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Tsallis entropy in phase-space quantum mechanics

Parvin Sadeghi, Siamak Khademi,* and Amir H. Darooneh
Department of Physics, University of Zanjan, ZNU, 45371-38791, Zanjan, Iran
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In this paper we define the quantum version of the Tsallis entropy in terms of quantum phase space distribution functions. The quantum Tsallis entropy is compared with Kenfack's nonclassicality indicator, for different systems, such as the Schrödinger cat state, the thermal state, a superposition of the ground and the first excited number states, and the harmonic oscillator state. These comparisons indicate that the Wigner representation gives us complete information about the state with the nonextensivity parameter $q = 1$, while the Husimi representation hides some information with the nonextensivity parameter $q < 1$.

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I. INTRODUCTION

In 1932 Wigner introduced the concept of the quantum distribution function in phase space to further the study of quantum statistical mechanics in a quasiclassical manner [1]. In the phase space formalism of quantum mechanics, we deal with the ordinary functions instead of the tedious operator algebra. In addition to the Wigner proposal for the quantum distribution function, several other representations were presented. The most well-known among them are Glauber's [2], Husimi's [3], Margenau *et al.*'s [4], Kirkwood's [5], P and Q [2,6], positive P [7], and gauge P [8]. Although these representations differ in their specific ordering rule for noncommutative operators when we pass from the Hilbert space to the phase space formalism [9], they are not independent. Lee proposed an integral transformation to show the relations between the different representations in phase space quantum mechanics [10].

Sobouti and Nasiri introduced a new derivation of phase space quantum mechanics in the extended phase space [11,12]. They showed that all representations of the quantum distribution function are related to any others by an extended canonical transformation [11,12]. The phase space formalism for quantum mechanics has found many applications in various fields of physics, e.g., in fundamental quantum mechanics [10,13], quantum statistical mechanics [14], condensed matter [15], Bose-Einstein condensates (BECs) [16], quantum optics [17], quantum information [18], and so on.

Unlike the classical distribution function, the Wigner distribution function is not positive definite; the negativities of the quantum distribution functions may be interpreted as an indicator of their nonclassical behavior in physical systems [19–21]. However the negativity of the Wigner distribution function can be removed by a smoothing method; the result is a positive definite Husimi distribution function [10]. The entropy is a key concept even in quantum statistical mechanics. In the Hilbert space formalism of quantum mechanics, the entropy of a system is usually defined as a function of its density operator $\hat{\rho}$ [22]. One of the common forms for quantum entropy is the von Neumann entropy [23]. Linear entropy has also found many applications in quantum information theory [24]. The quantum Rényi entropy [25] and the quantum Tsallis entropy [26] are one-parameter extensions of the von Neumann entropy and are widely used in quantum optics and quantum communication [27]. They are also one of the best candidates for the description of quantum dissipative systems [28,29].

In 1979 Wehrl extended the von Neumann entropy into the phase space [30]. The Wehrl entropy is the Shannon information measure associated with the Husimi distribution function. He used the Husimi representation to avoid the negativity problem. In another effort, the linear entropy is developed into phase space quantum mechanics in the Wigner and Husimi representations [31] and is used in many applications in quantum optics [32] and the study of entanglement [33], decoherence [34], purity of quantum states [35], squeezing [36], chaos [37], and so on.

Aims of this paper are:

(i) To develop the Tsallis entropy into phase space quantum mechanics in the Wigner and Husimi representations.

(ii) To introduce a relation between the quantum phase space Tsallis entropy and the nonclassicality indicator δ (δ is defined by Kenfack *et al.*). In quantum phase space one finds a reasonable relation between the quantum entropy and the quantum uncertainty relation. A correspondence between the uncertainty relation and the nonclassicality indicator is also investigated by Sadeghi *et al.* [21]. Therefore, we expect a correspondence between the nonclassicality indicator δ and the quantum Tsallis entropy.

(iii) All the information about the state under consideration maybe extracted in the Wigner representation while the Husimi representation hides some of the information about the states. This is shown by a comparison between the values of the nonextensivities of the Tsallis entropy in the Wigner and Husimi representations and the corresponding nonclassicality indicator δ.

(iv) Although the Manfredi and Feix [31] method which shows some information is hidden in the Husimi representation works only for the Tsallis entropy with $q = 2$, our method shows the same results without this limitation.

(v) The idea that the Husimi representation hides some information about the state cannot be extended for all entropies with nonextensivities $q \neq 2$. We show that in our method the value of the nonextensivity parameter q is a more suitable indicator for the hidden information in different representations that we investigated. Therefore even for the extensive systems the Husimi representation gives us $q < 1$ and hides some information, while the Wigner representation

*siamakkhademi@yahoo.com; skhademi@znu.ac.ir

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gives us $q = 1$ and is more suitable to apply to the study of the extensive systems. All investigated systems in this paper are extensive, and the application of our method for the nonextensive systems is an open problem.

As an application, the quantum Tsallis entropy is applied to study the quantum properties of the Schrödinger cat state, the thermal state, a superposition of ground and first excited number states, and the harmonic oscillator. We recognize a similar behavior between the Tsallis entropy for special nonextensivity parameters and the nonclassicality indicator which is introduced by Benedict and Czirják [19] or Kenfack and Życzkowski [20]. In the Wigner representation we find a nonextensivity parameter $q = 1$ in the quantum Tsallis entropy for the best coincidence between the quantum Tsallis entropy and nonclassicality indicator. These are not significantly equal in the Husimi representations. According to the incomplete information theory, for the nonextensivity parameter $q = 1$, whole information is accessible, while for the nonextensivity parameter $q \neq 1$, some information is hidden [38–40]. Therefore one may interpret that the Husimi quantum distribution function hides some information, because the nonextensivity parameter is not equal to 1, but whole information about the state is accessible in the Wigner representation. In the next section, the Tsallis entropy is defined in terms of the Wigner and Husimi distribution functions. In Secs. III, VI, V, and VI the phase space Tsallis entropy is applied to the Schrödinger cat state, the thermal state, a superposition of the ground and first excited number states, and the harmonic oscillator in the Wigner and Husimi representations. The quantum Tsallis entropy is compared with the Kenfack and Życzkowski nonclassicality parameter to find a suitable nonextensivity parameters q. The last section is devoted to the conclusions.

II. THE DEVELOPMENT OF THE TSALLIS ENTROPY IN PHASE SPACE QUANTUM MECHANICS

In 1988 C. Tsallis proposed a new form of entropy,

$$S_q = \frac{1}{q-1}\left[1 - \sum_{i=1}^{\Omega} P_i^q\right], \tag{1}$$

where P_i stands for the probability of the ith microstate, Ω is the number of accessible microstates of the system, and q is a positive real parameter. Many complex systems in nature with long range interactions are successfully described by this entropy [27]. The Tsallis entropy is, in general, nonextensive. This means that the entropy of the combination of two identical systems is not equal to the sum of the individual entropies:

$$S_q(A \otimes B) = S_q(A) + S_q(B) + (1-q)S_q(A)S_q(B). \tag{2}$$

The parameter $q \neq 1$ indicates the nonextensivity of a system. The Tsallis entropy will be reduced to the Shannon entropy $S = -\sum_{i=1}^{\Omega} P_i \ln P_i$, where $q = 1$. In the framework of information theory, the deviation of the nonextensivity parameter from 1 is interpreted as a consequence of incomplete information about the system [38–40], or in other words we cannot extract all aspects of a system out of its corresponding distribution function. The Tsallis entropy in quantum statistical mechanics for nonextensive systems is given by [26]

$$S_q = \frac{1}{q-1}[1 - \mathrm{Tr}(\hat{\rho}^q)]. \tag{3}$$

For $q = 1$ and $q = 2$, it reduces to the von Neumann entropy $S_1 = -\mathrm{Tr}(\hat{\rho}\ln\hat{\rho})$, and the linear entropy $S_2 = 1 - \mathrm{Tr}(\hat{\rho}^2)$, respectively [24]. Some authors have attempted to develop the concept of quantum entropy into quantum phase space [30,31]. In the following, the ordinary Tsallis entropy, Eq. (3), is generalized into quantum phase space:

$$S_q = \frac{1}{q-1}\left[1 - \int_{-\infty}^{\infty} |F(x,p)|^q dx dp\right], \tag{4}$$

where $|F(x,p)|$ is the absolute value of a distribution function in phase space. The phase space Tsallis entropy, Eq. (4), for $q = 1$ reduces into the Wehrl entropy in the Husimi representation:

$$S_{\mathrm{Husimi}} = -\int H(x,p)\ln H(x,p) dx dp. \tag{5}$$

The Shannon entropy is an approximation limit of the Wehrl entropy when $\hbar \to 0$ [41,42]. In the Wigner representation, Eq. (4) reduces to

$$S_{\mathrm{Wigner}} = -\int |W(x,p)| \ln[|W(x,p)|] dx dp, \tag{6}$$

where $q = 1$. Also the Manfredi-Feix entropy is obtained from the phase space Tsallis entropy, Eq. (4):

$$S_2 = 1 - \int W^2(x,p) dx dp, \tag{7}$$

where $q = 2$. Some applications of the Manfredi-Feix and Wehrl entropies are found in the Refs. [32–34,36,37,43–45].

Manfredi and Feix claim that for all the Tsallis entropies $q = 2$, the value of the entropies in the Husimi representation is more than the corresponding entropies in the Wigner representation [31]. Therefore the Husimi representation has less information and hides some of the information. Their interesting conclusion is valid just for $q = 2$, but it is violated for the Tsallis entropies with $q \neq 2$.

In the next sections, for an application of the phase space Tsallis entropy and to find a method to recognize the hiding information by the Husimi representation for the cases which $q \neq 2$, we study some properties of the phase space Tsallis entropy for the Schrödinger cat state, the thermal state, and a superposition of the ground and first excited number states. We investigate the nonextensivities, $q = 1$ and $q = 2$, for the phase space Tsallis entropy. In addition, these entropies are compared with the nonclassicality indicator δ, and a suitable nonextensivity parameter, which makes the best coincidence between the Tsallis entropy and nonclassicality indicator, is chosen in the Wigner and Husimi representations. The results indicate that in the Husimi distribution function always $q \neq 1$; therefore it hides some information about the investigated systems. But in the Wigner representation the best nonextensivity is obtained from $q = 1$ for all the considered systems in this paper. Therefore all information is, in general, extractable. This representation does not hide any information.

III. THE PHASE SPACE TSALLIS ENTROPY FOR THE SCHRÖDINGER CAT STATE

In this section the properties of the phase space Tsallis entropies for the Schrödinger cat state in the Wigner and Husimi representations are obtained. Suppose a coherent state field $|\alpha\rangle$ is interacted with a microscopic superposition of two atomic states, $|\psi\rangle_{\text{atom}} = 1/\sqrt{2}[|0\rangle + |1\rangle]$. An out of the resonance interaction Hamiltonian between the atom and the field is given by

$$\hat{H}_{\text{eff}} = \frac{-\hbar g^2}{\triangle}\left[\hat{\sigma}_z\hat{a}^\dagger\hat{a} + \frac{1}{2}(\hat{\sigma}_z + 1)\right], \quad (8)$$

where $g,\triangle,\hat{\sigma}_z$, and $\hat{a}(\hat{a}^\dagger)$ are, respectively, the atom-field coupling constant, the detuning parameter, the atomic operator, and the annihilation (creation) operators for a cavity mode [46]. The atom-field interacting system is obtained from the Schrödinger equation $\hat{H}_{eff}|\Psi\rangle = i\hbar(\partial/\partial t)|\Psi\rangle$, where $|\Psi\rangle$ is an atom-field state and the atom-field density operator is given by $\hat{\rho}_{A-F} = |\Psi\rangle\langle\Psi|$. Making a measurement of the basis of the atomic state gives us the density operator of the field state that is called the Schrödinger cat state [14]. The Schrödinger cat state is given by

$$|\psi\rangle_{\text{field}} = \frac{1}{\sqrt{2}}[|\alpha e^{i\varphi}\rangle + |\alpha e^{-i\varphi}\rangle], \quad (9)$$

where $\varphi = g^2 t/\Delta$ and t is the interaction time. It is a coherent superposition of two Gaussian wave packets with different phases. The separation between two coherent states for the Schrödinger cat in configuration space is denoted by $x_0 = 2|\alpha|\sin\varphi$ [14]. The total wave function for the Schrödinger cat state is

$$\psi_{\text{field}}(x) = \frac{N}{2}[\psi_+(x) + \psi_-(x)], \quad (10)$$

where $\psi_\pm(x) = (\frac{1}{\pi})^{1/4}\exp[-\frac{1}{2}(x \pm x_0)^2 + ip_0(x \pm x_0)]$ is the wave function for the Schrödinger cat state. The normalization coefficient is $N = [1 + \cos(2x_0p_0)e^{-x_0^2}]^{-1/2}$ and $p_0 = 2|\alpha|\cos\varphi$. Therefore, the Wigner distribution function in the phase space [20] for the Schrödinger cat state is obtained as

$$W(x,p) = \frac{N^2}{2\pi}[e^{-(x+x_0)^2-(p-p_0)^2} + e^{-(x-x_0)^2-(p-p_0)^2} + 2\cos(2px_0)e^{-x^2-(p-p_0)^2}]. \quad (11)$$

The Husimi distribution function is also obtained by a Gaussian smoothing of the Wigner distribution function [10] as

$$H(x,p) = \frac{1}{\pi}\int d\acute{x}d\acute{p}W(\acute{x},\acute{p})e^{-(x-\acute{x})^2-(p-\acute{p})^2}. \quad (12)$$

From the Eqs. (11) and (12), the Husimi distribution function is given for the Schrödinger cat state:

$$H(x,p) = \frac{N^2}{4\pi}\{e^{-\frac{1}{2}(x+x_0)^2-\frac{1}{2}(p-p_0)^2} + e^{-\frac{1}{2}(x-x_0)^2-\frac{1}{2}(p-p_0)^2} + 2e^{-\frac{1}{2}x^2-\frac{1}{2}x_0^2-\frac{1}{2}(p-p_0)^2}\cos[x_0(p+p_0)]\}. \quad (13)$$

According to Manfredi and Feix's claim, the Tsallis entropy in the Husimi representation is always more than the corresponding Tsallis entropy in the Wigner representation for $q = 2$. But their claim, in general, is not valid for other nonextensive parameters. For example, one can choose $q = 1$ in the phase space Tsallis entropy, Eqs. (5) and (6), to obtain the Wehrl entropy. Figure 1 shows the Wehrl entropy (the Tsallis entropy for $q = 1$ in Husimi representation) for the Schrödinger cat state in the Wigner and Husimi representations for different phases p_0. There are some points at which the entropy in the Husimi representation is less than the corresponding entropy in the Wigner representation. Figure 1 shows the violation of Manfredi and Feix's claim for $q = 1$.

The Wigner distribution function, Eq. (11), is used to determine the nonclassicality indicator

$$\delta = \int |W(x,p)|dxdp - 1, \quad (14)$$

which measures its negativity [20]. This nonclassicality indicator, which is defined by Kenfack and Życzkowski, is plotted in Fig. 2 for the Schrödinger cat state in terms of x_0.

In Fig. 2 the nonclassicality indicator δ and the phase space Tsallis entropy S_q in the Wigner and Husimi representations are plotted for the Schrödinger cat state in terms of x_0 for different p_0. The nonclassicality indicator and the phase space Tsallis entropy in the Wigner representation coincide closely for $q = 1$. The behavior of the phase space Tsallis entropy in the Husimi representation is very similar to the behavior of the nonclassicality indicator but is not as close as the phase space Tsallis entropy in the Wigner representation. In Fig. 2, the Tsallis entropy is normalized to 1, to be comparable with the nonclassicality indicator. The best coincidence occurs for the nonextensivity parameters $q = 1$ in the Wigner representation. The best coincidence for the Husimi representation occurs for $q = 0.37$ and $q = 0.50$, corresponding to $p_0 = 0$ and $p_0 = 7$, respectively. Their standard deviations in the Wigner and Husimi representations are 5.6×10^{-5} and 2.4×10^{-2}, respectively. The nonextensivity parameter for the Wigner representation is equal to 1; therefore all information about the system in the Wigner representation is extractable. But the nonextensivity parameter for the Husimi representation is less than 1. This means that some information (e.g., the interference patterns as a quantum signature [21]) are hidden in the Husimi representation. So complete information about the systems is not extractable in the Husimi representation. The correspondence between the nonclassicality indicator and uncertainty principle for the Schrödinger cat state is shown by Sadeghi *et al.* [21]. The similarity between the nonclassicality indicator and the phase space Tsallis entropy leads us to another correspondence between the phase space Tsallis entropy and the uncertainty principle. Thus the more the value of the entropy corresponds to the more the value of the uncertainty and the less the information, and vice versa.

IV. THE PHASE SPACE TSALLIS ENTROPY FOR THE THERMAL STATE

Another interesting example is the application of the phase space Tsallis entropy to the thermal state. Consider the nonlinear interaction of the microscopic superposition of an entangled two-mode harmonic oscillator,

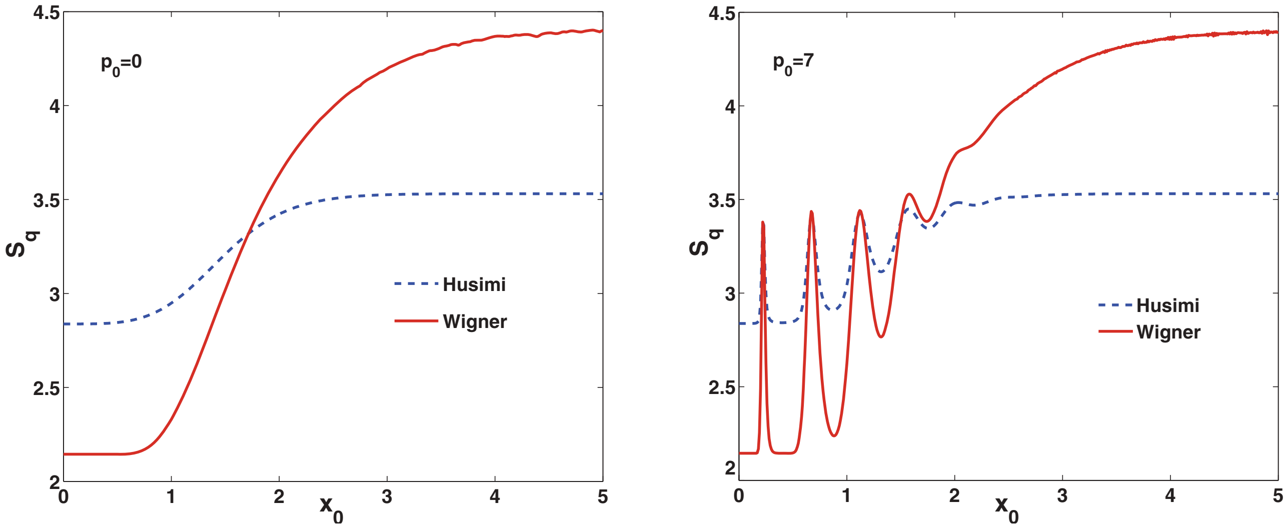

FIG. 1. (Color online) The phase space Tsallis entropy is shown for $q = 1$ in the Wigner and Husimi representations for the Schrödinger cat state with $p_0 = 0$ and $p_0 = 7$. The Tsallis entropy, p_0, and x_0 are dimensionless quantities.

$|\psi\rangle_{\text{atom}} = 1/\sqrt{2}[|0\rangle + |1\rangle]$, with a thermal state

$$\rho^{th}(v,d) = \int d^2\alpha p^{th}(v,d)|\alpha\rangle\langle\alpha|, \tag{15}$$

where $p^{th}(v,d) = [2/\pi(v-1)]\exp[(-2|\alpha - d|^2)/(v-1)]$, and $|\alpha\rangle$ is a coherent state [47]. The displacement in the phase space and variance which are shown by d and v are dependent on the field strength and temperature T, respectively. The nonlinear interaction Hamiltonian corresponding to the cross-Kerr nonlinearity is $\hat{H}_{\text{int}} = \hbar\lambda\hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b}$, where λ is the nonlinear strength, and $\hat{a}^\dagger(\hat{a})$ and $\hat{b}^\dagger(\hat{b})$ are the creation (annihilation) operators of the oscillator and field, respectively [47]. Therefore, the Wigner distribution function for the thermal state is given by

$$W(x,p) = N[W^{th}(x,p;d) + W^c(x,p;d) + W^c(x,p;d)^* + W^{th}(x,p;de^{i\varphi})], \tag{16}$$

where N is the normalization coefficient, $\varphi = \lambda t$, and t is the interaction time. The different terms in Eq. (16) are given by

$$W^{th}(x,p;d) = \frac{2}{\pi v}\exp\left[-\frac{(x-\sqrt{2}d)^2 + p^2}{v}\right], \tag{17}$$

$$W^{th}(x,p;de^{i\varphi}) = \frac{2}{\pi v}\exp\left\{-\frac{1}{v}[(x-\sqrt{2}d\cos\varphi)^2 + (p-\sqrt{2}d\sin\varphi)^2]\right\}, \tag{18}$$

and

$$W^c(x,p;d) = \frac{2}{\pi JK}\exp\left[-\frac{2d^2}{K}(1-e^{i\varphi}) - \frac{1}{2J}(x^2+p^2) + \frac{\sqrt{2}dx}{JK}(1+e^{i\varphi}) + \frac{\sqrt{2}dpi}{JK}(1-e^{i\varphi}) - \frac{4d^2}{JK^2}e^{i\varphi}\right], \tag{19}$$

where $K = 2 + (v-1)(1-e^{i\varphi})$ and $J = [\sin(\varphi/2) + iv\cos(\varphi/2)]/[2v\sin(\varphi/2) + 2i\cos(\varphi/2)]$. Using Eqs. (12) and (16) to (19) to obtain the Husimi distribution function for the thermal state [21],

$$H(x,p) = N[H^{th}(x,p;d) + H^c(x,p;d) + H^c(x,p;d)^* + H^{th}(x,p;de^{i\varphi})], \tag{20}$$

where the different terms in Eq. (20) are given by

$$H^{th}(x,p;d) = \frac{2}{\pi(v+1)}\exp\left[-\frac{(x-\sqrt{2}d)^2+p^2}{v+1}\right], \tag{21}$$

$$H^{th}(x,p;de^{i\varphi}) = \frac{2}{\pi(v+1)}\exp\left\{-\frac{1}{v+1}[(x-\sqrt{2}d\cos\varphi)^2 + (p-\sqrt{2}d\sin\varphi)^2]\right\}, \tag{22}$$

and

$$H^c(x,p;d) = \frac{2}{\pi(2J+1)K}\exp\left[-\frac{2d^2}{K}(1-e^{i\varphi}) - \frac{(x^2+p^2)}{2J+1} + \frac{\sqrt{2}dx}{(2J+1)K}(1+e^{i\varphi}) + \frac{\sqrt{2}dpi}{(2J+1)K}(1-e^{i\varphi}) + \frac{8d^2e^{i\varphi}}{(2J+1)K^2}\right]. \tag{23}$$

In Fig. 3 the phase space Tsallis entropy for $q = 1$ is shown in the Wigner and the Husimi representations. It is shown that in spite of Manfredi and Feix's claim for $q = 2$, the Tsallis entropy in the Husimi representation is not in total more than corresponding entropy in the Wigner representation.

In Fig. 4 the nonclassicality indicator δ is shown and compared with the phase space Tsallis entropy which is plotted for the thermal state in the Wigner and Husimi representations. The best coincidence for the nonclassicality indicator δ and the phase space Tsallis entropy is obtained for the nonextensivity parameter $q = 1$ in the Wigner representation and $q = 0.191$ for the Husimi representation, and their standard deviations are 9.604×10^{-5} and 3.684×10^{-2}, respectively. In Fig. 4 the entropy and nonclassicality indicator are close to each other and indistinguishable. As well as the Schrödinger cat

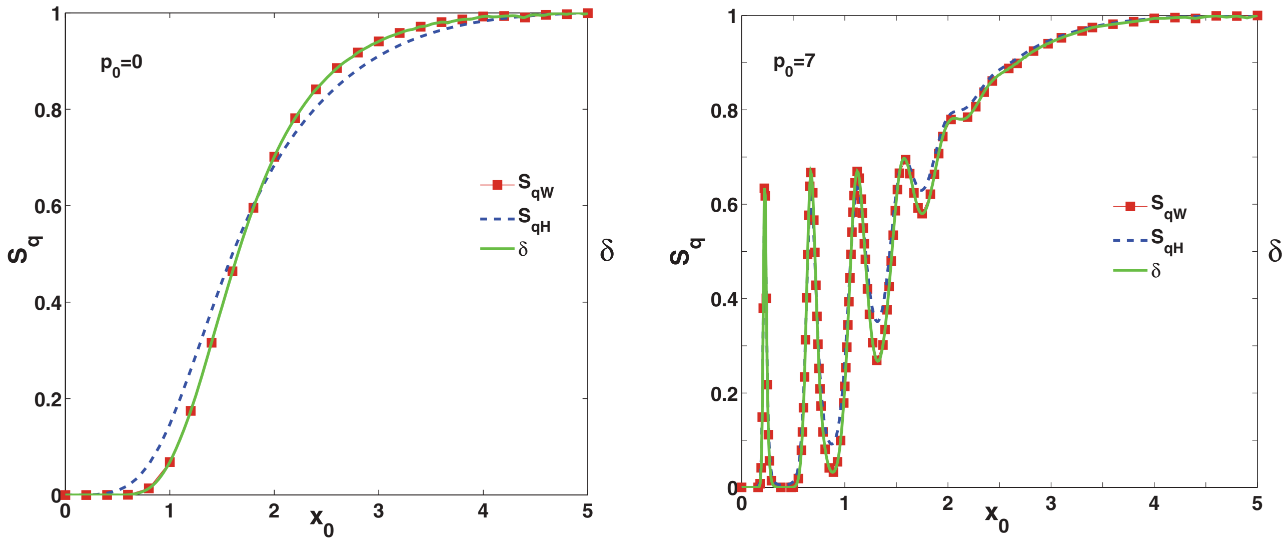

FIG. 2. (Color online) The Tsallis entropy is shown in the Wigner and Husimi representations for the Schrödinger cat state, where $p_0 = 0$ and $p_0 = 7$. The best coincidence happened between the Tsallis entropy and the nonclassicality indicator δ for $q = 1$ in the Wigner representation and $q = 0.37$ and $q = 0.50$ in the Husimi representation corresponding to $p_0 = 0$ and $p_0 = 7$, respectively. The Tsallis entropy and the (dimensionless) nonclassicality indicator δ are normalized to 1, for a comfortable comparison.

state, the thermal state in the Wigner representation shows complete information about the system. However, the Husimi representation hides some information about the system.

V. THE PHASE SPACE TSALLIS ENTROPY FOR A SUPERPOSITION STATE

Another example for the application of the phase space Tsallis entropy is the superposition of two levels of number states,

$$|\psi\rangle = (\sqrt{1-a^2}|0\rangle + a|1\rangle), \tag{24}$$

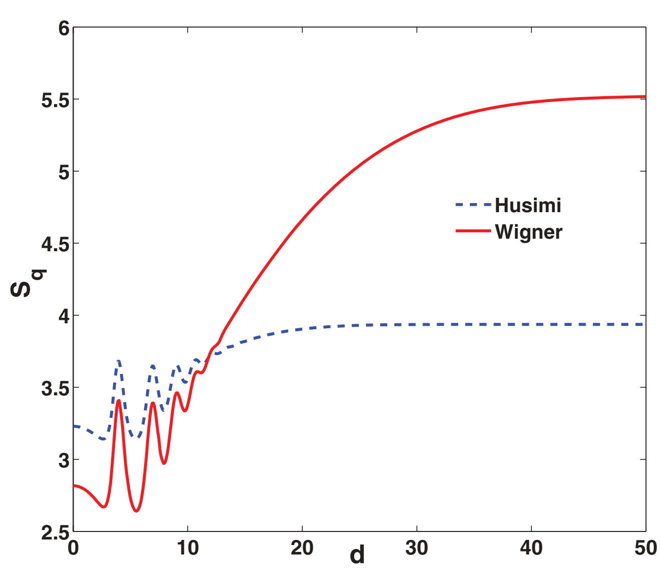

FIG. 3. (Color online) The Tsallis entropy is plotted in terms of the displacement d for the thermal state where $\varphi = \pi/16$ and $\upsilon = 2$ for $q = 1$ in the Wigner and Husimi representations. Quantities d and υ are dimensionless.

where a is the real probability amplitude. The states $|0\rangle$ and $|1\rangle$ are the ground and first excited number states, respectively. Figure 5 shows that the Tsallis entropy in the Husimi representation is not in total more than the corresponding Tsallis entropy in the Wigner representation, for $q = 1$. Therefore the generalization of Manfredi and Feix's claim for $q \neq 2$ is violated, and one cannot recognize the nonextensivity of the system.

The nonclassicality indicator and the Tsallis entropy in the Wigner and Husimi representations are plotted in Fig. 6 in terms of the probability a^2. The best coincidence between the

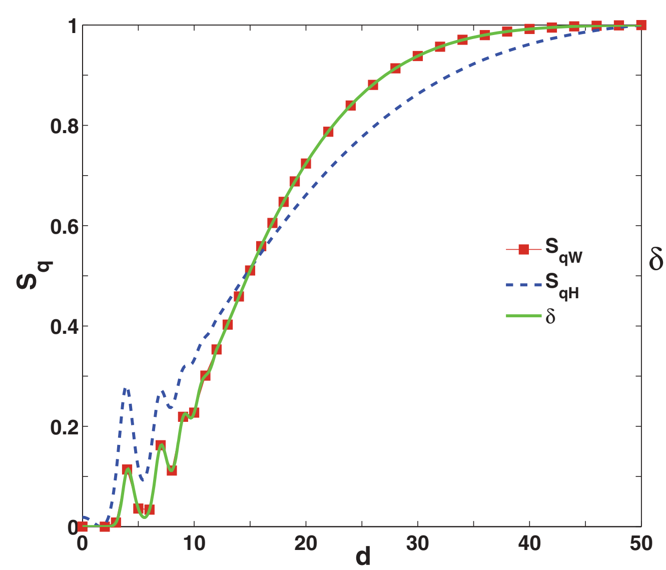

FIG. 4. (Color online) The Tsallis entropy and the nonclassicality indicator are plotted in terms of the displacement d for the thermal state where $\varphi = \pi/16$ and $\upsilon = 2$ in the Wigner and Husimi representations with the nonextensivity parameters $q = 1$ and $q = 0.191$, respectively.

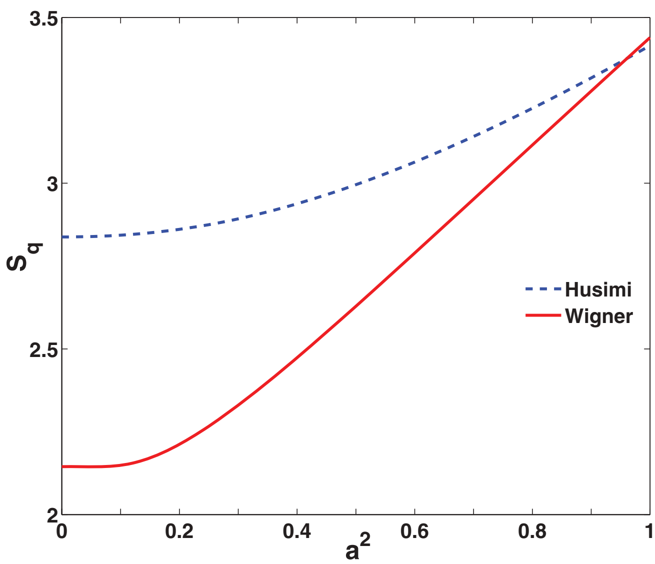

FIG. 5. (Color online) The Tsallis entropy for $q = 1$ is plotted versus the probability a^2 for the superposition of the ground and the first excited number states in the Wigner and Husimi representations.

Tsallis entropy and the nonclassicality indicator δ occurs for $q = 1$ and $q = 0.687$ in the Wigner and Husimi representations, respectively. Their corresponding standard deviations are 5.026×10^{-5} and 3.424×10^{-3}.

According to the values of the suitable nonextensivity parameters in different representations, one finds that the Husimi representation hides some information for the Schrödinger cat and the thermal and superposition states. In spite of the Husimi representation, all information about the considered system is available in the Wigner representation.

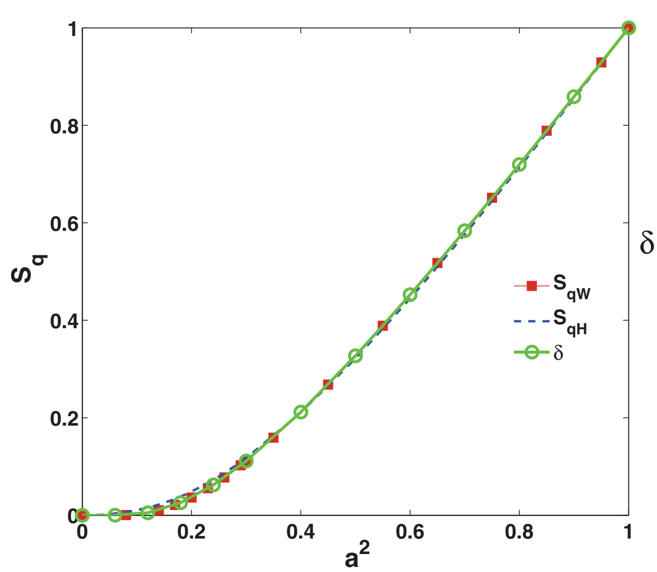

FIG. 6. (Color online) The Tsallis entropy and the nonclassicality indicator δ are plotted versus the probability a^2 for the superposition of the ground and the first excited number states in the Wigner and Husimi representations for the nonextensivity parameters $q = 1$ and $q = 0.687$, respectively.

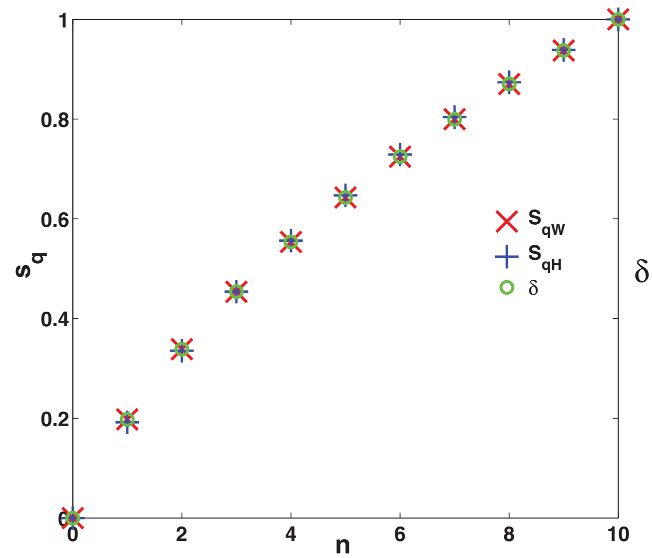

FIG. 7. (Color online) The Tsallis entropy and the nonclassicality indicator δ are plotted versus the integer quantum number n for the harmonic oscillator, in the Wigner and Husimi representations. We obtain $q = 1$ and $q = 0.23$ by coincidence between the Tsallis entropy and the nonclassicality indicator, in the Wigner and Husimi representations, respectively.

VI. THE PHASE SPACE TSALLIS ENTROPY FOR THE HARMONIC OSCILLATOR

In this section the eigenstates of the harmonic oscillator are applied to the phase space Tsallis entropy. The Wigner function for the harmonic oscillator state $|n\rangle$ is given by

$$W(x,p) = \frac{(-1)^n}{\pi} \exp[-x^2 - p^2] L_n[2(x^2 + p^2)], \quad (25)$$

where L_n is the Laguerre polynomial of the n-th order.

Quantum uncertainty increases with the increasing quantum numbers n. Furthermore, increasing the uncertainty reduces the information about the systems and increases their entropy. Figure 7 shows the relation between the phase space Tsallis entropy and the nonclassicality indicator δ.

In Fig. 7 we make the best coincidence of quantum Tsallis entropy in the phase space and nonclassicality indicator δ for nonextensivity parameters $q = 1$ and $q = 0.23$, in the Wigner representation (with standard deviation 5.336×10^{-5}) and the Husimi representations (with standard deviation 1.923×10^{-3}), respectively. The nonextensivity of the Tsallis entropy for the Husimi representation is less than 1; therefore, according to the incomplete information theory [38–40], we cannot extract complete information about the systems in this representation while in the Wigner representation all information is extractable. These results are obtained for the systems investigated in this paper.

In Fig. 8 the Tsallis entropy for $q = 1$ is plotted versus the integer quantum numbers n in the Wigner and Husimi representations. The value of the entropy in the Husimi representation for $n = 0$ is more than the value of entropy in

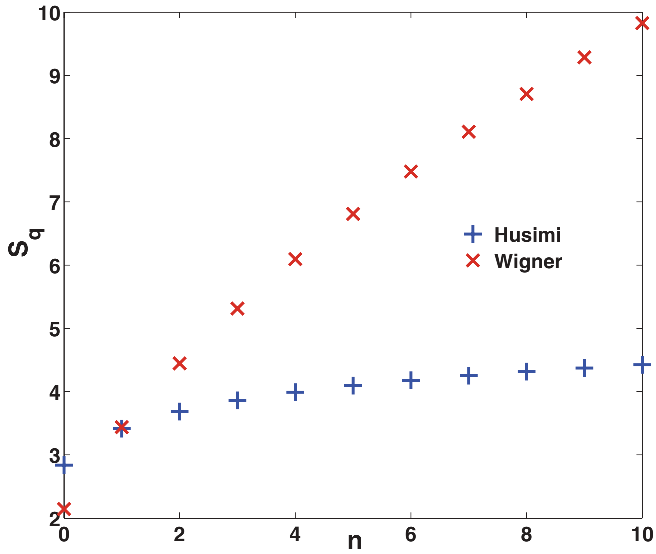

FIG. 8. (Color online) The Tsallis entropy for $q = 1$ is plotted versus the integer quantum number n for the harmonic oscillator, in the Wigner and Husimi representations.

the Wigner representation. Therefore, the idea that the Husimi representation hides some information about the state cannot be extended for all entropies with nonextensivities $q \neq 2$. In our method, the value of the nonextensivity parameter q is a more suitable indicator of the hidden information in different investigated representations. Therefore even for the extensive systems, the Husimi representation gives us $q < 1$ and hides some information while the Wigner representation gives us $q = 1$ and is more suitable to apply to the study of the extensive systems.

VII. CONCLUSION

In this paper we developed the Tsallis entropy in phase space quantum mechanics. The quantum Tsallis entropy reduces to the previously known Wehrl and Manfredi-Feix phase space entropies for the nonextensivity parameters $q = 1$ and $q = 2$, respectively.

As an application, our method is applied to the Schrödinger cat state, the thermal state, a superposition of states, and the harmonic oscillator state. It is known that the entropy measures the amount of information about these systems. Furthermore the uncertainty principle and nonclassicality indicator δ have similar behavior. It is also shown that the entropy and nonclassicality indicators have similar behavior. It is expected reasonably that the amount of information is also measured by the nonclassicality indicator. In this paper we set the nonextensivity parameter q to make the best coincidence between the Tsallis entropy and the nonclassicality indicator δ, in the Wigner and Husimi representations. The nonextensivity parameter is obtained at $q = 1$ for all investigated systems in the Wigner representations, but in the Husimi representation the nonextensivity parameter has different values which are all less than 1, $q < 1$. According to the incomplete information theory, for the nonextensivity parameter $q = 1$, the whole information is accessible, while for $q \neq 1$ some information is hidden. Therefore in the Wigner representation for our investigated systems, complete information is accessible while the Husimi representation hides some of the information. So the value of the nonextensivity parameters, in our method, is a more suitable indicator to show the hidden information in the Husimi representation. It is clear that the Wigner representation of Wehrl entropy (the Tsallis entropy with $q = 1$ in the Wigner representation) is more suitable with respect to the corresponding entropy in the Husimi representation, especially for inherently extensive systems.

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