PHYSICAL REVIEW A **85**, 062119 (2012)

Weak value of dwell time for quantum dissipative spin-1/2 systems

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(Received 13 April 2012; published 25 June 2012)

The dwell time is calculated within the framework of time-dependent weak measurement considering dissipative interaction between a spin-$\frac{1}{2}$ system and the environment. Caldirola and Montaldi's method of retarded Schrödinger equation is used to study the dissipative system. The result shows that inclusion of dissipative interaction prevents zero time tunneling.

DOI: 10.1103/PhysRevA.85.062119 PACS number(s): 03.65.Xp, 03.65.Yz

I. INTRODUCTION

The recent experimental results [1] on superluminal tunneling speed have raised a lot of controversy among the community. The problem of defining tunneling times has a long history [2], simultaneously with the fundamental problem of introducing time as a quantum-mechanical observable and, in particular, of a definition (in quantum mechanics) of the collision durations. In fact, experiments on transmitting information containing features of an optical pulse across the "fast light" medium, in which the group velocity exceeds the vacuum speed of light c, have renewed interest in the so-called "superluminal" propagation phenomenon. This superluminality has been predicted in connection with various quantum systems propagating in a forbidden zone. Aharonov *et al*. [3], along with other authors [4], dealt with the problem of tunneling time from the context of weak measurement. The notion of the weak value of a quantum-mechanical observable was originally introduced by Aharonov *et al*. [5–7]. This quantity is the statistical result of a standard measurement procedure performed upon a preselected and postselected (PPS) ensemble of quantum systems when the interaction between the measurement apparatus and each system is sufficiently weak. Unlike the standard strong measurement of a quantum-mechanical observable which sufficiently disturbs the measurement system, a weak measurement of an observable for a PPS system does not appreciably disturb the quantum system and yields the weak value as the measured value of the observable. Aharonov *et al*. [3] have shown that in their approach, tunneling time corresponds to superluminal velocity. On the other hand, experiments with photonic band-gap structures [8,9] also showed apparent superluminal barrier traversal. These observations as well as the theoretical predictions lead towards the phenomena of superluminal barrier traversal. In the case of superluminality, Winful suggested [10–14] an explanation of faster-than-light phenomena by the concept of energy storage and release in the barrier region. He argued that the group delay, which is directly related to the dwell time with an additive self-interaction delay [15], is actually the lifetime of stored energy (or stored particles) leaking through both ends of the barrier. Our aim is to incorporate dissipation in the framework of time-dependent weak value and calculate the dwell time in that context. Here dissipation means loss of energy of the tunneling entity to the atomic modes of the medium in the barrier region. So in this particular case some of the energy of the tunneling entity is absorbed by the interacting medium of the barrier region. We consider the time-dependent quantum weak value of a certain operator as described by Davies [16]. Then we will include dissipative interaction via the decay constant using the method of retarded Schrödinger equation developed by Caldirola and Montaldi [17] and arrive at an expression of finite nonzero dwell time. To start with, the concept of dwell time is discussed within the context of weak measurement theory in Sec. II. In Sec. III we discuss the time-dependent weak values for two-level systems. Based on this framework we calculate the dwell time for dissipative environment in Sec. IV, and finally some concluding remarks are made in Sec. V.

II. DWELL TIME AND WEAK MEASUREMENT

One of the commonly cited problems of measuring how much time it takes a quantum particle to cross a potential barrier is the nonexistence of a quantum-mechanical time operator. However, it is possible to construct an operator

$$\Theta_{(0,L)} = \Theta(x) - \Theta(x - L), \tag{2.1}$$

where $\Theta(x)$ and $\Theta(x-L)$ represent Heaviside functions. This operator measures whether the particle is in the barrier region or not. Such a projection operator is Hermitian and corresponds to a physical observable. It has eigenvalues 1 for the region $0 \leqslant x \leqslant L$ and 0 otherwise. Its expectation value simply measures the integrated probability density over the region of interest. It is the expectation value divided by the incident flux, which is referred to as the dwell time [2,18]. Ideally, transmission and reflection times τ_T and τ_R would, when weighted by the transmission and reflection probabilities $|T|^2$ and $|R|^2$, yield the dwell time

$$\tau_D = |T|^2\tau_T + |R|^2\tau_R. \tag{2.2}$$

In the past two decades a new approach to measurement theory in quantum mechanics has been developed by Aharonov and coworkers [5,6]. This approach of "weak measurement" differs from the standard "von Neumann measurement" [19] in that the interaction between the measuring apparatus and the measured system is too weak to trigger a collapse of the wave function. Although the individual weak measurement of an observable has no meaning, one can of course obtain the expectation value to any desired accuracy by averaging a sufficiently large number of such individual results. In

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the standard approach of quantum mechanics, measurement comprises a collapse of the wave function which occurs instantaneously [20]. Avoiding wave function collapse allows the simultaneous measurement of noncommuting observables, though no violation of the uncertainty principle occurs because the individual measurements of each observable are very imprecise. Moreover, since it allows the system to evolve after the measurement as if unperturbed, it is possible to define the average of a quantity conditioned to a given final state of the system. So if we are interested in the duration of some process, we can correspond this to a typical weak measurement extended in time, i.e., the interaction between the measuring probe and the system is not impulsive, but has a finite duration. Steinberg has shown [21] that these features make weak measurement theory a very promising background for the study of tunneling time in quantum mechanics.

Let us now first turn our attention to the theory of weak measurement. Weak value theory is a special consequence of the time symmetric reformulation of quantum mechanics. Whereas standard quantum mechanics describes a quantum system at time t using a state evolving forward in time from the past to t, weak value theory also uses a second state evolving backward in time from the future to t using the notion of pre- and postselection [5–7]. Consider the system prepared in an initial state $|i\rangle$ at a given initial time. At a given final time, the system is found to be in a final state $|f\rangle$. This means that a measurement performed at a particular initial time selects only the systems in the preselected state $|i\rangle$, performs the weak measurement, and at the final time, again the measurement is performed to test whether the system is in the postselected state $|f\rangle$. Measurement is nothing but interaction with a measuring device having a particular initial state. (Usually the position representation of the device wave function is taken as a Gaussian.) The system is made to interact with a pointer degree of freedom Q, via the interaction Hamiltonian

$$H_{\text{int}} = g(t)PA, \tag{2.3}$$

where P is the conjugate momentum variable to the pointer position Q. It is convenient to take the function $g(t)$ impulsive and $g_0 = \int g(t)dt = 1$; it is nonzero only in a small interval. The initial state of the pointer variable is described by the Gaussian wave function

$$\Phi_i(Q) = (\Delta^2\pi)^{-\frac{1}{4}} e^{-\frac{Q^2}{2\Delta^2}} \tag{2.4}$$

and the initial state of the system is given by

$$|\psi_i\rangle = \sum_k a_k|a_k\rangle. \tag{2.5}$$

The initial state of the system is the eigenvector of the observable A. After the measurement interaction, the composite state of the system and the measuring device is given by

$$(\Delta^2\pi)^{-\frac{1}{4}} \sum_k a_k|a_k\rangle e^{-\frac{(Q-a_k)^2}{2\Delta^2}}. \tag{2.6}$$

In an ideal measurement, the relative shifts corresponding to different eigenvalues of the observable A are large compared with the initial uncertainty in the pointer's position (given by the width Δ), and the resulting lack of overlap between the final states leads to the irreversible collapse between different eigenstates of A. It is then found very close to the position corresponding to a particular eigenvalue of A. In weak measurement, the initial position of the pointer has a large uncertainty (i.e., large Δ) so that the overlap between the pointer states remains close to unity, and hence the measurement does not constitute a collapse. The fact that the uncertainty in position measurement is large means that the momentum is more or less well defined, so it does not impart an uncertain kick to the particle. The measurement is weak in the sense that it disturbs the state of the particle as little as possible between the state preparation and postselection. Since the spread is large, the inaccuracy in measurement has to be compensated by large statistics (by averaging over a subensemble). For postselection of state $|f\rangle$ of the system, the pointer wave function at the final time is given by

$$\Phi_f(Q) = (\Delta^2\pi)^{-\frac{1}{4}} \sum_k a_k\langle f|a_k\rangle e^{-\frac{(Q-a_k)^2}{2\Delta^2}}. \tag{2.7}$$

After some mathematical analysis [3] we find that

$$\Phi_f(Q) \approx (\Delta^2\pi)^{-\frac{1}{4}} e^{-\frac{(Q-A_w)^2}{2\Delta^2}}, \tag{2.8}$$

where

$$A_w = \frac{\langle\psi_f|A|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle} \tag{2.9}$$

is denoted as the weak value of the observable A. We must note that the expression of this weak value may generally be complex. However, the physical significance of the real and imaginary parts is quite clear. The real part of the weak value corresponds to the mean shift of the pointer position, and the imaginary part constitutes a shift in the pointer momentum. So even though the imaginary part carries important physical significance, it does not play any part in the measurement outcome since it does not correspond to spatial translation of the pointer. It is also worthwhile to mention that besides being generally complex, the magnitude of the weak value can lie outside the range of the eigenvalues [5].

Now let us discuss the calculation of dwell time on the basis of weak measurement. The time taken by a particle to traverse a certain potential barrier is measured by a clock consisting of an auxiliary system which interacts weakly with the particle as long as it stays in a given region. Aharonov *et al.* [3] considered the interaction Hamiltonian as

$$H_{\text{int}} = P_m\Theta_{(0,L)}, \tag{2.10}$$

where m is the degree of freedom:

$$\Theta_{(0,L)} = \begin{cases} 1 & \text{if} \quad 0 < x < L \\ 0 & \text{otherwise.} \end{cases} \tag{2.11}$$

This is the projection operator as we discussed earlier. It is the effective form of the potential, seen by a particle in the S_z state, in the Stern-Gerlach experiment where $(0, L)$ is the region of magnetic field. We obtain the dwell time by calculating the weak value of the projection operator $\Theta_{(0,L)}$. Now the weak value of any operator A is expressed by Eq. (2.9). If we divide the measurement into many short ones, $(\Delta t)\ A \simeq \sum_{j=-\infty}^{\infty} A_j$, we get [3]

$$\langle A_j\rangle^w = C\Delta t \frac{\langle\psi_f(j\Delta t)|A|\psi_i(j\Delta t)\rangle}{\langle\psi_f(j\Delta t)|\psi_i(j\Delta t)\rangle},$$

where C is an arbitrary constant and can be set as $\frac{1}{\Delta t}$. In the limit $\Delta t \to 0$,

$$A_w = \frac{\int_{-\infty}^{\infty} \langle \psi_f(t)|A|\psi_i(t)\rangle dt}{\langle \psi_f(0)|\psi_i(0)\rangle}. \tag{2.12}$$

For $A = \Theta_{0,L}$, this formula leads to

$$\langle \tau \rangle^w = \frac{\int_{-\infty}^{\infty} dt \int_0^L \psi_f^*(x,t)\psi_i(x,t)dx}{\int_{-\infty}^{\infty} \psi_f^*(x,0)\psi_i(x,0)dx}. \tag{2.13}$$

Aharonov *et al.* [3] argued that direct calculation of the dwell time can be made using Eq. (2.13). It shows that it tends to zero in the low-energy limit. So irrespective of the length of the barrier, the particle traverses it in no time.

III. TIME-DEPENDENT WEAK VALUES OF A TWO-STATE SYSTEM

Now let us concentrate on time-dependent pre- and postselected states with the emphasis on decay of excited states. Let us consider the time evolution of a quantum-mechanical state as

$$|\psi(t)\rangle = U(t - t_0)|\psi(t_0)\rangle, \tag{3.1}$$

where $U(t - t_0) = e^{-iH(t-t_0)}$ is the time evolution operator.

In the light of the time evolution, the weak value of an operator A at a time t, $t_i < t < t_f$, preselected at t_i and postselected at t_f, can be defined as [16]

$$A_w = \frac{\langle \psi_f|U^\dagger(t - t_f)AU(t - t_i)|\psi_i\rangle}{\langle \psi_f|U^\dagger(t - t_f)U(t - t_i)|\psi_i\rangle}. \tag{3.2}$$

Let us consider an electron of charge e at rest in a magnetic field $\mathbf{B}$. The interaction Hamiltonian is

$$H = -\mu \cdot \mathbf{B}, \tag{3.3}$$

where

$$\mu = -\frac{e\hbar \mathbf{S}}{m} \tag{3.4}$$

and

$$\mathbf{S} = \tfrac{1}{2}(\sigma_x,\sigma_y,\sigma_z) \tag{3.5}$$

σ_i are the Pauli spin matrices. For simplicity, let us suppose the magnetic field lies in the z direction. Then the Hamiltonian looks like

$$H = \hbar\omega\sigma_z. \tag{3.6}$$

The time evolution operator in this case looks like

$$U(t) = \begin{pmatrix} e^{i\omega t/2} & 0 \\ 0 & e^{-i\omega t/2} \end{pmatrix}. \tag{3.7}$$

From this we can get

$$UU^\dagger = U^\dagger U = I \tag{3.8}$$

and

$$U(t_1 - t_2)U(t_2 - t_3) = U(t_1 - t_3), \tag{3.9}$$

so the unitary and evolution properties hold.

Consider that at an initial time t_i the state is polarized in the positive x direction. Then

$$|\psi_i\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{3.10}$$

The projection operator onto the eigenstate (3.10) is

$$P_+ = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}. \tag{3.11}$$

Now we come to the case of the decay of an excited state by considering an initial excited two-level atom coupled to a bath of $2N$ number of other two-level atoms initially in their ground states. Due to the interaction with the bath atoms, the concerning system loses energy to the bath modes. Choosing the ground-state energies of all atoms to coincide and be set to zero, and setting the excited states E_n to satisfy the relation

$$E_n - E_0 = n\Delta E, \quad -N \leqslant n \leqslant N, \tag{3.12}$$

the excited states are shown to be equispaced and distributed symmetrically about the excited state of the reference atom, labeled by $n = 0$. For the simplicity of the problem, it is assumed that the reference atom is equally coupled to each of the atoms of the bath; the interaction is described by the real constant Hamiltonian H.

The Schrödinger equation is equivalent to the coupled differential equations

$$\dot{a_0} = -i\sum_n Ha_n e^{-in\Delta Et}, \tag{3.13}$$

$$\dot{a_n} = -iHa_0 e^{in\Delta Et}, \tag{3.14}$$

where a_n is the amplitude of the excited state and we set $\hbar = 1$. According to Davies [16], Eqs. (3.13) and (3.14) can be solved exactly by the method of Laplace transformation. Without going into the details of the calculations, which can be found in Ref. [16], we find that

$$a_0(t) = e^{-\gamma(t-t_i)}, \tag{3.15}$$

where γ is the decay constant. We discuss this decay constant much more elaborately in the next section. The evolution operators $U(t)$ can also be found. If we consider that one atom at the time of the bath is excited, the evolution operator of the relevant subspace of the full Hilbert space of states will be a $(2N + 1) \times (2N + 1)$ dimensional matrix, the components of which may be calculated from Eqs. (3.13) and (3.14). From Eq. (3.15) it can be found that

$$U_{00} = e^{-\gamma t} \tag{3.16}$$

in the limit $\Delta E \to 0$. Using this limiting solution, from Eqs. (3.13) and (3.14) it is found that

$$U_{n0} = iH\left[\frac{e^{-\gamma t + in\Delta Et} - 1}{\gamma - in\Delta E}\right], \tag{3.17}$$

which is also in the limit $\Delta E \to 0$. Using the relation $U^\dagger(t) = U(-t)$, we get the time-dependent weak value of an operator A as

$$A_w = \frac{\langle \psi_f|U(t_f - t)AU(t - t_i|\psi_i\rangle}{\langle \psi_f|U(t_f - t_i)|\psi_i\rangle}. \tag{3.18}$$

Consider the operator A to be chosen as the projection operator P_+ onto the excited state at time t, given that it is preselected in the excited state at time t_i and postselected to have decayed at time t_f. Let the possible choice of the final state be

$$|\psi_f\rangle = |\psi_k\rangle. \tag{3.19}$$

This corresponds to the scenario that the atom is in the ground state and a photon of energy $E_k = k\Delta E$ has been emitted. This emitted photon may be absorbed by the bath modes due to the presence of the coupling. It can be shown that after some simple calculations the weak value gives

$$P_w = \frac{U_{k0}(t_f - t)U_{00}(t - t_i)}{U_{k0}(t_f - t_i)}. \tag{3.20}$$

Using (3.16) and (3.17) it can be shown that

$$P_w = e^{-\gamma(t-t_i)}\left[\frac{1 - e^{-\gamma(t_f-t)+ik\Delta E(t_f-t)}}{1 - e^{-\gamma(t_f-t_i)+ik\Delta E(t_f-t_i)}}\right]. \tag{3.21}$$

In the case of $E_k = E_0$, Eq. (3.21) reduces to the simple expression

$$P_w = e^{-\gamma(t-t_i)}\left[\frac{1 - e^{-\gamma(t_f-t)}}{1 - e^{-\gamma(t_f-t_i)}}\right]. \tag{3.22}$$

This is for the state that the system approaches asymptotically as $t \to \infty$. For the postselection of the state at a finite time t_f, according to Ref. [16], Eq. (3.22) changes as

$$P_w = e^{-\gamma(t-t_i)}\left[\frac{1 - e^{-2\gamma(t_f-t)}}{1 - e^{-2\gamma(t_f-t_i)}}\right]. \tag{3.23}$$

If we divide the measurement into many short ones, as we have discussed in the previous section, in comparison with Eq. (2.12) the weak value gives

$$P_w = \int_{t_i}^{t_f} e^{-\gamma(t-t_i)}\left[\frac{1 - e^{-2\gamma(t_f-t)}}{1 - e^{-2\gamma(t_f-t_i)}}\right]dt. \tag{3.24}$$

Since the pre- and postselection are done at t_i and t_f, respectively, correspondingly the limits of the integration are taken in the same manner. Consequently this projection operator P_+ can be understood as the projection operator $\Theta_{0,L}$ as described in the previous section. Then Eq. (3.24) gives the weak value of the operator $\Theta_{0,L}$, which in turn gives us the weak value of dwell time of the particle in the region of the magnetic field. Understanding the integral of the weak survival probability as dwell time also conforms with the understanding of Winful [10]. As we mentioned in the Introduction, group delay (τ^G) is understood as the lifetime of the energy storage in the barrier region, and it is directly related to the dwell time (τ^D) with an additive self-interference term (τ^I),

$$\tau^G = \tau^D + \tau^I. \tag{3.25}$$

When the reflectivity is high, the incident pulse spends much of its time dwelling in front of the barrier as it interferes with itself during the tunneling process. This excess dwelling is interpreted as the self-interference delay. Winful successfully disentangled this term from the dwell time [15]. If the surroundings of the barrier are dispersionless, then the self-interference term vanishes, resulting in the equality of the group delay and dwell time [13]. In that case, the dwell time will give a lifetime of energy storage in the barrier region. In our case the barrier region is dissipative (absorptive), so the integrated weak survival probability will give us a lifetime of the remaining unabsorbed energy leaking through the barrier. Moreover, this version of dwell time includes the history of the interaction with the environment through the coupling term γ, as stated previously, as the decay constant [16]. Therefore,

$$\tau_w = P_w = \int_{t_i}^{t_f} e^{-\gamma(t-t_i)}\left[\frac{1 - e^{-2\gamma(t_f-t)}}{1 - e^{-2\gamma(t_f-t_i)}}\right]dt. \tag{3.26}$$

Before calculating the dwell time explicitly, we want to investigate the decay constant γ much more elaborately. Since this γ represents the coupling between the bath modes and the concerning system, this is the signature of dissipation.

IV. DERIVATION OF DWELL TIME IN A DISSIPATIVE ENVIRONMENT

The approach we discuss here, to incorporate dissipation in the dynamics of quantum system, was developed by Caldirola and Montaldi [17] and Caldirola [22], introducing a discrete time parameter (δ) that could, in principle, be calculated from the properties of environment such as its temperature and composition. It is used to construct a retarded Schrödinger equation describing the dynamics of the states in the presence of environmentally induced dissipation, which is given by

$$H|\psi\rangle = i\frac{[|\psi(t)\rangle - |\psi(t-\delta)\rangle]}{\delta}. \tag{4.1}$$

Expanding $|\psi(t-\delta)\rangle$ in Taylor series, Eq. (4.1) can be written as

$$H|\psi\rangle = i\frac{\left[1 - e^{-\delta\frac{\partial}{\partial t}}\right]|\psi(t)\rangle}{\delta}. \tag{4.2}$$

Setting the trial solution as

$$|\psi(t)\rangle = e^{-\alpha t}|\psi(0)\rangle, \tag{4.3}$$

we solve for α to get

$$\alpha = \frac{1}{\tau}\ln(1 + iH\delta). \tag{4.4}$$

Substituting α in Eq. (4.2) we find that even the ground-state decays. To stabilize the ground state, Caldirola and Montaldi [17] rewrite Eq. (4.1) as

$$(H - H_0)|\psi\rangle = i\frac{[|\psi(t)\rangle - |\psi(t-\delta)\rangle]}{\delta}, \tag{4.5}$$

where H_0 represents the ground state. In this case we get

$$\alpha = \frac{1}{\tau}\ln\left[1 + i(H - H_0)\delta\right]. \tag{4.6}$$

For a spin-$\frac{1}{2}$ system in a magnetic field ($\mathbf{B}$), the Hamiltonian is

$$H = \frac{e}{m}S_z B. \tag{4.7}$$

For the eigenvalues of (4.7) we have

$$E_+ = \frac{eB}{2m}, \quad |\psi\rangle = \begin{pmatrix}1\\0\end{pmatrix}, \tag{4.8}$$

$$E_- = -\frac{eB}{2m}, \quad |\psi\rangle = \begin{pmatrix}0\\1\end{pmatrix}. \tag{4.9}$$

Now let us take the state to be initially polarized in the x direction as shown by Eq. (3.10). Following Wolf [23], to generate the states at time t we use

$$|\psi(t)\rangle = \exp\left[-\frac{t}{\delta}\ln[1+i(H-H_0)\delta]\right]|\psi(0)\rangle, \quad (4.10)$$

where

$$H_0 = -\frac{eB}{2m}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \quad (4.11)$$

and

$$H = \frac{eB}{2m}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}. \quad (4.12)$$

Therefore

$$H - H_0 = \frac{eB}{m}\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}, \quad (4.13)$$

so we find the state at time t as

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}\exp\left[-\frac{t}{\delta}\ln\left(1+\frac{ieB\delta}{m}\right)\right]\\ 1\end{pmatrix}. \quad (4.14)$$

Expanding the logarithmic term up to third order,

$$\ln\left(1+\frac{ieB\delta}{m}\right) = \frac{ieB\delta}{m} + \frac{e^2B^2\delta^2}{2m^2} - \frac{ie^3B^3\delta^3}{3m^3}, \quad (4.15)$$

the time evolution takes the form

$$\exp\left[-i\left(\frac{eB}{m} - \frac{e^3B^3\delta^2}{3m^3}\right)t - \frac{e^2B^2\delta}{2m^2}t\right]. \quad (4.16)$$

From (4.16) we find the modified precession frequency

$$\omega' = 2\omega\left(1 - \frac{e^2B^2\delta^2}{3m^2}\right), \quad (4.17)$$

where $\omega = \frac{eB}{2m}$ is the unmodified precession frequency. We also find the decay rate as

$$\gamma = \frac{e^2B^2\delta}{2m^2}. \quad (4.18)$$

Again, from Eq. (4.17) we find the time scale δ as

$$\delta = \frac{1}{2\omega}\sqrt{3\left(1-\frac{\omega'}{2\omega}\right)}, \quad (4.19)$$

so the decay constant takes the form

$$\gamma = \omega\sqrt{3\left(1-\frac{\omega'}{2\omega}\right)}. \quad (4.20)$$

Putting this into Eq. (3.26) and integrating, we find the dwell time to be

$$\tau_w = \frac{1}{\omega\sqrt{3\left(1-\frac{\omega'}{2\omega}\right)}}\coth\left[\frac{\omega T}{2}\sqrt{3\left(1-\frac{\omega'}{2\omega}\right)}\right], \quad (4.21)$$

where $T = t_f - t_i$. Here we arrive at the expression of dwell time for a spin-$\frac{1}{2}$ particle traversing through a magnetic potential barrier in the presence of dissipation, which was preselected in a state with energy ω at an initial time t_i and postselected in a state with energy ω' at a final time t_f. Let us consider a particular case where the particle is preselected in the spin-up state with $\omega = \omega_+ = \frac{eB}{2m}$ and postselected in the spin-down state with $\omega' = \omega_- = -\frac{eB}{2m}$. Then the dwell time takes the form

$$\tau_w = \frac{\sqrt{2}}{3\omega}\coth\left(\frac{3\omega T}{2\sqrt{2}}\right). \quad (4.22)$$

Similarly, we can find the dwell time for a spin-$\frac{1}{2}$ particle traversing through a similar kind of barrier, preselected and also postselected in the spin-up state. If we consider the interpretation of dwell time as stated by Winful, these finite nonzero dwell times will give us the lifetime of the decaying states in the magnetic barrier region, depending on the pre- and postselection of the states.

V. CONCLUSION

From the result of our calculation, it is evident that the presence of dissipative interaction with the bath modes precludes zero time tunneling, i.e., the instantaneous release of energy. This can be explained in terms of an efficient energy transfer from the particle motion to the environmental modes and loss of memory of the original tunneling direction. Here the barrier is acting as a lumped capacitor with coupling to the environment providing an effective dissipation. The delay is caused by the energy storage in the barrier region. It is also worth mentioning that a connection between dwell time and realistic examples of lifetimes of decaying states was established in some works of Kelkar *et al.* [24,25], where they have dealt with the phenomenon of α decay by investigating the quantum time scales of tunneling. They have formulated the half-life of the decaying state in terms of the transmission dwell time. They found that the major bulk of the half-life of a medium or super heavy radioactive nucleus is spent in front of the barrier before tunneling. The time spent inside the barrier is much smaller, though it is not negligible. If the barrier is considered to be dissipative, we can heuristically argue that the energy transfer to the nuclear modes will result in the loss of memory of the original tunneling direction and hence cause an extra delay, enhancing the dwell time inside the barrier. The α particles or clusters formed inside the nucleus can undergo some restructuring within the nucleus. Work has been done on the derivation of necessary formulas accounting for the final-state interaction between the knocked-out cluster and the residual nucleus [26]. This kind of interaction may account for the dissipation. Admitting this to be a heuristic argument on the subject of α decay, we can consider it an important subject for future work.

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