Medium effects close to *s*- and *p*-wave Feshbach resonances in atomic Fermi gases

Renyuan Liao[1,2] and Khandker F. Quader[3]

[1]*College of Physics and Energy, Fujian Normal University, Fuzhou 350108, China*

[2]*National Laboratory for Condensed Matter Physics, Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China*

[3]*Physics Department, Kent State University, Kent, Ohio 44240, USA*

(Received 10 April 2012; published 10 July 2012)

Many-body effects may influence properties, such as scattering parameters, nature of pairing, etc., close to a Feshbach resonance in the fermion BEC-BCS crossover problem. We study effects such as these using a tractable crossing-symmetric approach. This method allow us to include quantum fluctuations, such as density, current, spin, spin-current, and the higher-order fluctuations in a self-consistent fashion. The underlying fermion interaction is reflected in the "driving" term. We perform calculations here on both Bose-Einstein condensate (BEC) and BCS sides and taking the driving term to be finite range and of arbitrary strength. These are related to two-body singlet and triplet scattering parameters and can be connected with experimental *s*- and *p*-wave Feshbach resonances. We include the $\ell = 0$ density and spin fluctuations as well as $\ell = 1$ current and spin-current fluctuations. We calculate renormalized scattering amplitudes, pairing amplitudes, nature of pairing, etc., on both the BEC and BCS sides. We then compare our results qualitatively with experiments.

DOI: 10.1103/PhysRevA.86.012704 PACS number(s): 34.50.Cx, 71.10.Ay, 05.30.Fk, 03.75.Ss

I. INTRODUCTION

The impact of ultracold atomic and molecular quantum gases on present-day physics is associated with the extraordinary degree of control that such systems offer to investigate the fundamental behavior of quantum matter under various conditions [1]. Recent experimental achievements in the field of ultracold Fermi gases are based mainly on the possibility of tuning the scattering length a_s, in particular to values much large than the mean interatomic distance, by changing an external magnetic field [2]. This situation exists near the so-called Feshbach resonances.

Resonances in general refer to the energy-dependent enhancement of interparticle scattering cross section due to the existence of a metastable state. For Feshbach resonances, the metastable state is described in terms of coupling of a bound state of a subsystem to its environment. By tuning a magnetic field, it is possible to obtain a quasidegeneracy between the relative energy of two colliding atoms and that of a weakly bound molecular state. As the quasibound state passes through a threshold, the scattering length can be varied, in principle, from positive to negative infinity. The Feshbach resonances were observed in bosons [3–6], in fermions between distinct spin states [7–9], and in a single-component Fermi gas [10]. In this manner the interactions between the atoms can be strongly enhanced by an external magnetic bias field, giving rise to the BEC-BCS crossover phenomena [11–13]. As a result of the atomic physics of the Feshbach resonance, the nature of the Cooper pairs in the BEC-BCS crossover is, however, not solely determined by the interaction strength or scattering length but, in principle, also depends on the width of the Feshbach resonance. In the limit of an infinitely broad resonance, the properties of the gases can be described by a single-channel theory that requires only the resonant scattering length as an experimental input. In general, however, a two-channel theory is needed. This is, in particular, true for the description of the wave function of the Cooper pairs that plays an important role in the BEC-BCS crossover.

Near a resonance, fluctuations can be quite important since the system is no longer in the dilute limit, and therefore one may expect contributions from higher angular momenta and from quantum fluctuations or influence of collective modes. Generally, inclusion of many-body effects in these systems is not easy, and one usually stops at the level of random phase approximation (RPA).

In this paper, we aim at presenting a different scenario which emphasizes the role of the exchange particle-hole fluctuations in addition to RPA in the direct particle-hole channel. Our calculation is based on the induced interaction model of Babu and Brown [14,15], subsequently generalized and termed as the "crossing-symmetric approach" [16]. This takes into account a properly antisymmetrized, effective two-body interaction that reproduces the correct low-energy physics and also suppresses any spurious ground states. The physics is described in terms of the Landau interaction parameters and scattering amplitudes to be discussed. In a previous study Gaudio *et al*. [17] considered *s*-wave scattering by including $\ell = 0$ density and spin fluctuation. We shall study *s*- and *p*-wave scattering and include both $\ell = 0$ density and spin fluctuations and $\ell = 1$ current and spin-spin fluctuations.

II. THEORETICAL APPROACH

A. The crossing-symmetric method

The crossing-symmetric method [14–16] was formulated to calculate the effective quasiparticle interactions in Fermi systems. Due to an appropriate compromise between microscopic and phenomenological approaches, it has been successfully applied to a number of Fermi systems: liquid ^{3}He [14,16,18], ^{3}He-^{4}He mixtures [19], paramagnetic metals [16], heavy-fermion systems [20], nuclear matter [21], and ultracold atomic Fermi gases [17]. It has been known [14,20,22,23] that a consistent Fermi-liquid theory cannot be formulated in terms of short-range effective interactions alone; collective excitations generated by these must be exchanged between quasiparticles.

The main point is that the contributions to Landau interaction $f^{\sigma\sigma'}_{\mathbf{pp'}}$ can be separated into two parts [15]:

$$f^{\sigma\sigma'}_{\mathbf{pp'}} = d^{\sigma\sigma'}_{\mathbf{pp'}} + I^{\sigma\sigma'}_{\mathbf{pp'}}\left[f^{\sigma\sigma'}_{\mathbf{pp'}}\right], \tag{1}$$

where the induced part $I^{\sigma\sigma'}_{\mathbf{pp}'}$, a function of the Landau interactions $f^{\sigma\sigma'}_{\mathbf{pp}'}$ themselves, is particle-hole reducible in the exchange particle-hole (u) channel, whereas the direct part $d^{\sigma\sigma'}_{\mathbf{pp}'}$ is not particle-hole reducible in either the direct particle-hole (t) channel or the crossed particle-hole (u) channel. It is important to note that the direct interaction is model dependent, as it gives information about the underlying Hamiltonian, and that the induced interaction is a purely quantum effect, arising from the exchange diagrams required to antisymmetrize the effective two-body scattering amplitude.

For a two-component fermionic system, the Landau interaction can be expressed as

$$F^{\sigma\sigma'}_{\mathbf{pp}'} = F^s_{\mathbf{pp}'} + F^a_{\mathbf{pp}'}\vec{\sigma}\cdot\vec{\sigma}', \tag{2}$$

where $F^s_{\mathbf{pp}'}$ and $F^a_{\mathbf{pp}'}$ can be related to induced-interaction equations as follows [15]:

$$F^s_{\mathbf{pp}'} = D^s_{\mathbf{pp}'} + \frac{1}{2}\frac{F^s_0 U_0(q')F^s_0}{1+F^s_0U_0(q')} + \frac{3}{2}\frac{F^a_0 U_0(q')F^a_0}{1+F^a_0U_0(q')} + \frac{1}{2}\left[1-\frac{q'^2}{4k_F^2}\right]\left[\frac{F^s_1U_1(q')F^s_1}{1+F^s_1U_1(q')} + 3\frac{F^a_1U_1(q')F^a_1}{1+F^a_1U_1(q')}\right], \tag{3}$$

$$F^a_{\mathbf{pp}'} = D^a_{\mathbf{pp}'} + \frac{1}{2}\frac{F^s_0 U_0(q')F^s_0}{1+F^s_0U_0(q')} - \frac{1}{2}\frac{F^a_0 U_0(q')F^a_0}{1+F^a_0U_0(q')} + \frac{1}{2}\left[1-\frac{q'^2}{4k_F^2}\right]\left[\frac{F^s_1U_1(q')F^s_1}{1+F^s_1U_1(q')} - \frac{F^a_1U_1(q')F^a_1}{1+F^a_1U_1(q')}\right]. \tag{4}$$

The momentum transfer in the crossed particle-hole channel is $q' = |\mathbf{p}-\mathbf{p}'| = k_F\sqrt{1-\cos\theta_L}$, with the Landau angle $\theta_L = \cos^{-1}(\mathbf{p}\cdot\mathbf{p}')$. $U_0(q')$ and $U_1(q')$ are the Lindhard functions, or density-density and current-current correlation functions, respectively. The first term in Eqs. (3) and (4) is the so-called direct interaction. The direct term is designed to convey the fact that two quasiparticles can directly scatter via some effective potential and repeatedly so, as in a T matrix. It is of short range and contains information about the underlying Hamiltonian of the system under consideration. Thus, it is the "driving" term. The induced term is of somewhat longer range since two particles can scatter via an interaction mediated by another particle. Equations (3) and (4) are nonlinear coupled equations. To solve these, we need to do Legendre projections:

$$F^{s,a}_{\mathbf{pp}'} = \sum_l F^{s,a}_l P_l(\cos\theta_L), \tag{5}$$

$$D^{s,a}_{\mathbf{pp}'} = \sum_l D^{s,a}_l P_l(\cos\theta_L). \tag{6}$$

Ainsworth *et al.* [16] treated D^s_0, D^a_0, and D^s_1 phenomenologically so as to reproduce the empirical Landau parameters F^s_0, F^a_0, F^s_1, and F^a_1 and predicted the higher-order $F^{s,a}_l$'s ($l \geqslant 1$). Then effective pairing interaction can be obtained [24]:

$$g^{\rm eff}_s = \left[A^s_0 - 3A^a_0 - \left(A^s_1 - 3A^a_1\right)\right]/4 = A_s/4, \tag{7}$$

$$g^{\rm eff}_t = \left[A^s_0 + A^a_0 - \left(A^s_1 + A^a_1\right)\right]/12 = A_t/12, \tag{8}$$

where

$$A^{s,a}_l = \frac{F^{s,a}_l}{1+F^{s,a}_l/(2l+1)} \tag{9}$$

is the A^s_l stands for the l-partial-wave symmetric scattering amplitude, and A^a_l stands for the l-partial wave antisymmetric scattering amplitude and A_s and A_t are the pairing-channel singlet and triplet scattering amplitudes, respectively. The vanishing of forward scattering of two particles of equal spin yields the Landau sum rule $\sum_l(A^s_l + A^a_l) = 0$, which provides a test for obtained Landau parameters.

B. The driving term near a Feshbach resonance

In the crossing-symmetric approach, the form of the direct interaction used to derive the induced interactions must be determined. In general, it represents the sum of all particle-hole irreducible interactions. A self-consistent calculation could be performed starting with a T-matrix direct interaction if a more general interaction were used to derive the T-matrix calculation. According to the proposal by Bedell and Ainsworth [25], the direct interaction is the Fourier transform of an effective quasiparticle potential. From this potential the quasiparticle scattering amplitude $f_{\mathbf{k}}(\phi)$ is given by

$$f_{\mathbf{k}}(\phi) = \frac{-m^*}{4\pi}\int e^{i\mathbf{q}\cdot\mathbf{r}}V_{\rm eff}(\mathbf{r},\mathbf{k})d^3\mathbf{r}, \tag{10}$$

where $\hbar = 1, q^2 = |\mathbf{k}-\mathbf{k}'|^2 = 2k^2(1-\cos\phi), 2k^2 = k_F^2(1-\cos\theta)$, and the quasiparticle mass $m^* = m(1+F^s_1/3)$. The relative momentum of the incoming (outgoing) particles is $\mathbf{k}(\mathbf{k}')$, and the angle between the incoming and scattering plane is ϕ. Equation (10) is restricted to the Fermi surface; therefore $f_{\mathbf{k}}(\phi)$ depends on only two variables, θ and ϕ. According to the effective range expansion, the effective potential $V_{\rm eff}(\mathbf{r},\mathbf{k})$ is, in general, nonlocal and can be expanded in powers of $\mathbf{k}^2$,

$$V_{\rm eff}(\mathbf{r},\mathbf{k}) = U(\mathbf{r}) + \tfrac{1}{3}\mathbf{k}^2\mathbf{r}^2W(\mathbf{r}) + \cdots, \tag{11}$$

where $U(\mathbf{r})$ and $W(\mathbf{r})$ are local potentials. Keeping order of $\mathbf{k}$ to 2 yields a three-parameter approximation to $f_{\mathbf{k}}(\phi)$,

$$f_{\mathbf{k}}(\phi) \simeq \frac{m^*}{m}[-a_s + 3k^2a_t(1-\cos\phi) - 3\mathbf{k}^2b_t], \tag{12}$$

where $a_s = m\int \mathbf{r}^2U(\mathbf{r})d^3\mathbf{r}$, $a_t = (m/9)\int \mathbf{r}^4U(\mathbf{r})d^3\mathbf{r}$, and $b_t = (m/9)\int r^4W(\mathbf{r})d^3\mathbf{r}$. As pointed out in Ref. [25], the triplet quasiparticle scattering volume a_t and the nonlocal part of the effective potential b_t are p wave in nature and thus sample relatively little of the repulsive core of the bare interaction. Therefore they should not have a strong density dependence. The a_t is a finite-range correction to the contact interaction; this allows some interaction between particles of the same spin. The nonlocal piece of the direct interaction results from the coupling of quasiparticle currents. The direct interaction for particles of parallel spin is

$$d^{\uparrow\uparrow}(\theta,\phi) = -\frac{4\pi}{m^*}\left[f_{\mathbf{k}}(\phi) - f_{\mathbf{k}}(\phi+\pi)\right] = \frac{12\pi}{m}k_F^2a_t(1-\cos\theta)\cos\phi. \tag{13}$$

Similarly, for particles of opposite spin,

$$d^{\uparrow\downarrow}(\theta,\phi) = -\frac{4\pi}{m^*}f_{\mathbf{k}}(\phi) = \frac{4\pi}{m}\left[a_s - \frac{3}{2}k_F^2a_t(1-\cos\theta) \times(1-\cos\phi) + \frac{3}{2}k_F^2b_t(1-\cos\theta)\right]. \tag{14}$$

The standard Landau parameters are the $\mathbf{q} = 0$ values of momentum-dependent functions $F_l^{s,a}(\mathbf{q})$. In the limit of $\mathbf{q} \to 0$, $\cos\theta = \cos\theta_L$, $\cos\phi = 1$. $d^{\uparrow\uparrow}$, $d^{\uparrow\downarrow}$ could be rewritten as

$$d_{\mathbf{pp}'}^{\uparrow\uparrow} = d_0^{\uparrow\uparrow} + d_1^{\uparrow\uparrow} P_1(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}') = \frac{12\pi}{m} k_F^2 a_t (1-\cos\theta_L),$$
$$d_{\mathbf{pp}'}^{\uparrow\downarrow} = d_0^{\uparrow\downarrow} + d_1^{\uparrow\downarrow} P_1(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}') = \frac{4\pi}{m}\left[a_s + \frac{3}{2}k_F^2 b_t(1-\cos\theta_L)\right].$$

Solving the above equations, we obtain

$$\begin{aligned} d_0^{\uparrow\uparrow} &= \frac{12\pi}{m}k_F^2 a_t,\\ d_0^{\uparrow\downarrow} &= \frac{4\pi}{m}a_s + \frac{6\pi}{m}k_F^2 b_t,\\ d_1^{\uparrow\uparrow} &= -\frac{12\pi}{m}k_F^2 a_t,\\ d_1^{\uparrow\downarrow} &= -\frac{6\pi}{m}k_F^2 b_t. \end{aligned} \quad (15)$$

The direct interactions D^s and D^a are linear combinations of $d^{\uparrow\uparrow}$ and $d^{\uparrow\downarrow}$:

$$D^s = \frac{N(0)}{2}(d^{\uparrow\uparrow} + d^{\uparrow\downarrow}), \quad (16)$$
$$D^a = \frac{N(0)}{2}(d^{\uparrow\uparrow} - d^{\uparrow\downarrow}), \quad (17)$$

where $N(0) = k_F m^*/\pi^2$ is the density of states at Fermi surface. For a pure s-wave resonance, we set $a_t = 0, b_t = 0$:

$$D_0^s = \frac{2\pi}{m}a_s N(0) = U_0/2, D_1^s = 0, \quad (18)$$
$$D_0^a = -\frac{2\pi}{m}a_s N(0) = -U_0/2, D_1^a = 0. \quad (19)$$

For a pure p-wave resonance, we set $a_s = 0$:

$$D_0^s = \frac{3\pi k_F^2}{m}(2a_t + b_t)N(0), D_1^s = -D_0^s; \quad (20)$$
$$D_0^a = \frac{2\pi k_F^2}{m}(2a_t - b_t)N(0), D_1^a = -D_0^a. \quad (21)$$

C. Connection to the usual scattering parameters

The two-body scattering amplitude is

$$\begin{aligned} f(\theta) &= \frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)(e^{2i\eta_l}-1)P_l(\cos\theta)\\ &= \sum_{l=0}^{\infty}(2l+1)f_l(k)P_l(\cos\theta), \end{aligned} \quad (22)$$

where the lth angular momentum channel is given by [26]

$$f_l(k) = \frac{k^{2l}}{-a_l^{-1} + r_l k^2 - ik^{2l+1}}. \quad (23)$$

According to the effective range expansion, we have

$$k^{2l+1}\cot\eta_l = -\frac{1}{a_l} + r_l k^2 + \cdots. \quad (24)$$

From the induced-interaction description, as $q = 0$, $2k^2 = k_F^2(1-\cos\theta)$. Equation (12) becomes

$$f_{\mathbf{k}}(\theta) = \frac{m^*}{m}\left[-a_s - \frac{3}{2}k_F^2 b_t + \tfrac{3}{2}k_F^2 b_t\cos\theta\right]. \quad (25)$$

Combining Eqs. (22)–(25), we reach

$$\frac{m^*}{m}\frac{1}{2}k_F^2 b_t = \left|\frac{k^2}{-a_1^{-1} + r_1 k^2 - ik^3}\right| = \frac{k^2}{\sqrt{(k^3\cot\eta_1)^2 + k^6}}. \quad (26)$$

III. CALCULATIONS AND RESULTS

A. Close to an s-wave resonance

In the vicinity of a Feshbach resonance, the s-wave scattering length $a(B)$ is described approximately by [28]

$$a(B) = a_{bg}\left(1 - \frac{\Delta B}{B - B_0}\right), \quad (27)$$

where B is the applied field, a_{bg} is the background scattering length, and B_0 is the field at which the resonance occurs. The resonance width ΔB is proportional to the strength of the coupling between the open and close channels. For ^{40}K, $a_{bg} = 174a_0$ [29], where a_0 is the Bohr radius. Based on this, we construct the driving terms and solve Eqs. (3) and (4). We find that the scattering length tends to smooth out as it approaches the resonance, as shown in Fig. 1. On the BCS side far from the resonance, the many-body results give exactly two-body physics. When the system is driven to the resonance, two-body scattering gives a diverging unrenormalized (bare) scattering length. Many-body exchange fluctuations greatly suppress the divergence of the scattering length. In this region, the exchange fluctuations are quite strong and act as a feedback to the system. On the Bose-Einstein condensate (BEC) side far from the resonance, the medium effects reduce the effect of the interaction. This is similar to what occurs

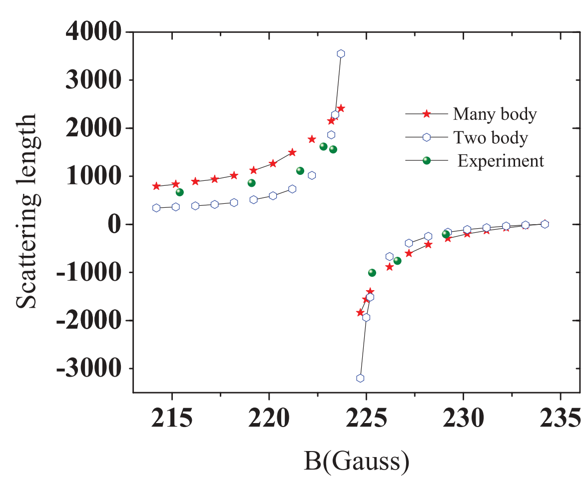

FIG. 1. (Color online) The s-wave scattering length as a function of the magnetic field B on both sides of the Feshbach resonance in ^{40}K, using data from Regal and Jin [27]. The density here is $n = 5.8\times10^{13}$ cm^{-3}, with $\Delta B = 9.7$ G, and the Feshbach resonance occurs at $B_0 = 224.21$ G. The scattering length is measured in terms of the Bohr radius.

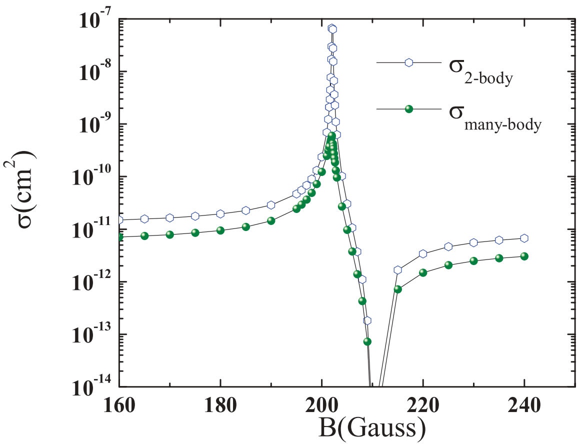

FIG. 2. (Color online) The scattering cross section of an s-wave resonance between ^{40}K atoms in $|f=9/2,m_f=-9/2\rangle$ and $|f=9/2,m_f=-7/2\rangle$ states. The data used [10] for calculation are as follows: number density is $n_{pk}=1.5\times10^{13}$ cm^{-3}, with $\Delta B=7.8$ G, and the Feshbach resonance occurs at $B_0=202.1$ G.

on the BCS side far from the resonance. The suppression of divergence close to the resonance is also suggested in the many-body renormalized scattering cross section, as shown in Fig. 2, where scattering occurs between two hyperfine species, namely, $|f=9/2,m_f=-9/2\rangle$ and $|f=9/2,m_f=-7/2\rangle$. Here f is the total angular momentum, and m_f is the corresponding magnetic quantum number. As can be seen, away from the resonance, the medium effects are small. However, close to resonance, the medium effects strongly modify the scattering cross section.

B. Close to a *p*-wave resonance

A p-wave resonance is distinct from an s-wave ($l=0$) resonance in that the atoms must overcome a centrifugal barrier to couple to the bound state. It is sensitive to the temperature and the magnetic field. We need two parameters to characterize the p-wave resonance: scattering volume a_t and effective range r_1. The magnetic field dependence of a_t and r_1 is obtained from the fitting formula given by Ticknor *et al.* [30]. Therefore we can construct the driving terms and solve Eqs. (3) and (4). The p-wave resonance could be tuned by the magnetic field to occur between two atoms in the hyperfine states of $|f,m_f\rangle=|9/2,-9/2\rangle$ and $|f,m_f\rangle=|9/2,-7/2\rangle$. The joint state of the atom pair will be written as $|f_1m_{f_1}\rangle|f_2m_{f_2}\rangle|\ell m_\ell\rangle$. The many-body effects in a p wave are less severe than in an s wave, as shown in Fig. 3. In Fig. 3, we plot the scattering cross section for a p-wave resonance for ^{40}K atom pairs in the state $|f=9/2,m_f=-7/2\rangle|\ell=1,m_\ell=0\rangle$. Here $\ell=1$ is the orbital angular momentum quantum number, and $m_\ell=0$ is the corresponding magnetic quantum number. Away from the resonance, the many-body corrections due to medium effects are small. Distinct correction only appears at a region close to the Feshbach resonance. Figure 4 shows the temperature dependence of the many-body scattering cross section. We can see that the lower the temperature is, the higher the resonance peak is. As the temperature rises, the resonance cross section broadens. The position of the resonance changes slightly with temperature due to the

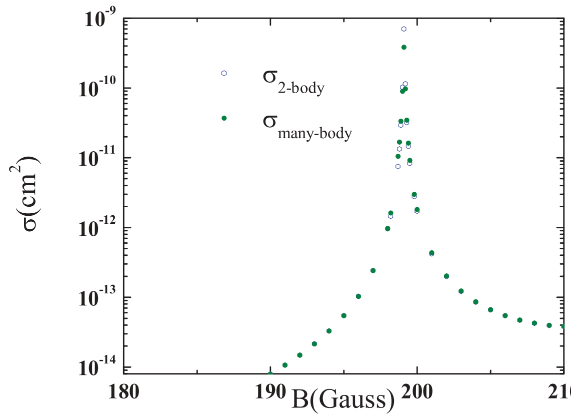

FIG. 3. (Color online) The scattering cross section of a p-wave resonance between ^{40}K atom pairs in the state $|f=9/2,m_f=-9/2\rangle|f=9/2,m_f=-7/2\rangle|\ell=1,m_\ell=0\rangle$ at $T=3.2$ μK.

temperature dependence of the scattering cross section. The resonance at $T=5.0$ μK (green dots) shows a double-peak feature resulting from the strong energy dependence of the cross section. The pairing-channel scattering amplitudes (A_s and A_t) are shown in Fig. 5. The triplet scattering amplitude A_t is negative on the BCS side, indicating a p-wave superfluid pairing. On the BEC side, the singlet scattering amplitude is negative, indicating the formation of s-wave molecules. The p-wave Feshbach resonances offer a means to experimentally study anisotropic interactions in systems other than identical fermions. On resonance the p-wave cross section becomes comparable to the background s-wave scattering. This means that it could have an equally important role in determining the collisional behavior and mean-field interaction of the quantum gases. Utilizing p-wave Feshbach resonances, the Joint Institute for Laboratory Astrophysics (JILA) group has successfully created p-wave molecules [31].

C. Close to a *p*-wave resonance with an *s*-wave background

For a p-wave resonance, the JILA group [10] found that there exists non-negligible off-resonant scattering in the ultracold Fermi gas of ^{40}K. Our model can be employed

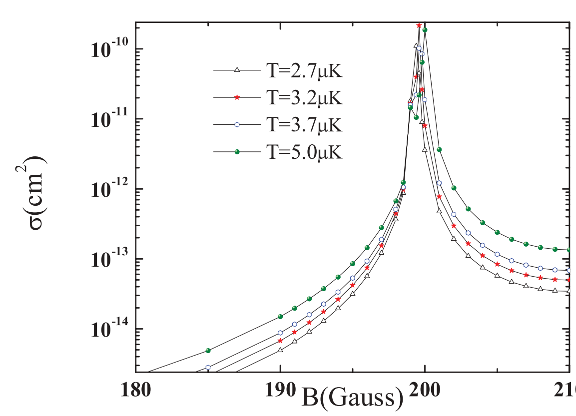

FIG. 4. (Color online) The many-body scattering cross section of a p-wave resonance for ^{40}K atom pairs in the $|f=9/2,m_f=-9/2\rangle|f=9/2,m_f=-7/2\rangle|\ell=1,m_\ell=0\rangle$ state at various temperatures.

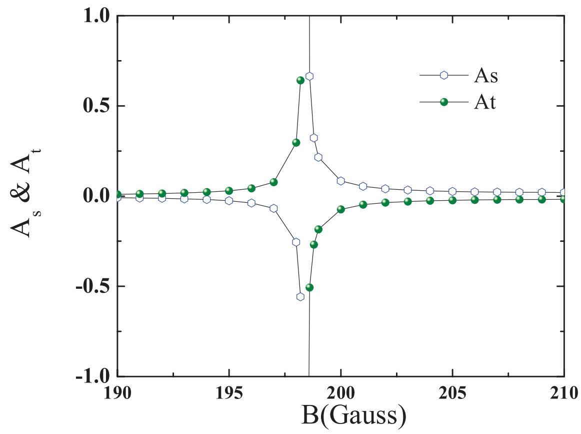

FIG. 5. (Color online) The singlet and triplet scattering amplitudes (A_s and A_t) for a p-wave resonance between ^{40}K atom pairs in the state $|f = 9/2, m_f = -9/2\rangle|f = 9/2, m_f = -7/2\rangle|\ell = 1, m_\ell = 0\rangle$ at $T = 0.1\ \mu$K.

to study the situation when the singlet correlation (s-wave scattering) and the triplet correlation (p-wave scattering) are both present. This situation can occur when the resonant magnetic field for p waves and s waves are well separated. By tuning the magnetic field around p-wave resonance one can realize a p-wave resonance with an s-wave background. The driving terms can then be modeled as

$$\begin{aligned} D_0^s &= \frac{3\pi}{m} k_F^2 (2a_t + b_t) N(0) + \frac{2\pi}{m} a_s N(0), \\ D_0^a &= \frac{3\pi}{m} k_F^2 (2a_t - b_t) N(0) - \frac{2\pi}{m} a_s N(0), \\ D_1^s &= -\frac{3\pi}{m} k_F^2 (2a_t + b_t) N(0), \\ D_1^a &= -\frac{3\pi}{m} k_F^2 (2a_t - b_t) N(0). \end{aligned} \tag{28}$$

Here s-wave background interaction is characterized by $U_0 = \frac{4\pi}{m} a_s N(0)$. By fixing U_0 and varying the parameters $1/k_F^3 a_t$ and $1/k_F^3 b_t$, we can evaluate the pairing-channel scattering amplitudes (A_s and A_t) in full parameter space through solving crossing-symmetric equations (3) and (4). From the obtained Landau parameters $F_l^{s,a}$, by s-p approximation [24], one can straightforwardly construct pairing-channel scattering amplitudes via

$$\begin{aligned} A_{\text{singlet}} &= A_0^s - 3A_0^a - A_1^s + 3A_1^a, \\ A_{\text{triplet}} &= A_0^s + A_0^a - A_1^s - A_1^a. \end{aligned} \tag{29}$$

The calculated singlet scattering amplitude is plotted in Fig. 6. When there is no background s-wave scattering, the singlet scattering amplitude becomes very large when it crosses the unitary limit (strongly interacting regime) of either parameter $1/k_F^3 a_t$ or $1/k_F^3 b_t$, as suggested in Fig. 6(a). To investigate the effects of the s-wave background on the p-wave resonance, we choose $U_0 = -1.5$ for illustration such that it has a noticeable effect. In the presence of background s-wave scattering, the singlet scattering amplitude becomes negative and shows a pronounced peak when it crosses the unitary limit of both parameters $1/k_F^3 a_t$ and $1/k_F^3 b_t$. Away from the unitarity limit, the singlet scattering amplitude A_s is mainly

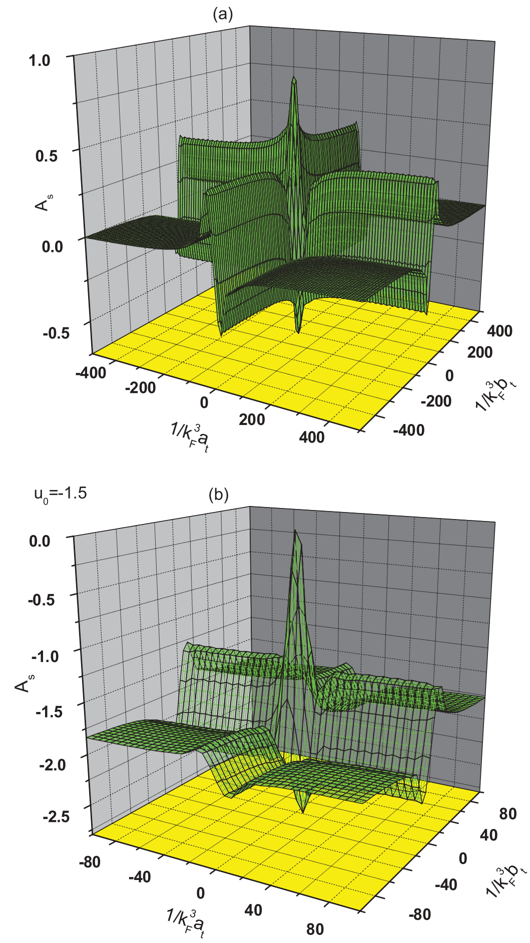

FIG. 6. (Color online) The singlet scattering amplitude A_s for a p-wave resonance: (a) without s-wave background and (b) with s-wave background where $U_0 = -1.5$.

negative, manifesting trends of background s-wave pairing. The purpose of introducing U_0 is to introduce background singlet pairing interaction. The competition between the p-wave scattering and background s-wave scattering determines the sign of the singlet scattering amplitude. When the p-wave interaction is weak, the properties of the system are dominated by s-wave behavior with a constant negative singlet pairing amplitude, as can be seen in Fig. 6(b).

To investigate p-wave pairing, we plot the triplet scattering amplitude A_t in Fig. 7. Interestingly, the triplet scattering

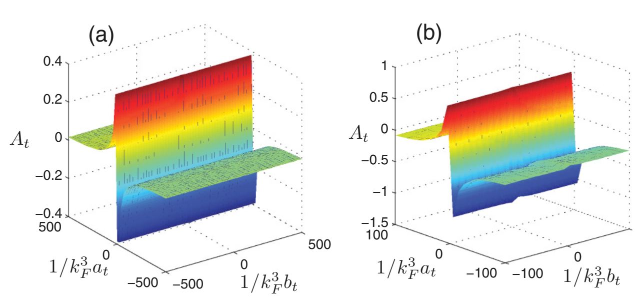

FIG. 7. (Color online) The triplet scattering amplitude A_t for a p-wave resonance: (a) without s-wave background; (b) with s-wave background where $U_0 = -1.5$.

amplitude A_t is sensitive to parameter $1/k_F^3 a_t$ but insensitive to parameter $1/k_F^3 b_t$ except building up a bump near the unitarity regime in the presence of background s-wave scattering. Remarkably, in the absence of background s-wave scattering A_s is antisymmetric with respect to parameter $1/k_F^3 a_t$ while symmetric with respect to parameter $1/k_F^3 b_t$. These properties are closely related to the parametrization and symmetrical features in Eq. (28), where s-wave scattering only contributes to the $\ell = 0$ channel in the driving terms. The background s-wave scattering shows its existence by shifting the triplet scattering amplitude in total toward the negative side. In addition, it enhances the effect of parameter $1/k_F^3 b_t$, especially near resonance. The antisymmetry of A_t with respect to parameter $1/k_F^3 a_t$ is lost; in contrast, the symmetry of A_t with respect to parameter $1/k_F^3 b_t$ is still preserved. It is necessary to point out that when both A_s and A_t are negative, the actual pairing symmetry depends on their relative magnitude.

IV. CONCLUDING REMARKS

By using the crossing-symmetric method to treat many-body Fermi systems, one generally solves nonlinear coupled crossing-symmetric equations for four-point vertex functions in particle-particle, particle-hole, and exchange particle-hole channels. In appropriate limits on the Fermi surfaces, these vertex functions become the Landau quasiparticle interaction $F(q)$ and the scattering amplitude $A(q)$. For isotropic systems, these can be expressed in Legendre polynomials giving Landau parameters F_l^s and F_l^a in spin-symmetric (s) and spin-antisymmetric (a) channels. From the obtained Landau parameters, one can calculate several thermodynamic, transport, and pairing properties of a system.

We study s- and p-wave Feshbach resonance with the crossing-symmetric method. Our findings are as follows: (1) many-body exchange effects may be important close to a Feshbach resonance. Renormalized physical quantities get smoothed out at the Feshbach resonance. In particular, we find that the particle-hole exchange fluctuations introduce an effective scattering length which has been substantially reduced close to resonances. (2) For a p-wave resonance, the triplet scattering amplitude is negative on the BCS side, indicating a triplet (p-wave) superfluid pairing. (3) Background off-resonant scattering has some effects on the singlet and triplet scattering amplitudes, which may influence the pairing symmetry of the ground state.

ACKNOWLEDGMENTS

We are grateful for stimulating discussions with K. S. Bedell and S. Gaudio. This work was supported by the NSFC under Grants No. 10934010 and No. 60978019, the NKBRSFC under Grants No. 2009CB930701, No. 2010CB922904, No. 2011CB921502, and No. 2012CB821300, and the NSFC-RGC under Grants No. 11061160490 and No. 1386-N-HKU748/10.

[1] C. Chin, R. Grimm, P. Julienne, and E. Tiesinga, Rev. Mod. Phys. **82**, 1225 (2010).
[2] S. Giorgini, L. P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. **80**, 1216 (2008).
[3] S. Inouye, M. Andrews, J. Stenger, H. Miesner, D. M. Stamper, and W. Ketterle, Nature (London) **392**, 151 (1998).
[4] P. Courteille, R. S. Freeland, D. J. Heinzen, F. A. van Abeelen, and B. J. Verhaar, Phys. Rev. Lett. **81**, 69 (1998).
[5] J. L. Roberts, N. R. Claussen, J. P. Burke, C. H. Greene, E. A. Cornell, and C. E. Wieman, Phys. Rev. Lett. **81**, 5109 (1998).
[6] A. Marte, T. Volz, J. Schuster, S. Durr, G. Rempe, E. G. M. van Kempen, and B. J. Verhaar, Phys. Rev. Lett. **89**, 283202 (2002).
[7] T. Loftus, C. A. Regal, C. Ticknor, J. L. Bohn, and D. S. Jin, Phys. Rev. Lett. **88**, 173201 (2002).
[8] K. M. O'Hara, S. L. Hemmer, S. R. Granade, M. E. Gehm, J. E. Thomas, V. Venturi, E. Tiesinga, and C. J. Williams, Phys. Rev. A **66**, 041401 (2002).
[9] K. Dieckmann, C. A. Stan, S. Gupta, Z. Hadzibabic, C. H. Schunck, and W. Ketterle, Phys. Rev. Lett. **89**, 203201 (2002).
[10] C. A. Regal, C. Ticknor, J. L. Bohn, and D. S. Jin, Phys. Rev. Lett. **90**, 053201 (2003).
[11] D. M. Eagles, Phys. Rev. **186**, 456 (1969).
[12] A. J. Leggett, *Modern Trends in the Theory of Condensed Matter* (Springer, Berlin, 1980).
[13] P. Nozieres and S. Schmitt-Rink, J. Low Temp. Phys. **59**, 195 (1985).
[14] S. Babu and G. E. Brown, Ann. Phys. (NY) **78**, 1 (1973).
[15] K. F. Quader, K. S. Bedell, and G. E. Brown, Phys. Rev. B **36**, 156 (1987).
[16] T. L. Ainsworth, K. S. Bedell, G. E. Brown, and K. F. Quader, J. Low Temp. Phys. **50**, 317 (1983).
[17] S. Gaudio, J. Jackiewicz, and K. S. Bedell, Philos. Mag. Lett. **87**, 713 (2007).
[18] K. F. Quader and K. S. Bedell, J. Low Temp. Phys. **58**, 89 (1985).
[19] L. Yi, K. S. Bedell, and T. L. Ainsworth, Bull. Am. Phys. Soc. **28**, 655 (1983).
[20] K. S. Bedell and K. F. Quader, Phys. Rev. B **32**, 3296 (1985).
[21] O. Sjoberg, Ann. Phys. (NY) **78**, 39 (1973).
[22] K. S. Bedell and K. F. Quader, Phys. Lett. **96A**, 91 (1983).
[23] K. S. Bedell and K. F. Quader, Phys. Rev. B **30**, 2894 (1984).
[24] B. R. Patton and Z. Zaringhalam, Phys. Lett. A **55**, 95 (1975).
[25] T. L. Ainsworth and K. S. Bedell, Phys. Rev. B **35**, 8425 (1987).
[26] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics* (Permagon, Oxford, 1994).
[27] C. A. Regal and D. S. Jin, Phys. Rev. Lett. **90**, 230404 (2003).
[28] S. J. J. M. F. Kokkelmans, J. N. Milstein, M. L. Chiofalo, R. Walser, and M. J. Holland, Phys. Rev. A **65**, 053617 (2002).
[29] M. Greiner, C. A. Regal, and D. S. Jin, Nature (London) **426**, 537 (2003).
[30] C. Ticknor, C. A. Regal, D. S. Jin, and J. L. Bohn, Phys. Rev. A **69**, 042712 (2004).
[31] J. P. Gaebler, J. T. Stewart, J. L. Bohn, and D. S. Jin, Phys. Rev. Lett. **98**, 200403 (2007).