Quasi-one-dimensional atomic gases across wide and narrow confinement-induced resonances

Xiaoling Cui

Institute for Advanced Study, Tsinghua University, Beijing, 100084, People's Republic of China and Department of Physics, The Ohio State University, Columbus, Ohio 43210, USA

(Received 30 January 2012; revised manuscript received 1 July 2012; published 17 July 2012)

We study quasi-one-dimensional atomic gases across wide and narrow confinement-induced resonances (CIRs). We show using the virial expansion that, by tuning the magnetic field, the repulsive scattering branch initially prepared at low fields can continuously go across CIRs without decay; instead, the decay occurs when the noninteracting limit is approached. The interaction properties essentially rely on the resonance width of the CIR. Universal thermodynamics holds for the scattering branch right at a wide CIR, but is smeared out in a narrow CIR due to the strong energy dependence of the coupling strength. In wide and narrow CIRs, the interaction energy of the scattering branch shows different types of strong asymmetry when the decay is approached from opposite sides of the magnetic field. Finally, we discuss the stability of the repulsive branch for a repulsively interacting Fermi gas in different trapped geometries at low temperatures.

DOI: 10.1103/PhysRevA.86.012705 PACS number(s): 34.50.−s, 03.65.Nk

I. INTRODUCTION

Quasi-one-dimensional (1D) atomic gases across scattering resonances can be realized in laboratories by utilizing confinement-induced resonance (CIR) [1]. By initially preparing the system at a high or low magnetic field and sweeping the field properly, the quasi-1D system can evolve on the attractive branch with molecules [2] or on the repulsive scattering branch that is free of molecules [3,4]. Quasi-1D atomic gases have many fascinating properties that are very different from those of 3D gases. For instance, in quasi-1D a two-body bound state exists for an arbitrary s-wave scattering length a_s [1,2]. The three-body recombination rate of 1D bosons is efficiently suppressed in the Tonks-Girardeau (TG) regime with strong repulsion [3,5]. A long-lived metastable quantum phase in the super-TG regime [6,7] has been realized in the scattering branch of a 1D bosonic system with strong attraction [4].

Apart from dimensionality, the resonance width is another important ingredient affecting many-body properties. Take (3D) Feshbach resonance (FR) for example. In a wide FR, where the width is much larger than the typical energy scale $\mathcal{E}^*$ [8], the system exhibits universal thermodynamics (UT) right at resonance where a_s diverges [9,10]. UT means that the thermodynamic potential is a universal function of the temperature and density, regardless of any detail of the interparticle interactions. However, in a narrow FR, where the width is much smaller than $\mathcal{E}^*$, the universality is not evident due to the considerable effective-range effect [11]. Another interesting property in a narrow FR is that the interaction energy shows strong asymmetry when the resonance is approached from different sides [12], as has recently been observed in a ^{6}Li Fermi gas [13]. Considering the facts that the quasi-1D geometry is reduced from 3D by confinement and the 1D resonance originates from the 3D s-wave interaction, it is natural to expect that the effective-range effect in 3D will also influence the interaction properties of the quasi-1D system.

In this work, from the analysis of two-body solutions and high-temperature virial expansions, we study the scattering property and thermodynamics of quasi-1D atomic gases across CIRs. We shall show that the quasi-1D geometry greatly modifies the stability of the repulsive scattering branch compared to the 3D case. As in FRs, CIRs can also be classified as wide or narrow according to the resonance width. We find very different thermodynamic properties between wide and narrow CIRs due to the energy dependence of the coupling strength. The stability of the repulsive branch in other trapped geometries, such as isotropic 3D traps or anisotropic quasi-low-dimensional trapped systems, is also discussed in combination with recent developments on cold Fermi gases in the laboratories. We use $\hbar = k_B = 1$ throughout the paper.

The paper is organized as follows. In Sec. II, we introduce the effective quasi-1D scattering, from which the wide and narrow CIRs are defined and the corresponding thermodynamics is presented. In Sec. III we carry out a high-T virial expansion for an effectively 1D system, and present a detailed study of the stability and thermodynamic properties of the repulsive branch. An extensive discussion of the stability of repulsive the branch for a cold Fermi gas in various trapped geometries is given in Sec. IV. Finally we summarize the paper in Sec. V.

II. EFFECTIVE ONE-DIMENSIONAL SCATTERING AND THERMODYNAMICS

The Schrödinger equation for the relative motion of two atoms moving in quasi-1D is

$$H_0\psi(\mathbf{r}) + \frac{4\pi a_s(E)}{m}\delta(\mathbf{r})\frac{\partial}{\partial r}[r\psi(\mathbf{r})]|_{r\to 0} = E\psi(\mathbf{r}); \qquad (1)$$

here the noninteracting part is $H_0 = -\nabla_{\mathbf{r}}^2/m + m\omega_\perp^2(x^2 + y^2)/4$, $\mathbf{r} = (x,y,z)$, and $r = |\mathbf{r}|$; In the pseudopotential part, we use the energy-dependent s-wave scattering length obtained from a renormalization procedure [14,15]:

$$a_s(E) = a_{\rm bg}\left(1 + \frac{W}{E/\delta\mu - (B - B_0)}\right). \qquad (2)$$

$a_s(E)$ physically describes both wide and narrow FRs, with background scattering length $a_{\rm bg}$, magnetic field B, resonance position B_0, width W, and magnetic moment difference $\delta\mu$ between the atom and the closed molecular state.

The reduced quasi-1D scattering from the 3D s-wave interaction has been analyzed by Olshanii and co-workers [1]. For low-energy scattering with $E = \omega_\perp + k^2/m$ and $k^2/m \ll$

$2\omega_\perp$, the wave function at large interparticle distance is frozen in the lowest transverse mode, and its even-parity part is phase shifted as $\Psi_{\rm even}(\mathbf{r}) \sim \exp[-(x^2+y^2)/(2a_\perp^2)]\cos(k|z|+\delta_k)$, with $a_\perp = \sqrt{2/(m\omega_\perp)}$ and

$$\cot\delta_k = -\frac{ka_\perp}{2}\left[\frac{a_\perp}{a_s(E)} - C_0 + o\left(\frac{k^2a_\perp^2}{4}\right)\right], \tag{3}$$

where $C_0 = 1.4603$. Hereafter we neglect the small correction from the last term in Eq. (3). δ_k in turn determines the 1D energy-dependent coupling strength $g(\bar{E}) = 2k\tan\delta_k/m$ with $\bar{E} = k^2/m$, as

$$g(\bar{E}) = g_{\rm bg}\left(1 + \frac{W_{\rm 1D}}{\bar{E}/\delta\mu - (B - B_{\rm 1D})}\right), \tag{4}$$

where $g_{\rm bg} = 2\gamma\omega_\perp a_{\rm bg}$, $W_{\rm 1D} = \gamma W$, $B_{\rm 1D} = B_0 - (\gamma - 1)W - \omega_\perp/\delta\mu$, with $\gamma = (1 - C_0 a_{\rm bg}/a_\perp)^{-1}$ (see also [15]). Equation (4) explicitly shows all realistic parameters describing CIRs, namely, the background coupling $g_{\rm bg}$, the resonance position $B_{\rm 1D}$, and the width $W_{\rm 1D}$. Near a CIR ($B \sim B_{\rm 1D}$) and for $\bar{E} \ll \delta\mu W_{\rm 1D}$, one can construct an effective-range model to formulate the 1D interaction,

$$\frac{1}{g(\bar{E})} = \frac{1}{g_{\rm 1D}} - \frac{m}{2\gamma^2\omega_\perp} r_0 \bar{E}, \tag{5}$$

with $g_{\rm 1D}$ the zero-energy coupling strength and $r_0 = -1/(ma_{\rm bg}\delta\mu W)$ the effective range characterizing the E dependence in $a_s(E)$ [16]. To this end, Eqs. (4) and (5) show the reduced effective-range effect (or E dependence of the coupling strength) in going from a 3D to a quasi-1D system.

In tight transverse confinements and the low-atomic-density (n) limit, $na_\perp \ll 1$, we consider an effective 1D system with interaction given by Eq. (4). Generally, the pressure takes the form

$$P = \mu(2m\mu)^{1/2}\mathcal{F}\left(\frac{T}{\mu}, \left\{\frac{\delta\mu(B - B_{\rm 1D})}{\mu}, \frac{\delta\mu W_{\rm 1D}}{\mu}, \frac{E_{\rm bg}}{\mu}\right\}\right), \tag{6}$$

where μ is the chemical potential, $E_{\rm bg} = mg_{\rm bg}^2$, and $\mathcal{F}$ is a dimensionless function.

For wide CIRs, $\delta\mu W_{\rm 1D}$ ($\gg 2\omega_\perp$) $\gg n^2/m$, the E dependence in Eqs. (4) and (5) is negligible, and the interaction parameters in braces of Eq. (6) can be replaced by a single $g_{\rm 1D}$. The pressure is then reduced to

$$P = \mu(2m\mu)^{1/2}\mathcal{F}\left(\frac{T}{\mu}, \frac{\mu}{mg_{\rm 1D}^2}\right). \tag{7}$$

In wide CIRs ($g_{\rm 1D} = \infty$), P is just a function of T and μ (or T and n), indicating UT for the scattering branch [17]. Particularly at $T = 0$, UT can be established by noting that the bosons and spin-1/2 fermions with infinite repulsion are fully fermionalized, with energy identical to that of an ideal single-species Fermi sea [7,18]. However, in narrow CIRs ($B = B_{\rm 1D}$), Eq. (6) still essentially relies on other interaction parameters ($W_{\rm 1D}, g_{\rm bg}$) and thus UT is absent. More explicitly, UT can be identified by virial expansions at high temperatures.

III. HIGH-TEMPERATURE VIRIAL EXPANSION

At high temperatures, $n^2/m \ll T \ll 2\omega_\perp$, we carry out virial expansions on the effectively 1D system [19]. The pressure can be expanded using the small fugacity $z = e^{\mu/T}$ as $P = \alpha\frac{T}{\lambda}\sum_{n\geqslant 1} b_n z^n$, where $\lambda = \sqrt{2\pi/(mT)}$ is the thermal wavelength, and α is 1 for spinless bosons and 2 for an equal mixture of spin-1/2 fermions. Compared with the noninteracting case (with superscript 0),

$$P = P^{(0)} + \alpha\frac{T}{\lambda}\sum_{n\geqslant 2}(b_n - b_n^{(0)})z^n. \tag{8}$$

Here the difference $b_n - b_n^{(0)}$ characterizes the interaction effect in the n-body cluster, which is generally a function of $\{\frac{\delta\mu(B-B_{\rm 1D})}{T}, \frac{\delta\mu W_{\rm 1D}}{T}, \frac{E_{\rm bg}}{T}\}$. For wide CIRs, $b_n - b_n^{(0)}$ depends only on a single parameter $1/(\lambda m g_{\rm 1D})$, which is free of parameters at $g_{\rm 1D} = \infty$ for any order of the virial expansion and leads to UT according to Eq. (8). This also justifies us in examining UT within the second-order virial expansion. Consideration of higher-order expansions will not change the conclusions, except for a negligible correction (of higher order in z or $n\lambda$) to the thermodynamic quantities.

Due to the interaction effect, the second virial coefficient $\Delta b_2 = (b_2 - b_2^{(0)})/\sqrt{2}$ can be written as $\Delta b_2 = \sum_l [e^{-E_l/T} - e^{-E_l^{(0)}/T}]$ (here l is the energy level for relative motion of two atoms). Given $P(T,\mu)$ in Eq. (8), it is straightforward to obtain the density $n = \partial P/\partial\mu$ and entropy density $s = \partial P/\partial T$, and finally the energy densities $\mathcal{E} = \mu n + Ts - P$, for spinless bosons ($b$) and spin-1/2 fermions (f) as

$$\mathcal{E}^b = \frac{nT}{2}\left[1 + \frac{n\lambda}{2^{3/2}}(-1 + 2\epsilon_{\rm int}) + o((n\lambda)^2)\right], \tag{9}$$

$$\mathcal{E}^f = \frac{nT}{2}\left[1 + \frac{n\lambda}{2^{5/2}}(1 + 2\epsilon_{\rm int}) + o((n\lambda)^2)\right], \tag{10}$$

with the dimensionless interaction energy

$$\epsilon_{\rm int} = -\Delta b_2 + 2T\frac{\partial\Delta b_2}{\partial T}. \tag{11}$$

In the following we derive Δb_2 in strictly 1D by enumerating the energy levels of two interacting particles in a tube ($[-L/2, L/2]$). For simplicity, we first consider the scattering branch without inclusion of any bound state. The discretized wave vector ($k > 0$) is determined by the boundary condition

$$k_l L/2 + \delta_l = (l + 1/2)\pi \quad (l = 0,1,\ldots). \tag{12}$$

By comparing to the noninteracting $k_l^{(0)}$ where $\delta_l = 0$, we obtain

$$\Delta b_2 = \sum_l \left\{\exp\left[-k_l^2/(mT)\right] - \exp\left[-k_l^{(0)2}/(mT)\right]\right\}, \tag{13}$$

which can be transformed to an integral as $2/(mT)\int_0^\infty dk\, k\delta_k e^{-k^2/(mT)}$ and further to

$$\Delta b_2^{\rm sc} = -\frac{1}{2} + \frac{1}{\pi}\int_0^\infty dk\, e^{-k^2/mT}\frac{d\delta_k}{dk}. \tag{14}$$

Note that to obtain Eq. (14) we extrapolate $\delta_{l=0}$ to $\delta_{k=0}$ in the thermodynamic limit, and set $\delta_{k=0} = -\pi/2$ considering $\delta_{l=0} < 0$ as well as Eq. (3). When considering a bound state (with $l = 0$ occupied), the lowest available l for the scattering state should be $l = 1$. This implies that $\delta_{k=0}$ is upshifted by π

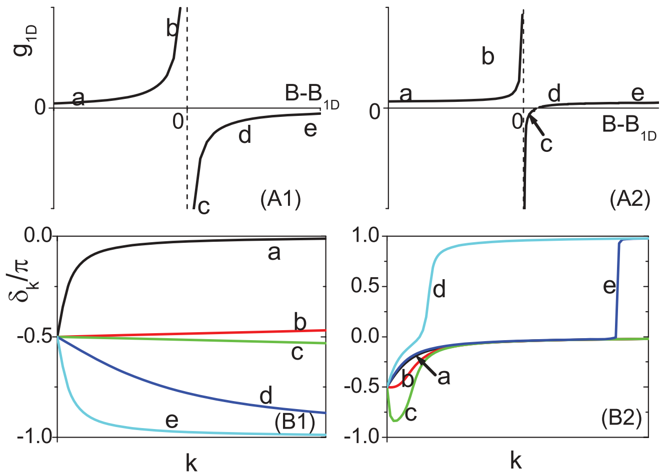

FIG. 1. (Color online) Upper panel: Schematic plot of zero-energy coupling strength g_{1D} across wide (A1) and narrow (A2) CIRs. The labels a–e correspond to $B \ll B_{1D}$ (a); $B \to B_{1D} - 0^+$ (b); $B \to B_{1D} + 0^+$ (c); $B > B_{1D}$ (d); $B \gg B_{1D}$ (e). Lower panel: Phase shift δ_k versus k across wide (B1) and narrow (B2) 1D resonances, with each label (a,b,c,d,e) corresponding to a specific g_{1D} as marked in (A1) and (A2).

as revealed by Levinson's theorem. In this case,

$$\Delta b_2^{\rm bd} = e^{-|E_b|/T} - \frac{1}{2} + \frac{1}{\pi}\int_0^\infty dk\, e^{-k^2/mT}\frac{d\delta_k}{dk}. \tag{15}$$

In this way Δb_2 is obtained for both the repulsive scattering branch [Eq. (14)] and the attractive branch [Eq. (15)]. Remarkably, compared with 3D [9], Δb_2 in 1D has an additional term $(-1/2)$ resulted from the zero-energy phase shift and the unique scattering properties of a 1D system. Equations (14) and (15) are consistent with results obtained from Bethe-ansatz solution [20] and analyses of real-space wave functions [21].

For a quasi-1D system, the virial expansions are carried out by setting $k^\Lambda = 2/a_\perp$ as the upper limit of the integrals in Eqs. (14) and (15). In the rest of this section, we shall mainly focus on the scattering branch [cf. Eq. (14)] which might exhibit UT as discussed above.

A. Wide CIR

With $\delta\mu W_{1D}\,(\gg 2\omega_\perp) \gg n^2/m$, we replace the E-dependent $g(\bar{E})$ by a constant g_{1D}. g_{1D} is schematically plotted in Fig. 1(A1), giving the phase shift (δ_k) of the scattering branch in Fig. 1(B1). Here we have excluded the existence of a bound state for any B field; thus δ_k all start from $-\pi/2$ at $k=0$. By increasing B across the CIR (from a to e), the amplitude of δ_k at finite k gradually becomes enhanced, implying more repulsive energies in the system. In particular, δ_k is uniformly $-\pi/2$ for all k right at the CIR (between b and c), leading to universal values of $\Delta b_2^{\rm sc}$ and $\epsilon_{\rm int}^{\rm sc}$ as shown below.

For a strictly 1D system with constant g_{1D}, Eqs. (14) and (15) can be analytically solved, for example,

$$\Delta b_2^{\rm sc} = -\frac{1}{2} + \frac{{\rm sgn}(g_{1D})}{2}\exp\left(\frac{1}{x^2}\right)\left[1 - {\rm erf}\left(\frac{1}{x}\right)\right]; \tag{16}$$

here $x = 2\sqrt{2\pi}/(m|g_{1D}|\lambda)$; sgn( ) is the sign function and erf( ) is the error function. In the weak-coupling limit ($x \to \infty$), we obtain

$$\Delta b_2^{\rm sc} = -1/(\sqrt{\pi}x), \quad \epsilon_{\rm int}^{\rm sc} = 2/(\sqrt{\pi}x) \tag{17}$$

for the scattering branch at $g_{1D} \to 0^+$ (corresponding to the solid lines in small B field in Fig. 2); and

$$\Delta b_2^{\rm bd} = 1/(\sqrt{\pi}x), \quad \epsilon_{\rm int}^{\rm bd} = -2/(\sqrt{\pi}x) \tag{18}$$

for the attractive branch at $g_{1D} \to 0^-$ [23] (dashed lines in large B field in Fig. 2). In the strong-coupling limit ($x \to 0$), we obtain

$$\Delta b_2^{\rm sc} = -\frac{1}{2} \pm \frac{1}{2\sqrt{\pi}}\left(x - \frac{x^3}{2}\right), \tag{19}$$

$$\epsilon_{\rm int}^{\rm sc} = \frac{1}{2} \mp \frac{x^3}{2\sqrt{\pi}}; \tag{20}$$

the universal values at $x=0$, $-\Delta b_2^{\rm sc} = \epsilon_{\rm int}^{\rm sc} = 1/2$, are direct consequences of the k-independent phase shift $(-\pi/2)$ as mentioned above.

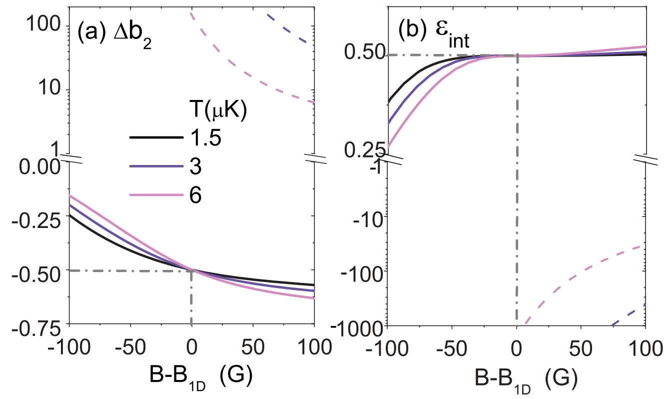

FIG. 2. (Color online) Δb_2 (a) and $\epsilon_{\rm int}$ (b) for two-species ^{6}Li fermions across a wide CIR at T (μK) = 1.5 (dark black), 3 (medium purple), and 6 (light pink). We consider FR at $B_0 = 834.1$ G with width $W = -300$ G [22]. Transverse confinements are generated by optical lattices with lattice spacing $a_L = 500$ nm and depth $V_0 = 25E_R$ [$E_R = (1/2m)(\pi/a_L)^2$], giving $\omega_\perp = (2\pi)300$ kHz. CIR occurs at $B_{1D} = B_0 - 152.2$ G with $W_{1D} = -147.9$ G, which satisfies $\delta\mu W_{1D} \gg 2\omega_\perp \gg T$. Solid and dashed lines are respectively for the scattering [Eq. (14)] and attractive [Eq. (15)] branches. Dash-dotted lines denote the universal values $-\Delta b_2^{\rm sc} = \epsilon_{\rm int}^{\rm sc} = 1/2$ at the CIR.

In Fig. 2, we plot Δb_2 and $\epsilon_{\rm int}$ for two-species ^{6}Li fermions across a wide CIR. For the scattering branch, we see that all curves of $\Delta b_2^{\rm sc}$ (or $\epsilon_{\rm int}^{\rm sc}$) at different T intersect at a single point in the $\Delta b_2^{\rm sc}$-B (or $\epsilon_{\rm int}^{\rm sc}$-$B$) plane [24], demonstrating the UT of the scattering branch right at a wide CIR. The scattering system on the strongly repulsive side of a CIR can smoothly evolve to the strongly attractive side with even higher energy. This is consistent with previous theoretical predictions of a super-TG phase [6,7] and its recent experimental realization in a bosonic gas [4]. Virial expansion also shows that the scattering branch will achieve the strongest repulsion as $g_{1D} \to 0^-$ at large B field, with $-\Delta b_2^{\rm sc}, \epsilon_{\rm int}^{\rm sc} \to 1$. All the above properties can be clearly seen by tracing any individual energy level of two scattering atoms in a tube, as shown in Fig. 3(a).

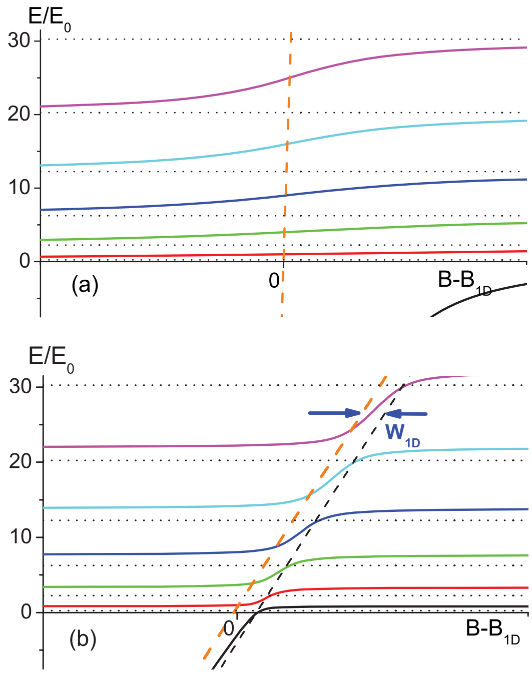

FIG. 3. (Color online) Two-body energy levels in the center-of-mass frame for a quasi-1D system confined in a tube ($[-L/2,L/2]$) across wide (a) and narrow (b) CIRs. The orange and black dashed lines denote $\pi/2$ and 0 phase shifts [corresponding to $g(E)=\infty$ and 0]. $E_0=(2\pi/L)^2/m$. The dotted lines denote noninteracting energy levels with $E^{(0)}/E_0=(l+1/2)^2$, $l=0,1,2\ldots$.

Here we remark on the stability of the scattering branch. In the framework of two-body clusters in the virial expansion, the decay of the scattering branch manifests itself in the discontinuity of thermodynamic quantities, due to the relabeling of scattering states when the underlying bound state converts to the lowest scattering state. This is why in 3D the decay occurs right at the FR where the bound state converts to a scattering state at $a_s=\infty$ [9]. In 1D, however, the conversion is at $g_{1D}=0$ instead of at resonance, and therefore the scattering branch can extend far away from the CIR until the zero-coupling limit is approached. A more comprehensive discussion of the stability of the scattering branch in other trapped geometries will be given in Sec. IV.

At the end of this section we briefly discuss the second-order virial expansion in a 1D harmonic trap, which can be carried out given the two-body spectrum under the coupling strength of Eq. (4). In particular, at a wide CIR with $g_{1D}=+\infty$, the spectrum is $E_l=(2l+3/2)\omega_z$ compared with $E_l^{(0)}=(2l+1/2)\omega_z$; this gives $\Delta b_{2,\mathrm{trap}}^{\mathrm{sc}}=-1/(2\sqrt{2})$ compared with $-1/2$ in the homogeneous case. In fact, based on the local density approximation (as used in a 3D trapped system in Ref. [25]), we have

$$\Delta b_{n,\mathrm{trap}}^{\mathrm{sc}}=\frac{1}{\sqrt{n}}\Delta b_{n,\mathrm{hom}}^{\mathrm{sc}} \tag{21}$$

for the scattering branch right at a wide CIR. This shows a more rapid convergence of virial expansions in a trapped 1D system than in the homogeneous case.

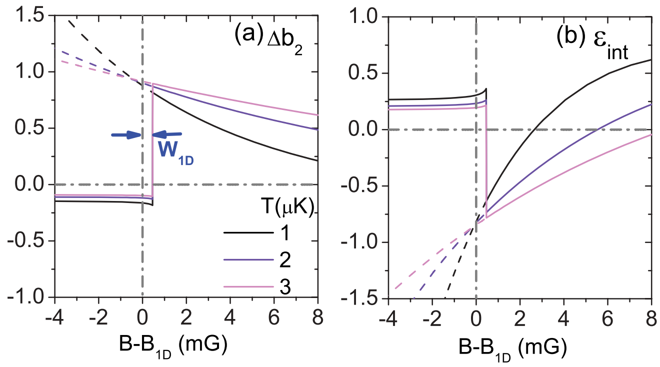

FIG. 4. (Color online) Δb_2 (a) and ϵ_{int} (b) for ^{87}Rb bosons across a narrow CIR at T (μK) = 1 (dark black), 2 (medium purple), and 3 (light pink). We consider a FR at $B_0=406.2$ G with width $W=0.4$ mG [22]. The optical lattice is the same as that for ^{6}Li in Fig. 2, giving $\omega_\perp=(2\pi)80$ kHz. CIR occurs at $B_{1D}=B_0-27.1$ mG with $W_{1D}=0.5$ mG, which satisfies $\delta\mu W_{1D}\ll T\ll 2\omega_\perp$. On decreasing B across $B_{1D}+W_{1D}$, the scattering branch (solid lines) continuously evolves to the attractive branch (dashed lines) with a bound state emerging at threshold.

B. Narrow CIR

With $\delta\mu W_{1D}\ll n^2/m$ ($\ll\omega_\perp$), we take the full form of $g(\bar{E})$ [Eq. (4)] due to the strong E dependence. Assuming a positive background a_{bg}, we give a schematic plot of g_{1D} in Fig. 1(A2) and δ_k for the scattering branch in Fig. 1(B2). In Fig. 4, we show Δb_2 and ϵ_{int} for the ^{87}Rb system across an extremely narrow CIR.

Compared with the wide-CIR case, the scattering branch in a narrow CIR shows many distinct properties. First, δ_k is no longer universal at the CIR; instead, it sensitively depends on k and is quite small at finite k [see b and c in Fig. 1(B2)]. This can be attributed to the strong E dependence in Eq. (4): $g(\bar{E})$ is far off resonance at finite $\bar{E}$, even its zero-energy value $g_{1D}\to\infty$. Accordingly, as plotted in Fig. 3(b) the two-body levels at finite energies are just shifted by a small amount although the lowest level is shifted halfway. As a result, there is no UT at a narrow CIR, and the scattering branch generally has very weak repulsion even close to CIR (see also Fig. 4).

Second, shortly beyond the CIR, the scattering branch goes through a decay at $B=B_{1D}+W_{1D}$ as manifested by discontinuous Δb_2^{sc} and $\epsilon_{\mathrm{int}}^{\mathrm{sc}}$ there (see Fig. 4). This is exactly the place where g_{1D} evolves from 0^- to 0^+ and the bound state transforms to a scattering state.

Third, after the decay, i.e., for $B>B_{1D}+W_{1D}$ and $g_{1D}>0$, we see from Fig. 1(B2) that δ_k will complete a continuous change from $-\pi/2$ to nearly π within an energy window $\Delta\bar{E}\approx B-B_{1D}$. Due to the large π shift for all energies larger than $\Delta\bar{E}$, we see a large and negative $\epsilon_{\mathrm{int}}^{\mathrm{sc}}$ in Fig. 4(b) despite the positive g_{1D}. As for a narrow FR in 3D [12], we expect that the negative $\epsilon_{\mathrm{int}}^{\mathrm{sc}}$ in an extremely narrow CIR will extend to much larger B field, until $\delta\mu(B-B_{1D})$ approaches the typical energy scale of the system [8].

On the whole, the scattering branch in a narrow CIR has weak repulsion ($\epsilon_{\mathrm{int}}^{\mathrm{sc}}\to 0^+$) or strong attraction [$\epsilon_{\mathrm{int}}^{\mathrm{sc}}\to(-1)^+$] when B approaches the decay position ($B=B_{1D}+W_{1D}$) from the small- or large-field side. The asymmetry here differs from that in wide CIRs, where $\epsilon_{\mathrm{int}}^{\mathrm{sc}}$ approaches 1^- or 0^+, respectively.

It is also helpful to compare these features in quasi-1D with those in 3D systems [9,12]. For a wide FR in 3D, the amplitudes of interaction energies are symmetric for systems evolving in different branches and approaching the FR from different sides [9], which is in contrast with what we find in a quasi-1D system across a wide CIR. For a narrow FR, the interaction effects are greatly suppressed for the repulsive branch but greatly enhanced for the attractive branch [12], the same features as revealed above in the quasi-1D system across a narrow CIR. In cold-atom experiments, all these features in quasi-1D systems can be detected using the technique of rf spectroscopy, as has been successfully applied to a quasi-2D Fermi gas [26] and a 3D Fermi gas across a narrow FR [13].

IV. STABILITY OF REPULSIVE FERMI GASES IN TRAPPED GEOMETRIES

Recently, the metastable repulsive branch of atomic gases has attracted much research interest, in the context of the experiment of Jo *et al.* on itinerant ferromagnetism for a repulsively interacting Fermi gas [27]. The same researchers later claimed the absence of itinerant ferromagnetism from a measurement of the spin susceptibility, and attributed this to the instability of the repulsive branch against molecule formation for a 3D Fermi gas near a Feshbach resonance [28]. Similarly, the instability of a repulsive Fermi gas has also been observed in a quasi-2D Fermi gas [26], but at the negative-a_s side.

There have also been quite a few theoretical studies as to why the repulsive Fermi gas in a 3D homogenous system is unstable close to a Feshbach resonance [29–31]. For instance, the instability has been attributed to a shifted resonance in the background of a Fermi sea [29], the pairing instability dominating over the ferromagnetism instability [30], or the vanishing zero-momentum molecule due to the Pauli-blocking effect of Fermi-sea atoms [31]. All these studies can lead to the same conclusion at low temperatures, i.e., the 3D Fermi gas becomes unstable at the place where a_s is comparable to the interparticle distance ($1/k_F$), or equivalently, the two-body binding energy ($E_b \sim 1/ma_s^2$) is comparable to the Fermi energy ($E_F \sim k_F^2/m$). The scattering branch can be stable only when $E_b > E_F$, where the deep molecule cannot be absorbed by the Fermi-sea atoms [29–31]. In other words, in this parameter regime the existence of a deep two-body bound state effectively stabilizes a many-body system on the metastable repulsive branch. Since the physics behind this criterion does not depend on any detail of the dimension or trapping geometry, it should be equally applicable to other cases besides the homogeneous 3D system. In the following, we will use this criterion to study the stability of a repulsive Fermi gas in various trapping geometries at low temperatures.

Typically we consider three different types of trapping potential, namely, an isotropic or nearly isotropic 3D trap ($\omega_x \sim \omega_y \sim \omega_z$), an extremely anisotropic quasi-2D trap ($\omega_z \gg \omega_x,\omega_y$), and a quasi-1D trap ($\omega_z \ll \omega_x,\omega_y$). To facilitate the discussion, we consider the system across a wide resonance (with a single interaction parameter), while the extension to a narrow resonance should be straightforward.

In a trapped system, a two-body bound state is always supported no matter how weak is the attractive interaction [1,32,33]. For a 3D isotropic trap ($\omega_x \sim \omega_y \sim \omega_z \sim \omega$), however, it should be noted that the two-body binding energy E_b at the $a_s < 0$ side is less than the order of ω, i.e., the level spacing of all scattering states [32]. As a result, in the thermodynamic limit with atom number $N \gg 1$, E_b ($< \omega$) at the $a_s < 0$ side is negligible compared with the Fermi energy $E_F \sim N\omega$. The system is then expected to behave as in the homogeneous case, in the sense that the repulsive branch is stable only with positive a_s where the bound state is visibly deep ($E_b \sim N\omega$). This is consistent with what has been observed in experiments [27,28]. On the contrary, for an anisotropic quasi-2D or quasi-1D trap, the energy spacing of the scattering state is generally of the order of trapping frequency of the shallow confinement, while the binding energy can be of the order of trapping frequency of the tight confinement even at $a_s < 0$ side [1,33]. For example, for a quasi-2D trapped system at $a_s = \infty$, $E_b \geqslant \omega_z \gg \omega_x,\omega_y$, and the existence of a deep molecule would be possible to stabilize the repulsive branch at $a_s = \infty$ as long as $E_b > E_F \sim N\omega_{x,y}$. In this case, the stable scattering branch can even extend to the negative-a_s side, as shown in the experiment with a quasi-2D Fermi gas [26]. The same conclusion can be drawn in the quasi-1D trapped case ($\omega_z \ll \omega_x,\omega_y$), which is also consistent with the high-temperature result presented in the last section.

In short summary of this section, at low temperatures, the stability of the repulsive Fermi gas in a trapped geometry relies not only on the existence of a two-body bound state, but more importantly on the value of its binding energy compared with the typical energy scale of a many-body system. In other words, here the two-body physics should be evaluated in a many-body background. Generally, the repulsive branch in quasi-low-dimensional systems is expected to be more stable than that in an isotropic 3D system. Therefore the low-dimensional system provides us a more favorable platform to realize possible itinerant ferromagnetism in repulsively interacting Fermi gases.

V. SUMMARY

In this paper, we have studied quasi-1D atomic gases across wide and narrow CIRs. Our main results are summarized as follows.

First, from high-temperature virial expansions we obtain the following:

(1) By tuning the magnetic field across the CIR, the repulsive scattering branch of a quasi-1D system can evolve continuously across the CIR, from the $g_{1D} = +\infty$ to the $g_{1D} = -\infty$ side.

(2) Universal thermodynamics is identified for the repulsive scattering branch right at a wide CIR, but is found to be washed away at a narrow CIR by the strong energy dependence of the coupling strength.

(3) The decay of the quasi-1D repulsive branch occurs when $g_{1D} \to 0$. The interaction energy shows different types of strong asymmetry for wide and narrow CIRs, when the decay position is approached from opposite sides of the magnetic field.

Moreover, the second-order virial expansion presented in this paper can also serve as a benchmark for testing future experiments on 1D atomic gases.

Second, we have discussed the stability of the repulsive branch for a repulsively interacting Fermi gas at low temperatures in different trapped geometries. By evaluating the two-body bound state in the presence of a Fermi sea, we conclude that the system can generally be more stable in a quasi-low-dimensional trapped system than in a 3D isotropic trap. This should shed light on the current experiments seeking for ferromagnetism in more stable and strongly interacting Fermi gases in low dimensions.

ACKNOWLEDGMENTS

The author is grateful to Tin-Lun Ho for stimulating discussions. This work is supported in part by Tsinghua University Basic Research Young Scholars Program and Initiative Scientific Research Program and NSFC under Grant No. 11104158, and in part by NSF Grant No. DMR-0907366 and by DARPA under the Army Research Office Grants No. W911NF-07-1-0464 and No. W911NF0710576.

[1] M. Olshanii, Phys. Rev. Lett. **81**, 938 (1998); T. Bergeman, M. G. Moore, and M. Olshanii, *ibid.* **91**, 163201 (2003).
[2] H. Moritz, T. Stöferle, K. Güenter, M. Köhl, and T. Esslinger, Phys. Rev. Lett. **94**, 210401 (2005).
[3] E. Haller, Mohamed Rabie, Manfred J. Mark, Johann G. Danzl, Russell Hart, Katharina Lauber, Guido Pupillo, and Hanns-Christoph Nägerl, Phys. Rev. Lett. **107**, 230404 (2011).
[4] E. Haller, M. Gustavsson, M. J. Mark, J. G. Danzl, R. Hart, G. Pupillo and H.-C. Näerl, Science **325**, 1224 (2009).
[5] D. M. Gangardt and G. V. Shlyapnikov, Phys. Rev. Lett. **90**, 010401 (2003).
[6] G. E. Astrakharchik, J. Boronat, J. Casulleras, and S. Giorgini, Phys. Rev. Lett. **95**, 190407 (2005).
[7] M. T. Batchelor, M. Bortz, X. W. Guan, and N. Oelkers, J. Stat. Mech.: Theory Exp. (2005) L10001; S. Chen, L. Guan, X. Yin, Y. Hao, and X.-W. Guan, Phys. Rev. A **81**, 031609(R) (2010).
[8] The typical energy scale $\mathcal{E}^*$ is the temperature T at high T and the Fermi energy E_F at low T.
[9] T.-L. Ho, Phys. Rev. Lett. **92**, 090402 (2004); T.-L. Ho and E. J. Mueller, *ibid.* **92**, 160404 (2004).
[10] J. E. Thomas, J. Kinast, and A. Turlapov, Phys. Rev. Lett. **95**, 120402 (2005).
[11] D. S. Petrov, Phys. Rev. Lett. **93**, 143201 (2004); M. Jona-Lasinio and L. Pricoupenko, *ibid.* **104**, 023201 (2010); S.-G. Peng, S.-Q. Li, P. D. Drummond, and X.-J. Liu, Phys. Rev. A **83**, 063618 (2011).
[12] T.-L. Ho, X. Cui, and W. Li, Phys. Rev. Lett. **108**, 250401 (2012).
[13] E. L. Hazlett, Y. Zhang, R. W. Stites, and K. M. O'Hara, Phys. Rev. Lett. **108**, 045304 (2012).
[14] S. J. J. M. F. Kokkelmans, J. N. Milstein, M. L. Chiofalo, R. Walser, and M. J. Holland, Phys. Rev. A, **65**, 053617 (2002). $a_{bg}\delta\mu W$ and $\delta\mu(B-B_0)$ respectively correspond to the physical coupling $\bar{g}^2$ and detuning $\bar{\nu}$ therein.
[15] V. A. Yurovsky, Phys. Rev. A **71**, 012709 (2005).
[16] In the low-energy limit and near a FR, one can expand $1/a_s(E)$ as $1/a_s(0) - mr_0E$.
[17] UT is absent for the attractive branch at $g_{1D} = \infty$, where atoms tend to form deep molecules with typical size as small as $a_\perp$ [1,2]. The corresponding energy scale is not within the scope of this paper.
[18] L. Guan and S. Chen, Phys. Rev. Lett. **105**, 175301 (2010).
[19] All higher transverse modes ($n > 0$) contribute a negligible correction, which is of the order of $e^{-2\omega_\perp/T}$.
[20] S. Servadio, J. Math. Phys. **12**, 2413 (1971).
[21] W. G. Gibson, Phys. Rev. A **36**, 564 (1987).
[22] C. Chin, R. Grimm, P. Julienne, and E. Tiesinga, Rev. Mod. Phys. **82**, 1225 (2010).
[23] Combined with Eqs. (9) and (10), this reproduces the mean-field interaction energy as $g_{1D}n^2/2$ for bosons and $g_{1D}n^2/4$ for fermions.
[24] The deviation, due to the finite cutoff ($k^\Lambda = 2/a_\perp$) in a quasi-1D system, is negligible if $T \ll 2\omega_\perp$.
[25] The same analysis can be found in the 3D case; see X.-J. Liu, H. Hu, and P. D. Drummond, Phys. Rev. Lett. **102**, 160401 (2009).
[26] B. Fröhlich, M. Feld, E. Vogt, M. Koschorreck, W. Zwerger, and M. Köhl, Phys. Rev. Lett. **106**, 105301 (2011).
[27] G.-B. Jo, Y.-R. Lee, J.-H. Choi, C. A. Christensen, T. H. Kim, J. H. Thywissen, D. E. Pritchard, and W. Ketterle, Science **325**, 1521 (2009).
[28] C. Sanner, E. J. Su, W. Huang, A. Keshet, J. Gillen, and W. Ketterle, Phys. Rev. Lett. **108**, 240404 (2012).
[29] R. Combescot, Phys. Rev. Lett. **91**, 120401 (2003).
[30] D. Pekker, M. Babadi, R. Sensarma, N. Zinner, L. Pollet, M. W. Zwierlein, and E. Demler, Phys. Rev. Lett. **106**, 050402 (2011).
[31] V. B. Shenoy and T.-L. Ho, Phys. Rev. Lett. **107**, 210401 (2011).
[32] T. Busch, B.-G. Englert, K. Rzazewski, and M. Wilkens, Found. Phys. **28**, 549 (1998).
[33] D. S. Petrov, M. Holzmann, and G. V. Shlyapnikov, Phys. Rev. Lett. **84**, 2551 (2000); D. S. Petrov and G. V. Shlyapnikov, Phys. Rev. A. **64**, 012706 (2001).