Fast periodic modulations in the photon correlation of single-mode vertical-cavity surface-emitting lasers

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We report on the observation of periodic modulations in the second-order photon correlation of single-mode vertical-cavity surface-emitting lasers (VCSELs) in steady-state operations. We realized the fast photon correlation measurement of VCSELs by using a streak camera operated in a photon-counting mode. Fast modulation signals were observed in second-order photon correlations at and around the threshold. The calculations using photon rate equations that include a gain medium dynamics shows that the oscillating signals originate from relaxation oscillations.

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I. INTRODUCTION

From the viewpoint of the practical application of lasers, one of the most important properties is the smallness of intensity noises (or intensity fluctuations). The theory of laser noise has been well established based on lasers with a large cavity volume and small gain, such as He:Ne lasers [1]. However, most of the currently used lasers are semiconductor and small solid-state lasers. In particular, the sizes of semiconductor diode lasers are continuously being reduced. These lasers have high gain media and high-Q cavities with a small mode volume. Therefore, their noise studies show unique properties, including cavity quantum electrodynamics effects [2,3], photon-number squeezed light generation [4], and semiconductor effects [5]. Moreover, there is growing interest in a new type of photon source called cavity-polariton Bose-Einstein condensate [6,7], which has a different mechanism for the generation of coherent photons. The photon correlation measurement is a method that can evaluate the intensity noise property of lasers in the time domain. The coherence time of semiconductor diode lasers around a threshold is relatively short, and thus a photon-counting detector that has picosecond time resolution is required. For this purpose, a photon-counting streak camera was developed about 25 years ago. Using this device, Ueda *et al.* [8] demonstrated photon correlation measurements on a picosecond time scale. Furthermore, several authors [9–12] have recently employed a photon-counting streak camera for photon correlation measurements of lasers.

In this paper, we report on the fast second-order photon correlation measurements of single-mode vertical-cavity surface-emitting lasers (VCSELs) using a photon-counting streak camera. The evaluation of the photon correlation properties of VCSELs is important from both practical and scientific viewpoints. The noise property of VCSELs in a steady-state operation is an important property for the channel capacity of optical communications. In terms of scientific interest, VCSELs are many-body correlated electron-hole systems that interact with photons in a small cavity. Therefore, their dynamical property on a fast time scale could elucidate some new aspects of laser physics. The second-order correlation function $g^{(2)}(\tau)$ is defined as

$$g^{(2)}(\tau) = \frac{\langle \mathcal{T} : \hat{n}(0)\hat{n}(\tau) :\rangle}{\langle \hat{n}\rangle^2}, \qquad (1)$$

where $\hat{n}(t) = \hat{a}^\dagger(t)\hat{a}(t)$ is the photon number operator and $\langle \mathcal{T} ::\rangle$ represents the time and normal order of the operators. The second-order photon correlation function at zero delay $g^{(2)}(0)$ reflects the photon statistical property of a light source. For a light with super Poissonian statistics like a thermal light, $g^{(2)}(0) > 1$, while for a light with Poissonian statistics, $g^{(2)}(0) = 1$. Lasers behave as thermal light sources below the threshold with $g^{(2)}(0) > 1$, while they emit a light with Poissonian statistics above the threshold with $g^{(2)}(0) = 1$. More insight is gleaned from the dynamic evolution of the second-order correlation $g^{(2)}(\tau)$, which represents the dynamical property of a light field.

II. EXPERIMENT

We used a streak camera (C5680 Hamamatsu), for which the best time resolution is 2 ps in a minimum dynamic range. We used the photon-counting mode of the streak camera. The key point is the time-resolved recording of photon arrival times. Photoelectrons are generated when photons hit a photocathode. The photoelectrons are swept by time-dependent voltages and hit a high-gain two-dimensional (2D) image intensifier with a phosphorus screen. The afterglow of the phosphorus screen is recorded by the CCD camera as a streak image (Fig. 1). The streak camera and the CCD camera are synchronized with an external signal generator. The exposure time of the CCD camera is 6 ms. The streak image is in the form of a 2D picture (Fig. 1) whose vertical axis represents the times at which photons reach the camera, while the horizontal axis has spatial information. For every streak image, the times and positions of single-photon events are obtained and are then registered by finding the center of the afterglow spot in the streak image (see the inset in Fig. 1). Furthermore, we allocate the photons taken in the streak images into time bins. The second-order photon correlation

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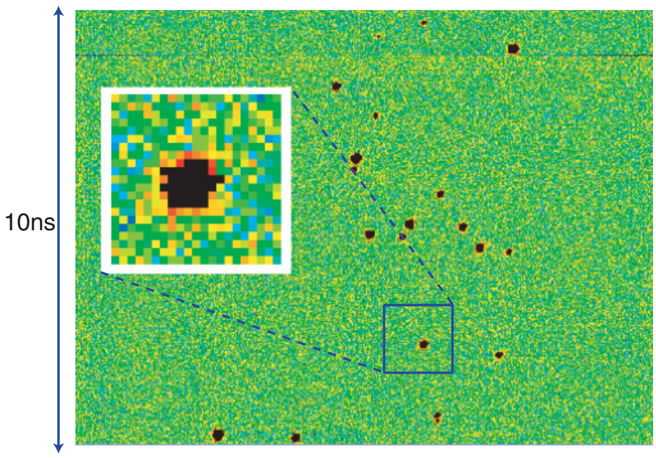

FIG. 1. (Color online) Streak image of the emission from VCSELs. This 2D image consists of 640×480 pixels. Black dots represent the afterglows of the photons on a phosphorus screen. The vertical axis represents the arrival times of photons. The horizontal axis represents the spatial position of photons. The inset is the magnified view of a single-photon afterglow.

function $g^{(2)}(\tau)$ can be calculated from the time-binned records of the stream of photons. With the present method, the photon correlations of a continuous-wave light source can be measured; on the other hand, the method reported by Wiersig *et al*. [10,11] is applied to pulsed light sources that are synchronized with the scanning of the streak camera.

We examined a commercial red VCSEL device (Optwell SS67-4U001) with a gain region of GaInP multi-quantum wells. The lasing threshold of the VCSEL was identified as 2.44 mA from the input-output curve (Fig. 2). Using a spectrometer, we confirmed that the VCSEL operates, both below and above the threshold, in a single longitudinal mode (see the inset in Fig. 2). A single-mode optical fiber was used in order to single out a TEM_{00} transverse mode from the emission of the VCSEL. In addition, we have confirmed that the emission of the VCSEL is linearly polarized. All experiments were performed at room temperature. We measured the second-order photon correlation functions of the VCSEL by increasing the injection current from 2.47 to 3.00 mA. A neutral density (ND) filter was used to reduce the intensity of the emission from the VCSEL. This attenuation process and the quantum efficiency of the streak camera do not change the value of $g^{(2)}(\tau)$ because of the linear loss independence of $g^{(2)}(\tau)$ [13]. Photon correlation measurements were not available for injection currents below 2.47 mA. This is because the output intensity of the VCSEL was too small and the average number of counted photons per a single streak image was much less than 1.

About 300 000 streak images were used to calculate the second-order correlations. The frame rate of the CCD camera was 150 Hz. We set the time bin to 125 ps. By changing the time bin around 125 ps, we confirmed that this choice does not significantly affect the value of the second-order photon correlation functions. We have measured $g^{(2)}(\tau)$ of a single-mode He:Ne laser and thus confirmed the reliability of our measurement method.

III. RESULT AND DISCUSSION

Figure 3 shows $g^{(2)}(0)$ as a function of the injection current; $g^{(2)}(0)$ decreased with an increase of the injection current. From Fig. 3, we can clearly see that the emission of the VCSEL changes from thermal to coherent with an increase in the injection current. Figure 4 shows photon correlation functions at three different injection currents. At 2.47 mA, we observed a modulation of $g^{(2)}(\tau)$ with a fast decay. At around 2.72 mA, the duration of the modulation increased. Above 3.00 mA, no oscillations were observed. Now, we show that these fast periodic modulations around the threshold can be explained in the framework of relaxation oscillations. Relaxation oscillations of lasers are usually measured as a transient turn-on dynamics [14]. However, in our experiment the VCSEL was in a steady-state operation. Even under the steady-state operation, the intensity of a laser is constantly driven by noise. The cavity feedback of such perturbed fields

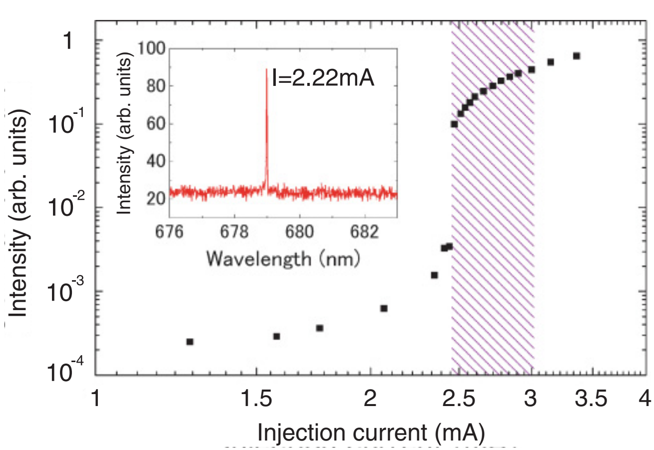

FIG. 2. (Color online) Input output characteristics of the VCSEL. The inset shows the emission spectrum at 2.22-mA current. The shaded area highlights the region where photon correlation measurements were performed (from 2.47 to 3.00 mA).

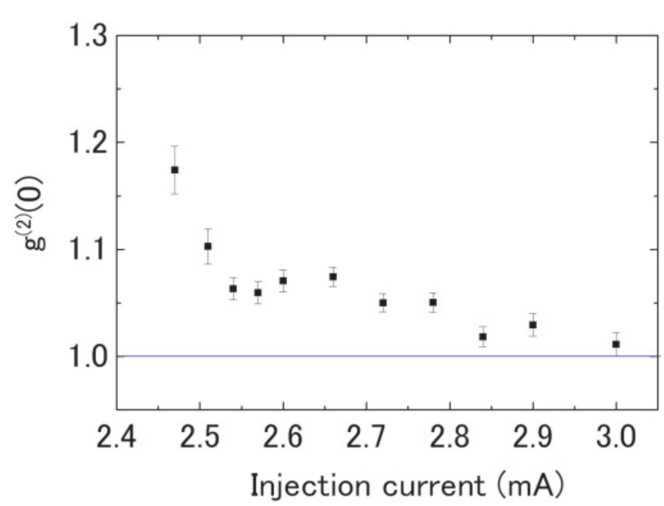

FIG. 3. (Color online) Second-order photon correlation with zero delay $g^{(2)}(0)$ as a function of the injection current.

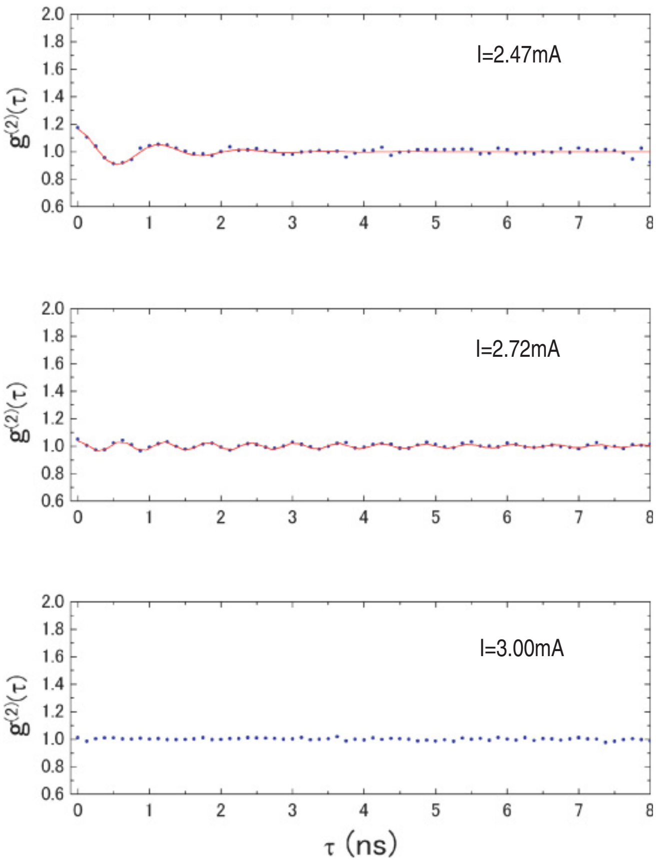

FIG. 4. (Color online) Second-order photon correlation function $g^2(\tau)$ for various injection currents.

induces instability and triggers a relaxation oscillation. In such situations, periodic modulations appear in the second-order correlations [15,16].

We may quantitatively model our system using two coupled rate equations for the photon number n and the population inversion N [3,16–19]:

$$\begin{aligned} \dot{n} &= -\Gamma_c n + \beta\gamma_\parallel N n + R_{sp}, \\ \dot{N} &= P - \gamma_\parallel N - \beta\gamma_\parallel N n, \end{aligned} \tag{2}$$

where P is the pump rate, β is the fraction of spontaneous emission going into the lasing mode, and $R_{sp} = N\beta\gamma_\parallel$ is the spontaneous emission rate. Γ_c and $\gamma_\parallel$ represent the cavity photon decay rate and the population-inversion decay rate, respectively. The lasers described by these rate equations are called class-B lasers [17]. We assume that Γ_c is much larger than $\gamma_\parallel$. We write $n = n_0 + \delta n$ and $N = N_0 + \delta N$, where δn and δN are the photon and population-inversion number fluctuations, respectively. Linearizing Eq. (2) around the steady state, one can find the expressions for δn and δN:

$$\frac{d}{dt}\begin{pmatrix} \delta n \\ \delta N \end{pmatrix} = \begin{pmatrix} -\gamma_n & \beta\gamma_\parallel(n_0+1) \\ -\beta\gamma_\parallel N_0 & -\gamma_N \end{pmatrix}\begin{pmatrix} \delta n \\ \delta N \end{pmatrix}, \tag{3}$$

where $\gamma_n = \Gamma_c/(n_0+1)$ and $\gamma_N = \gamma_\parallel(1+\beta n_0)$. One can easily find the solution of Eq. (3). The photon number fluctuation δn can be expressed as

$$\delta n(t) = \delta n(0) e^{-\gamma_r t}\cos(\omega_r t + \psi_0), \tag{4}$$

where

$$\begin{aligned} \omega_r &= \sqrt{\beta\gamma_\parallel\Gamma_c n_0 - (\gamma_n - \gamma_N)^2/4} \\ &\approx \sqrt{\beta\gamma_\parallel\Gamma_c n_0 - \gamma_n^2/4} \quad (\Gamma_c \gg \gamma_\parallel) \end{aligned} \tag{5}$$

is the resonance frequency and

$$\gamma_r = (\gamma_n + \gamma_N)/2 \tag{6}$$

is the damping rate of the relaxation oscillation [16]. $\delta n(0)$ and ψ_0 are determined from the initial conditions. Using Eq. (4), we can obtain the second-order photon correlation function as

$$\begin{aligned} g^{(2)}(\tau) &= 1 + \frac{\langle \delta n(0)\delta n(\tau)\rangle}{n_0^2} && (7) \\ &= 1 + \frac{\langle \delta n(0)^2\rangle}{n_0^2} e^{-\gamma_r \tau}\cos(\omega_r\tau + \psi_0). && (8) \end{aligned}$$

These equations indicate that the second-order correlation function experiences a damped oscillation. The observed periodic modulations in Fig. 4 are well fit by Eq. (8), which indicates that our experimental result can be interpreted in the framework of the relaxation oscillations presented above. In Eq. (8), the term $\langle \delta n(0)^2\rangle/n_0^2$ refers to the excess noise of the laser. As shown in Fig. 3, at around the threshold, $g^{(2)}(0)$ is larger than 1. This means that the large excess noise triggers a relaxation oscillation, which is the origin of the periodic modulation of $g^{(2)}(\tau)$ (see Fig. 4, $I = 2.71$ mA). Above the threshold, $g^{(2)}(0)$ tends to 1, which indicates that the excess noise becomes smaller; therefore, the periodic modulation in $g^{(2)}(\tau)$ disappears (see Fig. 4, $I = 3.00$ mA). The fitting of $g^{(2)}(\tau)$ by Eq. (8) allows us to obtain the resonance frequencies and damping rate of the relaxation oscillations at various injection currents and is summarized in Fig. 5. The resonance frequency increases from 0.86 to 1.65 GHz as injection current increases from 2.47 to 2.72 mA. The square of the resonance frequency has an approximate linear dependence on the injection current, which is a characteristic of relaxation oscillations above the threshold [see Fig. 4(a)] [20]. The damping rate decreases from 0.97 to 0.14 ns^{-1} as the injection current increases from 2.47 to 2.72 mA. Equation (6) indicates that with the increase in the photon number, the damping rate decreases and reaches a minimum slightly above the threshold and then increases again [19]. Thus, our experimental result is consistent with the photon rate equation analysis.

Finally, we would like to comment on the relationship between our experiments and relative intensity noise (RIN) measurements that are often used for the noise analyses of semiconductor lasers. RIN is connected to the second-order photon correlation function through the Fourier transformation as

$$\begin{aligned} S(\omega) &= \frac{e^2\eta\langle\hat{n}\rangle}{2\pi} + \frac{e^2\eta^2}{2\pi}\int_{-\infty}^{\infty}\langle \mathcal{T} : \delta\hat{n}(0)\delta\hat{n}(\tau) :\rangle e^{i\omega\tau} d\tau \\ &= \frac{e^2\eta\langle\hat{n}\rangle}{2\pi} + \frac{e^2\eta^2\langle\hat{n}\rangle^2}{2\pi}\int_{-\infty}^{\infty}[g^{(2)}(\tau) - 1] e^{i\omega\tau} d\tau, \end{aligned} \tag{9}$$

where e is the elementary electric charge and η is the quantum efficiency of a detector [21–23]. The large excess noise around

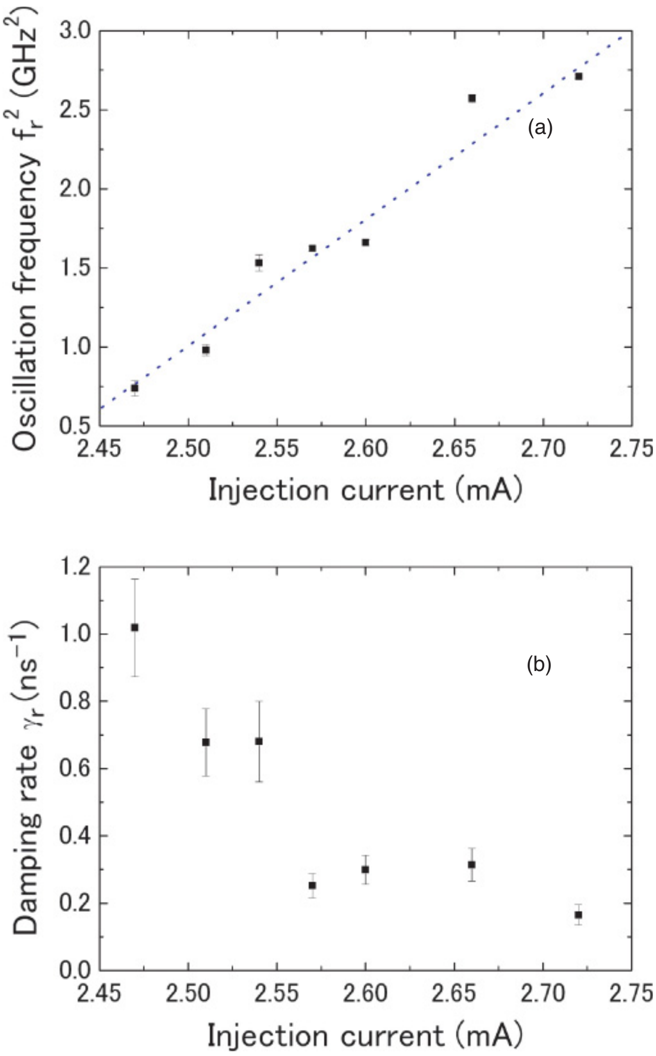

FIG. 5. (Color online) (a) Square of resonance frequency $f_r^2 = (\omega_r/\pi)^2$ vs injection current obtained from $g^{(2)}(\tau)$ measurements. The dashed line represents a linear fit. (b) Damping rate vs injection current for the same device.

the threshold of VCSELs and its decrease with increasing injection currents have been reported in RIN measurements [4]. In the RIN of VCSELs, relaxation oscillations are measured as peaks of a noise spectrum [20]. Thus, our observations agree with the preceding results in RIN measurements. Furthermore, Eq. (9) indicates the possibility that our technique can be used for measuring ultrafast relaxation oscillations that are not available due to the limited bandwidth of conventional photodiodes.

IV. CONCLUSION

In conclusion, we demonstrated the fast second-order photon correlation measurements of commercial single-mode VCSELs using a streak camera under photon-counting operation. Fast periodic modulations were observed in the second-order photon correlations of the VCSELs, which can be interpreted as relaxation oscillations. The relaxation oscillations evidence that the measured VCSELs are class-B lasers: the dynamics of the gain medium must be included in the rate equations. Although detailed discussions on the many-body correlation effects of carriers in VCSELs are beyond the scope of this paper, VCSELs have the potential to elucidate the physics of many-body correlated electron and hole systems that interact with photons [24,25]. The microscopic theory of relaxation oscillations of semiconductor lasers has been discussed recently [26,27]. A detailed comparison between photon correlations of VCSELs and theories beyond the linearized rate equations will reveal the nature of dynamically correlated electron-photon coupled systems.

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