Dynamical Casimir effect in two-atom cavity QED

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We study analytically and numerically the dynamical Casimir effect in a cavity containing two stationary two-level atoms that interact with the resonance field mode via the Tavis-Cummings Hamiltonian. We determine the modulation frequencies for which the field and atomic excitations are generated and study the corresponding dynamical behaviors in the absence of damping. It is shown that the two-atom setup allows for monitoring of photon generation without interrupting the growth, and different entangled states can be generated during the process.

Introduction. In view of the recent progress [1] in experiments on the observation of the so-called dynamical Casimir effect (DCE) [2], the problem of detecting photons generated from the initial vacuum state becomes quite actual. It was shown long ago [3] that the presence of a detector can change significantly the statistics (including the mean number) of created quanta, compared with the predictions made for an idealized empty cavity model. Therefore, it is necessary to study in detail different detection schemes. At least two main schemes were proposed until now. In the so-called MIR experiment, the quanta of the microwave field are to be detected by an antenna put inside the closed cavity [4]. Another idea was to use as detectors real Rydberg atoms passing through the cavity [3,5–7] or "artificial atoms" [8] in the case of Circuit QED systems, such as those described in Refs. [1,2]. The simplest solutions for a detector modeled as a single two-level atom were obtained in Refs. [3,5], and recently results of more detailed theoretical and numerical studies of the atom-field interaction during the DCE were presented in Refs. [9–11]. Three-level models of detectors were considered in Refs. [12,13]. It seems important to analyze different configurations to choose the optimal scheme.

Here we study how the DCE dynamics is affected by the presence of *two* two-level atoms (detectors) interacting with a single resonance cavity field mode. Our starting point is the Hamiltonian (we set $\hbar = 1$)

$$H_0 = \omega_t n + \sum_{j=1}^{2}\left[\frac{\Omega_j}{2}\sigma_j^z + g_j(a\sigma_j^+ + a^\dagger\sigma_j^-)\right] - i\chi_t(a^2 - a^{\dagger 2}),$$

where a ($a^\dagger$) is the cavity annihilation (creation) operator and $n \equiv a^\dagger a$ is the photon number operator. The Pauli operators are defined as $\sigma_j^z = |\overline{e}_j\rangle\langle\overline{e}_j| - |\overline{g}_j\rangle\langle\overline{g}_j|$, $\sigma_j^- = |\overline{g}_j\rangle\langle\overline{e}_j|$, $\sigma_j^+ = |\overline{e}_j\rangle\langle\overline{g}_j|$, where $|\overline{g}_j\rangle$ and $|\overline{e}_j\rangle$ are the ground and excited states of the jth atom ($j = 1,2$), respectively. Ω_j and g_j are the atomic transition frequencies and the atom-field coupling constants (assumed real for simplicity). If $\chi_t = 0$, then H_0 is the special case of the known Tavis-Cummings Hamiltonian [14] studied in numerous papers (see, e.g., Refs. [15–18] and references therein). Physical realizations of this Hamiltonian (which holds for $|g_j| \ll \Omega_j$) were demonstrated in Ref. [19] for trapped ions and in Ref. [20] for the Circuit QED systems.

The last term in H_0 describes the effect of photon creation (equivalent to squeezing) in a cavity whose fundamental eigenfrequency varies in time due to the motion of a boundary [3,21,22]. We suppose that the boundary performs harmonic oscillations at the modulation frequency η. Then the instantaneous cavity eigenfrequency depends on time as $\omega_t = \omega_0 + \varepsilon \sin(\eta t)$, where ε is the small modulation amplitude. Normalizing the unperturbed cavity frequency to $\omega_0 = 1$, we write the modulation frequency as $\eta = 2(1 + x)$, where x is a small resonance shift. For a weak modulation, $|\varepsilon| \ll 1$, we can write to the first order in ε: $\chi_t \equiv (4\omega_t)^{-1}d\omega_t/dt \simeq 2q\cos(\eta t)$ [3,21,22], where $q \equiv \varepsilon(1 + x)/4$. Moreover, the term $\omega_t n$ in H_0 can be replaced simply by n, as soon as the main effect of modulation is due to the presence of operators a^2 and $a^{\dagger 2}$ in the squeezing part of H_0 but not due to the photon number preserving part $\omega_t a^\dagger a$.

In the empty cavity, the resonance generation of many photons is achieved for $x = 0$ (being impossible if $|x| \gtrsim |\varepsilon|$ [23]). On the other hand, it was shown in Ref. [3] that no more than two photons can be created in the presence of a single atom if $|\varepsilon| \ll |g_1|$, and this can happen if $|x| \sim |g_1|$. Our aim is to find the resonance regimes in the presence of *two* atoms for different relations between the parameters ε, g_j, and Ω_j. We show that there are two types of resonances. For some distinguished values of $x \neq 0$, at most two photons can be created. But under certain conditions, the multiphoton generation becomes possible again for $x \approx 0$ (contrary to the one-atom case), even if $|\varepsilon| \ll |g_1|$. This interesting result is one of the main motivations for this publication.

The dynamics of the closed system (atoms + field mode) is governed (neglecting dissipation) by the Schrödinger equation $i\partial|\Psi(t)\rangle/\partial t = H_0|\Psi(t)\rangle$. To find analytical solutions we go to the *interaction picture*: $|\Psi(t)\rangle = \exp[-it(\eta/2)(n + \sigma_1^z/2 + \sigma_2^z/2)]|\psi(t)\rangle$, since the Hamiltonian acting upon the new wave function $|\psi(t)\rangle$ becomes time independent after the rotating wave approximation (RWA):

$$H_I = \sum_{j=1}^{2}\left[g_j(a\sigma_j^+ + a^\dagger\sigma_j^-) - \frac{\Delta_j + x}{2}\sigma_j^z\right]$$
$$- iq(a^2 - a^{\dagger 2}) - xn,$$

where $\Delta_j = 1 - \Omega_j$. We expand the wave function in the atom and Fock bases as follows:

$$
\begin{aligned}
|\psi(t)\rangle = \sum_{m=0}^{\infty} e^{ixmt} [& a_m(t) e^{-i(2x+\Delta_1+\Delta_2)t/2} |\overline{g}_1\rangle |\overline{g}_2\rangle |m\rangle \\
& + b_m(t) e^{-i(\Delta_1-\Delta_2)t/2} |\overline{g}_1\rangle |\overline{e}_2\rangle |m\rangle \\
& + c_m(t) e^{i(\Delta_1-\Delta_2)t/2} |\overline{e}_1\rangle |\overline{g}_2\rangle |m\rangle \\
& + d_m(t) e^{i(2x+\Delta_1+\Delta_2)t/2} |\overline{e}_1\rangle |\overline{e}_2\rangle |m\rangle].
\end{aligned} \tag{1}
$$

Then the Schrödinger equation with Hamiltonian H_I leads to the set of coupled differential equations

$$
\dot{a}_m = -ig_1\sqrt{m}\,c_{m-1}e^{i\Delta_1 t} - ig_2\sqrt{m}\,b_{m-1}e^{i\Delta_2 t} + q\hat{W}_m a_m \tag{2}
$$

$$
\begin{aligned}
\dot{b}_{m-1} = & -ig_1\sqrt{m-1}\,d_{m-2}e^{i\Delta_1 t} - ig_2\sqrt{m}\,a_m e^{-i\Delta_2 t} \\
& + q\hat{W}_{m-1}b_{m-1}
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
\dot{c}_{m-1} = & -ig_1\sqrt{m}\,a_m e^{-i\Delta_1 t} - ig_2\sqrt{m-1}\,d_{m-2}e^{i\Delta_2 t} \\
& + q\hat{W}_{m-1}c_{m-1}
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
\dot{d}_{m-2} = & -ig_1\sqrt{m-1}\,b_{m-1}e^{-i\Delta_1 t} - ig_2\sqrt{m-1}\,c_{m-1}e^{-i\Delta_2 t} \\
& + q\hat{W}_{m-2}d_{m-2},
\end{aligned} \tag{5}
$$

where $\hat{W}_m O_m \equiv \sqrt{m(m-1)}\,O_{m-2}e^{-2ixt} - \sqrt{(m+1)(m+2)}\,O_{m+2}e^{2ixt}$.

Weak modulation with atoms in resonance. This regime is defined by the inequality $|\varepsilon| \ll G \equiv \sqrt{g_1^2 + g_2^2}$. If two atoms are in resonance, $\Delta_1 = \Delta_2 = 0$, the solution to Eqs. (2)–(5) in the absence of external modulation ($q = 0$) is (for $m \geq 2$)

$$
a_m = \sum_{\alpha,\beta=+,-} \mathcal{F}_m^{\alpha\beta} \exp\left(\alpha i G L_m^\beta t\right), \tag{6}
$$

$$
d_{m-2} = -\sum_{\alpha,\beta=+,-} V_m^\beta \mathcal{F}_m^{\alpha\beta} \exp\left(\alpha i G L_m^\beta t\right), \tag{7}
$$

$$
\begin{aligned}
b_{m-1} = \frac{G}{(g_1^2-g_2^2)} \sum_{\alpha,\beta=+,-} & \alpha L_m^\beta \mathcal{F}_m^{\alpha\beta} \exp\left(\alpha i G L_m^\beta t\right) \\
& \times \left[g_2/\sqrt{m} + g_1 V_m^\beta/\sqrt{m-1} \right],
\end{aligned} \tag{8}
$$

$$
c_{m-1} = b_{m-1}[g_1 \to g_2; g_2 \to g_1], \tag{9}
$$

where $\mathcal{F}_m^{\alpha\beta}$ are constant coefficients,

$$
V_m^\pm = \frac{1 \mp 2R_m}{2\rho\sqrt{m(m-1)}}, \quad \rho = \frac{2g_1 g_2}{G^2},
$$

$$
R_m = \frac{1}{2}\sqrt{1 + 4\rho^2 m(m-1)}, \quad L_m^\pm = \sqrt{m - 1/2 \pm R_m}.
$$

Substituting now Eqs. (6)–(9) back into Eqs. (2)–(5) and assuming that $\mathcal{F}_m^{\alpha\beta}$ are slowly varying functions of time, one can verify that for specific values of the resonance shift x some of these functions become multiplied by imaginary exponentials with large arguments (compared to q), while others are multiplied by time-independent coefficients, so one is allowed to perform the RWA and obtain simplified effective dynamics. We find that for the initial zero-excitation state $|\overline{g}_1\rangle|\overline{g}_2\rangle|0\rangle$, at most two photons can be created whenever $G|L_4^\pm - L_2^\pm| \gg q$. The resonant regimes occur for $2x = -\alpha G L_2^\beta$ (with α,

$\beta = +,-$), when the only nonzero amplitudes (neglecting small terms of the order of ε/G) are $a_0 = \cos(qt R_\beta)$ (it does not depend on the sign of α) and $\mathcal{F}_2^{\alpha\beta} = R_\beta \sin(qt R_\beta)/\sqrt{2}$, where $R_\pm = \frac{1}{2}\sqrt{2 \pm R_2^{-1}}$.

For a single atom ($g_2 = 0$) one has $R_m \equiv 1/2$, so that $R_+ = 1$ and $R_- = 0$. Then the only resonances with a periodic creation of at most two photons happen for $x = \pm|g_1|/\sqrt{2}$. In this case $a_0 = \cos(qt)$, while the only other nonzero coefficients are $\mathcal{F}_2^{\mp+} = \sin(qt)/\sqrt{2}$ in accordance with Ref. [3]. In the presence of the second atom, new resonances become possible. If $|g_2| \ll |g_1|$, then these additional resonance frequencies have the values $x \approx \pm|g_1|/2$. However, since $R_- \approx \rho\sqrt{2} \ll 1$ in this case, the corresponding dynamic is quite slow and the probability of the photon creation is small, too.

The most interesting situation takes place if $|g_1| = |g_2|$. Then $R_m = m - 1/2$ and $L_m^- \equiv 0$. We still have the resonances at $x = \pm|g_1|\sqrt{3/2}$, when no more than two photons can be created from the initial ground state, since the only nonzero coefficients in this case are $a_0 = \cos(\sqrt{2/3}qt)$ and $\mathcal{F}_2^{\mp+} = r\sin(\sqrt{2/3}qt)/\sqrt{3}$, where $r = g_2/g_1 = \pm 1$. But two other resonances merge in the single one at $x = 0$. In this case, solving Eqs. (2)–(5) with $q = 0$, one can write (for $m \geq 2$)

$$
a_m = r[\mathcal{W}_m E_m^-(t) + \mathcal{X}_m E_m^+(t) + \mathcal{Y}_m],
$$

$$
b_{m-1} = \sqrt{1 - (2m)^{-1}}\,[\mathcal{W}_m E_m^-(t) - \mathcal{X}_m E_m^+(t)] + \mathcal{Z}_m,
$$

$$
c_{m-1} = r(b_{m-1} - 2\mathcal{Z}_m),
$$

$$
d_{m-2} = r a_m \sqrt{\frac{m-1}{m}} - \frac{2m-1}{\sqrt{m(m-1)}}\mathcal{Y}_m,
$$

where $E_m^\pm(t) = \exp[\pm i g_1 \sqrt{2(2m-1)}\,t]$. In the presence of additional terms proportional to the small parameter $q \ll G$ in Eqs. (2)–(5), the coefficients $\mathcal{W}_m$, $\mathcal{X}_m$, $\mathcal{Y}_m$, and $\mathcal{Z}_m$ become time-dependent. For the standard atomless DCE resonance $\eta = 2$, assuming that $|\mathcal{W}_m|, |\mathcal{X}_m| \ll 1$ for all m, we perform the RWA and find that $\mathcal{Z}_m(t) = 0$, meaning that $b_m(t), c_m(t) \approx 0$ for all times. Only functions $\mathcal{Y}_m$ vary slowly with time according to the equations

$$
\begin{aligned}
\dot{\mathcal{Y}}_m \simeq q\Big[& \sqrt{m(m-1)}\frac{2m-3}{2m-1}\mathcal{Y}_{m-2} \\
& - \sqrt{(m+1)(m+2)}\frac{m-1}{m+1}\frac{2m+1}{2m-1}\mathcal{Y}_{m+2} \Big],
\end{aligned} \tag{10}
$$

with the initial condition $\mathcal{Y}_m(0) = r\delta_{m0}$. Therefore, eventually all (even) coefficients $\mathcal{Y}_m$ become different from zero, so that many photons can be created from the initial vacuum state. Equation (10) has two remarkable properties. First, it does not contain the atomic coupling coefficients. Second, the fractions in its right-hand side tend to the unit values for $m \gg 1$, and in this limit Eq. (10) has the same form as the equation governing the evolution of the field amplitudes (in the Fock basis) in the empty cavity. Since the main contribution to the mean photon number $\langle n(t) \rangle$ is given by the coefficients $\mathcal{Y}_m$ with $m \gg 1$ if $\langle n \rangle \gg 1$, we can expect that after some transient time the photons will be steadily created with the same asymptotical rate $d\ln(\langle n \rangle)/d(\varepsilon t)$ as in the empty cavity. Moreover, since $|d_{m-2}(t)|^2 = [m/(m-1)]|a_m(t)|^2$, both atoms become excited simultaneously. Numerical calculations confirm these
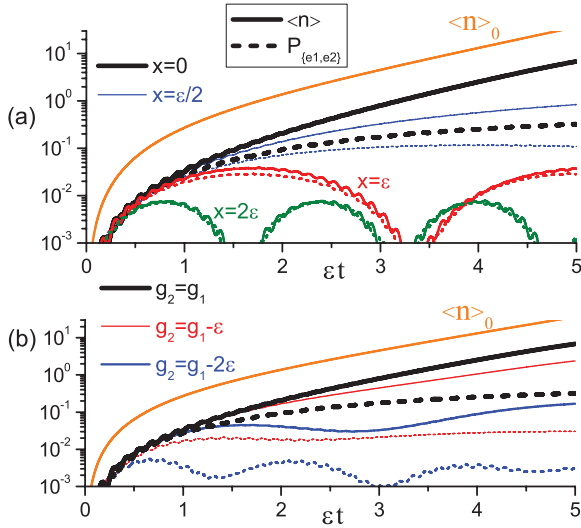
FIG. 1. (Color online) The mean photon number (solid lines) and atomic excitation probabilities (dashed lines) as functions of dimensionless time εt. (a) The influence of nonzero detuning x for $g_1 = g_2$. (b) The influence of disbalance $g_2 - g_1$ for $x = 0$. Numerical values of parameters are given in the text.

predictions, as shown in Fig. 1, where we plot the mean photon number $\langle n \rangle$ and the probability of double excitation $P_{\{e1,e2\}}$ for parameters $g_1 = 4 \times 10^{-2}$ and $\varepsilon = 2 \times 10^{-3}$.[1] Figure 1(a) shows the role of the detuning parameter x when $g_2 = g_1$: the photon creation and atomic excitations practically stop for $x \gtrsim \varepsilon$. Figure 1(b) shows the influence of disbalance $g_2 - g_1$ when $x = 0$; again, all effects practically disappear if $|g_2 - g_1| \gtrsim \varepsilon$.

The mean number of photons for $x = 0$ is smaller than that in the empty-cavity case, $\langle n_0(t) \rangle = \sinh^2(\varepsilon t/2)$, due to initial transient processes, when the atomic populations attain stationary values: one can see that the line $\langle n(t) \rangle$ can be obtained from $\langle n_0(t) \rangle$ by some positive shift in time. Therefore, the $x = 0$ resonance for $|g_1| = |g_2|$ is interesting from the point of view of detecting Casimir photons, since the atoms get excited simultaneously without interrupting the photon generation process.

If the second atom is in the dispersive regime, $|g_2| \ll |\Delta_2|$ (while $\Delta_1 = 0$), we define the dispersive shift $\delta_2 \equiv g_2^2/\Delta_2$ and repeating the previous steps we find that for $|\delta_2| \ll |g_1|$ the photon generation occurs for the resonance shifts $2x = (3/2)\delta_2 \pm G_2$ with $G_2 \equiv \sqrt{2g_1^2 + \delta_2^2/4}$. The resulting nonzero probability amplitudes read: $a_0 = \cos[qt\sqrt{1 \pm \delta_2/(2G_2)}]$,

$$a_2 = e^{-i(3/2)\delta_2 t}[\mathcal{W}e^{-iG_2 t} + \mathcal{X}e^{iG_2 t}],$$

$$c_1 = \frac{G_2 e^{-i(3/2)\delta_2 t}}{\sqrt{2}g_1}\{\mathcal{W}[1 - \delta_2/(2G_2)]e^{-iG_2 t}$$
$$- \mathcal{X}[1 + \delta_2/(2G_2)]e^{iG_2 t}\},$$

_____

[1] All numerical calculations have been performed for the initial Hamiltonian H_0 without any simplifications. The scheme of such calculations was described briefly in Ref. [10]. We verified that the analytical results according to Eq. (10) are indistinguishable from the numerical ones within the thicknesses of lines.

$$b_1 \simeq \sqrt{2}(g_2/\Delta_2)e^{-i\Delta_2 t}a_2, \quad d_0 \simeq (g_2/\Delta_2)e^{-i\Delta_2 t}c_1,$$

$$\binom{\mathcal{W}}{\mathcal{X}} = \frac{\sqrt{1 \pm \delta_2/(2G_2)}}{\sqrt{2}}\sin[qt\sqrt{1 \pm \delta_2/(2G_2)}].$$

At most two photons can be created in this case.

Dispersive regimes. Many photons can be generated from vacuum if both atoms are in the dispersive regime, $|g_j| \ll |\Delta_j|$. In this case, instead of solving coupled differential equations it is convenient to write the wavefunction $|\psi(t)\rangle$ as [11,13] $|\psi(t)\rangle = U^\dagger \exp(-iH_{ef}t)U|\psi(0)\rangle$, where the effective Hamiltonian $H_{ef} \equiv U H_I U^\dagger$ is defined by means of the unitary operator $U = \exp(Y)$. Choosing $Y = a^\dagger(\zeta_2\sigma_2^- + \zeta_1\sigma_1^-) - \text{H.c.}$ (where $\zeta_j = g_j/\Delta_j$ are small parameters, $|\zeta_j| \ll 1$, $j = 1,2$) and expanding the exponentials in Taylor's series we obtain to the second order in ζ_j [assuming $O(\zeta_1) \sim O(\zeta_2)$]

$$H_{ef} = -(x + \delta_1\sigma_1^z + \delta_2\sigma_2^z)n - \sum_{j=1}^{2}\frac{\Delta_j + x + \delta_j}{2}\sigma_j^z$$

$$- \zeta_1\zeta_2\left[\frac{\Delta_1 + \Delta_2}{2}\sigma_1^+\sigma_2^- - 2iq\sigma_1^+\sigma_2^+ + \text{H.c.}\right]$$

$$- iq[(1 + \zeta_1^2\sigma_1^z + \zeta_2^2\sigma_2^z)a^2 - \text{H.c.}]. \tag{11}$$

Here $\delta_j \equiv g_j^2/\Delta_j$ are the dispersive shifts ($j = 1,2$). In view of the perturbative expansion the effective Hamiltonian Eq. (11) is valid roughly for times $|\delta_1|t \ll 1$. For $q = 0$ it describes the indirect interaction between the two atoms via the cavity field [24]. Since the state $|\bar{g}_1\rangle|\bar{g}_2\rangle|0\rangle$ is the common eigenstate of Y and $(\sigma_1^+\sigma_2^- + \text{H.c.})$ with null eigenvalues, one has $U|\bar{g}_1\rangle|\bar{g}_2\rangle|0\rangle = |\bar{g}_1\rangle|\bar{g}_2\rangle|0\rangle$, so the term $(\sigma_1^+\sigma_2^- + \text{H.c.})$ can be dropped out if $|\psi(0)\rangle = |\bar{g}_1\rangle|\bar{g}_2\rangle|0\rangle$. Besides, if the coefficient in front of n in Eq. (11) is adjusted to zero, the photon generation term $iq(1 + \zeta_1^2\sigma_1^z + \zeta_2^2\sigma_2^z)a^{\dagger 2}$ becomes resonant, while the term $2iq\zeta_1\zeta_2(\sigma_1^+\sigma_2^+ - \text{H.c.})$ can be neglected for initial times. In this case the wave function for the resonance shift $x = \delta_1 + \delta_2$ reads as (neglecting a global phase) $|\psi(t)\rangle = U^\dagger\hat{\Lambda}(1 - \zeta^2)|\bar{g}_1\rangle|\bar{g}_2\rangle|0\rangle$, where the squeezing operator $\hat{\Lambda}(v) \equiv \exp[v qt(a^{\dagger 2} - a^2)]$ has the property [11,25] $\hat{\Lambda}^\dagger(v)a\hat{\Lambda}(v) = C_v a + S_v a^\dagger$, with $C_v = \cosh(2vqt)$, $S_v = \sinh(2vqt)$, and $\zeta^2 \equiv \zeta_1^2 + \zeta_2^2$.

Average values of the main observable quantities are as follows (to the second order in ζ_j):

$$\langle n(t) \rangle = (1 - \zeta^2)\sinh^2[2qt(1 - \zeta^2)],$$

$$P_{e1}(t) = \zeta_1^2\langle n(t) \rangle, \quad P_{e2}(t) = \zeta_2^2\langle n(t) \rangle,$$

$$\langle(\Delta X_\pm)^2\rangle = \frac{1}{2}\{\zeta^2 + (1 - \zeta^2)\exp[\pm 4qt(1 - \zeta^2)]\},$$

where $X_+ = (a + a^\dagger)/\sqrt{2}$ and $X_- = (a - a^\dagger)/(\sqrt{2}i)$ are the field quadratures. Moreover, for times $|\delta_1|t \ll 1$ the probability $P_{\{e1,e2\}}$ of detecting simultaneously both atoms in their excited states is proportional to ζ_1^4, so it is very small. Therefore, by measuring P_{e1} or P_{e2} one can estimate the mean photon number. In Fig. 2(a) we show the behavior of $\langle n \rangle$, P_{e1}, P_{e2}, and $P_{\{e1,e2\}}$ for parameters $\varepsilon = 2 \times 10^{-3}$, $g_1 = 4 \times 10^{-2}$, $g_2 = 3 \times 10^{-2}$, $\Delta_1 = 10g_1$, $\Delta_2 = 15g_2$, and $x = \delta_1 + \delta_2$. We see that many photons are created and the atomic populations are proportional to the mean photon
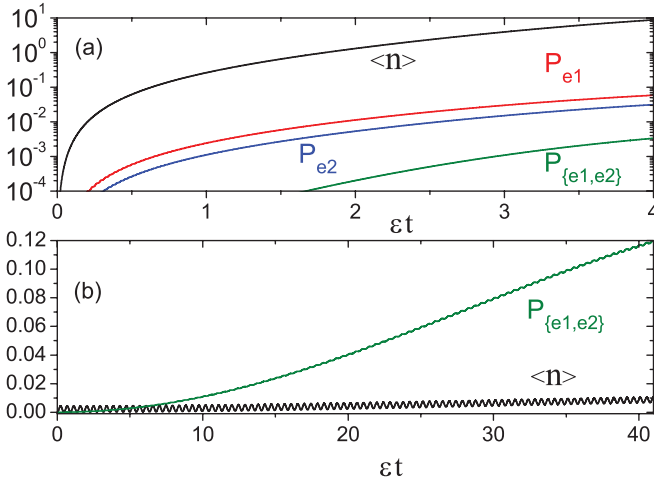
FIG. 2. (Color online) The mean photon number and atomic excitation probabilities versus the dimensionless time εt in the dispersive regimes: (a) $x = \delta_1 + \delta_2$; (b) $2x = -\sum_{j=1}^{2}(\Delta_j + \delta_j)$. Other parameters are specified in the text.

number, while the probability of double atomic excitation is very small.

If $|\sum_{j=1}^{2}(\Delta_j + 3\delta_j)| \gg q$ and the resonance shift is tuned to $2x = -\sum_{j=1}^{2}(\Delta_j + \delta_j)$, with $\Delta_1 \sim -\Delta_2$, then the photon generation term becomes off-resonant and the only resonant term $2iq\zeta_1\zeta_2(\sigma_1^+\sigma_2^+ - \text{H.c.})$ survives in the interaction part of the effective Hamiltonian Eq. (11) even to higher orders in ζ_1, extending its validity beyond the previous condition $|\delta_1|t \ll 1$. In this case only the atomic excitations are generated at a rather small rate $2q\zeta_1\zeta_2$ and the probability of detecting both atoms simultaneously in the excited states is $(1 - \zeta_1^2 - \zeta_2^2)\sin^2(2qt\zeta_1\zeta_2)$. In Fig. 2(b) we show the behavior of $\langle n \rangle$ and $P_{\{e1,e2\}}$ for parameters $g_1 = 4 \times 10^{-2}$, $g_2 = 3 \times 10^{-2}$, $\Delta_1 = 0.22$, $\Delta_2 = -0.2$, $\varepsilon = 2 \times 10^{-3}$, and $2x = -\sum_{j=1}^{2}(\Delta_j + \delta_j)$, where we see that double atomic excitations are created while essentially the field remains in the vacuum state.

Other regimes. If atom 1 is resonant ($\Delta_1 = 0$) and weakly coupled to the field ($|g_1| \ll \varepsilon$), while atom 2 is in the dispersive regime ($|g_2| \ll |\Delta_2|$), then we make the transformation with $Y = a^\dagger(\zeta_2\sigma_2^- + i\xi_1\sigma_1^+) - \text{H.c.}$, $\xi_1 = g_1/(2q)$, and $\zeta_2 = g_2/\Delta_2$. For the resonance shift $x = \delta_2$, the effective Hamiltonian describing parametric amplification reads (after RWA)

$$H_{ef} = -\frac{\Delta_2 + 2\delta_2}{2}\sigma_2^z - iq\big[\big(1 + \xi_1^2\sigma_1^z + \zeta_2^2\sigma_2^z\big)a^2 - \text{H.c.}\big]$$
$$- \delta_2\big(1 - 2\xi_1^2\sigma_1^z + \sigma_2^z\big)n - \frac{\delta_2}{2}\big(1 - 2\xi_1^2\big)\sigma_1^z.$$

For the initial state $|\bar{e}_1\rangle|\bar{g}_2\rangle|0\rangle$, one has $U|\bar{e}_1\rangle|\bar{g}_2\rangle|0\rangle = |\bar{e}_1\rangle |\bar{g}_2\rangle|0\rangle$, so $|\psi(t)\rangle = U^\dagger \hat{\Lambda}(1 + \xi_1^2 - \zeta_2^2)|\bar{e}_1\rangle|\bar{g}_2\rangle|0\rangle$ (up to a

global phase). This yields the following average values:

$$\langle n(t) \rangle = \big(1 - \xi_1^2 - \zeta_2^2\big)\sinh^2\big[2q\big(1 + \xi_1^2 - \zeta_2^2\big)t\big],$$

$$P_{g1} = \xi_1^2\langle n(t)\rangle, \quad P_{e2} = \zeta_2^2\langle n(t)\rangle,$$

$$\langle(\Delta X_\pm)^2\rangle = \frac{\xi_1^2 + \zeta_2^2}{2} + \frac{1 - \xi_1^2 - \zeta_2^2}{2}e^{\pm 4q(1+\xi_1^2-\zeta_2^2)t},$$

where P_{g1} is the ground state probability of atom 1. Besides, the probability $P_{\{g1,e2\}}$ of finding simultaneously atom 1 in the ground state and atom 2 in the excited state is zero [to the second order in $O(\xi_1)$, $O(\zeta_2)$].

Analogously, if both atoms are weakly coupled to the field, $G \ll |\varepsilon|$, then by performing the transformation with $Y = ia^\dagger(\xi_1\sigma_1^+ + \xi_2\sigma_2^+) - \text{H.c.}$ and $\xi_j = g_j/(2q)$ one obtains for $x = \Delta_1 = \Delta_2 = 0$ the effective Hamiltonian [to the second order in ξ_j, for $O(\xi_1) \sim O(\xi_2)$]

$$H_{ef} = iq\big[\big(1 + \xi_1^2\sigma_1^z + \xi_2^2\sigma_2^z\big)a^{\dagger 2} - 2\xi_1\xi_2\sigma_1^+\sigma_2^+ - \text{H.c.}\big].$$

In these cases many photons can be created as well, and the atoms may serve to monitor the photon generation.

Conclusions. We found that the two-atom nonstationary cavity QED is attractive from the point of view of producing different types of entangled states and detecting the DCE, because in specific regimes the atoms can acquire independent information about the field state without inhibiting the photon generation process. In particular, we showed that in the realistic case when the external modulation amplitude is much smaller than the atom-cavity coupling strengths, many photons, as well as atomic excitations, can be generated from the initial zero-excitation state even if both atoms are resonant with the unperturbed cavity field, contrary to the single two-level atom scenario. Moreover, simply by adjusting the modulation frequency, keeping the other parameters unaltered, one can achieve the regime in which at most two photons are generated. If the atoms are off-resonant, then for the zero-excitation initial state many photons can be created for a specific modulation frequency; yet by appropriately tuning the modulation frequency one can achieve the regime in which only atomic excitations are generated. Furthermore, one can explore the regime in which one atom is resonant but weakly coupled to the field, while the other atom is in the dispersive regime—in this case many photons can be created from vacuum and the atoms act as independent detectors of the process. This variety of possibilities can be useful for choosing optimal schemes of detecting the Casimir photons. In view of the results obtained, generalizations to the systems of three and more atoms could be quite interesting. But we leave this problem for another study.

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