

Coherent perfect absorbers for transient, periodic, or chaotic optical fields: Time-reversed lasers beyond threshold

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Recent works [Chong *et al.*, *Phys. Rev. Lett.* **105**, 053901 (2010); Wan *et al.*, *Science* **331**, 889 (2011)] have shown that the time-reversed process of lasing at threshold realizes a coherent perfect absorber (CPA). In a CPA, a lossy medium in an optical cavity with a specific degree of dissipation, equal in modulus to the gain of the lasing medium, can perfectly absorb coherent optical waves at discrete frequencies that are the time-reversed counterpart of the lasing modes. Here the concepts of time reversal of lasing and CPA are extended for optical radiation emitted by a laser operated in an arbitrary regime, i.e., for transient, chaotic, or periodic coherent optical fields. We prove that any electromagnetic signal $E(t)$ generated by a laser system S operated in an arbitrary regime can be perfectly absorbed by a CPA device S' which is simply realized by placing inside S a broadband linear absorber (attenuator) of appropriate transmittance. As examples, we discuss CPA devices that perfectly absorb a chaotic laser signal and a frequency-modulated optical wave.

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I. INTRODUCTION

A laser oscillator is a device that self-organizes to emit a narrowband coherent electromagnetic radiation when the pumping level exceeds a threshold value. Above the first lasing threshold, lasers are generally complex nonlinear systems [1] and may emit different kinds of output signals $E(t)$, ranging from the ideal monochromatic wave under single-mode continuous-wave operation to irregular or chaotic signals when multimode or other kinds of instabilities set in Refs. [1,2] or giant pulses or periodic trains of ultrashort optical pulses when operated in the Q -switching or mode-locking regimes [3,4]. Generally speaking, a “time-reversed” laser refers to a device that, rather than emitting the signal $E(t)$ propagating outgoing from the laser cavity, is capable of perfectly absorbing the same signal $E(t)$ that propagates backward into the cavity, without any reflection [5]. Such a device thus realizes a coherent perfect absorber (CPA) for the signal $E(t)$. For a continuous-wave field, it is known that a dissipative resonator can perfectly absorb incident light at the condition of critical coupling [6]. Recently, Douglas Stone and coworkers have suggested and proven rather generally the possibility to realize a CPA exploiting the time-reversed process of lasing *at threshold*. In this case, the laser system behaves as a linear one, and the time-reversal process is realized simply by the replacement $\epsilon(\mathbf{r}) \rightarrow \epsilon^*(\mathbf{r})$ for the complex dielectric constant ϵ of the medium. In this way, any *linear* absorbing medium of *arbitrary* shape behaves as a CPA at some discrete frequencies under appropriate coherent illumination (the time reverse of the output lasing modes) and provided that a precise amount of dissipation in the medium occurs (equal in modulus to the threshold gains for lasing). The CPA idea has received much interest and stimulated theoretical and experimental studies along different lines [7–16]. In particular, in Ref. [8] an experimental demonstration of interferometric control of the absorption based on CPA was reported using a thin slice of silicon illuminated by two beams, whereas, in Refs. [7,9], a laser-absorber device, which can operate as a CPA and as

a laser simultaneously, has been suggested, combining the CPA and \mathcal{PT} -symmetry concepts. Time reversal of other optical instabilities, such as time-reversal of optical parametric oscillation, have been proposed to realize a multicolor CPA in Ref. [11]. Also, extension of the concept of time-reversed laser and CPA to the spaser and plasmonic nanostructures has been suggested in Refs. [13,14]. Here the energy of the incoming wave is fully transferred into surface plasmon oscillations and evanescent electromagnetic fields. CPA-based devices may have potential applications to the realization of a new class of absorptive interferometers and nanosensors. In all previous studies, time reversal of lasing has been limited to consider either a laser at threshold or above threshold in steady-state operation. However, as previously mentioned, a laser can operate in rather complex or transient regimes, which are highly nonlinear. A major and foundational open question is whether there exists a device that realizes the time reversal of a laser operating in *any* (generally highly nonlinear) regime, i.e., capable of perfectly absorbs the field $E(t)$ emitted by a laser operating in any regime. If yes, how can we realize (at least, in principle) such an “anti-laser” device?

It is the aim of this work to answer to such two major questions. By considering an optical system with a single input or output channel and in the plane-wave approximation, we will prove rather generally that for any electromagnetic signal $E(t)$ generated by a laser system S operated in an arbitrary regime, i.e., emitting a transient, irregular, chaotic, or periodic signal, one can always construct a CPA device S' that perfectly absorbs the field $E(t)$ emitted by system S and that a possible simple realization of this system is obtained by placing inside S , near the output coupler, a broadband linear absorber (attenuator) of appropriate transmittance. As examples, we discuss two CPA devices that perfectly absorb the former, a chaotic signal emitted by a single-mode laser operated in the Lorenz-Haken instability regime, and the latter, a frequency-modulated (FM) optical wave emitted by a multimode FM-operated laser.

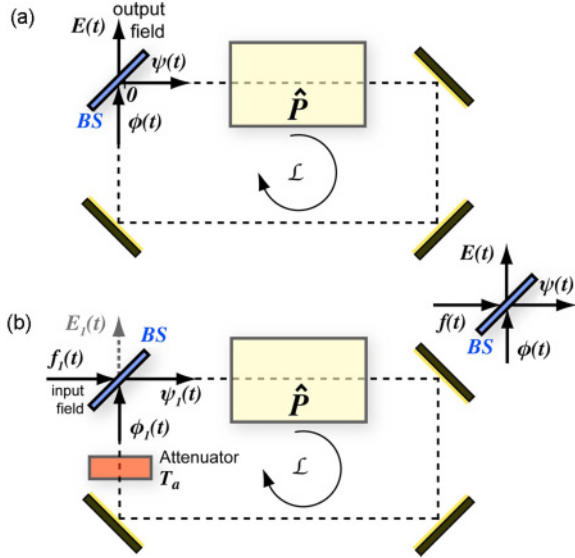


FIG. 1. (Color online) Schematic of (a) a laser system S , made of a ring-cavity of length \mathcal{L} with a single output coupler (lossless beam splitter BS with transmittance T and reflectance $R = 1 - T$), and (b) corresponding CPA system S' . Note that S' is simply obtained from S by the insertion of a linear absorber (attenuator) nearby the output coupler with transmittance $T_a = R^2$. The inset in (a) shows the scattering relations of the field amplitudes at the lossless BS.

II. BASIC CPA IDEA

Let us consider a rather general ring-cavity laser system S of length \mathcal{L} with a single output coupler, consisting of a lossless beam splitter BS, as shown in Fig. 1(a). The laser cavity contains a gain medium and possibly other optical elements or devices, such as saturable absorbers, amplitude or frequency modulators, etc., depending on the operating regime of the laser. The electric field $\mathcal{E}(z, t)$ circulating inside the cavity can be written as $\mathcal{E}(z, t) = A(z, t) \exp(ikz - \omega t)$, where ω is a reference frequency, $k = \omega/c$ is the wave number in vacuum, z is the longitudinal spatial coordinate along the ring, and $A(z, t)$ is a slowly varying envelope. Without loss of generality, ω is chosen to be a resonance frequency of the empty cavity, so $k\mathcal{L}$ is an integer multiple of 2π . After one cavity transit, the envelope $\phi(t) \equiv A(z = \mathcal{L}^-, t)$ at the plane $z = \mathcal{L}^-$ can be formally written as

$$\phi(t) = \hat{P}(\psi(t - T_R), t), \quad (1)$$

where $\psi(t) = A(z = 0^+, t)$ is the field envelope at the $z = 0$ plane, $T_R = \mathcal{L}/c$ is the cavity transit time, and \hat{P} is a functional of $\psi(t - T_R)$ that describes the field evolution over one cavity round trip. The specific form of $\hat{P}(\psi, t)$ depends on the optical elements and devices put inside the cavity and, in general, cannot be given explicitly, requiring to solve coupled equations that account for, e.g., polarization and population dynamics in the active medium or in saturable absorbers possibly present inside the laser cavity with the appropriate initial conditions for the population and polarization variables (see, for instance, Ref. [17]). Explicit forms of the operator \hat{P} can be given in some cases, for example, for mode-locked lasers [18]. However, for our purposes we do not need to explicitly specify the form of \hat{P} here. The time delay T_R in Eq. (1) is introduced

in such a way that for an empty cavity \hat{P} is the identity operator, i.e., $\hat{P}(\psi(t - T_R), t) = \psi(t - T_R)$. The evolution of $\psi(t)$ at successive cavity transits and the corresponding output laser field $E(t)$ are simply obtained by imposing the scattering relation between input and output channels at the beam splitter BS. For a lossless BS, the scattering matrix is unitary, and one can write (see the inset of Fig. 1)

$$\begin{pmatrix} \psi \\ E \end{pmatrix} = \begin{pmatrix} \sqrt{T} & \sqrt{R} \\ -\sqrt{R} & \sqrt{T} \end{pmatrix} \begin{pmatrix} f \\ \phi \end{pmatrix}, \quad (2)$$

where T and $R = 1 - T$ are the BS transmittance and reflectance, respectively, that are assumed to be spectrally flat. For the laser system S without an injected signal, one has $f(t) = 0$, $\phi(t) = (1/\sqrt{R})\psi(t)$ and thus one obtains the following equation for $\psi(t)$

$$\psi(t) = \sqrt{R} \hat{P}(\psi(t - T_R), t). \quad (3)$$

The signal $E(t)$ emitted by the laser system is then given by

$$E(t) = \sqrt{\frac{T}{R}} \psi(t). \quad (4)$$

Once the initial field distribution $\psi(t)$ is assigned in the interval $0 < t < T_R$, Eq. (3) can be used to determine the evolution of $\psi(t)$ at successive transits in the ring. To clarify this point, let us introduce a local time variable τ , with $0 < \tau < T_R$, and let us set $t = nT_R + \tau$, where $n = 0, 1, 2, 3, \dots$ is the round trip number in the ring. After setting $\psi^{(n)}(\tau) = \psi(t = nT_R + \tau)$, Eq. (3) can be written as

$$\psi^{(n)}(\tau) = \sqrt{R} \hat{P}(\psi^{(n-1)}(\tau), \tau + nT_R). \quad (5)$$

For an assigned initial condition $\psi^{(0)}(\tau)$, the map (5) enables us to determine recursively the field distributions $\psi^{(n)}(\tau)$ at successive round trips. The functional $\hat{P}(\psi^{(n-1)}(\tau), \tau + nT_R)$ is generally computed in the frame of an initial-boundary value (Goursat) problem of coupled differential equations (see, for instance, Ref. [17]; see also the example discussed in the appendix). The temporal evolution of the emitted field $\psi(t)$ depends on the operating regime of the laser. For example, it could describe a periodic train of ultrashort pulses when the laser is operated in the mode-locking regime; in this case, the self-consistent equation (3) has the form of a differential-delayed equation, the so-called master equation of mode-locking [3, 18]. But it could also describe a rather irregular waveform, such as for a laser operated in the chaotic regime (see, for instance, Refs. [2, 19]). The main question is now as follows: Is there an optical system S' that can perfectly absorb the optical field $E(t)$ emitted by the laser system S , i.e., can we in some way time reverse a laser that is operated in an arbitrary regime?

The answer is affirmative, and a possible realization of the system S' is depicted in Fig. 1(b). The laser system S' is basically obtained from S by just inserting, near the output beam splitter BS, a broadband linear absorber (attenuator) with a transmittance T_a given by

$$T_a = R^2. \quad (6)$$

In this way, it can be shown rather generally that S' behaves as a CPA when the signal $f_1(t)$ injected into the cavity is

given by

$$f_1(t) = \sqrt{R}E(t) = \sqrt{T}\psi(t), \quad (7)$$

where $E(t)$ is given by Eq. (4) and $\psi(t)$ is the solution to Eq. (3) with the appropriate initial conditions. Such an injected signal is basically the output of the laser S , attenuated by the factor \sqrt{R} [20].

To prove that S' time reverses the laser system S when Eqs. (6) and (7) are satisfied, let us inject into S' via the beam splitter port the signal $f_1(t)$. Let us then indicate by $\mathcal{E}_1(z,t) = A_1(z,t)\exp(ikz - i\omega t)$ the electric field that is established inside the cavity of the system S' after signal injection and by $E_1(t)$ the corresponding output field [see Fig. 1(b)]. In this case one can obviously write [compare with Eq. (1)]

$$\phi_1(t) = \sqrt{T_a}\hat{P}(\psi_1(t - T_R), t), \quad (8)$$

where we have set $\psi_1(t) = A_1(z = 0^+, t)$ and $\phi_1(t) = A_1(z = \mathcal{L}^-, t)$. In fact, the cavity round-trip operator \hat{P}_1 of the system S' is simply obtained by cascading the propagator \hat{P} of S with the transmission amplitude $\sqrt{T_a}$ of the added linear absorber, i.e., $\hat{P}_1 = \sqrt{T_a}\hat{P}$. On the other hand, the scattering relation of the field amplitudes at the beam splitter BS in the system S' relates $\psi_1(t)$, $\phi_1(t)$ and the injected signal $f_1(t)$ according to

$$\psi_1(t) = \sqrt{T}f_1(t) + \sqrt{R}\phi_1(t). \quad (9)$$

From Eq. (9) one obtains $\phi_1(t) = (1/\sqrt{R})\psi_1(t) - \sqrt{T/R}f_1(t)$, which, after substitution into Eq. (8), yields the following equation for the intracavity field $\psi_1(t)$ that is established inside the cavity at the plane $z = 0^+$

$$\psi_1(t) = \sqrt{T}f_1(t) + \sqrt{RT_a}\hat{P}(\psi_1(t - T_R), t). \quad (10)$$

The field $E_1(t)$ leaving S' is then obtained from the scattering relation at the beam splitter and reads explicitly $E_1(t) = -\sqrt{R}f_1(t) + \sqrt{T}\phi_1(t)$, which by means of Eq. (9) can be written as

$$E_1(t) = \sqrt{\frac{T}{R}}\psi_1(t) - \frac{f_1(t)}{\sqrt{R}}. \quad (11)$$

Similarly to the case of the laser system S in the absence of the injected signal, Eq. (10) can be written as a recursive relation

$$\psi_1^{(n)}(\tau) = \sqrt{T}f_1^{(n)}(\tau) + \sqrt{RT_a}\hat{P}(\psi_1^{(n-1)}(\tau), \tau + nT_R), \quad (12)$$

where we have set $\psi_1^{(n)}(\tau) = \psi_1(\tau + nT_R)$ and $f_1^{(n)}(\tau) = f_1(nT_R + \tau)$. Once the initial field distribution $\psi_1^{(0)}(\tau)$ is assigned, the map (12) enables one to determine the evolution of the intracavity field $\psi_1(t)$ at successive round trips. Let us now assume that the two systems S and S' are initially prepared in the same state, i.e., that $\psi_1^{(0)}(\tau) = \psi^{(0)}(\tau)$ and that atomic variables (population inversion and polarization) have the same initial values in the active medium (for example, their equilibrium values). In this case, one can readily show, using Eq. (5), that, if the injected signal $f_1(t)$ is chosen according to

$$f_1(t) = \frac{1 - \sqrt{T_a}}{\sqrt{T}}\psi(t), \quad (13)$$

the recursive relation (12) admits of the solution $\psi_1^{(n)}(\tau) = \psi^{(n)}(\tau)$ for any n . This implies that the intracavity fields

$\psi_1(t)$ and $\psi(t)$ established in the two systems S and S' are the same. Note that this result holds even if the map (12) shows different attractors (such as for a bistable system), provided that the initial conditions in S' belong to the basin of attraction of the CPA solution $\psi_1^{(n)}(\tau) = \psi^{(n)}(\tau)$. This ensures that, asymptotically, one has $\psi_1^{(n)}(\tau) \rightarrow \psi^{(n)}(\tau)$ as $n \rightarrow \infty$, even though $\psi_1^{(n)}(0) \neq \psi^{(n)}(0)$. But for some other initial conditions it might happen that $\psi_1^{(n)}(\tau)$ is not attracted toward $\psi^{(n)}(\tau)$, and the CPA scheme fails. However, like in any nonlinear dynamical system showing different attractors, one can switch the path from one stable attractor to another one by introducing large perturbations of the system parameters (e.g., by transiently changing the cavity losses or the gain in the medium). As discussed in Sec. III A with reference to a specific example, the result holds for transient or chaotic regimes as well, which are very sensitive to the initial state of the system.

The field $E_1(t)$ emitted by the system S' is obtained after substitution of Eq. (13) into Eq. (11). For $\psi_1(t) = \psi(t)$, one obtains

$$E_1(t) = \frac{\sqrt{T_a} - R}{\sqrt{TR}}\psi(t), \quad (14)$$

which vanishes if the transmittance T_a is chosen to satisfy Eq. (6). Correspondingly, the injected field $f_1(t)$, as obtained from Eq. (13) after setting $T_a = R^2$, is given by Eq. (7). Hence, provided that Eqs. (6) and (7) are met and the two systems S and S' are initially prepared in the same state, S' behaves as a CPA device for the signal emitted by S , regardless of its operational regime.

III. EXAMPLES OF CPA DEVICES

In this section we discuss two examples of CPA devices corresponding to a laser oscillator S operating in two nontrivial regimes. The former example is a CPA device for a chaotic optical signal emitted by a single-mode homogeneously broadened laser operated in the chaotic (Lorenz-Haken instability) regime [1,2,19]; the latter example is a CPA device for a frequency-modulated (FM) optical signal emitted by a FM-operated laser with an intracavity phase modulator [3,21,22].

A. CPA for a chaotic optical field

The first example we would like to discuss is a CPA device that perfectly absorbs a chaotic optical field emitted by a single-mode homogeneously broadened laser operated in the chaotic (Lorenz-Haken instability) regime [1,2,19]. In this case, the laser system S just contains a homogeneously broadened two-level medium of length l with a small-signal gain coefficient per unit length g , which is provided by population inversion in the medium. In the single longitudinal mode and uniform field approximations, the intracavity field $A(z,t)$ is assumed to be almost uniform along the ring (i.e. almost constant for $0 < z < \mathcal{L}$) and slowly varying in time over one cavity round trip [2]. In this case, map (5) can be effectively replaced by a set of three coupled differential equations describing the evolution of the intracavity field, polarization, and

population inversion in the two-level medium, the so-called Lorenz-Haken model of the single-mode homogeneously broadened laser (see the appendix for technical details). These equations read [2]

$$\begin{aligned}\frac{d\psi}{dt} &= -\kappa(\psi + 2C\Lambda) \\ \frac{d\Lambda}{dt} &= -\gamma_{\perp}[(1 + i\Delta)\Lambda + \psi n] \\ \frac{dn}{dt} &= -\gamma_{\parallel} \left[n - 1 - \frac{1}{2}(\psi\Lambda^* + \psi^*\Lambda) \right]\end{aligned}\quad (15)$$

where $\psi(t)$, $\Lambda(t)$, and $n(t)$ are the intracavity electric field, polarization, and population inversion, respectively, normalized as in Refs. [2,12]; γ_{\parallel} and γ_{\perp} are the population and dipole decay rates, respectively;

$$\kappa = \frac{Tc}{2\mathcal{L}} \quad (16)$$

is the cavity decay rate; $C = gl/T$; and $\Delta = (\omega_0 - \omega)/\gamma_{\perp}$ is the normalized detuning parameter between the cavity resonance frequency ω and the atomic transition frequency ω_0 . In particular, at resonance $\Delta = 0$ (i.e., for $\omega = \omega_0$), Λ and ψ can be taken to be real valued, and Eqs. (15) are analogous to the Lorenz model, developed for convective instabilities in hydrodynamics [1,2]. In the following, we will mainly consider the limit $\Delta = 0$. In this case, the laser threshold is attained for $2C > 1$, at which the nonlasing trivial solution ($\psi = 0, \Lambda = 0, n = 1$) to Eqs. (15) becomes unstable. Above the threshold, the laser equations admit of a steady-state solution, with two possible phases for ψ and Λ (0 and π), namely

$$\psi_{\pm} = \pm\sqrt{2C-1}, \quad \Lambda_{\pm} = \mp\frac{1}{2C}\sqrt{2C-1}, \quad n_{\pm} = \frac{1}{2C}. \quad (17)$$

Such solutions may undergo a subcritical Hopf instability in the bad cavity limit ($\kappa > \gamma_{\parallel} + \gamma_{\perp}$) and for large small-signal gains [1,2]. In such a regime chaotic self-pulsations can be observed. Here, the point $(\psi(t), \Lambda(t), n(t))$ in phase space never settles down but continually makes excursions about one of the two laser solutions (17) with what appear to be random jumps from circling one fixed point to circling the other. The strange set is known as the Lorenz attractor, and the output laser field $E(t) \simeq \sqrt{T}\psi(t)$ turns out to be strongly sensitive to the initial conditions [1,2]. As an example, in Fig. 2(a) we show the evolution of the intracavity laser field $\psi(t)$ as obtained by numerical solution of the Lorenz-Haken equations (15) for parameter values $\kappa/\gamma_{\perp} = 4$, $\gamma_{\parallel}/\gamma_{\perp} = 0.5$, $2C = 40$, $\Delta = 0$ and for the initial condition $\psi(0) = 0.001$, $\Lambda(0) = 0$, $n(0) = 1$. The corresponding projection of the phase-space trajectory in the (ψ, n) plane is depicted in Fig. 2(b), which shows the characteristic Lorenz attractor of the laser dynamics in the chaotic regime. It should be noted that in such a regime the established intracavity laser field (and, hence, the laser output field) is strongly sensitive to the initial conditions. For example, the dotted curve in Fig. 2(a) shows the intracavity field that one would observe for the slightly changed initial conditions $\psi(0) = 0.0011$, $\Lambda(0) = 0$, $n(0) = 1$.

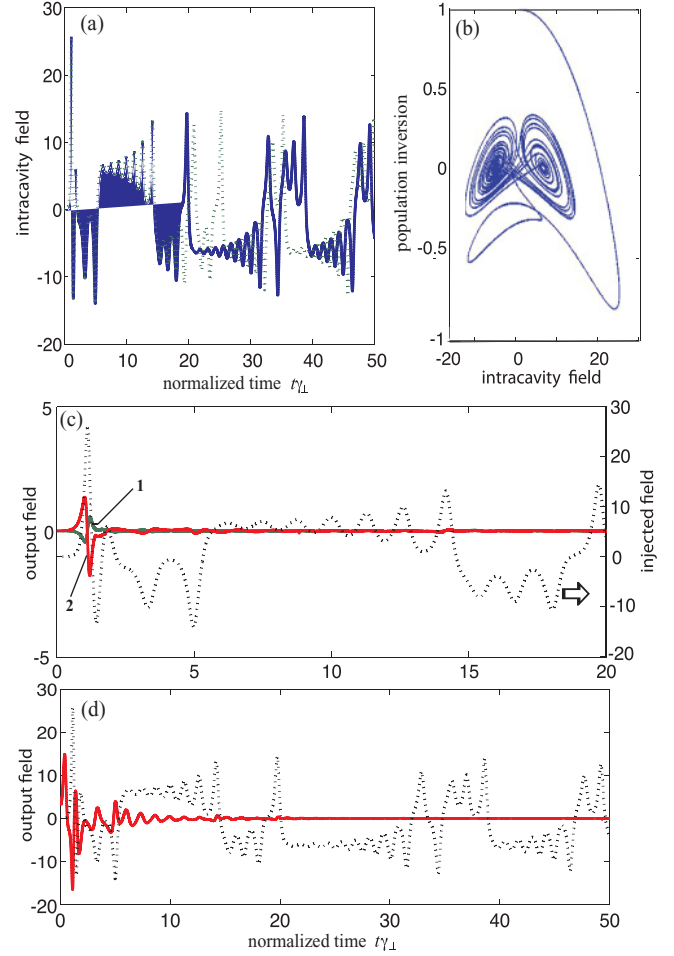


FIG. 2. (Color online) (a) Evolution of the intracavity field $\psi(t)$ in the laser system S as obtained by numerical solution of the Lorenz-Haken equations (15) for the initial condition $[\psi(0) = 0.001, \Lambda(0) = 0, n(0) = 1]$ (solid curve), and (b) corresponding projection of the phase-space trajectory in the (ψ, n) plane. Parameter values are $\kappa/\gamma_{\perp} = 4$, $\gamma_{\parallel}/\gamma_{\perp} = 0.5$, $2C = 40$, and $\Delta = 0$. In (a) the dotted curve shows the evolution of $\psi(t)$ as obtained by the slightly different initial condition $[\psi(0) = 0.0011, \Lambda(0) = 0, n(0) = 1]$. Note the strong sensitivity on the initial condition, which is a rather general feature of a chaotic attractor. In (c) we show the numerically computed evolution of the output field $E_1(t)$, normalized to \sqrt{T} [i.e., the behavior of $\psi_1(t) - \psi(t)$], in the CPA system S' with an injected field $f_1(t)$ and attenuator transmittance T_a satisfying Eqs. (6) and (7), for the initial conditions $[\psi_1(0) = 0.0001, \Lambda_1(0) = 0, n_1(0) = 1]$ (curve 1) and $[\psi_1(0) = 0.005, \Lambda_1(0) = 0, n_1(0) = 1]$ (curve 2). The dotted curve in (c) shows, for comparison, the behavior of the injected field $f_1(t)$, normalized to \sqrt{T} [i.e., the behavior of $\psi(t)$]. (d) Same as in (c) but for a large deviation of initial conditions $[\psi_1(0) = 5, \Lambda_1(0) = 0, n_1(0) = 1]$.

Let us now consider the CPA device S' associated to S , as discussed in Sec. II. In this case, in the single-longitudinal mode and mean-field approximations, the dynamical evolution of the intracavity field $\psi_1(t)$ is governed by a set of coupled equations similar to Eqs. (15), in which the first equation is modified to take into account the effects of the injected field $f_1(t)$ and of the intracavity absorber with transmittance T_a .

One obtains (see the appendix for technical details)

$$\begin{aligned}\frac{d\psi_1}{dt} &= -\kappa(\psi_1 + 2C\Lambda_1) - \kappa_1\psi_1 + \kappa\frac{2f_1(t)}{\sqrt{T}} \\ \frac{d\Lambda_1}{dt} &= -\gamma_{\perp}[(1+i\Delta)\Lambda_1 + \psi_1n_1] \\ \frac{dn}{dt} &= -\gamma_{\parallel}\left[n - 1 - \frac{1}{2}(\psi\Lambda^* + \psi^*\Lambda)\right],\end{aligned}\quad (18)$$

where

$$\kappa_1 = \frac{c(1 - \sqrt{T_a})}{\mathcal{L}} = 2\frac{(1 - \sqrt{T_a})}{T}\kappa. \quad (19)$$

The output field $E_1(t)$ from the CPA device is then given by Eq. (11). According to the general analysis presented in Sec. II, if the injected field is chosen to satisfy the condition (13), it can be readily shown that the solution to Eqs. (18) is given by $\psi_1(t) = \psi(t)$, $\Lambda_1(t) = \Lambda(t)$, $n_1(t) = n(t)$ provided that the same initial conditions $\psi_1(0) = \psi(0)$, $\Lambda_1(0) = \Lambda(0)$, $n_1(0) = n(0)$ are assumed for the field and atomic variables in the two systems S and S' . Furthermore, if the transmittance T_a of the absorber in S' is tuned to satisfy the condition (6), the output field $E_1(t)$ emitted by S' vanishes, i.e., S' behaves as a perfect CPA for the chaotic optical field $\psi(t)$. Note that for the perfect CPA device one has $\kappa_1 = 2\kappa$ and the dynamical system (18) reads

$$\begin{aligned}\frac{d\psi_1}{dt} &= -\kappa(\psi_1 + 2C\Lambda_1) - 2\kappa[\psi_1 - \psi(t)] \\ \frac{d\Lambda_1}{dt} &= -\gamma_{\perp}[(1+i\Delta)\Lambda_1 + \psi_1n_1] \\ \frac{dn_1}{dt} &= -\gamma_{\parallel}\left[n_1 - 1 - \frac{1}{2}(\psi_1\Lambda_1^* + \psi_1^*\Lambda_1)\right],\end{aligned}\quad (20)$$

where $\psi(t)$ is the solution to Eqs. (15) with the assigned initial conditions.

It should be noted that, in a practical case, the laser system S and the CPA system S' cannot be exactly prepared in the same state. In this case, the solution (ψ_1, Λ_1, n_1) to Eqs. (20) deviates from the solution (ψ, Λ, n) of Eqs. (15), just because the initial conditions of field and/or atomic variables differ. Correspondingly, the output field $E_1(t)$ emitted by the CPA device, given by

$$E_1(t) = \sqrt{\frac{T}{R}}[\psi_1(t) - \psi(t)] \simeq \sqrt{T}[\psi_1(t) - \psi(t)], \quad (21)$$

would not vanish, i.e., perfect absorption would be lost. Such a circumstance raises the question whether the CPA idea presented in Sec. II is actually of physical relevance. This objection is especially serious whenever the system dynamics is strongly sensitive to the initial conditions, such as for a chaotic laser, or if the system S' with injected signal may show two (or more) stable attractors, i.e., in the presence of bistability or multistability [23]. In this case, an initial condition of S' sufficiently far from that of S can bring the dynamical system (20) into a *different attractor* than $(\psi(t), \Lambda(t), n(t))$ [23]. Strong deviations of the initial conditions may occur, for example, whenever the pump parameter $2C$ is larger than 3 and system S' thus self-oscillates, i.e., it is above laser

threshold (in the absence of the injected signal) in spite of the attenuator put in the cavity. However, provided that the basin of attraction of $(\psi(t), \Lambda(t), n(t))$ for S' is sufficiently wide, a *large perturbation* can switch the path of the dynamical system toward the “right” CPA attractor $(\psi(t), \Lambda(t), n(t))$. Of course, the basin of attraction of $(\psi(t), \Lambda(t), n(t))$ for S' , as well as the kind and strength of the large perturbations requested to switch the nonlinear dynamics of Eqs. (20) from one attractor to another, should be considered on a case-by-case basis. Let us focus here our attention to the Lorenz-Haken model of laser chaos. In this regime, it is known that even a small change in the value of $\psi(0)$ may deeply modify the output waveform, as shown in Fig. 2(a). However, numerical results show that the basin of attraction of $(\psi(t), \Lambda(t), n(t))$ for Eqs. (20) is quite broad, i.e., asymptotically one has $\psi_1(t) \rightarrow \psi(t)$ for a quite broad range of initial conditions $(\psi_1(0), \Lambda_1(0), n_1(0))$ around $(\psi(0), \Lambda(0), n(0))$. The reason is that the dynamical system (20) [contrary to the Lorenz-Haken dynamical system (15)] is *driven* by the external field $\psi(t)$, which forces $\psi_1(t)$ toward $\psi(t)$ after an initial transient. This is clearly shown in Fig. 2(c), which depicts the output field $E_1(t)$, normalized to \sqrt{T} [i.e., the difference $\psi_1(t) - \psi(t)$; see Eqs. (21)] emitted by the CPA device S' , as obtained by numerical integration of Eqs. (20), for the two initial conditions $(\psi_1(0) = 0.0001, \Lambda_1(0) = 0, n_1(0) = 1)$ [curve 1 in Fig. 2(c)] and $(\psi_1(0) = 0.005, \Lambda_1(0) = 0, n_1(0) = 1)$ [curve 2 in Fig. 2(c)], which appreciably differ from $\psi(0) = 0.001, \Lambda(0) = 0, n(0) = 1$. For comparison, in the figure we show (but on a different scale) the field $f_1(t) = \sqrt{T}\psi(t)$ injected into the cavity, normalized to \sqrt{T} [i.e., the behavior of $\psi(t)$], where $\psi(t)$ is the field of S for the initial condition $(\psi(0) = 0.001, \Lambda(0) = 0, n(0) = 1)$ [i.e., the solid curve in Fig. 2(a)]. Note that, since the initial conditions in S' and S differ, the CPA output field does not exactly vanish in an initial transient [see curves 1 and 2 in Fig. 2(c)]; however, it remains much smaller than the injected field even though the initial conditions in the two systems appreciably differ. Most importantly, after an initial transient, the output field vanishes, and perfect absorption is attained. Such a behavior is observed even for large deviations of initial conditions, as shown in Fig. 2(d) as an example.

B. CPA for a frequency-modulated optical field

As a second example, we discuss the realization of a CPA device for an optical frequency-modulated signal $E(t)$, i.e., an optical field with a constant intensity but with a sinusoidally modulated optical phase. A coherent FM optical signal $E(t)$ is approximately generated by placing a phase modulator inside a laser device and sinusoidally driving it asynchronously as compared to the cavity round-trip time. In this way the laser operates in the so-called frequency-modulation (FM) regime [3,21,22]. The FM regime is a multimode regime in which several cavity axial modes are excited out of resonance with steady-state amplitudes but with time-varying phases, resulting in a constant intensity laser field but with a carrier laser frequency which is sinusoidally swept at the frequency impressed by the phase modulator [3]. As shown in Ref. [22], a pure FM signal at optical carrier can be generated provided that the phase modulation is sufficiently asynchronous such that the bandwidth of the FM signal is much smaller than

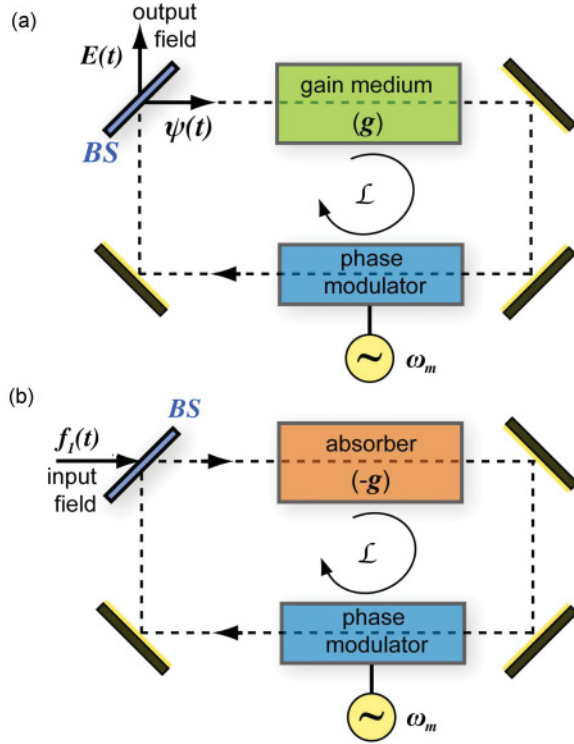


FIG. 3. (Color online) Schematic of (a) a FM-operated laser S , made of a ring-cavity containing a gain medium (gain coefficient g) and a phase modulator, driven at a frequency ω_m detuned from the frequency separation ω_{ax} of cavity axial modes, and (b) the corresponding CPA system S' , which is obtained from S by replacing the gain medium with an absorber with absorption coefficient $-g$.

the gain bandwidth of the active medium. A schematic of a FM-operated laser is shown in Fig. 3(a). The optical cavity contains a gain medium and a phase modulator, which is sinusoidally driven at a frequency ω_m which is detuned enough from the cavity axial mode separation $\omega_{ax} = 2\pi/T_R$, where T_R is the cavity photon transit time [3]. For a slow gain medium and neglecting dispersive and finite gain bandwidth effects, the cavity round-trip operator reads [22]

$$\hat{P}(t, \psi) = \exp(g) \exp[i \Delta \cos(\omega_m t)] \psi(t - T_R), \quad (22)$$

where g is the single-pass saturated gain in the active medium and Δ is the modulation depth impressed by the phase modulator. The self-consistent equation for the field $\psi(t)$ Eq. (3) then reads

$$\psi(t) = \exp[g - \gamma + i \Delta \cos(\omega_m t)] \psi(t - T_R), \quad (23)$$

where $\gamma = -\ln\sqrt{R}$ is the logarithmic loss of the cavity due to the output coupling. Equation (23) should be supplemented with a rate equation for the saturated gain g (see, for instance, Ref. [24]). After an initial transient, a steady-state operation is achieved, in which the saturated gain settles down to the stationary value $g = \gamma$ [24]. Correspondingly, the intracavity field $\psi(t)$ is given by one of the “modes” of the phase-modulated cavity (generally the one with the carrier frequency closest to the center of the gainline). Such modes are simply obtained as eigenfunctions of Eq. (23) and read

explicitly [22,24]

$$\psi(t) = \exp[i \Gamma \cos(\omega_m t + \varphi) + i l \omega_{ax} t]. \quad (24)$$

In Eq. (24), $l = 0, \pm 1, \pm 2, \dots$ is the mode index, Γ is the effective modulation index, and φ a phase offset. The values of Γ and φ are obtained after substitution of the ansatz (24) into Eq. (23) and equating the imaginary terms in the exponentials on the left- and right-hand sides of the equation so obtained. This yields

$$\Gamma = \frac{\Delta}{2 \sin(\pi \omega_m / \omega_{ax})}, \quad (25)$$

$$\cos \varphi = \frac{\Delta}{2\Gamma}. \quad (26)$$

The output field, $\sim \psi(t) e^{i \omega t}$, is thus a pure FM signal, i.e., a frequency comb, with spectral lines at frequencies $n \omega_m$ ($n = 0, \pm 1, \pm 2, \dots$) around the carrier frequency $\omega + l \omega_{ax}$ of amplitudes $\sim J_n(\Gamma)$, where J_n is the Bessel function of the first kind and of order n and ω is a reference cavity resonance frequency. According to Eq. (25), the effective modulation index Γ increases and diverges as the synchronous modulation condition $\omega_m = \omega_{ax}$ is attained. Indeed, in practical FM-operated lasers large modulation indices are usually obtained by tuning the modulation frequency ω_m close to the cavity axial mode spacing ω_{ax} (see, for instance, Refs. [21,25]). However, the solution (24) provides an accurate approximation to the output field of the FM-operated laser provided that the modulation frequency ω_m remains sufficiently detuned far apart from ω_{ax} in such a way that the spectral extent $\sim 2\Gamma \omega_m$ of the FM signal $\psi(t)$ is much smaller than the gain linewidth ω_g , i.e., provided that the following condition

$$|\omega_m - \omega_{ax}| \gg \frac{\omega_{ax} \omega_m \Delta}{\pi \omega_g} \quad (27)$$

is satisfied.

Indeed, as the synchronous modulation condition $\omega_m = \omega_{ax}$ is approached, laser operation switches into a pulsed regime (FM mode-locking), with the generation of a periodic train of short optical pulses [3,26].

Let us now discuss the possibility of perfectly absorbing the FM signal emitted by laser S . According to the analysis of Sec. II, a CPA system S' that perfectly absorbs the FM signal can be obtained from laser S by placing a linear absorber with a transmittance $T_a = R^2$ inside the cavity. Note that, as $g = \gamma = -\ln\sqrt{R}$ and since we neglected gain bandwidth and dispersion effects, the combined effect of the gain and absorber media in laser S' is equivalent to that of a single linear absorber placed in the cavity with an absorption coefficient $-g$, as shown in Fig. 3(b). In other words, for this special example we retrieve the simple rule of Ref. [5] that the CPA system is obtained from the lasing system by replacing the gain medium with an absorber with an amount of dissipation exactly opposite to the amplification factor in the lasing medium at threshold. We stress, however, that the CPA system S' that realizes the time reverse of the lasing system S , obtained by the general procedure outlined in Sec. II, does not generally correspond to the replacement of the gain medium with an absorber, as discussed for instance in the example of Sec. III A.

IV. CONCLUSION

In conclusion, in this work we have extended the concepts of time reversal of lasing and CPA, recently introduced in Ref. [5], to the case of optical radiation emitted by a laser operated in an arbitrary regime, i.e., for transient, chaotic, or periodic coherent optical fields. We have proven rather generally that any electromagnetic signal $E(t)$ generated by a laser system S operated in an arbitrary (and generally highly nonlinear) regime can be perfectly absorbed by a CPA device S' which is simply realized by placing inside S a broadband linear absorber (attenuator) of appropriate transmittance. As examples, we discussed CPA devices that perfectly absorb a chaotic laser signal and a frequency-modulated optical wave.

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APPENDIX: CPA FOR CHAOTIC OPTICAL FIELDS: DERIVATION OF THE MEAN-FIELD EQUATIONS

In this section we briefly derive the mean-field equations (15) and (18) given in the text from the general maps (5) and (12) in case where the functional \hat{P} describes field propagation in a homogeneously broadened two-level medium of length l . We will perform such a derivation for the more general case of the system S' of Fig. 1(b) with arbitrary injected signal $f_1(t)$ and transmittance T_a of the absorber; the laser system S of Fig. 1(a) is simply obtained after setting $f_1 = 0$, $T_a = 0$. Field propagation inside the ring cavity is described by the Maxwell-Bloch equations (see Sec. 4.3 of Ref. [2]; see also Ref. [12])

$$\begin{aligned}\frac{\partial A_1}{\partial z} &= -\frac{1}{c} \frac{\partial A_1}{\partial t} - g \Lambda_1 \\ \frac{\partial \Lambda_1}{\partial t} &= -\gamma_{\perp} [(1 + i\Delta)\Lambda_1 + n_1 A_1] \\ \frac{\partial n_1}{\partial t} &= -\gamma_{\parallel} \left[n_1 - 1 - \frac{1}{2}(A_1^* \Lambda_1 + A_1 \Lambda_1^*) \right],\end{aligned}\quad (\text{A1})$$

where $A_1(z, t)$, $\Lambda_1(z, t)$, $n_1(z, t)$ are the intracavity electric field envelope, polarization envelope, and population inversion, respectively, normalized as in Refs. [2,12]; γ_{\parallel} and γ_{\perp} are the population and dipole decay rates, respectively; g is the small-signal gain per unit length in the active medium; and $\Delta = (\omega_0 - \omega)/\gamma_{\perp}$ is the normalized detuning parameter between the cavity resonance frequency ω and the atomic transition frequency ω_0 . For the sake of definiteness, the gain medium (of length l) is placed at $0 < z < l$; similarly, we assume that the attenuator T_a is a thin plate placed close to the beam splitter BS [see Fig. 1(b)]. In this way field attenuation after passage across the absorber can be included in the scattering matrix of the beam splitter BS. This yields the following ring-cavity boundary condition for the field $A_1(z, t)$

at the planes $z = 0^+$ and $z = \mathcal{L}^-$,

$$A_1(0^+, t) = \sqrt{T} f_1(t) + \sqrt{RT_a} A_1(\mathcal{L}^-, t). \quad (\text{A2})$$

The output field $E_1(t)$ escaping from the cavity through the beam splitter BS is then given by

$$E_1(t) = \sqrt{\frac{T}{R}} A_1(0^+, t) - \frac{f_1(t)}{\sqrt{R}}. \quad (\text{A3})$$

Obviously, n_1 and Λ_1 in Eqs. (A1) are defined in the range $0 < z < l$, and for $z > l$ (i.e., in the empty cavity region) the first equation of (A1) is still valid provided that we assume $g = 0$. The field $A_1(\mathcal{L}^-, t)$ at $z = \mathcal{L}^-$ and time t entering in Eq. (A2) can be computed as a functional of the field $A_1(0^+, t - T_R)$ at plane $z = 0^+$ and time $t - T_R$, where $T_R = \mathcal{L}/c$ is the cavity transit time. In fact, after introduction of the new variables

$$\xi = z, \quad \eta = t - z/c \quad (\text{A4})$$

Eqs. (A1) take the form

$$\begin{aligned}\frac{\partial A_1}{\partial \xi} &= -g \Lambda_1 \\ \frac{\partial \Lambda_1}{\partial \eta} &= -\gamma_{\perp} [(1 + i\Delta)\Lambda_1 + n_1 A_1] \\ \frac{\partial n_1}{\partial \eta} &= -\gamma_{\parallel} \left[n_1 - 1 - \frac{1}{2}(A_1^* \Lambda_1 + A_1 \Lambda_1^*) \right].\end{aligned}\quad (\text{A5})$$

Equations (A5) can be integrated in the interval $0 < \xi < l$ and for $0 < \eta < T_R$ once the initial conditions $A_1(\xi = 0^+, \eta)$ and $\Lambda_1(\xi, \eta = 0)$, $n_1(\xi, \eta = 0)$ are assigned [initial-boundary value (Goursat) problem [17]]. Typically, for a medium initially at equilibrium in the absence of the field, one can take $\Lambda_1(\xi, \eta = 0) = 0$ and $n_1(\xi, \eta = 0) = 1$. In this way, one can compute $A(\xi = \mathcal{L}^-, \eta) = A(\xi = l, \eta)$ for $0 < \eta < T_R$. In the physical space-time variables (z, t) , this means that we know the value of $A_1(\mathcal{L}^-, t)$ in the time interval $T_R < t < 2T_R$. With such a solution, from Eq. (A2) one can then calculate $A_1(0^+, t)$ in the time interval $T_R < t < 2T_R$, which serves as an initial condition to integrate Eqs. (A5) in the interval $0 < \xi < l$ and for $T_R < \eta < 2T_R$ (second round trip) with the appropriate initial values of $\Lambda_1(\xi, \eta = T_R)$ and $n_1(\xi, \eta = T_R)$ computed at the previous step. Iteration of such a procedure enables one to calculate $A_1(0^+, t) = \psi_1(t)$ at any time $t > 0$, and, thus, the output field $E_1(t)$ according to Eq. (A3).

An important case is the one obtained by taking the limit $gl \rightarrow 0$. In this case, from Eqs. (A5) it follows that the variables A_1 , Λ_1 , and n_1 are almost constant functions with respect to the ξ variable, i.e., $A_1(\xi, \eta) \sim A_1(0, \eta)$, $\Lambda_1(\xi, \eta) \sim \Lambda_1(0, \eta)$, and $n_1(\xi, \eta) \sim n_1(0, \eta)$. The small change $A_1(\xi = \mathcal{L}^-, \eta) - A_1(\xi = 0^+, \eta)$ can be computed from the first of Eqs. (A5) as $A_1(\xi = \mathcal{L}^-, \eta) - A_1(\xi = 0^+, \eta) \simeq -gl\Lambda_1(\xi = 0, \eta)$, which in terms of the space-time physical variables (z, t) reads

$$A_1(\mathcal{L}^-, t + T_R) \simeq A_1(0^+, t) - gl\Lambda_1(0, t). \quad (\text{A6})$$

Indicating by $\psi_1(t) = A_1(z = 0^+, t)$, $\Lambda_1(t) = \Lambda_1(z = 0, t)$, and $n_1(t) = n_1(z = 0, t)$, from Eqs. (A2) and (A6) it follows that $\psi_1(t)$ satisfies the delayed equation

$$\psi_1(t + T_R) = \sqrt{T} f_1(t + T_R) + \sqrt{RT_a} [\psi_1(t) - gl\Lambda_1(t)], \quad (\text{A7})$$

which should be associated to the differential equations for $\Lambda_1(t)$ and $n_1(t)$

$$\begin{aligned} \frac{d\Lambda_1}{dt} &= -\gamma_{\perp}[(1 + i\Delta)\Lambda_1 + n_1\psi_1] \\ \frac{dn_1}{dt} &= -\gamma_{\parallel} \left[n_1 - 1 - \frac{1}{2}(\psi_1^*\Lambda_1 + \psi_1\Lambda_1^*) \right]. \end{aligned} \quad (\text{A8})$$

In their present form, Eqs. (A7) and (A8) represent a system of coupled differential-delayed equations that should be integrated with the initial conditions $n_1(0)$, $\Lambda_1(0)$, and $\psi_1(t)$ for $0 < t < T_R$. The common mean-field and single-mode laser model [1,2] is obtained from Eqs. (A7) and (A8) by taking the further limits $T = 1 - R \rightarrow 0$, $T_a \rightarrow 1$, with $C \equiv gl/T$ finite, and assuming that the injected and intracavity fields $f_1(t)$ and $\psi_1(t)$ vary slowly with respect to time over one cavity transit time T_R . Under such assumptions, the delayed

equation (A7) can be replaced by the following differential equation:

$$\psi_1(t) + T_R \frac{d\psi_1}{dt} \simeq \sqrt{T} f_1(t) + \sqrt{T_a} \psi_1(t) - \frac{T}{2} \psi_1(t) - gl\Lambda_1(t) \quad (\text{A9})$$

i.e.,

$$\frac{d\psi_1}{dt} \simeq -\kappa[\psi_1(t) + 2C\Lambda_1(t)] - \kappa_1\psi_1(t) + \kappa \frac{2f_1(t)}{\sqrt{T}}, \quad (\text{A10})$$

where κ and κ_1 are defined by Eqs. (16) and (19) given in the text. The system of coupled differential equations (A8) and (A10) is precisely the modified single-mode Lorenz-Haken laser model (18) given in the text. Note that the usual Lorenz-Haken laser model (15) is simply obtained after setting $f_1 = 0$ and $T_a = 1$ (i.e., $\kappa_1 = 0$).

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