Polarization property of the THz wave generated from a two-color laser-induced gas plasma

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(Received 15 December 2011; published 20 August 2012)

The recent experiments show that the polarization of the terahertz (THz) wave generated from two-color laser-induced gas plasma is a linear polarization, and the polarization angle of the THz wave can be controlled by the optical delay of the two laser beams. In this paper, the polarization property of the THz wave is studied theoretically via a classical and a quantum model, respectively. An analytical expression is obtained to reflect the relationship between the polarization angle of the THz wave and the relative phase of the two-color lasers.

DOI: 10.1103/PhysRevA.86.023833 PACS number(s): 42.50.−p, 13.88.+e

I. INTRODUCTION

The most efficient process for terahertz (THz) generation using laser pulses is based on optical rectification and difference-frequency generation in second-order nonlinear media [1,2]. However, intensive THz radiation could be obtained from a gas plasma channel induced by two-color femtosecond optical pulses. One pulse is the fundamental light (ω_1), and the other is the second-harmonic light (ω_2) [3–9]. It is well known that the process of the generation of the THz radiation in gases is described phenomenologically by a four-wave mixing process [6,10], but the complete explanation of the process and the underling mechanism are described by the nonperturbative asymmetric field-ionization models [11–14]. When the two light pulses are both circularly polarized or one is circularly polarized and the other is linearly polarized, the linearly polarized THz wave is obtained, and the azimuthal orientation of the THz polarization can be controlled by changing the optical delay of the two pulses ($\Delta\tau$), which produces a method for fast THz wave modulation and coherent control of nonlinear responses excited by an intensive THz wave [15,16]. These papers about the controllable THz polarization are very successful and are very interesting. They separately simulate the models that tunnel ionized electron wave-packet dynamics [15] and one-dimensional transient current [16] to explain these experimental results. However, there is no analytical expression to explain the relationship between the polarization angle of the THz wave and the optical delay. In this paper, based on the four-wave mixing process, by analyzing the function relation of θ_{THz} on $\Delta\varphi$, where θ_{THz} is the polarization angle of the THz wave and $\Delta\varphi$ is the relative phase, an analytic expression of θ_{THz} changing with $\Delta\varphi$ is obtained. The results demonstrate that θ_{THz} is equal to $-\Delta\varphi$, and $\Delta\varphi$ is in inverse proportion to $\Delta\tau$ when the pulses are both right-handed circularly polarized. The cases for the two lasers being left-circularly or linearly polarized are also discussed. Our results build a theoretical method for the analysis of the polarization property of the THz wave generated from two-color laser-induced gas plasma.

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II. PROBLEM FORMULATION FOR THE TWO-COLOR LASERS BEING RIGHT-CIRCULARLY POLARIZED

Let $\varphi_{\omega 1}$, $\varphi_{\omega 2}$, and φ_{THz} be the phases of the ω_1, ω_2 light pulses and the THz wave, respectively, which can be expressed as $\varphi_{\omega 1} = \omega_1 t + \varphi_1$, $\varphi_{\omega 2} = \omega_2 t + \varphi_2$, and $\varphi_{\mathrm{THz}} = \omega_{\mathrm{THz}} t$, where φ_1 and φ_2 are the initial phases. Because two ω_1 photons interact with one ω_2 photon in the plasma to generate one THz photon, i.e., $\hbar\omega_{\mathrm{THz}} = 2\hbar\omega_1 - \hbar\omega_2$, we have the following relation:

$$\Delta\varphi = 2\varphi_{\omega 1} - \varphi_{\omega 2} - \varphi_{\mathrm{THz}} = 2\varphi_1 - \varphi_2. \tag{1}$$

Experimental results have shown that θ_{THz} is a function of the optical delay $\Delta\tau$ [13,14]. The above-mentioned experiment has given the relationship $\Delta\tau = \Delta l(n_{\omega 2} - n_{\omega 1})$, whereas, the relative phase due to the step translation stage is $\Delta\varphi = 2\varphi_1 - \varphi_2 = 2\pi\ \Delta l(2n_{\omega 1}/\lambda_{\omega 1} - n_{\omega 2}/\lambda_{\omega 2}) = 2\pi\ \Delta l/\lambda_{\omega 2}(n_{\omega 1} - n_{\omega 2}) \propto -\Delta\tau$, here, Δl is the step translation stage, $n_{\omega 1}$ and $n_{\omega 2}$ are the refractive indices of the medium at ω_1 and ω_2, and $\lambda_{\omega 1}$ and $\lambda_{\omega 2}$ are the wavelengths of the ω_1 light and ω_2 light, respectively. Therefore, $\Delta\tau$ is in inverse proportion to $\Delta\varphi$. That means

$$\theta_{\mathrm{THz}} = f(\Delta\varphi). \tag{2}$$

In order to determine the form of this function, we perform an imaginative experiment. As shown in Fig. 1, x, y, and z axes are the space coordinates. The arrows along the circles describe the helicity state of the photon, which is right-circularly polarized.

At first, we rotate the whole experimental installation around the z axis with an angle θ, while we define the right circular rotation around the axis of the positive angle. Such an operation creates a rotation phase that is equal to the propagation phase owing to the helicity state of the photon. Thus, the phase relationship after the rotation is

$$\begin{aligned} &\varphi'_{\omega 1} = \omega_1 t + \varphi_1 - \theta, \quad \varphi'_{\omega 2} = \omega_2 t + \varphi_2 - \theta, \\ &\varphi'_{\mathrm{THz}} = \varphi_{\mathrm{THz}} = \omega_{\mathrm{THz}} t, \\ &\Delta\varphi' = 2\varphi'_{\omega 1} - \varphi'_{\omega 2} - \varphi'_{\mathrm{THz}} = 2\varphi_1 - \varphi_2 - \theta = \Delta\varphi - \theta. \end{aligned} \tag{3}$$

Note that the phase of the THz wave should stay the same because of the linear polarization. However, the azimuthal

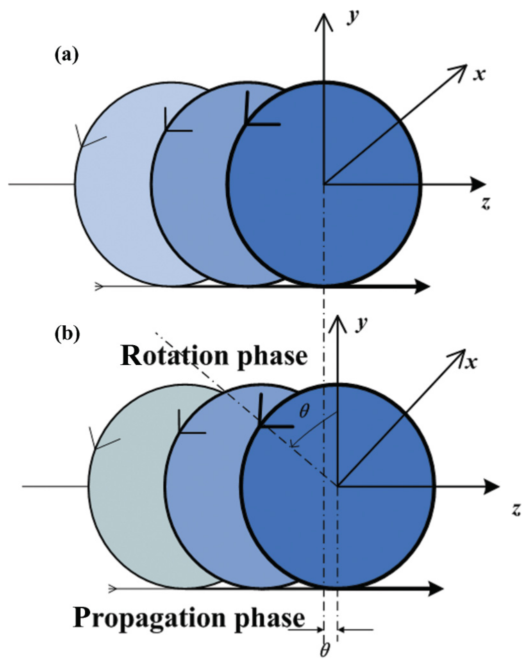

FIG. 1. (Color online) (a) The photon before rotation, which is right-circularly polarized. (b) After rotation by θ, the propagation phase is changed by the same angle.

polarization orientation angle of the THz wave after the rotation is

$$\theta'_{\mathrm{THz}} = f(\Delta\varphi') = f(\Delta\varphi - \theta). \tag{4}$$

On the other hand, in consideration of the isotropy of the space, θ'_{THz} relating to θ_{THz} would rotate by the same angle θ. Thus, we have

$$\theta'_{\mathrm{THz}} = \theta_{\mathrm{THz}} + \theta. \tag{5}$$

From Eqs. (2), (4), and (5), we can get

$$f(\Delta\varphi - \theta) = f(\Delta\varphi) + \theta. \tag{6}$$

The derivation of the function $f(x)$ is

$$f^{(1)} = \lim_{\theta\to 0} \frac{f(\Delta\varphi - \theta) - f(\Delta\varphi)}{-\theta} = -1. \tag{7}$$

By integration, we get the form of $f(x)$ as

$$f(x) = -x + x_0. \tag{8}$$

Assuming $\theta_{\mathrm{THz}} = f(\Delta\varphi = 0) = 0$, we have the constant $x_0 = 0$. From Eqs. (3) and (8), we have the following relation:

$$\theta_{\mathrm{THz}} = -\Delta\varphi. \tag{9}$$

This result represents that the azimuthal polarization orientation angle of the THz wave θ_{THz} is rigorous equal to the relative phase $-\Delta\varphi$.

III. DISCUSSION FOR THE TWO-COLOR LASERS BEING LEFT-CIRCULARLY OR LINEARLY POLARIZED

The analysis above is suitable for the case that both light pulses are right-handed circularly polarized, whereas, for the left-handed case, something is a little different. By using the method above, we get

$$\theta_{\mathrm{THz}} = \Delta\varphi = 2\varphi_1 - \varphi_2, \tag{10}$$

where both light pulses are left-handed circularly polarized. This conclusion is in agreement with the previous experimental result [15]. By the change in π, θ_{THz} denotes the same polarization angle for the THz wave. Thus, we have $\theta_{\mathrm{THz}} \pm n\pi = -\Delta\varphi$ and $\theta_{\mathrm{THz}} \pm n\pi = \Delta\varphi$ for the right- and left-handed cases, respectively, as shown in Fig. 2 where here, n is an integer.

The polarization of the THz wave $\hat{e}_{\mathrm{THz}}$ can be expressed as the superposition of the two spherical unit vectors: the right-circular polarization and the left-circular polarization [17]. That is,

$$\hat{e}_{\mathrm{THz}} = \hat{e}_{+1} e^{-ir} \cos\left(\alpha - \frac{\pi}{4}\right) + \hat{e}_{-1} e^{ir} \sin\left(\alpha - \frac{\pi}{4}\right), \tag{11}$$

where $\hat{e}_{+1}$ and $\hat{e}_{-1}$ are the spherical unit vectors and correspond with the positive and negative helicity states, α reflects the

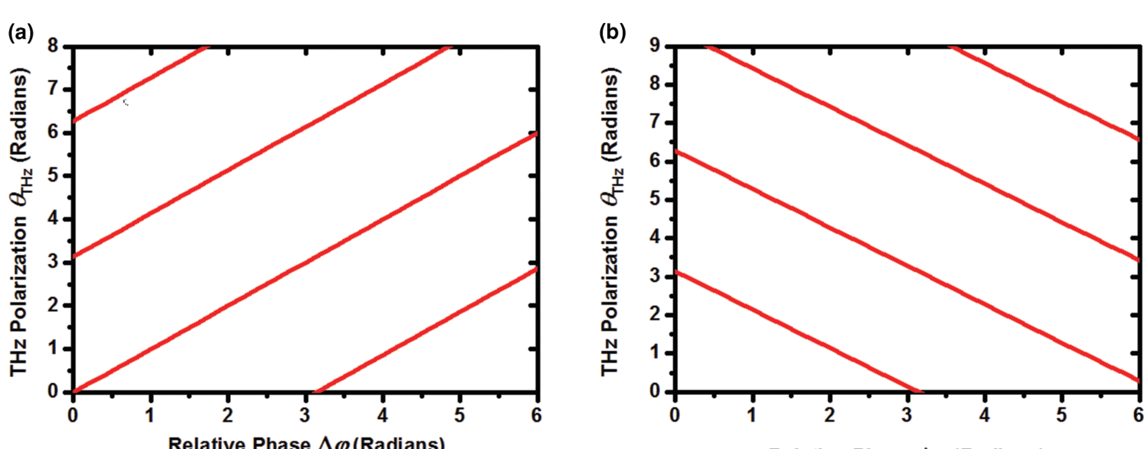

FIG. 2. (Color online) The polarization angles of the THz wave θ_{THz} changes with the relative phase $\Delta\varphi$ for both light pulses are (a) left- and (b) right-handed circularly polarized.

type of polarization, and r is the azimuthal orientation of the polarization. We refer to $\hat{e}_{+1}$ as the right-circular polarization, and we refer to $\hat{e}_{-1}$ as the left-circular polarization. Consequently, there should be two processes to generate THz waves in two-color laser-induced gas plasma. One is to generate right-circular polarization, and the other is to generate left-circular polarization. The probability amplitudes of the two processes are $e^{-ir}\cos(\alpha-\pi/4)$ and $e^{ir}\sin(\alpha-\pi/4)$, respectively. If $\alpha=0$, the probability amplitudes of the processes are equal to each other. In this case, the THz wave is linearly polarized with an azimuthal polarization orientation r. According to Eq. (9), we can specify that $r=-\Delta\varphi=-(2\varphi_1-\varphi_2)$. So, Eq. (11) could be written as

$$\hat{e}_{\mathrm{THz}}=P_I\hat{e}_+e^{i\,\Delta\varphi}+P_{II}\hat{e}_-e^{-i\,\Delta\varphi}, \tag{12}$$

where P_I and P_{II} are the probability amplitudes for the right-circularly polarized process and left-circularly polarized process. In order to obtain linear polarization, $|P_I|=|P_{II}|$.

The previous experimental results demonstrated that the polarization of the THz wave did not vary with the optical delay of the two pulses when the two pulses were both linearly polarized [15,16]. From the view above, we can explain that the polarization angle of the THz wave does not change with the relative phase when the two lasers are linearly polarized. Because the linear polarization could be expressed as the superposition of two spherical unit vectors, we should believe that the generation of the THz wave consists of two courses. One is that the two lasers are right-handed circularly polarized, and the other is that they are left-handed circularly polarized. For the first case, the polarization angle of the THz wave changes with $-\Delta\varphi=-(2\varphi_1-\varphi_2)$ based on Eqs. (1) and (9), whereas, for the second case, the polarization angle of the THz wave changes with $\Delta\varphi=2\varphi_1-\varphi_2$ based on Eq. (10). As a result, the polarization angle of the THz wave does not change with the relative phase.

When one laser is circularly polarized and the other is linearly polarized, the THz wave can also be controlled by the relative phase. Since linear polarization could be expressed as the superposition of two spherical unit vectors, it is the same case in which the two light pulses are both circularly polarized. Noteworthily, our results predict that the THz wave could not be obtained when one laser is left-circularly polarized and the other is right-circularly polarized.

IV. QUANTUM THEORY

In this section, based on a seven-level system, we build a quantum model to analyze the polarization property of the THz wave generated from two-color laser-induced gas plasma. The THz generation is described by the four-wave mixing process, which is described by a four-level system in quantum optics [18], and the nonlinear medium is the gas plasma induced by the two-color lasers. Energy levels of the system transform from the gas ones into the plasma ones when the two-color laser pulses are incident on the medium. The latter ones must coincide with the energy and the spin of the two-color laser photons because the plasma is formed by the two-color laser pulses.

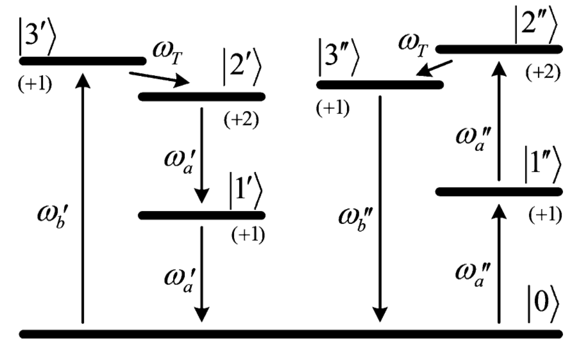

FIG. 3. The seven-level system to describe the polarization property of the THz wave generated from two-color laser-induced gas plasma. $|0\rangle$, $|1'\rangle$, $|1''\rangle$, $|2'\rangle$, $|2''\rangle$, $|3'\rangle$, and $|3''\rangle$describe the energy levels of the plasma excited by the two-color laser pulses. Here, $+1$ and $+2$ describe the spin of the levels excited by the right-circularly polarized photons. The left four-level system is to generate a left-circularly polarized THz photon; the right four-level system is to generate a right-circularly polarized THz photon.

In order to describe the polarization of the THz photon, there are two four-level systems involved. One system is to generate a right-circularly polarized THz photon, and the other is to generate a left-circularly polarized THz photon. The polarization of the THz photon is the superposition of the two processes. The two four-level systems share the same ground state, so altogether, it is a seven-level system (Fig. 3). The coherence of the two four-level systems reflects the superposition of the right- and left-circularly polarized THz photons and then reflects the polarization property of the THz photon generated from the two-color laser-induced gas plasma.

By treating the laser field as a classical electromagnetic wave, under the rotating-wave approximation and the dipole approximation, the Hamiltonian of the seven-level system used to describe the THz photon generation is [18]

$$\begin{aligned}H/\hbar=\;&\omega_1'|1'\rangle\langle 1'|+\omega_1''|1''\rangle\langle 1''|+\omega_2'|2'\rangle\langle 2'|+\omega_2''|2''\rangle\langle 2''|+\omega_3'|3'\rangle\langle 3'|+\omega_3''|3''\rangle\langle 3''|+\omega_T a_{+1}^{+}a_{+1}+\omega_T a_{-1}^{+}a_{-1}\\&+\tfrac{1}{2}[\Omega_a'e^{i(\omega_a't+\varphi_a)}(|0\rangle\langle 1'|+|1'\rangle\langle 2'|)+\Omega_b'e^{i(\omega_b't+\varphi_b)}|0\rangle\langle 3'|+\Omega_a''e^{i(\omega_a''t+\varphi_a)}(|0\rangle\langle 1''|+|1''\rangle\langle 2''|)+\Omega_b''e^{i(\omega_b''t+\varphi_b)}|0\rangle\langle 3''|\\&+\Omega_T a_{-1}^{+}|2'\rangle\langle 3'|+\Omega_T a_{+1}^{+}|3''\rangle\langle 2''|+\mathrm{H.c.}],\end{aligned} \tag{13}$$

where $|0\rangle$, $|1'\rangle$, $|1''\rangle$, $|2'\rangle$, $|2''\rangle$, $|3'\rangle$, and $|3''\rangle$ are the states of the seven-level system and 0, $\hbar\omega_1'$, $\hbar\omega_1''$, $\hbar\omega_2'$, $\hbar\omega_2''$, $\hbar\omega_3'$, and $\hbar\omega_3''$, respectively, are the eigenvalues of the corresponding states. Ω_a', Ω_a'', Ω_b', Ω'', and Ω_T are the Rabi frequencies, and ω_a', ω_a'', ω_b', ω_b'', and ω_T are the circular frequencies for which the subscripts a, b, and T denote the fundamental laser, the second-harmonic laser, and the THZ wave, respectively. a_{+1}^{+}, a_{+1} and a_{-1}^{+}, a_{-1} are the creation and annihilation operators for right- and left-circularly polarized photons. φ_a and φ_b are the phases of the pump lasers.

If the two-color lasers are both right-circularly polarized, the spin of states $|1'\rangle$, $|1''\rangle$, $|3'\rangle$, and $|3''\rangle$ is $+1$, and the spin of states $|2'\rangle$ and $|2''\rangle$ is $+2$ where we assume the spin of state $|0\rangle$ is zero. Thus, the spin of the THz photon generated from states $|2'\rangle$ and $|3'\rangle$ is -1, and the spin of the THz photon generated from states $|2''\rangle$ and $|3''\rangle$ is $+1$. It is worth noting that -1 denotes left-circular polarization and $+1$denotes right-circular polarization.

If we assume that $\omega_1' = \omega_a' + \Delta_{a1}'$, $\omega_2' = 2\omega_a' + (\Delta_{a1}' + \Delta_{a2}')$, $\omega_3' = \omega_b' + \Delta_b'$, $\omega_1'' = \omega_a'' + \Delta_{a1}''$, $\omega_2'' = 2\omega_a'' + (\Delta_{a1}'' + \Delta_{a2}'')$, and $\omega_3'' = \omega_b'' + \Delta_b''$, the Hamiltonian in the interaction picture with phase transformation becomes

$$\begin{aligned} H^I/\hbar = {} & \Delta_{a1}'|1'\rangle\langle 1'| + \Delta_{a1}''|1''\rangle\langle 1''| + (\Delta_{a1}' + \Delta_{a2}')|2'\rangle\langle 2'| + (\Delta_{a1}'' + \Delta_{a2}'')|2''\rangle\langle 2''| + \Delta_b'|3'\rangle\langle 3'| + \Delta_b''|3''\rangle\langle 3''| \\ & + \tfrac{1}{2}[\Omega_a'(|0\rangle\langle 1'| + |1'\rangle\langle 2'|) + \Omega_b'|0\rangle\langle 3'| + \Omega_a''(|0\rangle\langle 1''| + |1''\rangle\langle 2''|) + \Omega_b''|0\rangle\langle 3''| + \Omega_T e^{i(\omega_T + 2\omega_a' - \omega_b')t} e^{i(2\varphi_a - \varphi_b)} a_{-1}^{+}|2'\rangle\langle 3'| \\ & + \Omega_T e^{i(\omega_T - 2\omega_a' + \omega_b')t} e^{i(\varphi_b - 2\varphi_a)} a_{+1}^{+}|3''\rangle\langle 2''| + \text{H.c.}], \end{aligned} \tag{14}$$

where $\omega_T = \omega_b' - 2\omega_a' = 2\omega_a'' - \omega_b''$ is the THz frequency because of the conservation of energy. $\hbar\omega_1'$, $\hbar\omega_1''$, $\hbar\omega_2'$, and $\hbar\omega_2''$ are induced by the ω_1 light, whereas, $\hbar\omega_3'$ and $\hbar\omega_3''$ are induced by the ω_2 light. Because the two-color lasers are femtosecond optical pulses, the ranges of their spectrum widths are quite large. That is why $\hbar\omega_j'$ is different from $\hbar\omega_j''(j = 1,2,3)$, although it is induced by the same pump light. Because we have assumed that the seven-level system is induced by the pump lasers, we can get $\varphi_a = \varphi_1$and $\varphi_b = \varphi_2$. Thus, the Hamiltonian in Eq. (14) is

$$\begin{aligned} H^I/\hbar = {} & \Delta_{a1}'|1'\rangle\langle 1'| + \Delta_{a1}''|1''\rangle\langle 1''| + (\Delta_{a1}' + \Delta_{a2}')|2'\rangle\langle 2'| + (\Delta_{a1}'' + \Delta_{a2}'')|2''\rangle\langle 2''| + \Delta_b'|3'\rangle\langle 3'| + \Delta_b''|3''\rangle\langle 3''| \\ & + \tfrac{1}{2}[\Omega_a'(|0\rangle\langle 1'| + |1'\rangle\langle 2'|) + \Omega_b'|0\rangle\langle 3'| + \Omega_a''(|0\rangle\langle 1''| + |1''\rangle\langle 2''|) + \Omega_b''|0\rangle\langle 3''| + \Omega_T e^{i\,\Delta\varphi} a_{-1}^{+}|2'\rangle\langle 3'| \\ & + \Omega_T e^{-i\,\Delta\varphi} a_{+1}^{+}|3''\rangle\langle 2''| + \text{H.c.}]. \end{aligned} \tag{15}$$

The motion of the wave function is described by the Schrödinger equation,

$$i\frac{d\,|\Psi\rangle}{dt} = \frac{H^I}{\hbar}\,|\Psi\rangle\,. \tag{16}$$

The wave function is assumed to be

$$\begin{aligned} |\Psi\rangle = {} & A_1|n+1,n,3''\rangle + A_2|n,n,2''\rangle + A_3|n,n,1''\rangle + A_4|n,n,0\rangle + A_5|n+1,n,0\rangle + A_6|n,n+1,0\rangle + A_7|n,n,3'\rangle \\ & + A_8|n,n+1,2'\rangle + A_9|n,n+1,1'\rangle, \end{aligned} \tag{17}$$

where the first, second, and third quantum numbers in the Dirac symbols denote the right-circularly polarized THz photon, the left-circularly polarized THz photon, and the energy levels in the seven-level system, respectively, and $A_j(j = 1\text{–}9)$ is the superposition coefficient. The decay rates Γ_1, Γ_2, Γ_3, Γ_7, Γ_8, and Γ_9 can be incorporated in Eq. (16), and then we obtain

$$i\frac{dA_1}{dt} = (\Delta_b'' - i\Gamma_1)A_1 + \left(\sqrt{n+1}\frac{\Omega_T}{2}e^{-i\,\Delta\varphi}\right)A_2 + \left(\frac{\Omega_b''}{2}\right)A_5, \tag{18}$$

$$i\frac{dA_2}{dt} = \left(\sqrt{n+1}\frac{\Omega_T}{2}e^{i\,\Delta\varphi}\right)A_1 + (\Delta_{a1}'' + \Delta_{a2}'' - i\Gamma_2)A_2 + \left(\frac{\Omega_a''}{2}\right)A_3, \tag{19}$$

$$i\frac{dA_3}{dt} = \left(\frac{\Omega_a''}{2}\right)A_2 + (\Delta_{a1}'' - i\Gamma_3)A_3 + \left(\frac{\Omega_a''}{2}\right)A_4, \tag{20}$$

$$i\frac{dA_5}{dt} = \left(\frac{\Omega_b''}{2}\right)A_1 - i\Gamma_5 A_5, \tag{21}$$

$$i\frac{dA_6}{dt} = \left(\frac{\Omega_a'}{2}\right)A_9 - i\Gamma_6 A_6, \tag{22}$$

$$i\frac{dA_7}{dt} = \left(\frac{\Omega_b'}{2}\right)A_4 + (\Delta_b' - i\Gamma_7)A_7 + \left(\sqrt{n+1}\frac{\Omega_T}{2}e^{-i\,\Delta\varphi}\right)A_8, \tag{23}$$

$$i\frac{dA_8}{dt} = \left(\sqrt{n+1}\frac{\Omega_T}{2}e^{i\,\Delta\varphi}\right)A_7 + (\Delta_{a1}' + \Delta_{a2}' - i\Gamma_8)A_8 + \left(\frac{\Omega_a'}{2}\right)A_9, \tag{24}$$

$$i\frac{dA_9}{dt} = \left(\frac{\Omega_a'}{2}\right)A_6 + \left(\frac{\Omega_a'}{2}\right)A_8 + (\Delta_{a1}' - i\Gamma_9)A_9, \tag{25}$$

$$|A_1|^2 + |A_2|^2 + |A_3|^2 + |A_4|^2 + |A_5|^2 + |A_6|^2 + |A_7|^2 + |A_8|^2 + |A_9|^2 = 1. \tag{26}$$

The steady-state solutions at the resonance situation are

$$A_1 = \frac{i\Gamma_5\sqrt{n+1}\left(\frac{\Omega_a''}{2}\right)^2\left(\frac{\Omega_T}{2}\right)A_4 e^{-i\,\Delta\varphi}}{(n+1)\,\Gamma_3\Gamma_5\left(\frac{\Omega_T}{2}\right)^2+\left[\Gamma_1\Gamma_5+\left(\frac{\Omega_b''}{2}\right)^2\right]\left[\left(\frac{\Omega_b''}{2}\right)^2+\Gamma_2\Gamma_3\right]}, \tag{27}$$

$$A_2 = \frac{-\left(\frac{\Omega_a''}{2}\right)^2\left[\Gamma_1\Gamma_5+\left(\frac{\Omega_b''}{2}\right)^2\right]A_4}{(n+1)\Gamma_3\Gamma_5\left(\frac{\Omega_T}{2}\right)^2+\left[\Gamma_1\Gamma_5+\left(\frac{\Omega_b''}{2}\right)^2\right]\left[\left(\frac{\Omega_a''}{2}\right)^2+\Gamma_2\Gamma_3\right]}, \tag{28}$$

$$A_7 = \frac{-i\left(\frac{\Omega_b'}{2}\right)\left\{\Gamma_8\left[\left(\frac{\Omega_a'}{2}\right)^2+\Gamma_6\Gamma_9\right]+\Gamma_6\left(\frac{\Omega_a'}{2}\right)^2\right\}A_4}{\Gamma_6\Gamma_7\left(\frac{\Omega_a'}{2}\right)^2+\left[\left(\frac{\Omega_a'}{2}\right)^2+\Gamma_6\Gamma_9\right]\left[(n+1)\left(\frac{\Omega_T}{2}\right)^2+\Gamma_7\Gamma_8\right]}, \tag{29}$$

$$A_8 = \frac{-\sqrt{n+1}\left(\frac{\Omega_b'}{2}\right)\left(\frac{\Omega_T}{2}\right)\left[\left(\frac{\Omega_b'}{2}\right)^2+\Gamma_6\Gamma_9\right]e^{i\,\Delta\varphi}A_4}{\Gamma_6\Gamma_7\left(\frac{\Omega_a'}{2}\right)^2+\left[\left(\frac{\Omega_a'}{2}\right)^2+\Gamma_6\Gamma_9\right]\left[(n+1)\left(\frac{\Omega_T}{2}\right)^2+\Gamma_7\Gamma_8\right]}, \tag{30}$$

$$\langle\Psi|(a_{+1}^{+}|3''\rangle\langle 2''|)|\Psi\rangle = A_1^*A_2, \quad \langle\Psi|(a_{-1}^{+}|2'\rangle\langle 3'|)|\Psi\rangle = A_8^*A_7. \tag{31}$$

Here, we use the average of the operators $a_{+1}^{+}|3''\rangle\langle 2''|$ and $a_{-1}^{+}|2'\rangle\langle 3'|$ to reflect the coherence of the two polarizations at right-circular polarization and left-circular polarization as shown in Eq. (31). So, the polarization of the THz photon can be expressed as

$$\hat{e}_{\mathrm{THz}} = A_1^*A_2\hat{e}_{+1} + A_8^*A_7\hat{e}_{-1} = M_I e^{i\,\Delta\varphi} + M_{II}e^{-i\,\Delta\varphi}, \tag{32}$$

$$A_1^*A_2 = \frac{i\Gamma_5\sqrt{n+1}\left(\frac{\Omega_a''}{2}\right)^4\left(\frac{\Omega_T}{2}\right)\left[\Gamma_1\Gamma_5+\left(\frac{\Omega_b''}{2}\right)^2\right]A_4^2e^{i\,\Delta\varphi}}{\left\{(n+1)\Gamma_3\Gamma_5\left(\frac{\Omega_T}{2}\right)^2+\left[\Gamma_1\Gamma_5+\left(\frac{\Omega_b''}{2}\right)^2\right]\left[\left(\frac{\Omega_a''}{2}\right)^2+\Gamma_2\Gamma_3\right]\right\}^2} = M_I e^{i\,\Delta\varphi}, \tag{33}$$

$$A_8^*A_7 = \frac{i\sqrt{n+1}\left(\frac{\Omega_T}{2}\right)\left(\frac{\Omega_b'}{2}\right)^2\left[\left(\frac{\Omega_a'}{2}\right)^2+\Gamma_6\Gamma_9\right]\left\{\Gamma_8\left[\left(\frac{\Omega_a'}{2}\right)^2+\Gamma_6\Gamma_9\right]+\Gamma_6\left(\frac{\Omega_a'}{2}\right)^2\right\}A_4^2e^{-i\,\Delta\varphi}}{\left\{\Gamma_6\Gamma_7\left(\frac{\Omega_a'}{2}\right)^2+\left[\left(\frac{\Omega_a'}{2}\right)^2+\Gamma_6\Gamma_9\right]\left[(n+1)\left(\frac{\Omega_T}{2}\right)^2+\Gamma_7\Gamma_8\right]\right\}^2} = M_{II}e^{-i\,\Delta\varphi}. \tag{34}$$

Equation (32) has the same form as Eq. (12), which means we can use the seven-level system to describe the two-color laser-induced gas plasma. This result demonstrates that the relative phase can control the coherence of the two cyclically coupled four-level system and then can control the polarization angle of the THz photon generated from the plasma. If Ω_a'' or Ω_b' is zero, we can get the left- or right-circularly polarized THz wave, respectively.

V. CONCLUSION

We used a classical and a quantum model to describe the polarization property of the THz waves generated from two-color laser-induced gas plasma. The conclusion obtained from the models can clearly explain that the polarization angle of the THz wave is controllable by the relative phase of the two-color lasers. These models are promising for finding a way to obtain a THz wave with the desired polarization state. We can use the quantum model to study the physical characteristic of the plasma, for example, the nonlinear phenomenon.

ACKNOWLEDGMENTS

The authors gratefully acknowledge support from the National Natural Science Foundation of China under Grants No. 10974063 and No. 61177095, the Hubei Natural Science Foundation under Grant No. 2010CDA001, the Ph.D. Programs Foundation of Ministry of Education of China under Grant No. 20100142110042, and the Fundamental Research Funds for the Central Universities, HUST: Grants No. 2010MS041' and No. 2011TS001.

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