Generation of atomic and field squeezing by adiabatic passage and symmetry breaking

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We propose an efficient scheme for realizing squeezing for both an atomic ensemble and a cavity field via adiabatic evolution of the dark state of the atom-cavity system. Controlled symmetry breaking of the Hamiltonian ensures a unique dark state for the total system, in which the atomic system or cavity mode is squeezed depending upon the choice of the detunings. Since the generation of the atomic squeezed state requires neither the cavity mode nor the atomic system to be excited, the decoherence effects are effectively suppressed. The scheme is insensitive to the uncertainty in the atomic number and imperfect timing, and the time needed for the generation of the desired squeezed state decreases as the size of the system grows. The required experimental techniques are within the scope of what can be obtained in the present cavity QED setups.

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I. INTRODUCTION

In recent years, there has been growing interest in quantum states from both fundamental and practical points of view. Besides providing possibilities for testing fundamental quantum theory, nonclassical states have potential applications. Of special interest are the squeezed states of an electromagnetic field, whose quantum fluctuation in one quadrature is reduced below the vacuum level at the expense of amplifying the noise in the other quadrature [1]. Such states may be used to improve the signal-to-noise ratio in optical communications [2] and detect gravitational waves [3]. In correlated many-particle systems, spin squeezing can be defined as the reduction of fluctuation in one collective spin component below the standard quantum limit of the coherent spin state at the expense of amplifying another component. Such states are useful for atomic interferometers [4,5] and high-precision spectroscopy [6,7]. Recently, it has been found that spin squeezing is closely related to entanglement, which is the key resource for quantum communication [8–10] and quantum computation [11]. Ulam-Orgikh and Kitagawa have shown that spin squeezing implies pairwise entanglement [12], and Wang and Sanders have given a quantitative relation between the squeezing and concurrence for symmetric multispin states [13]. As the pairwise entanglement is manifested only in the collective properties of the multiqubit system, it is robust against the loss of coherence for a single qubit, which is important for quantum information processing. Schemes have been proposed for spin squeezing with an optical lattice [14] and Bose-Einstein condensates [15], and weak spin squeezing has been experimentally realized [16,17].

Cavity QED is a qualified candidate for quantum-state engineering and quantum information processing. The strong atom-cavity coupling achievable in a high-finesse cavity allows the generation of various nonclassical states before decoherence sets in. The high degree of control over single atoms [18–20] and atomic ensembles [21,22] in a resonator opens possibilities ranging from quantum-state engineering and quantum networking to quantum phase transitions. So far, schemes for preparing squeezed states in cavity QED have been based on either parametric down conversion [23–25] or quantum reservoir engineering [26–28]. To our knowledge, none of these schemes has been experimentally realized. In this paper, we propose an adiabatic passage scheme for generation of squeezed states for both the atomic system and cavity mode. Unlike previous schemes, our scheme is based on the symmetry breaking of the Hamiltonian for the combined atom-cavity system, which ensures a unique dark state of the interaction Hamiltonian, given by the product of the squeezed state of the atomic system with the vacuum state of the cavity mode or vice versa. The squeezing parameter is controllable via the intensities and phases of the classical driving fields. Compared with the previous schemes, this scheme has the following important features: (i) For the generation of the atomic squeezed state, neither the cavity mode nor the atomic system is excited, so that the model is robust against decoherence mechanisms and a high-fidelity squeezed state can be generated beyond the strong coupling regime, (ii) the method is immune to the uncertainty in the atomic number, (iii) the interaction time does not need to be accurately adjusted as long as the adiabatic condition is fulfilled, and (iv) the time needed to produce the state with a desired squeezing parameter decreases when the number of atoms increases. This scheme is feasible with current experimental technology.

II. EFFECTIVE HAMILTONIAN

We consider that N atoms are trapped in a single-mode cavity. The atomic level configuration is shown in Fig. 1. Each atom has two excited states, $|r\rangle$ and $|s\rangle$, and two ground states, $|e\rangle$ and $|g\rangle$. The cavity mode couples to the transitions $|g\rangle \longleftrightarrow |r\rangle$ and $|e\rangle \longleftrightarrow |s\rangle$ with coupling strengths g_1 and g_2, respectively. Meanwhile, one laser couples to the transition $|e\rangle \longleftrightarrow |r\rangle$ with Rabi frequency Ω_1 and phase ϕ_1 and another laser couples to $|g\rangle \longleftrightarrow |s\rangle$ with Rabi frequency Ω_2 and phase ϕ_2. As will be shown, these fields are used to drive two distinct Raman transitions between two atomic ground states, which leads to the competition between the annihilation and creation operators of the collective atomic mode or of the cavity mode, making the atomic or field squeezed state be the unique dark state of the effective two-mode coupling Hamiltonian depending upon the choice of the field detunings.

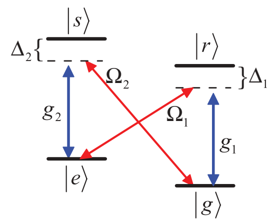

FIG. 1. (Color online) The atomic level configuration and excitation scheme. Each atom has two ground states, $|e\rangle$ and $|g\rangle$, and two excited states, $|r\rangle$ and $|s\rangle$. The transitions $|g\rangle \longleftrightarrow |r\rangle$ and $|e\rangle \longleftrightarrow |s\rangle$ are coupled to the cavity mode with the coupling strengths g_1 and g_2, respectively. Furthermore, the transitions $|e\rangle \longleftrightarrow |r\rangle$ and $|g\rangle \longleftrightarrow |s\rangle$ are driven by two classical fields with Rabi frequencies Ω_1 and Ω_2, respectively. The cavity mode, together with the laser fields, induces two Raman transitions between the two atomic ground states.

The Hamiltonian for the system is ($\hbar = 1$)

$$H = \omega_a a^\dagger a + \sum_{j=1}^{N} (g_1 a |r_j\rangle\langle g_j| + \Omega_1 e^{i\phi_1} e^{-i\omega_1 t} |r_j\rangle\langle e_j| + g_2 a |s_j\rangle\langle e_j| + \Omega_2 e^{i\phi_2} e^{-i\omega_2 t} |s_j\rangle\langle g_j| + \text{H.c.} + \omega_s |s_j\rangle\langle s_j| + \omega_r |r_j\rangle\langle r_j| + \omega_e |e_j\rangle\langle e_j|), \quad (1)$$

where ω_s, ω_r, and ω_e are the energies of levels $|s\rangle$, $|r\rangle$, and $|e\rangle$, respectively, ω_a is the frequency of the cavity mode, and ω_1 and ω_2 are the frequencies of the two classical fields. Here the energy of level $|g\rangle$ is set to zero. We now switch to the interaction picture with respect to

$$H_0 = \omega a^\dagger a + \sum_{j=1}^{N} (\omega_2 |s_j\rangle\langle s_j| + \omega |r_j\rangle\langle r_j| + \omega_e' |e_j\rangle\langle e_j|), \quad (2)$$

where $\omega = (\omega_1 + \omega_2)/2$ and $\omega_e' = (\omega_2 - \omega_1)/2$ are close to ω_c and ω_e, respectively. Then the Hamiltonian describing the atom-field interaction is

$$H_i = H_{i,0} + H_{i,1} + H_{i,2}, \quad (3)$$

where

$$H_{i,0} = (\omega_a - \omega) a^\dagger a + (\omega_e - \omega_e') \sum_{j=1}^{N} |e_j\rangle\langle e_j|,$$

$$H_{i,1} = A_{i,1} + A_{i,1}^\dagger + \Delta_1 \sum_{j=1}^{N} |r_j\rangle\langle r_j|,$$

$$H_{i,2} = A_{i,2} + A_{i,2}^\dagger + \Delta_2 \sum_{j=1}^{N} |s_j\rangle\langle s_j|, \quad (4)$$

$$A_{i,1} = \sum_{j=1}^{N} (g_1 a |r_j\rangle\langle g_j| + \Omega_1 e^{i\phi_1} |r_j\rangle\langle e_j|),$$

$$A_{i,2} = \sum_{j=1}^{N} (g_2 a |s_j\rangle\langle e_j| + \Omega_2 e^{i\phi_2} |s_j\rangle\langle g_j|),$$

and $\Delta_1 = \omega_r - \omega$ and $\Delta_2 = \omega_s - \omega_2$ are the detunings between the fields and the respective atomic transitions.

Under the large detuning condition, i.e., $\Delta_1, \Delta_2 \gg g_1, g_2, \Omega_1, \Omega_2, \omega_a - \omega, \omega_e - \omega_e'$, $H_{i,1}$ and $H_{i,2}$ can be respectively replaced by the effective Hamiltonians [29]

$$\begin{aligned} H_{\text{eff},1} &= \Delta_1 \sum_{j=1}^{N} |r_j\rangle\langle r_j| + \frac{[A_{i,1}, A_{i,1}^\dagger]}{\Delta_1} \\ &= (\Delta_1 + \eta_e + \xi_g a^\dagger a) \sum_{j=1}^{N} |r_j\rangle\langle r_j| \\ &\quad + \sum_{j,k=1}^{N} \xi_g |r_j\rangle\langle g_j| \otimes |g_k\rangle\langle r_k| - \eta_e (S_z + N/2) \\ &\quad - \xi_g a^\dagger a (N/2 - S_z) - (\lambda_1 e^{-i\phi_1} a S^+ + \text{H.c.}) \end{aligned} \quad (5)$$

and

$$\begin{aligned} H_{\text{eff},2} &= \Delta_2 \sum_{j=1}^{N} |s_j\rangle\langle s_j| + \frac{[A_{i,2}, A_{i,2}^\dagger]}{\Delta_2} \\ &= (\Delta_2 + \eta_g + \xi_e a^\dagger a) \sum_{j=1}^{N} |s_j\rangle\langle s_j| \\ &\quad + \sum_{j,k=1}^{N} \xi_e |s_j\rangle\langle e_j| \otimes |e_k\rangle\langle s_k| - \xi_e a^\dagger a (S_z + N/2) \\ &\quad - \eta_g (N/2 - S_z) - (\lambda_2 e^{i\phi_2} a^\dagger S^+ + \text{H.c.}), \end{aligned} \quad (6)$$

where $S^+ = \sum_{j=1}^{N} |e_j\rangle\langle g_j|$, $S_z = \frac{1}{2} \sum_{j=1}^{N} (|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|)$, $\eta_e = \frac{\Omega_1^2}{\Delta_1}$, $\eta_g = \frac{\Omega_2^2}{\Delta_2}$, $\xi_e = \frac{g_2^2}{\Delta_2}$, $\xi_g = \frac{g_1^2}{\Delta_1}$, $\lambda_1 = \frac{\Omega_1 g_1}{\Delta_1}$, and $\lambda_2 = \frac{\Omega_2 g_2}{\Delta_2}$. The atomic excitation number is not changed during the interaction since the atomic excitation number operator $\sum_{j=1}^{N} (|s_j\rangle\langle s_j| + |r_j\rangle\langle r_j|)$ commutes with the total effective Hamiltonian $H_{i,0} + H_{\text{eff},1} + H_{\text{eff},2}$. When all the atoms are initially in the ground states, they will remain in the ground states, i.e., the excited states $|r\rangle$ and $|s\rangle$ can be adiabatically eliminated. Since none of the operators $|r_j\rangle\langle r_j|$, $|s_j\rangle\langle s_j|$, $|g_k\rangle\langle r_k|$, and $|e_k\rangle\langle s_k|$ has any effect on the atomic ground states, the terms containing each of these operators can be discarded, and the effective Hamiltonians $H_{\text{eff},1}$ and $H_{\text{eff},2}$ reduce to

$$\begin{aligned} H_{\text{eff},1} &= -\eta_e (S_z + N/2) - \xi_g a^\dagger a (N/2 - S_z) \\ &\quad - (\lambda_1 e^{-i\phi_1} a S^+ + \text{H.c.}) \end{aligned} \quad (7)$$

and

$$\begin{aligned} H_{\text{eff},2} &= -\xi_e a^\dagger a (S_z + N/2) - \eta_g (N/2 - S_z) \\ &\quad - (\lambda_2 e^{i\phi_2} a^\dagger S^+ + \text{H.c.}). \end{aligned} \quad (8)$$

$H_{\text{eff},1}$ and $H_{\text{eff},2}$ describe two distinct Raman transitions between the two atomic ground states.

III. GENERATION OF SQUEEZED STATES

In the Holstein-Primakoff representation, the collective spin operators $\{S_z, S^\pm\}$ are associated with the bosonic annihilation and creation operators b and $b^\dagger$ via

$$S^+ = b^\dagger \sqrt{N - b^\dagger b}, \quad S^- = \sqrt{N - b^\dagger b}\, b, \quad S_z = b^\dagger b - N/2. \quad (9)$$

When the average number of atoms in state $|e\rangle$ is much smaller than total atomic number, i.e., $\langle b^\dagger b\rangle \ll N$, the collective spin operators are well approximated by $S^+ \simeq \sqrt{N}b^\dagger$, $S^- \simeq \sqrt{N}b$, and $S_z \simeq N/2$. In this case the atomic ensemble can be regarded as a bosonic system, and the transition of one atom from $|g\rangle$ to $|e\rangle$ corresponds to the creation of one quantum in the effective bosonic mode and vice versa. Then the effective Hamiltonians $H_{\mathrm{eff},1}$ and $H_{\mathrm{eff},2}$ approximate to

$$H_{\mathrm{eff},1} = -N\xi_g a^\dagger a - (\sqrt{N}\lambda_1 e^{-i\phi_1} ab^\dagger + \mathrm{H.c.}) \qquad (10)$$

and

$$H_{\mathrm{eff},2} = -N\eta_g - (\sqrt{N}\lambda_2 e^{i\phi_2} a^\dagger b^\dagger + \mathrm{H.c.}). \qquad (11)$$

The dynamics of the system is given by the total effective Hamiltonian,

$$\begin{aligned} H_{\mathrm{eff}} &= H_{i,0} + H_{\mathrm{eff},1} + H_{\mathrm{eff},2} = \delta_a a^\dagger a + \delta_b b^\dagger b \\ &\quad - [\sqrt{N}(\lambda_1 a e^{-i\phi_1} + \lambda_2 a^\dagger e^{i\phi_2})b^\dagger + \mathrm{H.c.}], \end{aligned} \qquad (12)$$

where $\delta_a = \omega_a - \omega - N\xi_g$, $\delta_b = \omega_e - \omega_e'$. We here have discarded the constant term. Set $\delta_b = 0$ and $\delta_a \neq 0$. In this case the effective Hamiltonian reduces to

$$H_{\mathrm{eff}} = \delta_a a^\dagger a - [\sqrt{N}a^\dagger(\lambda_1 e^{i\phi_1} b + \lambda_2 e^{i\phi_2} b^\dagger) + \mathrm{H.c.}]. \qquad (13)$$

Perform the unitary transformation $\widetilde{H}_{\mathrm{eff}} = S_b^\dagger(\xi) H_{\mathrm{eff}} S_b(\xi)$ with the atomic squeezing operator $S_b(\xi) = e^{(\xi^* b^2 - \xi b^{\dagger 2})/2}$, where $\xi = re^{i\theta}$. If we choose the squeezing strength $r = \tanh^{-1}\frac{\lambda_2}{\lambda_1}$ and squeezing phase $\theta = -(\phi_1 + \phi_2)$, the transformed Hamiltonian is given by

$$\widetilde{H}_{\mathrm{eff}} = \delta_a a^\dagger a - \mu(e^{-i\phi_1} ab^\dagger + \mathrm{H.c.}), \qquad (14)$$

where $\mu = \sqrt{N(\lambda_1^2 - \lambda_2^2)}$. This Hamiltonian describes the linear coupling between the field mode and the transformed collective atomic mode, with the total quantum number being conserved. The dark state (eigenstate with zero eigenenergy) of $\tilde{H}_{\mathrm{eff}}$ is the vacuum state $|0\rangle_a|0\rangle_b$. This implies that the dark state of the effective Hamiltonian H_{eff} is $S_b(\xi)|0\rangle_a|0\rangle_b$, which is the product of the squeezed atomic state with the vacuum field state. The squeezing strength and squeezing phase are controllable by the Rabi frequency Ω_2 and phase ϕ_2 of the second classical field. Suppose that the atom-cavity system is initially in the vacuum state $|0\rangle_a|0\rangle_b$ and the Rabi frequency Ω_2 is initially zero so that the initial state is identical to the dark state. When the Rabi frequency Ω_2 is slowly increased with the change rate much smaller than the energy scales, adiabatic theorem ensures that the system approximately follows the dark state, leading to the atomic squeezed state $S_b(\xi)|0\rangle_b$. During the adiabatic evolution, neither the atomic system nor the cavity mode is excited, and thus the method is robust against decoherence. Furthermore, the squeezing parameter is decided by the ratio between the two Raman coupling strengths and is independent of the number of atoms. This implies that the uncertainty in the atomic number does not affect the produced state.

Now we proceed to show how the squeezed state of the cavity mode can be generated. Setting $\delta_a = 0$ and $\delta_b \neq 0$, we obtain the effective Hamiltonian,

$$H_{\mathrm{eff}}' = \delta_b b^\dagger b - [\sqrt{N}(\lambda_1 a e^{-i\phi_1} + \lambda_2 a^\dagger e^{i\phi_2})b^\dagger + \mathrm{H.c.}] \qquad (15)$$

In this case we transform the effective Hamiltonian as $\tilde{H}_{\mathrm{eff}}' = S_a^\dagger(\zeta) H_{\mathrm{eff}}' S_a(\zeta)$ with the field squeezing operator $S_a(\zeta) = e^{(\zeta^* a^2 - \zeta a^{\dagger 2})/2}$, where $\zeta = re^{i(\phi_1+\phi_2)}$. We obtain the new engineered effective Hamiltonian,

$$\tilde{H}_{\mathrm{eff}}' = \delta_b b^\dagger b - \mu(e^{-i\phi_1} ab^\dagger + \mathrm{H.c.}). \qquad (16)$$

This Hamiltonian describes the linear coupling between the transformed field mode and the collective atomic mode. The squeezed state of the cavity mode can be produced from the vacuum state by adiabatically increasing the Rabi frequency Ω_2 from zero. It should be noted that if $\delta_a = \delta_b = 0$, both states $S_a(\zeta)|0\rangle_a|0\rangle_b$ and $S_b(\xi)|0\rangle_a|0\rangle_b$ are the eigenstates with null eigenenergy of the effective Hamiltonian. To ensure the required adiabatic change, it is necessary to lift the degeneracy. A nonzero detuning δ_a or δ_b breaks the symmetry of the effective Hamiltonian and renders $S_b(\xi)|0\rangle_a|0\rangle_b$ or $S_a(\zeta)|0\rangle_a|0\rangle_b$ the unique dark state.

IV. DISCUSSION AND CONCLUSION

We now address the experimental issues. We consider an ensemble of $N \sim 10^6$ ^{87}Rb atoms trapped in a ring cavity. The cavity mode is linearly polarized along an axis perpendicular to an applied magnetic field. States $|e\rangle$ and $|g\rangle$ can be the Zeeman sublevels $|F = 1, M_F = \pm 1\rangle$ of the ground state $|5^2S_{1/2}\rangle$. Take $g_1 \simeq g_2 \simeq 2\pi \times 50$ kHz, $\kappa = 2\pi \times 25$ kHz, and $\gamma = 2\pi \times 6$ MHz [30,31], where κ and γ are the cavity decay rate and the atomic spontaneous emission rate, respectively. For $\Omega_1/\Delta_1 = 1/200$ we have $\lambda_1 = 2\pi \times 0.25$ kHz. Slow variation of Ω_2/Δ_2 from 0 to 1/250 leads to the squeezing strength $r \simeq 1.1$. During the adiabatic evolution the effective linear coupling μ between the two bosonic modes is varied from $\mu_{\max} = 2\pi \times 250$ kHz to $\mu_{\min} = 2\pi \times 150$ kHz. For the generation of the atomic squeezed state the adiabatic approximation requires that $\delta_a T \gg 1$ and $\delta E T \gg 1$, where $\delta E = \sqrt{\mu^2 + \delta_a^2/4} - \delta_a/2$ is the energy gap between the dark state and the nearest states of the Hamiltonian $\tilde{H}_{\mathrm{eff}}$ with nonzero eigenenergy. We note that δE decreases as δ_a increases. To satisfy the adiabatic condition the value of δ_a should be moderate in comparison with that of μ. Choosing $\delta_a = \sqrt{\bar{\mu}^2 + \delta_a^2/4 - \delta_a/2}$, where $\bar{\mu} = (\mu_{\max} + \mu_{\min})/2$, leads to $\delta_a = \sqrt{\bar{\mu}/2} = 2\pi \times 100$ kHz. Then the energy gap δE is varied from $\delta E_{\max} = 2\pi \times 205$ kHz to $\delta E_{\min} = 2\pi \times 108$ kHz. If we set $T = 10/g \simeq 31.8$ μs, then the leakage error to the bright eigenstates is on the order of $P_b \sim 1/(\delta E_{\min} T)^2 + 1/(\delta_a T)^2 \simeq 4.64 \times 10^{-3}$, where $\delta\bar{E}$ is the average value of δE. This leads to the effective decoherence rate $\kappa_e \sim P_b \kappa \simeq 2\pi \times 0.116$ kHz of the cavity mode. The atoms are virtually excited during the evolution, and the effective decoherence rate due to the atomic spontaneous emission is $\gamma_e \sim \gamma\Omega_{2,\max}^2/(2\Delta_2^2) \simeq 2\pi \times 4.8 \times 10^{-2}$ kHz. The error induced by decoherence is about $(\kappa_e + \gamma_e)T \simeq 3.28 \times 10^{-2}$. Therefore, both the adiabaticity and neglect of decoherence can be perfectly satisfied. The result shows that the atomic squeezed state with a high fidelity can be generated even when the cooperativity parameter $g^2/2\gamma\kappa$ is as low as 10^{-2}. It should be noted that the interaction time does not need to be adjusted very accurately as long as the adiabatic condition is satisfied. For adiabatic following of the squeezed field state the cavity quality needs to be improved. In recent experiments with a Bose-Einstein condensate strongly coupled to an optical cavity [32,33], much higher cooperativity parameters have been achieved, indicating a high-fidelity squeezed state for the cavity can also be produced. An alternative way to generate the squeezed field state is to use the matter-light state-transfer scheme. After the atomic squeezed state has been produced, the resonant Raman transition induced by the cavity mode and the laser field Ω_1 can return all of the atoms to state $|g\rangle$ and transfer the state of the collective atomic mode to the cavity mode.

In conclusion, we have proposed a scheme for deterministically producing squeezed states for both the collective atomic mode and the cavity mode via adiabatic following of the dark state of the atom-cavity system by breaking the symmetry of the Hamiltonian, which renders the dark state unique. Compared with the previous methods, the present method offers potential practical advantages. For the generation of atomic squeezed states, neither the cavity mode nor the atomic system is excited, and the model is robust against decoherence mechanisms. The state evolution is unaffected by the uncertainty in the atomic number and imperfect timing. The scheme is within reach of current experiments and expands the range of possibilities for quantum-state preparation in continuous variable systems.

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