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Quantum-state transfer between a Bose-Einstein condensate and an optomechanical mirror

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We describe a scheme that allows for the transfer of a quantum state between a trapped atomic Bose condensate and an optomechanical end mirror mediated by a cavity field. Coupling between the mirror and the cold gas arises from the fact that the cavity field can produce density oscillations in the gas which in turn acts as an internal Bragg mirror for the field. After adiabatic elimination of the cavity field we find that the coherent dynamics of the atomic condensate–mirror hybrid system is described by an effective state transfer beam-splitter Hamiltonian. The state transfer fidelity is limited principally by the quantum noise associated with the intracavity field.

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Cavity optomechanics is rapidly developing into a major area of research. Several groups have now achieved cooling of the center-of-mass motion of micromechanical systems close to the ground state [1–3], and the coherent exchange of excitation between phonons and photons characteristic of the strong-coupling regime has also been demonstrated [1,4,5]. In a parallel development, Bose-Einstein condensates (BECs) trapped inside high-Q optical resonators have been shown to behave under appropriate conditions much as optically driven mechanical oscillators, offering an alternative, "bottom-up" route to study the optomechanical properties of mesoscopic systems [6–10]. Of particular interest are hybrid systems consisting of mechanical systems in the quantum regime coupled to atoms [11–15], molecules [16], or artificial atoms [1,17,18], as they merge the robust and scalable infrastructure provided by nanoelectromechanical system (NEMS) and microelectromechanical system (MEMS) devices with the remarkable precision measurement and quantum control capabilities of atomic physics.

A recent breakthrough in the study of quantum degenerate atomic gases is the ability to manipulate single atoms, opening up a number of possibilities in several frontier topics, including the control and simulation of strongly correlated quantum systems [19–22] and quantum information. Alternatively, one could think of generating macroscopic cat states in mechanical systems [1]. As such it would be of considerable interest to transfer quantum states between ultracold atomic systems and mechanical oscillators, as this would offer an intriguing route to study the quantum dynamics of truly macroscopic systems and the quantum-to-classical transition. One particularly attractive aspect of quantum-state transfer between micromechanical structures and atomic Schrödinger fields is that both subsystems can have extremely low dissipation and decoherence rates compared to optical fields in resonators.

While there are now well understood optomechanical quantum-state transfer protocols between optical and phonon fields and between electromagnetic fields of different frequencies [23–26], this is not yet the case for state transfer between Schrödinger fields and phonon fields (see, however, Refs. [27–29]). This Rapid Communication describes a scheme that achieves that goal for the case of single-mode fields in a hybrid system consisting of an atomic Bose-Einstein condensate (BEC) trapped inside a Fabry-Pérot cavity with a suspended end mirror or equivalent micromechanical analog. While most many-body states of interest in condensed matter physics involve multimode fields, achieving single-mode state transfer is an essential first step, and developments in cavity optomechanics are proceeding to the control of multimode fields in the near future.

A key result of our analysis is that under appropriate conditions our hybrid system can be described by an effective beam-splitter Hamiltonian with a quantum noise source due to the eliminated optical field. The beam-splitter Hamiltonian is well known as a paradigm for state transfer between subsystems and its appearance in our hybrid system opens the door to state transfer between a BEC and a micromechanical element.

The interaction between the oscillating end mirror, the (noninteracting) BEC, and the single-mode intracavity field is described by the Hamiltonian

$$H = \hbar\omega_c \hat{A}^\dagger \hat{A} + i\hbar\eta(\hat{A}^\dagger e^{-i\omega_l t} - \hat{A} e^{i\omega_l t}) + \int dx \hat{\psi}^\dagger(x)\left[-\frac{\hbar^2}{2m_a}\frac{d^2}{dx^2} + \frac{\hbar g^2 \hat{A}^\dagger \hat{A}}{\Delta_a}\cos^2(kx)\right]\hat{\psi}(x) + \frac{\hat{p}^2}{2m_m} + \frac{1}{2}m_m\Omega_m^2\hat{q}^2 - \hbar\xi\hat{A}^\dagger\hat{A}\hat{q} + H_{\rm d}. \quad (1)$$

Here $\hat{A}$ and $\hat{A}^\dagger$ are the bosonic annihilation and creation operators of the intracavity light field, $\hat{p}$ and $\hat{q}$ are the momentum and position operators of the mirror of effective mass m_m and frequency Ω_m, $\eta = \sqrt{P\kappa/\hbar\omega_l}$ describes the external driving of the optical cavity, where P and ω_l are the laser power and frequency, ω_c is the cavity frequency, L its length, and κ its decay rate, and $\xi = \omega_c/L$ is the optomechanical coupling constant. The second term in the square brackets describes the off-resonant dipole coupling between the condensate atoms and the intracavity light field of wavelength $\lambda = 2\pi/k$, in the form of an optical potential of period $\lambda/2$. Here, g is the resonant Rabi frequency and $\Delta_a = \omega_l - \omega_a$ is the detuning between the light field and the atomic transition, assumed large enough that the upper electronic state can be adiabatically eliminated. Finally $\hat{\psi}(x)$ is the Schrödinger field operator for the condensate of atoms of mass m_a, and $H_{\rm d}$ describes the coupling of the optical field, the condensate, and the optomechanical mirror to thermal reservoirs. In what follows we neglect the dissipation of the matter-wave and mechanical modes, as they are orders of magnitude slower than the optical decay rate.

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The cavity field propagating along the x axis can predominantly impart a photon recoil $2\hbar k$ to the initial zero-momentum cold atoms via Bragg scattering, and we assume that phase-matching limits the production of higher scattering orders. Restricting our analysis to one dimension (x) for simplicity we may expand the Schrödinger field as

$$\hat{\psi}(t) \approx \hat{c}_0\psi_0(x) + \hat{c}_2\psi_2(x), \tag{2}$$

where $\psi_0 = \sqrt{1/L}$ and $\psi_2 = \sqrt{2/L}\cos 2kx$, with $\hat{c}_0^\dagger\hat{c}_0 + \hat{c}_2^\dagger\hat{c}_2 = N_a$, the total number of atoms. Substituting this form into the atom-light part of the Hamiltonian (1) gives

$$H_{\text{a-l}} = \frac{\hbar(2k)^2}{2m_a}\hat{c}_2^\dagger\hat{c}_2 + \frac{\hbar g^2}{2\Delta_a}\hat{A}^\dagger\hat{A}(\hat{c}_0^\dagger\hat{c}_0 + \hat{c}_2^\dagger\hat{c}_2) + \frac{\hbar g^2}{\Delta_a\sqrt{8}}\hat{A}^\dagger\hat{A}(\hat{c}_0^\dagger\hat{c}_2 + \hat{c}_2^\dagger\hat{c}_0). \tag{3}$$

Assuming that the depletion of the zero-momentum component of the condensate is small, we treat it classically via the replacement $\hat{c}_0,\hat{c}_0^\dagger \to \sqrt{N_a}$. Then, neglecting unimportant constant terms, the total Hamiltonian becomes

$$H = \hbar\tilde{\omega}_c\hat{A}^\dagger\hat{A} + i\hbar\eta(\hat{A}^\dagger e^{-i\omega_l t} - \hat{A}e^{i\omega_l t}) + \hbar\Omega_m\hat{c}_m^\dagger\hat{c}_m + \hbar\Omega_2\hat{c}_2^\dagger\hat{c}_2 + \hbar\hat{A}^\dagger\hat{A}[-\xi_m(\hat{c}_m^\dagger + \hat{c}_m) + \xi_2(\hat{c}_2^\dagger + \hat{c}_2)] + H_{\text{d}}, \tag{4}$$

where $\tilde{\omega}_c = \omega_c + g^2N_a/2\Delta_a$ is the cavity frequency shifted by the presence of the atomic medium, $\hat{q} = \sqrt{\hbar/2m\Omega_m}(\hat{c}_m + \hat{c}_m^\dagger)$, $\Omega_2 = 2\hbar k^2/m_a$ is four times the recoil frequency of the atoms, $\xi_m = \sqrt{\hbar/2m\Omega_m}\xi$, and $\xi_2 = \hbar g^2\sqrt{2N_a}/(4\Delta_a)$.

The operator $(\hat{c}_2 + \hat{c}_2^\dagger)$ can be interpreted as the dimensionless "position" of the recoiled condensate side mode in Eq. (2). Hence, the last nondissipative term in Eq. (4) is an optomechanical term where the position of the recoiled condensate component is subjected to the radiation pressure of the intracavity light field. The Hamiltonian (4) therefore describes the interaction of the light field with two oscillating mirrors, one real and one effective. The sign difference between the optomechanical coupling of the suspended mirror and the condensate results from the fact that while the mirror is pushed by radiation pressure, the atoms in the condensate can be either attracted to regions of high-field intensity or of low-field intensity, depending on the laser's detuning from the atomic transition, as apparent from the definition of ξ_2.

In a frame rotating at laser frequency ω_l and with $\hat{a}(t) = \hat{A}(t)e^{i\omega_L t}$, the Heisenberg-Langevin equations of motion are

$$\frac{d\hat{a}}{dt} = (i\tilde{\Delta}_c + i\hat{\Phi} - \kappa/2)\hat{a} + \eta + \sqrt{\kappa}\hat{a}_{\text{in}}e^{i\omega_l t}, \tag{5}$$

$$\frac{d\hat{c}_m}{dt} = -i\Omega_m\hat{c}_m + i\xi_m\hat{a}^\dagger\hat{a}, \tag{6}$$

$$\frac{d\hat{c}_2}{dt} = -i\Omega_2\hat{c}_2 - i\xi_2\hat{a}^\dagger\hat{a}, \tag{7}$$

where $\tilde{\Delta}_c = \omega_l - \tilde{\omega}_c$ is the cavity detuning and

$$\hat{\Phi} \equiv [\xi_m(\hat{c}_m^\dagger + \hat{c}_m) - \xi_2(\hat{c}_2^\dagger + \hat{c}_2)] \tag{8}$$

is the combined optomechanical phase shift of the recoiled condensate and the moving mirror.

We now introduce the dimensionless position and momentum variables $\hat{x}_j = (\hat{c}_j + \hat{c}_j^\dagger)/2$ and $\hat{p}_j = i(\hat{c}_j^\dagger - \hat{c}_j)/2$, where $j = \{m,2\}$. In order to adiabatically eliminate the dynamics of the optical field, we proceed by first linearizing the system of operator equations around the classical steady state, with

$$\hat{x}_j \to \langle\hat{x}_j\rangle + \delta\hat{x}_j, \quad \hat{p}_j \to \langle\hat{p}_j\rangle + \delta\hat{p}_j, \quad \hat{a} \to \langle\hat{a}\rangle + \delta\hat{a}, \tag{9}$$

and $\hat{a}^\dagger\hat{a} \approx \langle a^\dagger a\rangle + \langle\hat{a}^\dagger\rangle\delta\hat{a} + \langle\hat{a}\rangle\delta\hat{a}^\dagger$. The equation of motion for the expectation value $\langle\hat{a}\rangle$ of the intracavity field is then

$$\frac{d\langle\hat{a}\rangle}{dt} = i\Delta'\langle\hat{a}\rangle + \eta, \tag{10}$$

with a steady-state value $\langle\hat{a}\rangle_s = -\eta/i\Delta'$, where we have introduced the complex detuning $\Delta' = \tilde{\Delta}_c + \langle\Phi\rangle + i\kappa/2$, which accounts for the optomechanical frequency shift. The fluctuations about the steady state are given in the usual input-output formalism by Ref. [30]

$$\frac{d\delta\hat{a}}{dt} = i\Delta'\delta\hat{a} + i\delta\hat{\Phi}\langle\hat{a}\rangle_s + \sqrt{\kappa}\hat{a}_{\text{in}}. \tag{11}$$

This equation can be formally integrated to give

$$\delta\hat{a}(t) = \delta\hat{a}(0)e^{i\Delta' t} + i\langle\hat{a}\rangle_s\int_0^t dt'\delta\hat{\Phi}(t')e^{i\Delta'(t-t')} + \sqrt{\kappa}\int_0^t dt'\hat{a}_{\text{in}}(t')e^{i\Delta'(t-t')}. \tag{12}$$

For times long compared to κ^{-1}, and a cavity decay rate much faster that the inverse response time of both the effective and mechanical mirror (characterized by their oscillation frequencies), the first term on the right-hand side of this equation decays to zero, and the operator $\delta\hat{\Phi}(t')$ can be evaluated at t to give

$$\delta\hat{a}(t) = \delta\hat{a}(0)e^{i\Delta' t} - \frac{i\eta(1 - e^{i\Delta' t})}{\Delta'^2}\delta\hat{\Phi}(t) + \hat{f}(t), \tag{13}$$

with $\hat{f}(t)$ being the last term in Eq. (12). Since $\hat{a}_{\text{in}}$ is a noise operator with $[\hat{a}_{\text{in}}(t),\hat{a}_{\text{in}}^\dagger(t')] = \delta(t - t')$ (we take the thermal photon number $n_{\text{th}} = 0$ for optical frequencies) we have, for $t_1 < t_2$,

$$[\hat{f}(t_1),\hat{f}^\dagger(t_2)] = e^{-i\Delta'(t_2-t_1)}(e^{-\kappa(t_2-t_1)} - e^{-\kappa t_2}), \tag{14}$$

with a similar form for $t_1 > t_2$. This commutator vanishes rapidly over the characteristic time scale of the mirror dynamics $(1/\Omega_m)$ for large κ, except for $t_1 = t_2$. Over that time scale, $\hat{f}(t)$ can therefore be thought of as a δ-correlated noise operator as far as the mirror motion is concerned, with

$$[\hat{f}(t_1),\hat{f}^\dagger(t_2)] \approx \frac{\kappa}{[(\tilde{\Delta}_c + \langle\hat{\Phi}\rangle)^2 + \kappa^2/4]}\delta(t_1 - t_2). \tag{15}$$

A more detailed analysis that includes resonator memory effects results in the familiar optical spring effect and cold damping description. In the Doppler regime $\Omega_m \ll \kappa$ considered here, cold damping is however negligible (of the order of 30 Hz for the examples discussed below), and likewise the optical spring effect does not significantly modify the rate of quantum-state transfer.

From now on we consider the situation where the steady-state value of the phase shift is $\langle\Phi\rangle = 0$. With $e^{-\kappa t/2} \to 0$, the linearization ansatz (9) then results in the equations of motion

$$\begin{aligned}\delta\dot{\hat{c}}_m &= -i\Omega_m\delta\hat{c}_m + i\xi_m(\langle\hat{a}\rangle_s\delta\hat{a}^\dagger + \langle\hat{a}^\dagger\rangle_s\delta\hat{a}),\\ \delta\dot{\hat{c}}_2 &= -i\Omega_2\delta\hat{c}_2 - i\xi_2(\langle\hat{a}\rangle_s\delta\hat{a}^\dagger + \langle\hat{a}^\dagger\rangle_s\delta\hat{a}),\end{aligned} \quad (16)$$

with

$$\langle\hat{a}\rangle_s\delta\hat{a}^\dagger + \text{H.c.} = -\frac{2|\langle\hat{a}\rangle_s|^2\tilde{\Delta}_c}{\tilde{\Delta}_c^2+\kappa^2/4}\delta\hat{\Phi}(t) + (\langle\hat{a}^\dagger\rangle_s\hat{f}(t) + \text{H.c.}).$$

Specifically, consider a system prepared such that the shifted frequencies of the two oscillators Ω'_m and Ω'_2 are equal, where $\Omega'_j = \Omega_j + \xi_j^2(2|\langle\hat{a}\rangle_s|^2\tilde{\Delta}_c)/(\tilde{\Delta}_c^2+\kappa^2/4)$, $j=\{m,2\}$. In an interaction picture with the time variation due to the shifted frequencies of the two oscillator operators removed, under the rotating-wave approximation, and neglecting constant terms, we can then describe the coupling between the mechanical oscillator and the BEC by the effective Hamiltonian

$$H_{\text{eff}} = -\hbar[\Omega_{\text{ST}}\delta c_m^\dagger\delta c_2 - \langle\hat{a}^\dagger\rangle_s\hat{f}(t)\delta\hat{\Phi} + \text{H.c.}] + H_{\text{d}}, \quad (17)$$

resulting in the equations of motion

$$\frac{d}{dt}\begin{bmatrix}\delta\hat{x}_2\\ \delta\hat{p}_2\\ \delta\hat{x}_m\\ \delta\hat{p}_m\end{bmatrix} = \Omega_{\text{ST}}\begin{bmatrix}0&0&0&-1\\0&0&1&0\\0&-1&0&0\\1&0&0&0\end{bmatrix}\begin{bmatrix}\delta\hat{x}_2\\ \delta\hat{p}_2\\ \delta\hat{x}_m\\ \delta\hat{p}_m\end{bmatrix} + \hat{\chi}'\begin{bmatrix}0\\-\xi_2\\0\\ \xi_m\end{bmatrix},$$

where $\Omega_{\text{ST}} = 2|\langle\hat{a}\rangle_s|^2\tilde{\Delta}_c\xi_2\xi_m/(\tilde{\Delta_c}^2+\kappa^2/4)$, and

$$\hat{\chi}' = 2[\langle\hat{a}^\dagger\rangle_s\hat{f}(t) + \langle\hat{a}\rangle_s\hat{f}^\dagger(t)]. \quad (18)$$

The coherent part of the Hamiltonian (17) has a beam-splitter form, resulting in the periodic exchange of correlations between the real and effective mirrors. The appearance of the beam-splitter Hamiltonian is the key result of this Rapid Communication and follows from the quantum fluctuations of the cavity field. The coupling between the real and effective mirrors is reminiscent of the Casimir force between two mirrors that arises from vacuum field fluctuations. In our case, however, there is no average net force between the mirrors but rather the cavity field fluctuations serve to dynamically exchange fluctuations in the quadratures of the two mirrors at the state transfer frequency Ω_{ST}. The term proportional to χ' is a noise term due to random momentum kicks arising from cavity field fluctuations. (Note that if the optical field is treated classically the quantum states of the two "mirrors" are uncoupled, although their oscillation frequencies depend on a common classical intracavity intensity, which in turn depends on the expectation value $\langle\hat{\Phi}\rangle$ of the optomechanical phase shift.)

Since both Ω_{ST} and the noise term depend on the same system parameters, these must be chosen carefully to optimize the fidelity of state transfer. For resonant frequencies $\Omega'_2 = \Omega'_{\text{m}}$, and equal optomechanical couplings $\xi_2 = \xi_m$, we find that $\langle\hat{\chi}'\hat{\chi}'^\dagger\rangle/\Omega_{\text{ST}} \approx \kappa/\tilde{\Delta}_c$.

As an example we consider an oscillating mirror of mass $m_m = 5$ ng and frequency $\Omega_m = 2\pi \times 100$ kHz forming the end mirror of a Fabry-Pérot cavity of length 190 μm and cavity decay rate $\kappa = 6\times 10^6$ rad/s^{-1}. The cavity is filled with a small ^{23}Na condensate with $N_a = 5\times 10^4$ atoms. The incident laser light, with $\tilde{\Delta}_c = 6\kappa$ and $\eta = 20\kappa$, is detuned by $\Delta_a = -2\pi\times 461$ GHz from the $D2$ transition line (whose recoil frequency is 25 kHz). These parameters result in $\xi_2 = \xi_m = 6.90\times 10^4$ Hz and a state transfer frequency of $\Omega_{\text{ST}} = 2.90$ kHz.

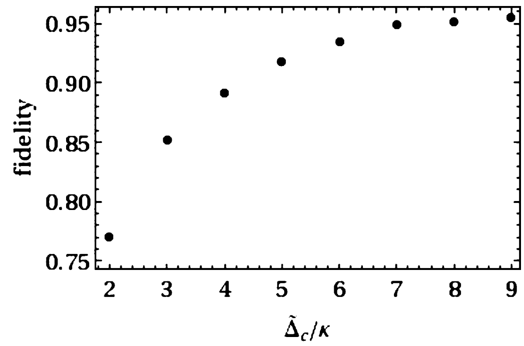

FIG. 1. State transfer fidelity of a condensate coherent state to an oscillating end mirror initially in its ground state of center-of-mass motion as a function of detuning $\tilde{\Delta}_c$ in units of κ. The system parameters are given in the text.

We first consider a Gaussian state described by the Wigner function,

$$W(x,p) = \frac{1}{2\pi\sigma_x\sigma_p}\exp\left[-\frac{(x-x_0)^2}{2\sigma_x^2} - \frac{(p-p_0)^2}{2\sigma_p^2}\right], \quad (19)$$

and evaluate specifically the state transfer fidelity of a coherent state of the BEC ($\sigma_x = \sigma_p = 0.5$) with $x_0 = 1, p_0 = 0$ as a function of $\tilde{\Delta}_c/\kappa$ by evaluating the overlap between this and the membrane state after time $t = \pi/(2\Omega_{\text{ST}})$ (see Fig. 1). While the fidelity increases for larger detunings, we note that eventually the state transfer frequency becomes very low (e.g., $\Omega_{\text{ST}} = 865$ Hz for $\tilde{\Delta}_c/\kappa = 9$). Also, care must be taken to avoid reaching a regime where the behavior of the mechanical system may become bistable. The example of Fig. 2 shows the state transfer from the mechanical membrane to the BEC side mode of the Schrödinger cat state $\frac{1}{\sqrt{\mathcal{N}}}(|\alpha\rangle + |-\alpha\rangle)$, with $\alpha = 2/x_{\text{zp}}$, $x_{\text{zp}} = \sqrt{\hbar/2m_m\Omega_m}$ being the width of the mirror ground state, and $\mathcal{N}$ a normalization constant. The upper plot is the initial Wigner distribution $W(x_m,p_m,t=0)$ for the mirror, and the lower plot is the corresponding Wigner distribution $W(x_2,p_2,t)$ of the BEC at time $t = \pi/(2\Omega_{\text{ST}})$. The similarity of the initial and transferred states indicates that the cat state nature of the initial state has been mostly preserved. The fidelity of the state transfer (the magnitude of the overlap between the two Wigner functions) is 0.835. Everything else being equal, the reduction in state transfer fidelity for a cat state, as compared to a coherent state, is a result of its faster decoherence from the quantum noise of the optical field. This suggests that in that case the fidelity could be improved by driving the resonator with a field with squeezed quantum fluctuations in the appropriate quadrature, which will be confirmed in a future publication [31]. In general, though, it is a nontrivial task to predict the dependence of state transfer fidelity on the specific quantum state under consideration. As would be intuitively expected, we also confirmed numerically that next to controlling quantum noise the most important condition to achieve a high state transfer fidelity is that $\Omega'_2 = \Omega'_m$. While in the specific example of ^{23}Na BEC considered here we have $\Omega_2 = \Omega_m$ and $\xi_2 = \xi_m$, this dual

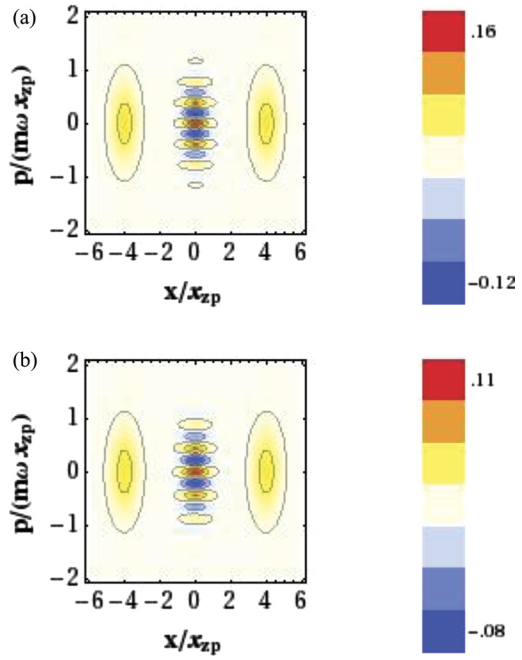

FIG. 2. (Color online) Transfer of cat states: Wigner distribution functions of (a) initial state of membrane $1/\sqrt{\mathcal{N}}(|\alpha\rangle + |-\alpha\rangle)$, where $\alpha = 2$ in our dimensionless units, and (b) BEC after an interaction time of $t = \pi/(2\Omega_{\rm ST})$.

equality does not have to be satisfied in general. The resonance condition $\Omega_2' = \Omega_m'$ can be realized by independently adjusting ξ_2 and ξ_m, e.g., by changing the atomic detuning, the length of the cavity, and/or the number of atoms (at the cost of unequal noise in the two mirrors). We finally remark that our analysis ignored the dissipation and decoherence of both the BEC and the mechanical oscillators, as well as the additional damping term resulting from the adiabatic elimination of the optical field. This is an appropriate approximation for high enough $\Omega_{\rm ST}$ and low bath temperatures, and in the Doppler regime $\kappa \gg \tilde{\Delta}_c$. A more detailed discussion of these noise mechanisms, including an extension of this work to the resolved sideband regime where they may become more important, as well as the analysis of the use of squeezed light to improve state transfer fidelity will be the subject of a future publication [31]. We also plan to extend these ideas to multimode state transfer as appropriate to condensed matter systems, to add many-body effects, and to use quantum control and dark state approaches for improving the fidelity of state transfer.

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