PHYSICAL REVIEW A **86**, 022520 (2012)

Temperature dependence of the Casimir force between a superconductor and a magnetodielectric

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(Received 21 June 2012; published 28 August 2012)

We investigate the dependence of the Casimir force between a superconducting plate (niobium) and a magnetodielectric plate (yttrium iron garnet) on the temperature near the transition point within a two-fluid model. For large separations between the plates, the direction of the Casimir force changes from attractive to repulsive as the temperature decreases below the transition temperature, because of an increase in superconducting current density. We show that this increase in the repulsive contribution to the Casimir force, which depends on the magnetic permeability of the magnetodielectric plate, can be experimentally verified by measuring the force acting on the magnetodielectric plate inserted midway between a superconducting plate and a normal conducting plate.

DOI: 10.1103/PhysRevA.86.022520 PACS number(s): 31.30.jh, 12.20.Ds, 74.90.+n, 42.50.Lc

I. INTRODUCTION

Casimir's theoretical discovery [1] of an attractive force between perfectly conducting plates, resulting from the zero-point fluctuations, has triggered interest in diverse fields ranging from nanotechnology to cosmology [2–4]. In the last few years, the Casimir force acting on magnetic materials has attracted particular attention in connection with the possibility of a repulsive Casimir force [5–12]. As is well known, Boyer has shown that the Casimir force between a perfectly conducting plate and a perfectly magnetic plate can be repulsive for any separation distance [13]. Here, a perfectly magnetic plate is an ideal material having infinite magnetic permeability. A superconductor at absolute zero is probably the most suitable candidate for the perfectly conductive plate. However, it is impossible to find practical materials suitable for the perfectly magnetic plate because the magnetic permeability becomes very small for high-frequency electromagnetic fields.

When seeking materials between which repulsive Casimir force is generated according to the Boyer's guiding principle, it should be considered that a magnetic plate must possess an inherently high magnetic permeability and a low dielectric permittivity. A possible candidate is a magnetodielectric such as yttrium iron garnet (YIG,$Y_3Fe_5O_{12}$) [7], which is a well-known garnet with a high magnetic permeability. Although the magnetic permeability of the YIG decreases with frequency, it has been recognized by recent theoretical studies that the contribution of the TE mode at zero frequency to the Casimir force is of considerable importance in determining the direction [14].

If the dielectric permittivity diverges near zero frequency in proportion to the inverse of the second power of frequency, as in the function used in the plasma model, the term at zero frequency of TE mode in the Lifshitz formula [15], which is referred to as the dc component of the TE mode in this study, can contribute to the Casimir force. On the contrary, if the dielectric permittivity diverges more moderately than the second-order divergence, as in the function used in the Drude model, the dc component of the TE mode does not contribute to the Casimir force. Current experiments using normal conducting metals support either the plasma [16–19] or the Drude model [20]. It should be noted that the results of the very recent experiment investigating the Casimir force between a ferromagnetic metal (Ni) and a nonmagnetic metal (Au) are in agreement with the results of the plasma model approach for investigating magnetic properties [12].

Bimonte *et al.* showed that the dc component of the TE mode contributes to the Casimir energy between superconducting plates [21,22]. Thus, the combination of the superconductor and the magnetodielectric may enable us to provide an answer to the question of the possibility of the repulsive Casimir force, because the dielectric permittivity near zero frequency changes significantly near the transition point. In the two-fluid model, the superfluid density increases as the temperature decreases below the transition temperature, and this causes the second-order divergence in the dielectric permittivity near zero frequency. Accordingly, the dc component of the TE mode, which contributes to the repulsive Casimir force, appears only below the transition temperature. Thus, we expect that the dc component of the TE mode can be detected using this phase transition.

The main aim of this study is to investigate the increase in the repulsive contributions of the Casimir force by the superconducting phase transition. Furthermore, we show that it may be measured for the combination of a niobium (Nb) plate and a YIG plate. This paper is structured as follows. In Sec. II, we calculate the dielectric permittivity of niobium along the imaginary frequency on the basis of the BCS theory. Further, in Sec. II, we briefly explain the ordinary Lifshitz formula at finite temperature and show the Casimir force between niobium in a normal-state and YIG. In Sec. III, we show the temperature dependence of the Casimir force between niobium in the superconducting state and YIG. In Sec. IV, we consider the Casimir force acting on the YIG located midway between a superconducting niobium plate and a normal conducting niobium plate and discuss the possibility of the detection of the repulsive contribution for the dc component of the TE mode. In Sec. V, we report our conclusions and remark that the several problems of the Casimir force can be solved by unique properties of superconductors.

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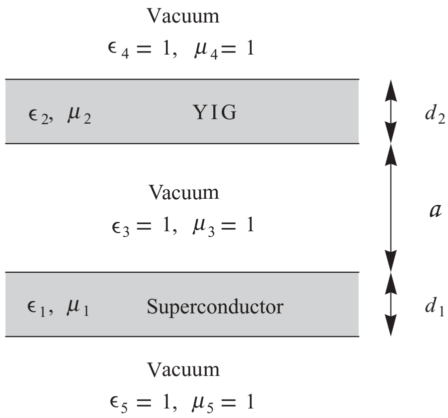

FIG. 1. Configuration of a superconducting plate (niobium) and a magnetodielectric plate (YIG).

II. DIELECTRIC PERMITTIVITY OF NIOBIUM AND YIG

We consider the Casimir force between a niobium plate of thickness d_1 and a YIG plate of thickness d_2 through the vacuum gap. These plates are arranged parallel to each other as depicted in Fig. 1. According to the Lifshitz theory [15], the Casimir force per unit area of the plate at finite temperature T depends on the separation distance between the plates, the temperature, and the dielectric permittivity and magnetic permeability along the imaginary frequency. Thus, the first point that we should consider is the temperature dependence of the dielectric permittivity of superconductor along the imaginary frequency. The dielectric permittivity of superconductor along the imaginary frequency $\epsilon_s(i\xi,T)$ is expressed by using the real part of the conductivity of the superconductor $\sigma_s'(\omega,T)$ for $\omega > 0$ at temperature T as

$$\epsilon_s(i\xi,T) = 1 + 8\int_0^\infty d\omega \frac{\sigma_s'(\omega,T)}{\xi^2+\omega^2}. \quad (1)$$

Here, the dependence of the dielectric permittivity on the wave number vector is neglected. We calculate the conductivity of niobium on the basis of BCS theory (see Refs. [21,23] in detail). The electrical properties of niobium used in the calculation, which are measured and evaluated by Pronin *et al.* [24], are summarized in Table I. Figure 2 shows the conductivity of niobium as a function of frequency at 4 K, 7 K, and 9 K near $2\Delta/\hbar = 4.5 \times 10^{12}$ rad/s. The real part of the conductivity has a δ function at zero frequency for $T < T_c$.

The conductivity in the normal state obeys the Drude model and it is expressed by

$$\sigma_D'(\omega) = \frac{1}{4\pi\epsilon_0}\frac{\omega_p^2}{1+\omega^2\tau^2}, \quad (2)$$

where ω_p is the plasma frequency and τ is the scattering time. The value of τ in Table I is evaluated at 9 K. Substituting Eq. (2) into Eq. (1), the dielectric permittivity along the imaginary frequency in the normal state is given by

$$\epsilon_D(i\xi) = 1 + \frac{\omega_p^2}{\xi(\xi+1/\tau)}. \quad (3)$$

We employed the following formula given by Bimonte *et al.* [21] to calculate the permittivity in the superconducting state:

$$\epsilon_s(i\xi,T) = \epsilon_D(i\xi) + \frac{\omega_s^2(\xi,T)}{\xi^2}, \quad (4)$$

where $\omega_s^2(\xi,T)$ is defined by

$$\omega_s^2(\xi,T) = -8\int_0^\infty d\omega \frac{\omega^2[\hat{\sigma}_s'(\omega,T) - \sigma_D'(\omega)]}{\xi^2+\omega^2}. \quad (5)$$

The dielectric permittivity along the imaginary frequency near zero frequency is expressed as a combination of the Drude model and the plasma model:

$$\epsilon_s(i\xi,T) \approx \epsilon_D(i\xi) + \frac{\omega_s^2(0,T)}{\xi^2}, \quad (6)$$

where $\omega_s^2(0,T)$ converge to zero as $T \to T_c$. Figure 3 shows the dielectric permittivity along the imaginary frequency at 4 K, 7 K, and 9 K. Since the energy gap $2\Delta(0)$ at zero temperature corresponds to 4.5×10^{12} rad/s, the dielectric permittivity along the imaginary frequency changes significantly below 10^{12} rad/s. The slope of the dielectric permittivity near zero frequency on a log-log scale changes from -1 (Drude model) to -2 (plasma model).

To calculate the dielectric permittivity along the imaginary axis of the YIG, we used the experimental results for dielectric permittivity obtained by Kahn *et al.* for 2.4 eV $< E \leqslant$ 5.8 eV [25] and by Kučera *et al.* for 5.8 eV $< E \leqslant$ 30 eV [26]. We use the following approximations. First, the imaginary part of the dielectric permittivity for E less than 2.4 eV is zero [27]. Next, the imaginary part for large frequencies is expressed by $(11.6/E)^4$ for $E >$ 30 eV [26,28].

TABLE I. Electric properties of niobium

Transition temperature	8.31 K
Energy gap 2Δ	2.93 meV
Plasma frequency $\hbar\omega_p$	5.8 eV
Scattering time τ	0.03 ps

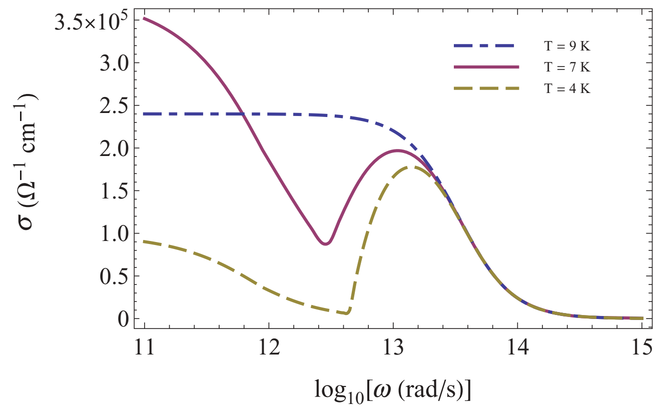

FIG. 2. (Color online) Frequency dependence of the real part of the conductivity at 4 K, 7 K, and 9 K in niobium. The transition temperature of niobium is 8.31 K.

III. CASIMIR FORCE BETWEEN NIOBIUM IN A NORMAL-STATE AND YIG

We begin by considering the Casimir force between normal conducting niobium and YIG. As shown in Fig. 1, let

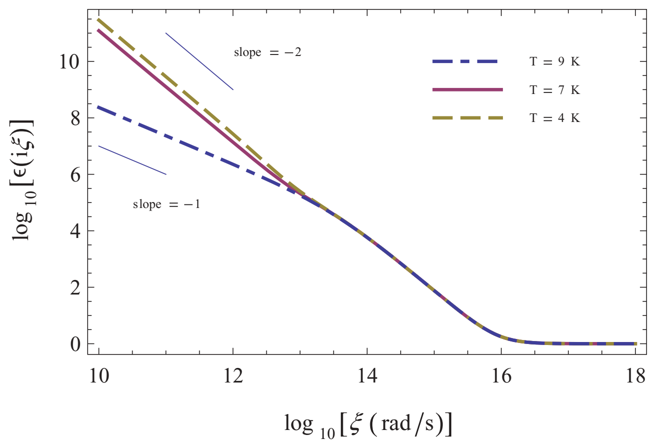

FIG. 3. (Color online) Plot of the dielectric permittivity of niobium along the imaginary frequency $\omega = i\xi$ at 4 K, 7 K, and 9 K. The slope of the permittivity on the log-log scale changes from -1 to -2 near zero frequency as the temperature decreases below the transition temperature.

the dielectric permittivity and magnetic permeability of the medium labeled by an integer j be $\epsilon^{(j)}$ and $\mu^{(j)}$, respectively. The vacuum gap between a niobium plate with $j = 1$ and a YIG plate with $j = 2$ is labeled by $j = 3$, and the upper and bottom vacuums are labeled by $j = 4$ and $j = 5$, respectively. According to the Lifshitz theory [15], the Casimir force between plates per unit area at temperature T can be expressed by a summation of the following four components:

$$P(a,T) = \sum_{p\in\{\mathrm{TM,TE}\}} P_0^p(a,T) + \sum_{p\in\{\mathrm{TM,TE}\}} P_{l>0}^p(a,T). \tag{7}$$

Here, P_0^{TM} and P_0^{TE} are contributions of vacuum fluctuation in the transverse magnetic (TM) and transverse electric (TE) modes, respectively, at zero frequency; similarly, $P_{l>0}^{\mathrm{TM}}$ and $P_{l>0}^{\mathrm{TE}}$ are contributions of vacuum fluctuations in the TM and TE modes, respectively, at positive frequencies. We note that the representation of the Casimir force in Eq. (7) is not unique, and the physical meanings are discussed later.

The contribution to the Casimir force of a vacuum fluctuation with mode p at positive frequencies is given by

$$P_{l>0}^p(a,T) = -\frac{k_{\mathrm{B}}T}{\pi}\sum_{l=1}^{\infty}\int_0^{\infty} k_l^{(3)}k_\perp dk_\perp \times \left[\frac{e^{2ak_l^{(3)}}}{R_p^{(1)}(i\xi_l,k_\perp)R_p^{(2)}(i\xi_l,k_\perp)} - 1\right]^{-1}, \tag{8}$$

where $\xi_l = 2\pi k_{\mathrm{B}}Tl/\hbar$ with positive integer l represents the Matsubara frequencies and

$$k_l^{(n)} \equiv k_l^n(k_\perp) = \sqrt{k_\perp^2 + \epsilon^{(n)}(i\xi_l)\mu^{(n)}(i\xi_l)\frac{\xi_l^2}{c^2}}. \tag{9}$$

Here k_{B} is the Boltzmann constant and $k_\perp$ is the modulus of the wave-vector projection on the plate. The reflection coefficients $R_p^{(j)}$ for positive frequencies are given by

$$R_p^{(1)}(i\xi_l,k_\perp) = \frac{r_p^{(3,1)} + r_p^{(1,5)}e^{-2k^{(1)}d_1}}{1 + r_p^{(3,1)}r_p^{(1,5)}e^{-2k^{(1)}d_1}}, \tag{10}$$

$$R_p^{(2)}(i\xi_l,k_\perp) = \frac{r_p^{(3,2)} + r_p^{(2,4)}e^{-2k^{(2)}d_2}}{1 + r_p^{(3,2)}r_p^{(2,4)}e^{-2k^{(2)}d_2}}, \tag{11}$$

where

$$r_{\mathrm{TM}}^{(n,m)}(i\xi_l,k_\perp) = \frac{\epsilon^{(m)}(\xi_l)k_l^n - \epsilon^{(n)}(\xi_l)k_l^{(m)}}{\epsilon^{(m)}(\xi_l)k_l^n + \epsilon^{(n)}(\xi_l)k_l^{(m)}}, \tag{12}$$

$$r_{\mathrm{TE}}^{(n,m)}(i\xi_l,k_\perp) = \frac{\mu^{(m)}(\xi_l)k_l^n - \mu^{(n)}(\xi_l)k_l^{(m)}}{\mu^{(m)}(\xi_l)k_l^n + \mu^{(n)}(\xi_l)k_l^{(m)}}. \tag{13}$$

We need to carefully consider the reflection coefficients at zero frequency. Since the dielectric permittivity of the superconductor diverges in the limit of $\xi \to 0$, we require the following limit defined by

$$C^{(n)} = \lim_{\xi\to 0}\epsilon^{(n)}(i\xi)\mu^{(n)}(i\xi)\frac{\xi^2}{c^2}. \tag{14}$$

If the dielectric permittivity of the plate diverges near zero frequency in the form described by the plasma model of $\epsilon_{\mathrm{pl}}^{(n)}(i\xi) = 1 + \omega_p^2/\xi^2$, the limit $C_{\mathrm{pl}}^{(n)}$ is given by

$$C_{\mathrm{pl}}^{(n)} = \mu^{(n)}(0)\frac{\omega_p^2}{c^2}. \tag{15}$$

On the other hand, if the dielectric permittivity of the plate obeys the Drude model, then C is zero, and the TE mode does not contribute to the Casimir force for the combination of nonmagnetic metal and magnetodielectric.

We present the Casimir force between niobium in the normal-state and YIG with static magnetic permeability in the form of a ratio,

$$\eta(a,T) \equiv \frac{P(a,T)}{P_\infty(a)}, \tag{16}$$

where the denominator is the Casimir force between perfectly conductive plates at zero temperature,

$$P_\infty(a) = -\frac{\pi^2\hbar c}{240a^4}. \tag{17}$$

Figure 4 shows the ratio at 9 K for a fixed thickness of niobium $d_1 = 100\ \mu$m and different thicknesses of YIG

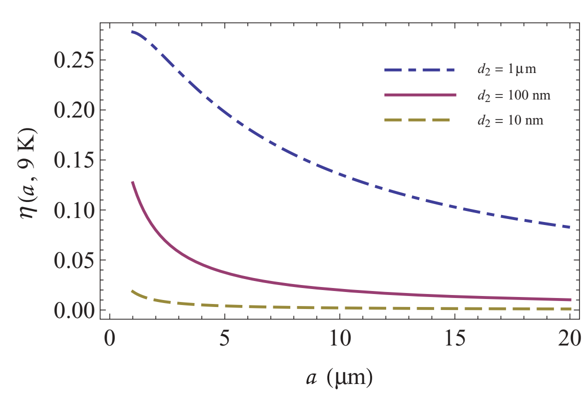

FIG. 4. (Color online) Dependence of the ratios of the Casimir force between a normal conducting niobium plate with a thickness of 100 μm and a YIG plate with a thickness of d_2 at 9 K to the Casimir force between perfectly conducting plates on the separation distance a. The positive sign of η means that the Casimir force is attractive.

$d_2 = 10$ nm, 100 nm, and 1 μm. In this computation, the dielectric permittivity of the superconductor is given by Drude's formula in Eq. (3), and the static magnetic permeability of YIG is $\mu(0) = 160$. Although the dielectric permittivity of YIG depends on the temperature, it changes near the transition temperature of niobium more smoothly than the change in the dielectric permittivity of niobium. Thus, the temperature dependence is mainly governed by the increase in the superfulid component of niobium.

The Casimir force between a normal conducting niobium plate and a YIG plate is always attractive, and its strength decreases as the thickness of the YIG decreases. Similarly, when the thickness of the YIG is fixed and the thickness of niobium is decreased, the strength of the Casimir force decreases. However, the change is not significant compared with the former case.

IV. CHANGE IN THE CASIMIR FORCE BELOW THE TRANSITION TEMPERATURE

In a two-fluid model for superconductors, the superfluid appears by cooling a conductor below the transition temperature, and it results in the addition of the divergent term obeying the inverse square law for frequency to the dielectric permittivity. Thus, the value of C in Eq. (15) is changed from zero to positive by decreasing the temperature. Assuming that the relative magnetic permittivity of niobium is $\mu^{(1)}(i\xi) = 1$, the reflection coefficients for the TE mode at $\xi = 0$ are explicitly given by

$$R_{\rm TE}^{(1)}(0,k_\perp) = \frac{\frac{k_\perp - \sqrt{k_\perp^2 + C_{\rm pl}}}{k_\perp + \sqrt{k_\perp^2 + C_{\rm pl}}}\left(1 - e^{-2k^{(1)}d_1}\right)}{1 - \left(\frac{k_\perp - \sqrt{k_\perp^2 + C_{\rm pl}}}{k_\perp + \sqrt{k_\perp^2 + C_{\rm pl}}}\right)^2 e^{-2k^{(1)}d_1}}, \tag{18}$$

$$R_{\rm TE}^{(2)}(0,k_\perp) = \frac{\frac{\mu^{(2)}(0)-1}{\mu^{(2)}(0)+1}\left(1 - e^{-2k^{(2)}d_2}\right)}{1 - \left(\frac{\mu^{(2)}(0)-1}{\mu^{(2)}(0)+1}\right)^2 e^{-2k^{(2)}d_2}}. \tag{19}$$

The contribution of the TE mode at zero frequency $P_0^{\rm TE}(a,T)$ is given by

$$P_0^{\rm TE}(a,T) = -\frac{k_B T}{2\pi}\int_0^\infty k_\perp^2 dk_\perp \left[\frac{e^{2k_\perp a}}{R_{\rm TE}^{(1)}(0,k_\perp)R_{\rm TE}^{(2)}(0,k_\perp)} - 1\right]^{-1}. \tag{20}$$

Since $R_{\rm TE}^{(1)}(0,k_\perp)R_{\rm TE}^{(2)}(0,k_\perp) \leqslant 0$ for any $k_\perp$, the contribution of $P_0^{\rm TE}(a,T)$ to the Casimir force is repulsive.

Figure 5 shows the ratios of the Casimir force between a superconducting niobium plate and a YIG plate at 7 K to that between a normal conducting niobium plate and a YIG plate at 9 K. Since the Casimir force at 9 K is attractive for any separation distance, the negative sign of the ratio means that the Casimir force at 7 K is repulsive. In Fig. 5, we see that the Casimir force between a superconducting niobium plate and a YIG plate can be repulsive for large separations. The separation distance at which the Casimir force vanishes depends on the thickness of the YIG, and the strength of the repulsive Casimir force at a fixed separation distance increases as the thickness of the YIG decreases [11]. To consider the dependence of four

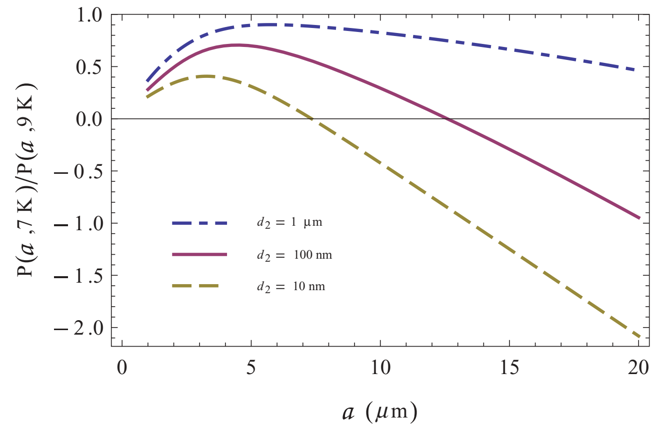

FIG. 5. (Color online) Ratios of the Casimir force between a superconducting niobium plate and a YIG plate with a thickness of d_2 at 7 K to that between a normal conducting niobium plate and a YIG plate at 9 K.

contributions $P_{l>0}^{\rm TE}$, $P_{l>0}^{\rm TM}$, $P_0^{\rm TE}$, and $P_0^{\rm TM}$ on the separation, we introduce ratios defined by

$$\eta_0^{(p)}(a,T) \equiv \frac{P_0^{(p)}(a,T)}{P_\infty(a)}, \tag{21}$$

$$\eta_{l>0}^{(p)}(a,T) \equiv \frac{P_{l>0}^{(p)}(a,T)}{P_\infty(a)}. \tag{22}$$

Figure 6 shows these ratios at $T = 7$ K for a YIG of 10 nm thickness. We clearly find that only the dc component of the TE mode contributes to the repulsive force. The increase in the contribution of $P_0^{\rm TE}$ to the Casimir force results in a reversal in the direction of the Casimir force.

We now focus on the change in the repulsive Casimir force for large separations by decreasing the temperature. Figure 7 shows the change in the ratios of the Casimir force between a superconducting niobium plate and a YIG

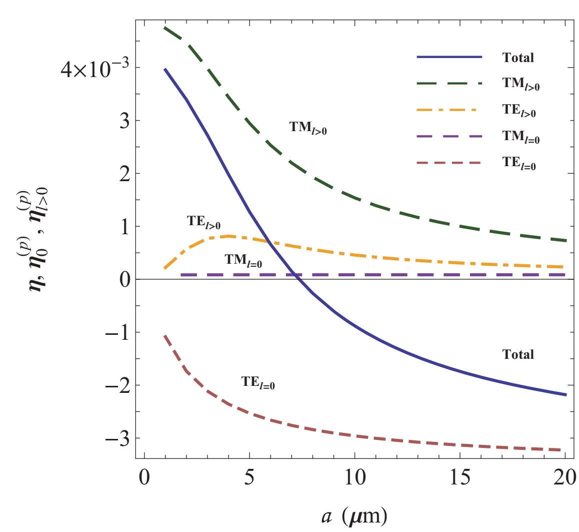

FIG. 6. (Color online) Plot of four contributions of $\eta_0^{\rm TE}$, $\eta_0^{\rm TM}$, $\eta_{l>0}^{\rm TE}$, $\eta_{l>0}^{\rm TM}$ and their summation at 7 K. Only the dc component of the TE mode $\eta_0^{\rm TE}$ contributes to the repulsive Casimir force, and it becomes dominant for large separations.

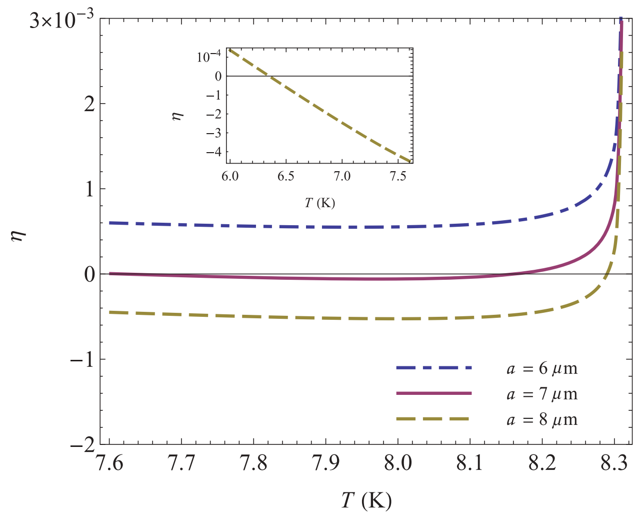

FIG. 7. (Color online) Temperature dependencies of the ratios of the Casimir force between a superconducting niobium plate with a thickness of 100 μm and a YIG plate with a thickness of 10 nm to the Casimir force between perfectly conducting plates for the separation distances 6, 7, and 8 μm. The inset is the plot of η for $a = 8$ μm between 6 K and 7.6 K.

plate of thickness of 10 nm to the Casimir force between perfectly conducting plates at three separation distances $a = 6$, 7, and 8 μm. As the temperature decreases from the transition temperature $T_c = 8.31$ K, the Casimir force rapidly decreases. In particular, the Casimir force changes from attractive to repulsive for the separation distance $a = 8$ μm within a 0.1 K decrease. The inset of Fig. 7 shows the change in the Casimir force at $a = 8$ μm for small T. The Casimir force changes from repulsive to attractive by decreasing the temperature. This is due to the decrease in the contribution of the dc component to the Casimir force. Let us recall the term in the Lifshitz formula at zero frequency given by Eq. (20). Since $|R_{\rm TE}^{(1)}(0,k_\perp)R_{\rm TE}^{(2)}(0,k_\perp)| \leqslant 1$, the contribution of $P_0^p(a,T)$ is bounded by the following inequality:

$$-\frac{\zeta(3)k_{\rm B}T}{8\pi a^3} \leqslant P_0^p(a,T) \leqslant \frac{3\zeta(3)k_{\rm B}T}{32\pi a^3}. \tag{23}$$

Thus, this contribution vanishes in the limit of $T \to 0$.

V. REPULSIVE COMPONENT OF THE CASIMIR FORCE

To consider the temperature dependence of the Casimir force, it is useful to assume that the Casimir force consists of the attractive component and the repulsive component. In the Lifshitz formula, the Casimir force is expressed by the infinite contributions depending on the frequency, and we cannot separately measure each term. However, for the combination of the superconductive niobium and YIG, only the repulsive component, that is, $P_0^{\rm TE}$, couples with nonzero magnetic permeability. If we use this property, we may measure the dc component of the TE mode.

If the repulsive Casimir force is measured between a niobium plate and a YIG plate, the existence of the repulsive component is very clear. However, the strength of the repulsive Casimir force is too small to be measured by current measuring devices. A possible indirect proof of the existence of the

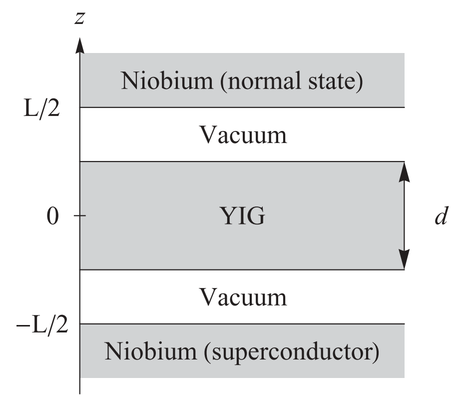

FIG. 8. Illustration of the three-layer system, which consists of a superconducting plate, a normal conducting plate, and a YIG plate. The YIG plate is located midway between two superconducting plates.

repulsive component can be obtained by measuring the change in the Casimir force by the phase transition of niobium. As shown in Fig. 7, the Casimir force decreases considerably when niobium changes from a normal conductor to a superconductor. Thus, the difference between the measured Casimir force and the theoretical value, which is obtained assuming the Drude model, can be considered as the repulsive component. However, this method is not persuasive, because we need to take into account decrease in the attractive component due to the decrease in the number of real photons, which obey Bose-Einstein statistics. Thus, let us consider a more direct measurement of the repulsive component by using a multilayered system [29,30]. We examine the Casimir force acting on the YIG plate located at the middle between niobium in a normal state, whose dielectric permittivity is given by the Drude model in Eq. (3) and niobium in a superconducting state (see Fig. 8). Here, the direction of the z axis is upwards. We expect that all terms except for $P_0^{\rm TE}$ almost cancel by locating the YIG plate midway between the niobium plates. If all plates are the same temperature, the Casimir force acting on a YIG plate can be calculated by generalizing the Lifshitz theory (see the Appendix) in a straightforward manner. Previously, the adhesion of a dielectric membrane in a similar configuration was experimentally studied by Bucks and Roukes [31].

We assume that all plates are in equilibrium with the environment at temperature T. Although the coexistence of a superconducting state and a normal conducting state at the same temperature is difficult to realize, doping the niobium with hydrogen may enable the transition temperature to be decreased by several degrees Kelvin due to the increase in the lattice constant without considerable change in the plasma frequency and scattering time, which mainly depend on electron density [32]. If this assumption is correct, the primary difference between a superconductor and a normal conductor near the transition temperature is that only the dielectric permittivity of the superconductor includes a term proportional to ξ^{-2}. Figure 9 shows the relation between the Casimir force acting on the YIG plate $P_3(T)$, which is given in Eq. (A13), for a niobium plate separation distance $L = 3$ μm and a YIG plate thickness $d_2 = 1$ μm, and the dc component of the TE mode $P_0^{\rm TE}(1\ \mu{\rm m},T)$. The good agreement between

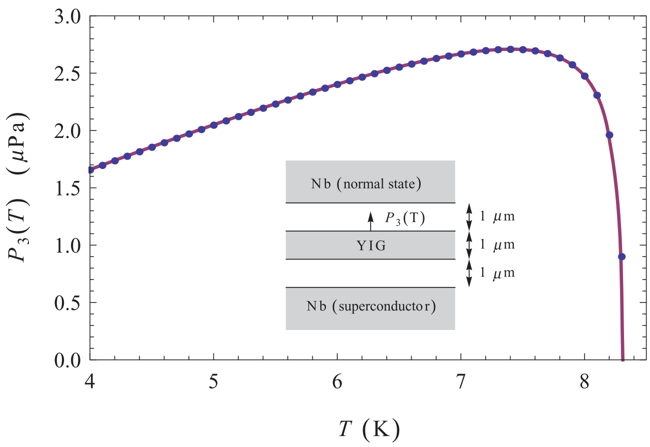

FIG. 9. (Color online) Comparison of the temperature dependence of the Casimir force acting on a YIG plate midway between a superconducting niobium plate and a normal conducting niobium plate (solid circles) with the contribution for the dc component of the TE mode to the Casimir force between a superconducting niobium plate and a YIG plate (solid line). The vacuum gap is 1 μm, and the thickness of the YIG plate is 1 μm.

them in Fig. 9 suggests that all components of the Casimir force acting on YIG plate are almost canceled except for $P_0^{\mathrm{TE}}(a,T)$. Accordingly, the Casimir force acting YIG plate at the midpoint becomes positive for any distance. If the contribution of $P_0^{\mathrm{TE}}(a,T)$ does not affect the Casimir force, the Casimir force acting on the YIG plate at the midpoint becomes negative. As a result, our numerical results suggest that the repulsive Casimir force is too weak to be measured, but the existence of the repulsive contribution P_0^{TE} may be measured.

VI. CONCLUSIONS

As already emphasized by Bimonte *et al.*, the unique properties of superconductors provide opportunities for the study of the Casimir effect [21,22]. In this paper, we have added that the superconductor is a suitable material for examining the possibility of a repulsive Casimir force. The possibility of a repulsive Casimir force has been studied, and many methods of generating it have already been proposed [8,33–36]. For instance, a repulsive Casimir force between metals in the vacuum gap can be realized by making the shapes of the objects complicated [37,38]. However, the repulsive force generated by this method is not stable, and the two metallic objects are considered to be getting closer if one of the objects shifts from the stable position. In contrast, the Casimir force between a superconductive plate and a magnetodielectric plate that is perpendicular to the surface is always repulsive for large separation gaps. This property is desirable for quantum levitation [39,40]. When considering a method for generating the repulsive Casimir force between parallel plates, the use of the dc component of the TE mode seems to be practical.

An essential condition of the repulsive Casimir force between the metallic plate and the magnetodielectric plate is that the dielectric permittivity diverges in proportion to the inverse-square of the frequency as the frequency approaches zero. Thus, if the dielectric permittivity of a normal conductive metal such as gold obeys the plasma model, the Casimir force between the ordinary metal and the magnetodielectric can be repulsive at room temperature [12]. However, we have not yet established theoretical bases of the model for the dielectric permittivity of metal used in the Lifshitz theorem. By comparison, the conductivity of the superconductor is well understood on the basis of BCS theory and an inverse square divergence of the dielectric permittivity near zero is a natural consequence of the BCS theory. Accordingly, the combination of the superconductor and magnetodielectric is a promising candidate for a test of the possibility of the repulsive Casimir force in the vacuum gap. The disadvantage of this system is that the repulsive contribution becomes small when the temperature is decreased, as mentioned in Sec. IV.

To calculate the Casimir force, we used the Lifshitz formula, which is expressed as an infinite summation over the Matsubara frequency. In this summation, the dc component of the TE mode plays an important role in the generation of the repulsive Casimir force. However, this does not mean that only the static electromagnetic field contributes to the repulsive Casimir force. If we use the representations of the Lifshitz formula using the real frequency of the Lifshitz formula [3], different explanations for the decrease in the Casimir force below the transition temperature are possible. The advantage of the Lifshitz formula using the imaginary frequency at finite temperature is that the pressure which depends only on static electromagnetic properties can be easily calculated, and it can be measured. We here emphasize that the repulsive component can be measured for any separation distance in principle. Although the Casimir force between a niobium plate and a YIG plate becomes repulsive only for large separations, the repulsive component exists for any separation distance. The strength of the repulsive component for small separations is much larger than the repulsive Casimir force. Thus, the measurement of the repulsive component is more practical than that of the repulsive Casimir force.

ACKNOWLEDGMENTS

The author thanks G. L. Klimchitskaya and K. Miura for the helpful comments. This research was partially supported by Science, Sports and Culture, Grant-in-Aid for Scientific Research(C), 22560054, 2012.

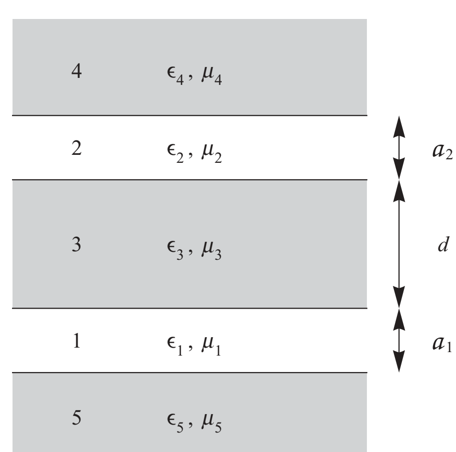

FIG. 10. Configuration of five parallel layers.

APPENDIX: THE LIFSHITZ FORMULA FOR THREE PLATES

We consider the Casimir force acting on a plate of thickness d sandwiched by two parallel plates, as shown in Fig. 10. The dispersion relation of the electromagnetic field in this system can be expressed for $p \in \{\mathrm{TM,TE}\}$ by

$$\Delta^{(p)}(\omega,k_\perp) = \sum_{i\in\{-1,1\}}\sum_{j\in\{-1,1\}}\sum_{k\in\{-1,1\}} C^{(p)}_{i,j,k} e^{iK_1a_1+jK_2a_2+kK_3d}, \tag{A1}$$

where

$$K_n(\omega,k_\perp) \equiv \sqrt{k_\perp^2 - \epsilon_n(\omega)\mu_n(\omega)\frac{\omega^2}{c^2}}. \tag{A2}$$

If the thicknesses of the plates labeled by 4 and 5 are infinite, $C^{(p)}_{i,j,k}$ is compactly given by

$$C^{(p)}_{i,j,k} \equiv C^{(p)}_{i,j,k}(\omega,k_\perp) = f^{(p)}_{4,1,i} f^{(p)}_{1,3,ik} f^{(p)}_{3,2,jk} f^{(p)}_{2,5,j}, \tag{A3}$$

where

$$f^{(\mathrm{TM})}_{i,j,s} \equiv f^{(\mathrm{TM})}_{i,j,s}(\omega,k_\perp) = s\epsilon_j K_i(\omega,k_\perp) + \epsilon_i K_j(\omega,k_\perp), \tag{A4}$$

$$f^{(\mathrm{TE})}_{i,j,s} \equiv f^{(\mathrm{TE})}_{i,j,s}(\omega,k_\perp) = s\mu_j K_i(\omega,k_\perp) + \mu_i K_j(\omega,k_\perp). \tag{A5}$$

In particular, since $K_n(i\xi,k_\perp) \geqslant 0$, the dispersion relation $\Delta^{(p)}(i\xi,k_\perp)$ along the imaginary frequency for large a_1 and a_2, which corresponds to the infinite gaps between plates 4 and 5, is given by

$$\Delta^{(p)}_\infty(i\xi,k_\perp) = (C_{1,1,1} + C_{1,1,-1}e^{-2K_3d})e^{K_1a_1+K_2a_2+K_3d}. \tag{A6}$$

We now obtain the free energy,

$$\mathcal{F}(a_1,a_2,d) = \frac{k_\mathrm{B}T}{2\pi}\sum_{l=0}^{\infty}{}' \int_0^\infty k_\perp dk_\perp \left\{ \ln\frac{\Delta^{(\mathrm{TM})}(i\xi_l,k_\perp)}{\Delta^{(\mathrm{TM})}_\infty(i\xi_l,k_\perp)} + \ln\frac{\Delta^{(\mathrm{TE})}(i\xi_l,k_\perp)}{\Delta^{(\mathrm{TE})}_\infty(i\xi_l,k_\perp)}\right\}, \tag{A7}$$

where the prime on the summation symbol implies that a factor 1/2 should be inserted if $l = 0$ (see Ref. [3]). The ratio Δ^p/Δ^p_∞ is explicitly expressed as

$$\begin{aligned}\frac{\Delta^p(i\xi_l,k_\perp)}{\Delta^p_\infty(i\xi_l,k_\perp)} = 1 &+ \frac{C^{(p)}_{-1,1,1} + C^{(p)}_{-1,1,-1}e^{-2K_3d}}{C^{(p)}_{1,1,1} + C^{(p)}_{1,1,-1}e^{-2K_3d}}e^{-2K_1a_1} + \frac{C^{(p)}_{1,-1,1} + C^{(p)}_{1,-1,-1}e^{-2K_3d}}{C^{(p)}_{1,1,1} + C^{(p)}_{1,1,-1}e^{-2K_3d}}e^{-2K_2a_2}\\ &+ \frac{C^{(p)}_{-1,-1,1} + C^{(p)}_{-1,-1,-1}e^{-2K_3d}}{C^{(p)}_{1,1,1} + C^{(p)}_{1,1,-1}e^{-2K_3d}}e^{-2K_1a_1-2K_2a_2}.\end{aligned} \tag{A8}$$

In the absence of the middle plate, by setting ϵ_n and μ_n to 1 for $n = 1,2,3$, we have the Casimir energy between two plates separated by a vacuum with a gap $(a_1 + a_2 + d)$:

$$\mathcal{F}(a_1,a_2,d) = \frac{k_\mathrm{B}T}{2\pi}\sum_{l=0}^{\infty}{}' \int_0^\infty k_\perp dk_\perp \big\{\ln\big[1 - r_4^{\mathrm{TM}} r_5^{\mathrm{TM}} e^{-2K_1(a_1+a_2+d)}\big] + \ln\big[1 - r_4^{\mathrm{TE}} r_5^{\mathrm{TE}} e^{-2K_1(a_1+a_2+d)}\big]\big\}, \tag{A9}$$

where

$$r_n^{\mathrm{TM}} = \frac{\epsilon_n(i\xi)K_3(i\xi,k_\perp) - K_n(i\xi,k_\perp)}{\epsilon_n(i\xi)K_3(i\xi,k_\perp) + K_n(i\xi,k_\perp)}, \tag{A10}$$

$$r_n^{\mathrm{TE}} = \frac{\mu_n(i\xi)K_3(i\xi,k_\perp) - K_n(i\xi,k_\perp)}{\mu_n(i\xi)K_3(i\xi,k_\perp) + K_n(i\xi,k_\perp)}. \tag{A11}$$

On the other hand, if the thickness of the middle plate is infinite, the free energy is given by

$$\begin{aligned}\mathcal{F}(a_1,a_2,d) = \frac{k_\mathrm{B}T}{2\pi}\sum_{l=0}^{\infty}{}' \int_0^\infty k_\perp dk_\perp \big\{ &\ln\big[\big(1 - r_4^{\mathrm{TM}} r_3^{\mathrm{TM}} e^{-2K_2a_2}\big)\big(1 - r_5^{\mathrm{TM}} r_3^{\mathrm{TM}} e^{-2K_1a_1}\big)\big]\\ &+ \ln\big[\big(1 - r_4^{\mathrm{TE}} r_3^{\mathrm{TE}} e^{-2K_2a_2}\big)\big(1 - r_5^{\mathrm{TE}} r_3^{\mathrm{TE}} e^{-2K_1a_1}\big)\big]\big\}.\end{aligned} \tag{A12}$$

This is equal to a summation of the free energy between plate 4 and plate 3 and that between plate 5 and plate 3; both these free energies can be calculated by the Lifshitz formula for a two-layer system. Accordingly, if the thickness of the middle plate is large, the Casimir force is approximately given by a resultant force of the Casimir force between plate 4 and plate 3 in the vacuum gap and that between plate 5 and plate 3 in the vacuum gap.

We fix the distance between plate 1 and plate 2 to $L = a_1 + a_2 + d$. If the separation distance between plate 1 and plate 3 is a, the Casimir force acting on plate 3 per unit area is given by

$$P_3(a,T) = -\frac{\partial}{\partial a}\mathcal{F}(a,L-a-d,d). \tag{A13}$$

This calculation is straightforward, but the explicit expression is lengthy and it is omitted.

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