed band calculated via Eq. (13). There are 99 peaks in each band, so they can hardly been resolved on the scale used. Moreover, in Fig. 2(b) the parameter $\cos(\beta\Lambda)$ is plotted versus the frequency ω for this periodic system. The first gap and the first allowed band have been shown for better comprehension.

A systematic numerical simulation of a realistic system with 50 000 layers has been carried out. The parameters are the same as in the previous example. In Fig. 3 we represent the localization length ξ versus the wavelength λ for different values of the disorder parameter δ (shown in the legend of the figure). The dashed line corresponds to "total disorder," that is, an arrangement of layers with random boundaries and alternating indices of refraction n_1 and n_2. Several features are evident in the figure. For long wavelengths, one observes quadratic asymptotic behavior, as can be compared with the dotted line [16–19]. An in-depth numerical analysis of the coefficient characterizing this dependence has been performed. To this aim, 20 different H stacks were considered and the following expression for the localization length was found:

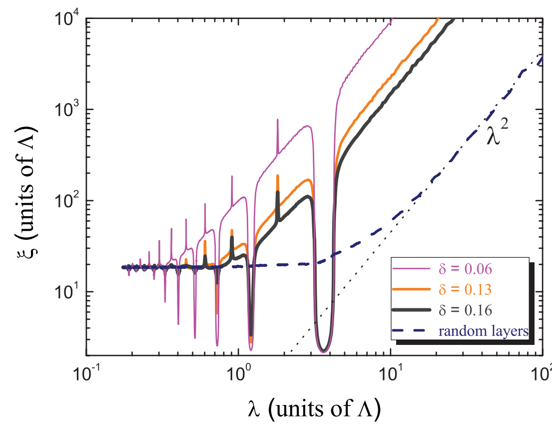

FIG. 3. (Color online) Localization length ξ vs the wavelength λ for different values of the disorder parameter δ. The H stack corresponds to the arrangement represented in Fig. 2 but now 50 000 layers have been considered. The dashed line stands for the "total disorder" case. All lengths are expressed in units of the grating period Λ.

$$\xi \simeq 0.063\frac{\lambda^2}{\Lambda_{op}^2 r^2\delta^2}, \quad \text{for} \quad \lambda \to \infty \quad (14)$$

where $\Lambda_{op} = n_1L_1 + n_2L_2$ is the optical path across one grating period Λ. All the lengths in Eq. (14) are expressed in units of Λ. In the opposite limit of short λ, the localization length ξ saturates to a constant value [15,28]. Our numerical results have shown that this constant is proportional to the inverse of the reflection coefficient between alternating layers $|r|^2$, that is,

$$\xi \simeq \frac{1}{r^2}, \quad \text{for} \quad \lambda \to 0. \quad (15)$$

Izrailev *et al.* [24,25] have developed a perturbative theory up to second order in the disorder to calculate analytically the localization length in both homogeneous and mixed stacks. This model is quite general and is valid for both quarter stack medium (that is, arrangements of two types of layers with the same optical path) and systems with different optical widths. Assuming uncorrelated disorder and random perturbations with the same amplitude in both layers (the main considerations in our numerical calculations), one can easily derive the following analytical expression for ξ at long wavelengths from Izrailev's formulation:

$$\xi = \frac{Z_1Z_2}{(Z_1 - Z_2)^2}\frac{2\lambda^2}{\pi^2(n_1^2 + n_2^2)\delta^2}. \quad (16)$$

For similar values of the layer impedances $Z_1 \simeq Z_2$, the first term in Eq. (16) can be approximated by $1/4r^2$ and $n_1^2 + n_2^2 \simeq 2\Lambda_{op}^2$, so Eq. (16) reduces to

$$\xi = \left(\frac{1}{4\pi^2}\right)\frac{\lambda^2}{\Lambda_{op}^2 r^2\delta^2} \simeq 0.025\frac{\lambda^2}{\Lambda_{op}^2 r^2\delta^2}, \quad \text{for} \quad \lambda \to \infty, \quad (17)$$

which is similar to our numerical expression, Eq. (14).

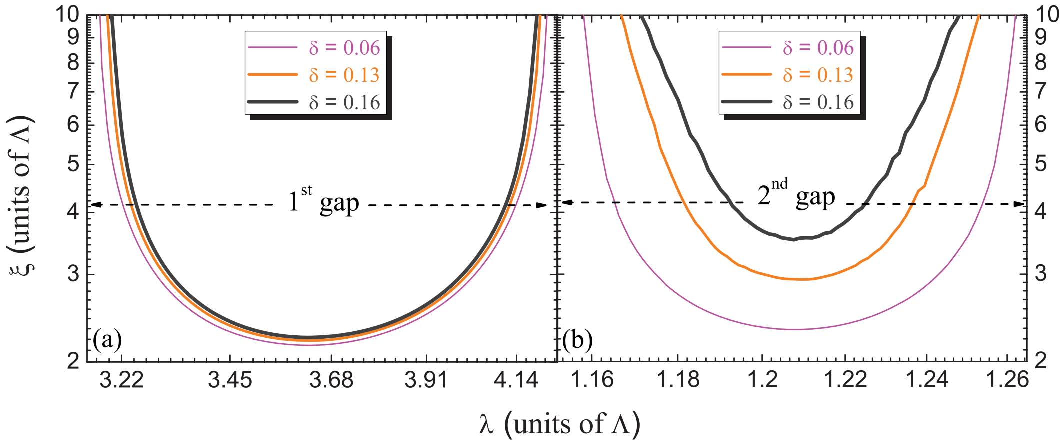

FIG. 4. (Color online) Localization length ξ vs the wavelength λ for (a) the first and (b) the second gaps depicted in Fig. 3.

The randomness only partially affects the periodicity of the system, which manifests in the existence of bands and gaps. The localization length depends on the position in the band and on the disorder. The modulation of ξ by the bands can be clearly appreciated in Fig. 3. These results are consistent with other published works on this topic [29,30]. Recently, Mogilevtsev *et al*. [29] reported that the photonic gaps of the corresponding periodic structure are not completely destroyed by the presence of disorder, while Luna-Acosta *et al*. [30] showed that the resonance bands survive even for relatively strong disorder and large number of cells.

Having a close look into the first gap in Fig. 3, one observes that the localization length is practically independent of the disorder δ. In order to visualize this effect, Fig. 4 represents (a) the first and (b) the second gaps depicted in Fig. 3. As mentioned, the dependence of ξ with the disorder is almost negligible in the first gap. When the wavelength is similar to the grating period Λ, the influence of the disorder is greater, as can be easily deduced from simple inspection of Fig. 4(b).

Let us now focus on the allowed bands and study in detail the behavior of the localization length in these regions. To this aim, three-dimensional (3D) graphs of ξ versus the wavelength λ and the disorder δ have been plotted in Fig. 5 for (a) the first and (b) the third allowed bands (see again Fig. 2). All magnitudes have been normalized to the grating period Λ. The localization length ξ is enhanced in a small region around the center of each allowed band. A similar result was found by Hernández-Herrejón *et al*. [31], who obtained a resonant effect of ξ close to the band center in the Kronig-Penney model with weak compositional and positional disorder. This increase in the localization length is due to emergence of the Fabry-Perot resonances associated with multiple reflections inside the layers from the interfaces [24,25,32]. In particular, for homogeneous quarter stack systems, the Fabry-Perot resonances arise exactly in the middle of each allowed band where β vanishes [24,25]. The saturation of ξ for short wavelengths is also appreciated in these 3D images.

Up to now, H stacks with the same optical path in layers of both types have been considered, that is, arrangements

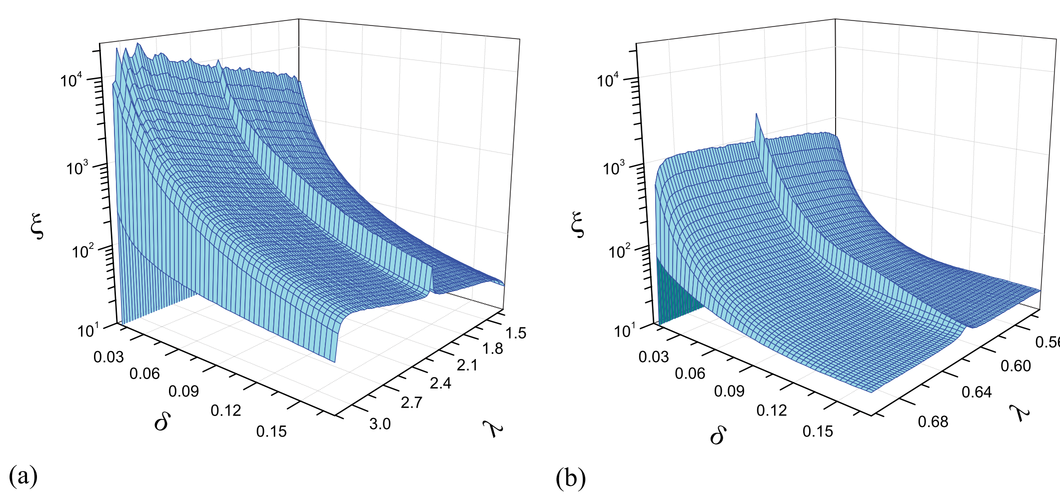

FIG. 5. (Color online) Three-dimensional graphs of the localization length ξ vs the wavelength λ and the disorder δ for (a) the first and (b) the third allowed bands. The H stack is the same as in Fig. 2. All lengths are expressed in units of the grating period Λ.

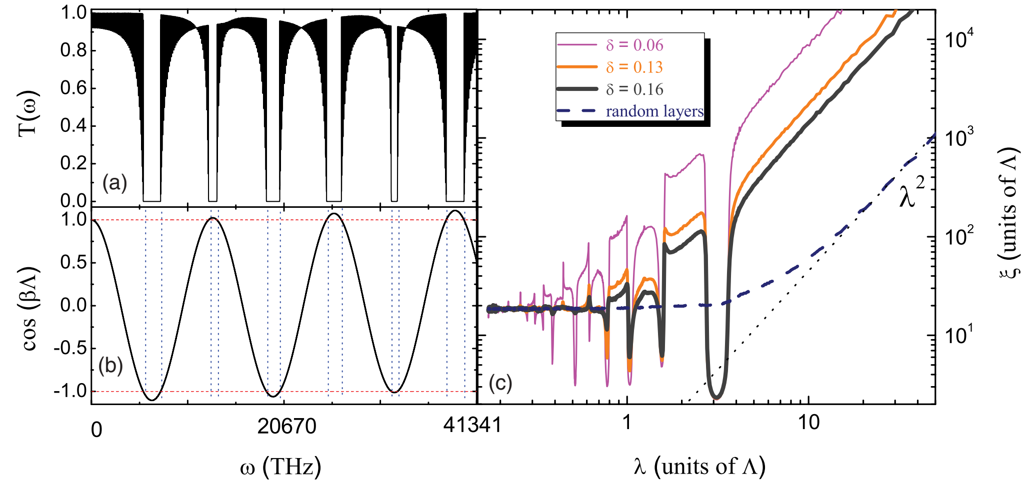

FIG. 6. (Color online) (a) The transmission coefficient T and (b) the parameter $\cos(\beta\Lambda)$ vs the frequency ω for the asymmetric periodic H stack described in the main text (99 layers) and (c) the corresponding localization length ξ vs the wavelength λ for different disorder parameters δ (50 000 layers).

verifying the condition $|n_1L_1| = |n_2L_2|$ in the absence of disorder. As a consequence, the transmission spectrum T of the corresponding periodic system presented a symmetric distribution of allowed bands and gaps (as previously shown in Fig. 2). What happens in the case of a nonsymmetric band distribution, that is, when the condition $|n_1L_1| = |n_2L_2|$ is not satisfied? To answer this question, we have plotted the transmission coefficient T [Fig. 6(a)] and the parameter $\cos(\beta\Lambda)$ [Fig. 6(b)] versus the frequency ω for a periodic H stack formed by 50 layers of length $L_1 = 52.92$ nm and index of refraction $n_1 = 1.58$ alternating with 49 layers of length $L_2 = 28.80$ nm and $n_2 = 2.12$. Note that the condition $|n_1L_1| = |n_2L_2|$ is no longer held, so the band structure is asymmetric. Accordingly, the localization length ξ shown in Fig. 6(c) presents an irregular form in the allowed and forbidden bands. As in the symmetric case, no band modulation exists for high disorders and the quadratic asymptotic behavior for long wavelengths is also verified. Moreover, the peaks in the localization length due to Fabry-Perot resonances still can be appreciated, although they are no longer in the center of the bands [24,25]. A total number of 50 000 layers was considered in our localization length calculations.

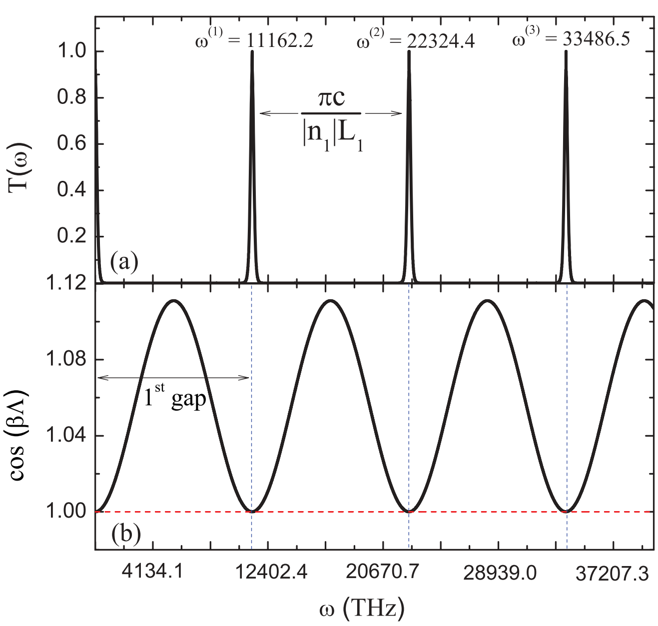

FIG. 7. (Color online) (a) The transmission coefficient T and (b) the parameter $\cos(\beta\Lambda)$ vs the frequency ω for the mixed periodic system described in the text (99 layers).

IV. LOCALIZATION LENGTH FOR MIXED STACKS

Now that we have analyzed in detail the behavior of the localization length ξ for homogeneous systems, let us deal with M stacks composed of alternating LH and RH layers.

In our numerical calculations we have considered a periodic M stack formed by 50 layers of length $L_1 = 52.92$ nm and index of refraction $n_1 = -1.58$ alternating with 49 layers of length $L_2 = 39.38$ nm and $n_2 = 2.12$. Again, the condition $|n_1L_1| = |n_2L_2|$ has been imposed. Note that this arrangement has similar parameters to the one depicted in Sec. III, but now

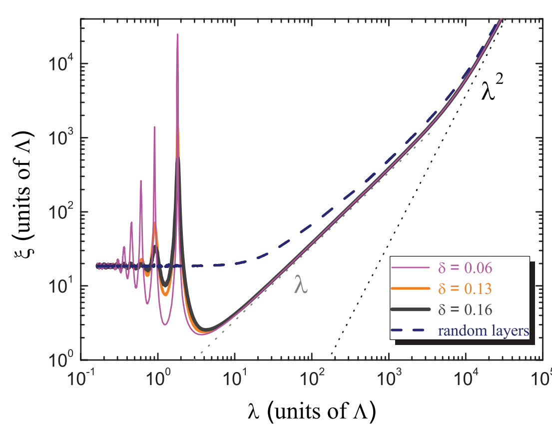

FIG. 8. (Color online) Localization length ξ vs the wavelength λ for different disorder parameters δ. The M stack corresponds to the one represented in Fig. 7 but here 50 000 layers have been considered. The dashed line stands for the "total disorder" case.

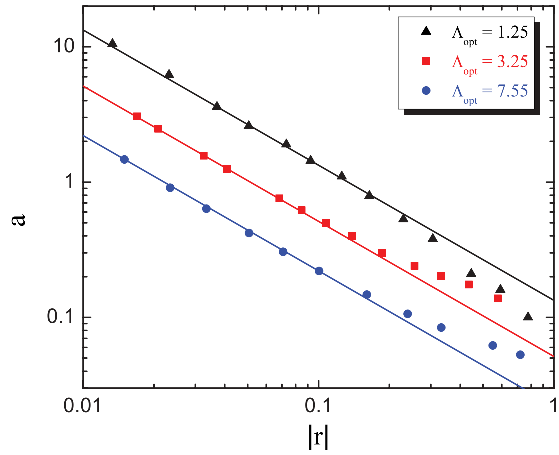

FIG. 9. (Color online) Numerical calculations of the slope a vs $|r|$ for several values of Λ_{op} (expressed in units of the grating period Λ). The solid lines correspond to the results obtained via Eq. (18).

n_1 is negative. This change of sign results in a severe modification of the transmission coefficient T. For this the periodic system, Fig. 7 represents (a) the transmission coefficient T and (b) the parameter $\cos(\beta\Lambda)$ versus the frequency ω of the incident light. Unlike the H stack case, no allowed bands exist and practically the entire transmission spectrum is formed by gaps. A set of periodically distributed Lorentzian resonances is found instead. The position of the center of each resonance is given by Eq. (13), that is, the center of the allowed bands in homogeneous systems.

In respect to the localization length, positional disorder was introduced as explained in Sec. II. As previously considered, the total number of layers in our numerical calculations was 50 000 and the number of disordered configurations to average the logarithm of the transmission coefficient was 800. The result is shown in Fig. 8, where the localization length ξ is represented versus the wavelength λ for different values of the disorder parameter δ. The dashed line corresponds to the "total disorder" case. Again, for long wavelengths a quadratic asymptotic behavior of ξ is found, but now a region where the localization length is proportional to λ exists. We turn to this point in the next figure to quantify the slope of this linear dependence. The Lorentzian resonances associated with multiple reflections in the layers modulate the shape of ξ and this modulation decreases as the disorder increases. Moreover, the saturation of the localization length for low wavelengths can also be appreciated. As in the H stack case, the constant where ξ saturates is proportional to the inverse of the reflection coefficient between alternating layers $|r|^2$.

The linear dependence of ξ with the wavelength λ has been exhaustively studied by our group to find a simple analytical expression for the localization length in this region. More than 30 different M stacks have been simulated and we have arrived at the following empirical equation:

$$\xi = \frac{\lambda}{6\Lambda_{\text{op}}|r|} = a\lambda, \tag{18}$$

where ξ, λ, and Λ_{op} are expressed in units of the grating period Λ. In Fig. 9, our numerical calculations of the slope a versus $|r|$ have been plotted for several values of Λ_{op}: triangles (1.25), squares (3.25), and circles (7.55). The solid lines correspond to the results obtained via Eq. (18). One notices a good degree of validity for a wide range of $|r|$ values.

Finally, let us now consider an asymmetrical M stack where the condition $|n_1L_1| = |n_2L_2|$ is no longer satisfied. In Fig. 10 we have represented (a) the transmission coefficient T and (b) the parameter $\cos(\beta\Lambda)$ versus the frequency ω for a periodic M stack formed by 50 layers of length of length $L_1 = 52.92$ nm and index of refraction $n_1 = -1.58$ alternating with 49 layers of length $L_2 = 28.80$ nm and $n_2 = 2.12$. Note the strong difference between this transmission spectrum and the symmetrical one [see Fig. 7(a)], where a set of periodically distributed Lorenztian resonances exists. Despite this fact, the localization length ξ shown in Fig. 10(c) presents a region of linear dependence with the wavelength, as in the symmetric case. However, Eq. (18) cannot be used to evaluate the localization length in this region.

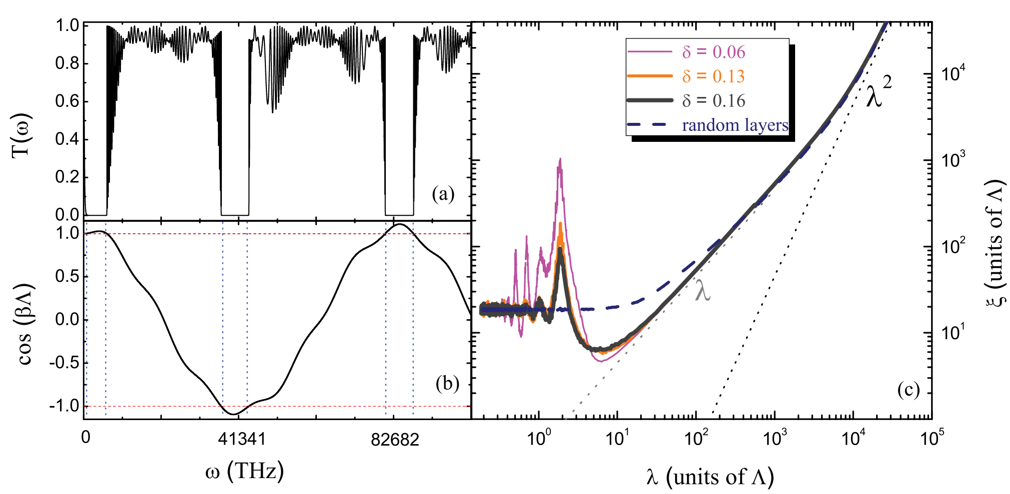

FIG. 10. (Color online) (a) The transmission coefficient T and (b) the parameter $\cos(\beta\Lambda)$ vs the frequency ω for the asymmetric periodic M stack described in the main text (99 layers) and (c) the corresponding localization length ξ vs the wavelength λ for different disorder parameters δ (50 000 layers).

V. DISCUSSION AND CONCLUSIONS

We have analyzed numerically the localization length of light ξ for homogeneous and mixed stacks of layers with index of refraction $\pm|n_1|$ and thickness L_1 alternating with layers of index of refraction $|n_2|$ and thickness L_2. The positions of the layer boundaries have been randomly shifted with respect to ordered periodic values. The refraction indices n_1 and n_2 present no disorder.

For H stacks, the parabolic behavior of the localization length in the limit of long wavelengths, previously found in purely disordered systems [16–19], has been recovered. On the other hand, the localization length ξ saturates for very low values of λ. The transmission bands modulate the localization length ξ and this modulation decreases with increasing disorder. Moreover, the localization length is practically independent of the disorder δ at the first gap; that is, it has a very low tendency in this region. We have also characterized ξ in terms of the reflection coefficient of alternating layers $|r|^2$ and the optical path across one grating period Λ_{op}. Equation (14) has been proved to be valid for a wide range of $|r|^2$ values, that is, from transparent to opaque H stacks. It has also been shown (see Fig. 5) that the localization length ξ is enhanced at the center of each allowed band.

When left-handed metamaterials are introduced in our system, the localization length behavior presents some differences with respect to the traditional stacks, formed exclusively by right-handed materials. For low-disordered M stacks and wavelengths of several orders of magnitude greater than the grating period Λ, the localization length ξ depends linearly on λ with a slope inversely proportional to the modulus of the reflection amplitude between alternating layers $|r|$ [see Eq. (18)]. As in the H case, ξ saturates for low wavelengths, with this saturation constant proportional to the inverse of $|r|^2$.

If we take into account losses, there is an absorption term whose absorption length ξ_{abs} is [15]

$$\xi_{abs} = \frac{\lambda}{2\pi\sigma}, \tag{19}$$

where σ is an absorption coefficient. The inverse of the total decay length is the sum of the inverse of the localization length ξ plus the inverse of the absorption length ξ_{abs}. Note that ξ_{abs} is proportional to λ, so for low-disordered M stacks and weak absorption metamaterials, the final expression for the localization length ξ in the linear region can be written as

$$\xi = \frac{\lambda}{6\Lambda_{op}|r| + 2\pi\sigma}. \tag{20}$$

In the case of both homogeneous and mixed stacks with nonsymmetric band distribution, that is, when the condition $|n_1L_1| = |n_2L_2|$ is not satisfied, the localization length ξ presents an irregular form in the transmission spectrum. These changes in ξ are more sensitive in mixed stacks than in homogeneous structures.

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