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Optimal fidelity for a quantum channel may be attained by nonmaximally entangled states

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We consider the problem of establishing a two-qubit entangled state of optimal fidelity across a noisy quantum channel when only a single use of the channel and local postprocessing by trace-preserving operations are allowed. We show that the optimal fidelity is obtained only when part of an appropriate nonmaximally entangled state is transmitted through the channel. The entanglement of this state can be vanishingly small when the channel becomes very noisy. Moreover, in the optimal case no further local processing is required to enhance the fidelity. We further show that local postprocessing can enhance fidelity if and only if the amount of noise is larger than a critical value and entanglement of the transmitted state is bounded from below. A notable consequence of these results is that the ordering of states under an entanglement monotone can be reversed even when the states undergo the same local interaction via a trace-preserving, completely positive map.

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Introduction. Quantum entanglement [1] between two distant observers (Alice and Bob) has now been established as a physical resource for quantum information processing. It enables tasks such as quantum teleportation [2], superdense coding [3], quantum cryptography [4], and distributed quantum computation [5] that would otherwise be impossible classically. Shared entanglement, however, is not a *given* resource and must be prepared *a priori* by sending pure entanglement across quantum channels that are typically noisy. The mixed states thus obtained are subsequently subjected to local processing to enhance their purity [6–9] so that they can be useful for tasks such as teleportation. Thus the problem of establishing an entangled state of high purity through a noisy quantum channel is of fundamental interest in quantum information theory.

The purity of a mixed state ρ is expressed by its fidelity or fully entangled fraction [7,9,10]. It is defined as the maximum overlap of the state with a maximally entangled state:

$$F(\rho) = \max_{\Psi} \langle \Psi | \rho | \Psi \rangle, \tag{1}$$

where the maximization is taken over all maximally entangled states $|\Psi\rangle$. Fidelity also assumes a central role in quantum teleportation and entanglement distillation. For two-qubit systems $F(\rho)$ is related to the optimal teleportation fidelity $f(\rho)$ via the following relation [10]:

$$f(\rho) = \frac{2F(\rho)+1}{3}. \tag{2}$$

Let us note that without shared entanglement the best possible fidelity for teleportation (classical fidelity) of a completely unknown qubit is given by 2/3 [11]. Therefore, to outperform a classical strategy with shared entanglement ρ, the condition $F(\rho) > 1/2$ must be satisfied. In the context of entanglement distillation the same condition, i.e., $F(\rho) > 1/2$, determines whether ρ can be distilled by the existing distillation protocols [7,9,12].

Typically, questions related to entanglement distillation and fidelity presuppose that Alice and Bob already share a single copy of a mixed entangled state ρ or many copies of it. In this Rapid Communication we take a step backward and ask the following: Given a quantum channel Λ, what is the maximum achievable fidelity and what is the best strategy to establish an entangled state for which this optimal fidelity is attained? We consider these questions when only a single use of the channel and local postprocessing by trace-preserving operations are allowed. The first condition implies that we are only interested in establishing a *single* copy of an entangled state, and the second condition ensures that there is no particle loss under local operations. The purpose of this Rapid Communication is to explicitly demonstrate the counterintuitive nature of the answers that may be obtained in this setting.

Before we get to our results it is necessary to recall some very useful results on fidelity. For separable states it is known that $F = 1/2$. Surprisingly there exist entangled states for which $F \leqslant 1/2$ [13–15], implying that such states are not directly useful for teleportation. Nevertheless, by local filtering, fidelity of such entangled states can be brought above 1/2 so that they become useful for both teleportation and distillation [6]. Local filtering [16,17], however, is not trace preserving: It succeeds only with some nonzero probability and in case of a failure the state becomes separable. Interestingly, in Refs. [14,15] examples of mixed entangled states with $F \leqslant 1/2$ were given whose fidelity can be increased beyond 1/2 by trace-preserving local operations and classical communication (TP LOCC). Subsequently, it was proved that a state of two qubits is entangled if and only if under TP LOCC its fidelity exceeds 1/2 [18]. This led the authors in Ref. [18] to define the maximum achievable fidelity $F^*(\rho)$ for any $2 \otimes 2$ density matrix ρ as

$$F^*(\rho) = \max_{\text{TP LOCC}} F(\rho) \geqslant F(\rho). \tag{3}$$

While the exact analytical expression $F^*(\rho)$ is not known, it can be obtained by solving a convex semidefinite program [18]. Moreover, $F^*(\rho)$ was shown to be an entanglement monotone; in particular, it quantifies the minimal amount of mixing required to destroy the entanglement of ρ [18]. Here one should

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note that fidelity $F(\rho)$ is not an entanglement monotone as it can increase under TP LOCC.

Formulation of the problem. To answer the questions raised in the beginning of this Rapid Communication it is necessary to consider a two-step process. In the first step, Alice prepares a two-qubit pure entangled state, say, $|\chi\rangle$, and sends the second qubit through the quantum channel Λ. This results in a mixed state, possibly entangled, $\rho(\chi,\Lambda)$ shared between them. As it is possible to enhance the fidelity $F(\rho(\chi,\Lambda))$ of this state by TP LOCC, the second step constitutes Alice and Bob performing optimal trace-preserving local operations to attain the maximum fidelity. Let us therefore define the quantity of interest:

$$\mathcal{F}(\Lambda) = \max_{|\chi\rangle} F^*(\rho(\chi,\Lambda)). \tag{4}$$

We call the quantity $\mathcal{F}(\Lambda)$ maximum achievable fidelity or optimal fidelity for the channel Λ. Clearly, given a quantum channel Λ, the objective of Alice and Bob is to maximize $F(\rho(\chi,\Lambda))$ over all TP LOCC and $|\chi\rangle$. Here we want to emphasize that it is important to distinguish $\mathcal{F}(\Lambda)$ from the channel fidelity considered in Ref. [10]. From Eq. (2) we can also obtain the optimal teleportation fidelity for a single use of the channel Λ:

$$f(\Lambda) = \frac{2\mathcal{F}(\Lambda)+1}{3}.$$

Amplitude damping channel. The quantum channel considered in this work is the amplitude damping channel. The action of an amplitude damping channel Λ on a qubit σ is given by

$$\sigma \to \Lambda(\sigma) = M_0\sigma M_0^\dagger + M_1\sigma M_1^\dagger, \tag{5}$$

where M_0 and M_1 are the Krauss operators defined by

$$M_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{bmatrix}, \quad M_1 = \begin{bmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{bmatrix}, \tag{6}$$

with the real parameter $0 \leqslant p \leqslant 1$ characterizing the strength of the channel. The channel is trace preserving, that is, $\sum_{i=0,1} M_i^\dagger M_i = \mathcal{I}$. For the noise-free case $p = 0$, otherwise $0 < p \leqslant 1$. For $p = 1$ the channel is entanglement breaking [19]. Therefore, throughout this Rapid Communication we only consider values of $0 < p < 1$. We note that $\mathcal{F}(\Lambda)$ is a function of p alone.

Summary of the results. Intuition suggests that for any channel Λ the best strategy to obtain optimal fidelity is to send part of a maximally entangled state across the channel plus local postprocessing, i.e., the relation

$$\mathcal{F}(\Lambda) = F^*(\rho(\Phi^+,\Lambda)), \tag{7}$$

where $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ should be true. But, as will be demonstrated here, the above relation does *not* hold in general. We show that the maximum achievable fidelity $\mathcal{F}(\Lambda)$ is attained for nonmaximally entangled states for all p, $0 < p < 1$; i.e.,

$$\mathcal{F}(\Lambda) = F^*(\rho(\chi_0,\Lambda)) > F^*(\rho(\Phi^+,\Lambda)), \tag{8}$$

where $|\chi_0\rangle$ is a nonmaximally entangled state. And when the channel is very noisy, that is, $p \approx 1$, the entanglement of $|\chi_0\rangle$ becomes vanishingly small, and yet it gives the optimal value for fidelity over all transmitted states, including maximally entangled under trace-preserving local operations. Surprisingly, we find that to achieve the optimal value, local postprocessing is not be required: i.e.,

$$\mathcal{F}(\Lambda) = F^*(\rho(\chi_0,\Lambda)) = F(\rho(\chi_0,\Lambda)). \tag{9}$$

Thus the preprocessed fidelity obtained simply by sending one half of the *appropriate* nonmaximally entangled state through the channel is actually optimal.

A consequence of the first result is that the ordering of entangled states under some entanglement monotone can be reversed even though the states undergo identical local interaction via a trace-preserving, completely positive map. The argument goes as this. Before the second qubit underwent interaction with the channel Λ, we had trivially $F^*(\Phi^+) \geqslant F^*(\chi_0)$. Now after the interaction our first result implies that

$$F^*(\rho(\chi_0,\Lambda)) > F^*(\rho(\Phi^+,\Lambda)). \tag{10}$$

The conclusion now follows by noting that F^* is an entanglement monotone. It is interesting that the ordering does not change for any pair of transmitted states under concurrence. For example, we find that $C(\rho(\Phi^+,\Lambda)) > C(\rho(\chi_0,\Lambda))$, where C is the concurrence [20].

We further show that local trace-preserving operations can enhance the fidelity of the states $\rho(\chi,\Lambda)$ if and only if $p_0 < p < 1$ and $C(\chi(q)) < C(\chi) \leqslant 1$, where q is a function of p. The first condition implies that if $p \leqslant p_0$, then $F(\rho(\chi,\Lambda))$ cannot be increased by TP LOCC for any $|\chi\rangle$. The second condition, on the other hand, shows that when $p > p_0$, fidelity can be increased only for a subset of states $\rho(\chi,\Lambda)$: in particular, those resulting from the transmission of states $|\chi\rangle$ with relatively higher entanglement.

Remark. In the above results both $\mathcal{F}(\Lambda)$ and $|\chi_0\rangle$ are functions of the channel parameter p. This means that for different values of p different optimal values of fidelity are obtained. The corresponding nonmaximally entangled states are different as well.

Details of the results. We shall now prove the results. Alice prepares a two-qubit pure entangled state $|\chi\rangle = \alpha|00\rangle + \beta|11\rangle$, where α,β are real and satisfy the conditions $\alpha \geqslant \beta > 0$ and $\alpha^2 + \beta^2 = 1$. She sends the second qubit through the amplitude damping channel defined by Eq. (5). We therefore have

$$\rho(\chi) \to \rho(\chi,\Lambda) = \sum_{i=0,1} (\mathcal{I}\otimes M_i)\rho(\chi)(\mathcal{I}\otimes M_i^\dagger), \tag{11}$$

where $\rho(\chi) = |\chi\rangle\langle\chi|$. We first obtain the fidelity $F(\rho(\chi,\Lambda))$ before any postprocessing is performed. Define a real 3×3 matrix T whose elements are given by $t_{ij} = \mathrm{Tr}[\rho(\chi,\Lambda)\sigma_i \otimes \sigma_j]$, where σ_i's are the Pauli matrices. In our case T is diagonal and $\det T$ is negative. For the states with diagonal T and $\det T < 0$, F is given by [14]

$$F = \frac{1}{4}\left(1 + \sum_i |t_{ii}|\right), \tag{12}$$

which in our case turns out to be

$$F(\rho(\chi,\Lambda)) = \tfrac{1}{2}(1 + 2\alpha\beta\sqrt{1-p} - p\beta^2). \tag{13}$$

The concurrence [20] of $\rho(\chi,\Lambda)$ is given by $C = 2\alpha\beta\sqrt{1-p}$. It is easy to check that F is not always greater than $1/2$ even though $C(\rho(\chi,\Lambda))$ is always nonzero as long as $p \neq 1$. For example, if $|\chi\rangle = |\Phi^+\rangle$, then for all values $p \geqslant 2(\sqrt{2}-1)$, $F \leqslant 1/2$.

The useful observation to be made here is that the maximum of F (for any $p, 0 < p < 1$) is not obtained when $|\chi\rangle = |\Phi^+\rangle$. In particular,

$$F_{\max} = F(\rho(\chi_0,\Lambda)) = 1 - \frac{p}{2}, \tag{14}$$

where

$$|\chi_0\rangle = \frac{1}{\sqrt{2-p}}|00\rangle + \sqrt{\frac{1-p}{2-p}}|11\rangle. \tag{15}$$

It is worth noting that $F_{\max}$ is the maximum eigenvalue of the density matrix $\rho(\Phi^+,\Lambda)$, and $|\chi_0\rangle$ is the corresponding eigenstate. Indeed, for any quantum channel \$, the maximum pre-processed fidelity is given by the maximum eigenvalue of the density matrix $\rho(\Phi^+,\$)$ and is obtained by sending one half of the corresponding eigenstate through the channel (see Ref. [21] for details).

Equation (14), while surprising, is not conclusive because the maximum achievable fidelity $\mathcal{F}$ may still be obtained for $|\chi\rangle = |\Phi^+\rangle$ *after* Alice and Bob perform trace-preserving LOCC: i.e., the possibility of $\mathcal{F}(\Lambda) = F^*(\rho(\Phi^+,\Lambda))$ cannot be ruled out immediately. The following proposition, however, negates this possibility.

Proposition 1. $\mathcal{F}(\Lambda) > F^*(\rho(\Phi^+,\Lambda))$ for any p, where $0 < p < 1$.

Proof. The result can be proved by computing $F^*(\rho(\Phi^+,\Lambda))$ [see Eqs. (18) and (19)]. Here we give an alternative proof which does not require computing it explicitly. We first note that by definition $\mathcal{F}(\Lambda) \geqslant F_{\max}$, where $F_{\max}$ is given by (14). Now, for any density matrix ρ, $F^*(\rho) \leqslant \frac{1}{2}(1 + N(\rho))$, where $N(\rho) = \max[0, -2\lambda_{\min}(\rho^\Gamma)]$ and ρ^Γ is partial transpose of ρ [13]. Importantly, the equality is achieved if and only if the eigenvector corresponding to the negative eigenvalue of ρ^Γ is maximally entangled [13]. It can be easily checked that the eigenvector corresponding to the negative eigenvalue of $\rho^\Gamma(\Phi^+,\Lambda)$ is not maximally entangled unless $p = 0$. It therefore follows that

$$\begin{aligned} F^*(\rho(\Phi^+,\Lambda)) &< \frac{1}{2}[1 + N(\rho(\Phi^+,\Lambda))] \\ &= 1 - \frac{p}{2} = F_{\max} \leqslant \mathcal{F}(\Lambda). \end{aligned} \tag{16}$$

This concludes the proof.

Remark. As we have explained before, the above result shows that a trace-preserving, completely positive map can reverse the ordering of entangled states for the entanglement monotone F^*. Here we simply note that this reversal is not present when the entanglement measure is concurrence. It is easy to see that for any pair of pure states $|\chi_1\rangle, |\chi_2\rangle$, if $C(\chi_1) \geqslant C(\chi_2)$, then after the interaction $C(\rho(\chi_1,\Lambda)) \geqslant C(\rho(\chi_2,\Lambda))$, where $C(\rho(\chi,\Lambda)) = 2\alpha\beta\sqrt{1-p}$.

We will now obtain an exact expression for $F^*(\rho(\chi,\Lambda))$ for any $|\chi\rangle$. In Ref. [18] it was shown that for any given $2 \otimes 2$ density matrix ρ the maximum achievable fidelity $F^*(\rho)$ by TP LOCC can be found by solving the convex semidefinite program: Maximize

$$F^* = \tfrac{1}{2} - \mathrm{Tr}(X\rho^\Gamma) \tag{17}$$

under the constraints

$$0 \leqslant X \leqslant \mathcal{I}_4, \quad -\frac{\mathcal{I}_4}{2} \leqslant X^\Gamma \leqslant \frac{\mathcal{I}_4}{2},$$

where X is a 4×4 matrix and Γ denotes partial transposition. Moreover, the optimal X is of rank 1. Solving the above in our case using the symmetries of the state $\rho(\chi,\Lambda)$, we obtain the following expressions for maximum achievable fidelity:

$$F^*(\rho(\chi,\Lambda)) = F_1^* = \frac{1}{2}(1 + 2\alpha\beta\sqrt{1-p} - p\beta^2)$$
$$\text{if} \quad \frac{p^2}{1-p+p^2} \leqslant \alpha^2 < 1, \tag{18}$$
$$F^*(\rho(\chi,\Lambda)) = F_2^* = \frac{1}{2}\left(1 + \alpha^2\frac{1-p}{p}\right)$$
$$\text{if} \quad \frac{1}{2} \leqslant \alpha^2 < \frac{p^2}{1-p+p^2}. \tag{19}$$

Maximum achievable fidelity for any ordered pair $(p,|\chi\rangle)$ can be obtained from the above equations. Let $g(p) = \frac{p^2}{1-p+p^2}$. We first observe that the cases corresponding to F_2^* arise only when $g(p) > \frac{1}{2}$, or equivalently $p > \frac{1}{2}(\sqrt{5}-1) = p_0$. Therefore, when $p \leqslant p_0$, then for any state $|\chi\rangle$, we have $F^* = F_1^* = F$, where the last equality follows by comparing Eqs. (13) and (18). In these cases, therefore, there is no benefit from local processing of the states $\rho(\chi,\Lambda)$. On the other hand, when $p > p_0$, the question of enhancing the fidelity of $\rho(\chi,\Lambda)$ depends on entanglement of the state $|\chi\rangle$. For any p, where $p_0 < p < 1$, the transmitted states $|\chi\rangle$ fall in two distinct classes: (a) those satisfying $\frac{1}{2} \leqslant \alpha^2 < g(p)$ or equivalently $C(g(p)) < C(\chi) \leqslant 1$, and (b) those for which $g(p) \leqslant \alpha^2 < 1$ or equivalently $0 < C(\chi) \leqslant C(g(p))$. Now every state in class (a) is *more* entangled than every state in class (b). Therefore, when $p > p_0$, the fidelity of the resulting mixed states can only be increased if the transmitted state belongs to class (a), that is, the class of states with relatively higher entanglement. Summarizing the above we have the next proposition.

Proposition 2. Local trace-preserving operations can enhance the fidelity of the states $\rho(\chi,\Lambda)$ if and only if $p_0 < p < 1$ and $C(\chi(q)) < C(\chi) \leqslant 1$, where $q = g(p)$.

Equations (18) and (19) contain all information that we need to know to obtain $\mathcal{F}(\Lambda)$. Let us denote

$$\mathbb{F}_1(\Lambda) = \max_{|\chi\rangle} F_1^*,$$

where the maximum is taken over all pure states $|\chi\rangle$, satisfying the condition $g(p) \leqslant \alpha^2 < 1$, and

$$\mathbb{F}_2(\Lambda) = \max_{|\chi\rangle} F_2^*,$$

where the maximum is taken over all pure states $|\chi\rangle$, satisfying the condition $\frac{1}{2} \leqslant \alpha^2 < g(p)$. Thus the optimal fidelity for the channel is given by

$$\mathcal{F}(\Lambda) = \mathbb{F}_1(\Lambda) \quad \text{if} \quad p \leqslant p_0, \tag{20}$$

$$\mathcal{F}(\Lambda) = \max\{\mathbb{F}_1(\Lambda), \mathbb{F}_2(\Lambda)\} \quad \text{if} \quad p > p_0, \tag{21}$$

where $p_0 = \frac{1}{2}(\sqrt{5}-1)$.

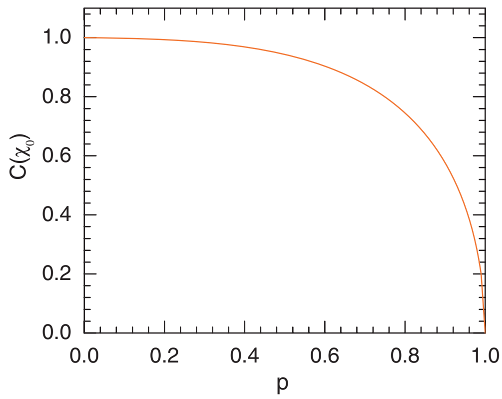

FIG. 1. (Color online) Concurrence of $|\chi_0\rangle$ vs channel parameter p.

Proposition 3. The maximum achievable fidelity $\mathcal{F}(\Lambda)$ is given by $F_{\max} = 1 - \frac{p}{2}$ for all p, $0 < p < 1$.

Proof. From Eqs. (20) and (21) it is clear that two cases have to be considered. We first consider the case when $p \leqslant p_0$. First observe that $F_1^* = F(\rho(\chi,\Lambda))$. Therefore,

$$\mathbb{F}_1(\Lambda) = \max_{|\chi\rangle} F_1^* = \max_{|\chi\rangle} F(\rho(\chi,\Lambda)),$$

where the maximum is taken over all pure states $|\chi\rangle$ such that $\alpha^2 \geqslant g(p)$. But $F_{\max}$ is obtained for the state $|\chi_0\rangle$ given by Eq. (15), which already satisfies the condition $\alpha^2 = \frac{1}{2-p} > g(p)$ for any p, $0 < p < 1$. Thus we have proven that, for $p \leqslant p_0$,

$$\mathcal{F}(\Lambda) = F_{\max} = 1 - \frac{p}{2}.$$

We now consider the case when $p > p_0$. From Eq. (19) we can get an upper bound on $\mathbb{F}_2(\Lambda)$,

$$\mathbb{F}_2(\Lambda) < \frac{1}{2}\left(1 + g(p)\frac{1-p}{p}\right).$$

It is now easy to check that $F_{\max} > \frac{1}{2}(1 + g(p)\frac{1-p}{p})$ for every p, $0 < p < 1$. Thus $F_{\max} > \mathbb{F}_2(\Lambda)$. This implies that if $p > p_0$, the optimal fidelity is not attained by any pure state satisfying $\frac{1}{2} \leqslant \alpha^2 < g(p)$. Instead the optimal fidelity is obtained, once again, for the state $|\chi_0\rangle$. Noting that $\mathbb{F}_1 = F_{\max}$, we have therefore proven that, for $p > p_0$,

$$\mathcal{F}(\Lambda) = F_{\max} = 1 - \frac{p}{2}.$$

This concludes the proof.

Remark. The maximum achievable fidelity $\mathcal{F}(\Lambda)$ being equal to $F_{\max}$ shows that postprocessing by TP LOCC is not necessary to achieve the optimal value as long as the appropriate nonmaximally entangled state $|\chi_0(p)\rangle$ is transmitted. This also suggests that enhancing of fidelity by TP LOCC is possibly a suboptimal phenomenon. While TP LOCC can certainly increase fidelity for some states, it may not be the case that the optimal fidelity for the channel is obtained that way.

Remark. The concurrence of $|\chi_0\rangle$ for which the optimal fidelity is obtained is given by $C(\chi_0) = 2\sqrt{1-p}/(2-p)$. Because $C(\chi_0)$ is a monotonically decreasing function of p, this shows that if the channel is very noisy, that is, $p \approx 1$, the concurrence of the state $|\chi_0\rangle$ becomes arbitrarily close to zero. Perhaps more interesting is the behavior of $C(\chi_0)$ with p. Figure 1 shows that the concurrence decreases with p rather slowly until p enters the "very noisy" domain, wherein it starts to fall quite rapidly. For example, for $p = 0.75$, $C(\chi_0) = 0.8$, whereas for $p = 0.999$, $C(\chi_0) = 0.063$.

Discussions. Several interesting questions arise in the context of the results reported. For example, for which other quantum channels can similar results be observed? A possible way to explore this is to characterize the quantum channels where the maximum fidelity (before any postprocessing by TP LOCC) is obtained by nonmaximally entangled states. The channels that show this behavior are those with the property that the eigenvector corresponding to the maximum eigenvalue of $\rho(\Phi^+,\$)$ (\$ is a quantum channel) is not maximally entangled [21]. The amplitude damping channel belongs to this class but phase damping and depolarizing channels do not. Thus if the channel is phase damping or depolarizing, then the maximum preprocessed fidelity is always attained by sending part of a maximally entangled state through the channel. Despite these observations, a complete characterization of channels exhibiting properties such as the ones presented here should be an interesting problem for future studies. Another question of interest is whether similar results can be observed in the regime of finite copies. The regime of finite copies is nonasymptotic but of considerable practical interest because such cases may be realized experimentally.

Conclusions. To conclude, we have investigated the question of optimal fidelity for a given quantum channel and what is the best protocol to achieve the optimal value. While the results presented in this Rapid Communication illustrate many interesting features that go against conventional intuition, it is likely that they are not generic features of quantum channels. Nevertheless we certainly hope that they would contribute to our understanding of quantum channels and fidelity.

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