PHYSICAL REVIEW A **85**, 053613 (2012)

Frustration and time-reversal symmetry breaking for Fermi and Bose-Fermi systems

Krzysztof Sacha, Katarzyna Targońska, and Jakub Zakrzewski
Instytut Fizyki imienia Mariana Smoluchowskiego and Mark Kac Complex Systems Research Center, Uniwersytet Jagielloński, ulica Reymonta 4, PL-30-059 Kraków, Poland
(Received 13 December 2011; published 11 May 2012)

The modulation of an optical lattice potential that breaks time-reversal symmetry enables the realization of complex tunneling amplitudes in the corresponding tight-binding model. For a superfluid Fermi gas in a triangular lattice potential with complex tunnelings, the pairing function acquires a complex phase, so the frustrated magnetism of fermions can be realized. Bose-Fermi mixtures of bosonic molecules and unbound fermions in the lattice also show interesting behavior. Due to boson-fermion coupling, the fermions become enslaved by the bosons and the corresponding pairing function takes the complex phase determined by the bosons. In the presence of bosons the Fermi system can reveal both gapped and gapless superfluidity.

DOI: 10.1103/PhysRevA.85.053613 PACS number(s): 03.75.Lm, 67.85.Pq, 74.20.Fg

I. INTRODUCTION

Cold atoms in optical lattices provide a unique medium for mimicking effects known from other areas of physics. This is primarily due to the great flexibility and the possibility of precise manipulation of cold atomic systems [1–3]. Atoms of a fermionic or bosonic character may be placed in an optical lattice potential whose geometry may be easily controlled by changing the directions and/or polarizations of laser beams. Interactions between atoms may be controlled via magnetic, optical, or microwave Feshbach resonances [4,5]. The change in the depth of the optical lattice modifies primarily the tunneling between lattice sites (having also an effect on the effective interaction strength), enabling, e.g., the superfluid–Mott-insulator quantum phase transition in the optical realization of the Bose-Hubbard model as proposed by Jaksch *et al.* [6] and subsequently demonstrated in Ref. [7].

Another spectacular way of controlling the tunneling has been proposed by Eckardt, Weiss, and Holthaus [8]. Fast periodic modulations of the optical lattice allow for an effective, time-averaged tunneling to be totally suppressed, keeping the depth of the lattice potential unchanged. By varying the strength of the modulation one can induce the superfluid–Mott-insulator quantum phase transition [8] as was verified experimentally a few years later [9,10]. Importantly, not only the magnitude, but also the sign of the tunneling amplitudes can be altered using this approach. This concept has been utilized in a recent proposition to create frustrated magnetism with cold bosons in a triangular lattice [11], later implemented in fascinating experiments of Struck and co-workers [12].

The effective tunneling caused by periodic lattice modulations is adequately explained in the framework of the Floquet theory [8] for periodically time-dependent Hamiltonians. The properties of the corresponding quasienergy spectra are known to depend on the global symmetries of the Hamiltonian [13–15]. Employing a similar idea, we demonstrate that periodic perturbations that break time-reversal invariance (TRI) not only can change the sign of the tunneling amplitudes, but may also induce them to have complex values.

In the following, we concentrate on superfluid fermions in the Bardeen-Cooper-Schrieffer (BCS) regime with broken TRI. We show that typically a pairing function for s-wave interactions acquires a complex phase which may be controlled by the TRI-breaking mechanism considered in the present paper (for a discussion of p-wave orbitals see [16]). We also point out that in a Bose-Fermi mixture with broken TRI the complex ground state of bosons affects the Fermi pairing function. The effect is reminiscent of the disorder-induced phase control in such mixtures discussed by one of us recently [17]. In our case the phase control is not due to disorder but to control over the tunneling mechanism and the TRI breaking.

II. BREAKING TIME-REVERSAL SYMMETRY

A. One-dimensional optical lattice

Let us begin, for simplicity, with a single particle in a one-dimensional (1D) optical lattice potential driven by a double harmonic perturbation. The Hamiltonian of the system reads

$$H_0 = \frac{p^2}{2m} + V(x) + K_1 x\cos(\omega t) + K_2 x\cos(2\omega t + \varphi), \qquad (1)$$

where $V(x) = V(x+a)$ is an optical lattice potential with the lattice constant a, and $K_{1,2}$ stand for strengths of the driving at the basic frequency ω and its second harmonic, respectively. The Hamiltonian is time periodic, i.e., $H_0(t + 2\pi/\omega) = H_0(t)$, and the Floquet theorem [18–20] guarantees that the so-called Floquet Hamiltonian

$$\mathcal{H} = H_0 - i\hbar\partial_t \qquad (2)$$

is diagonalized by periodic functions. Eigenvalues of $\mathcal{H}$ are referred to as quasienergies of the system, analogous to quasimomenta in solid state physics. They are defined modulo $\hbar\omega$, and it is sufficient to consider a single Floquet zone (an analog of the Brillouin zone). Periodic eigenfunctions of the Floquet Hamiltonian can be expanded in a Fourier series, i.e., in a basis where time t can be considered as an additional *degree of freedom*. Let us define the basis vectors

$$\phi_{j,m}(x,t) = \exp\left\{-ix\left[\frac{K_1}{\omega}\sin(\omega t) + \frac{K_2}{2\omega}\sin(2\omega t+\varphi)\right]\right\} \times \exp(im\omega t)W_j(x), \qquad (3)$$

which fulfill

$$\langle\langle\phi_{j',m'}|\phi_{j,m}\rangle\rangle = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} dt \int dx\, \phi^*_{j',m'}\phi_{j,m} = \delta_{j',j}\,\delta_{m',m}, \tag{4}$$

where m denotes a Fourier component and $W_j(x) = W(x - x_j)$ is a Wannier function of the lowest energy band localized on the jth lattice site. The first phase factor on the right-hand side (RHS) of Eq. (3) corresponds to a unitary transformation which allows us to switch from the length gauge to the velocity gauge using quantum optics language. The matrix of the Floquet Hamiltonian consists of diagonal m-blocks in the basis (3). The blocks are very weakly coupled among themselves provided the driving frequency ω is very high. Using the tight-binding approximation and taking into an account nearest-neighbor tunneling only, the diagonal blocks become

$$\langle\langle\phi_{j',m}|\mathcal{H}|\phi_{j,m}\rangle\rangle = -J_{\text{eff}}\delta_{j',j+1} - J^*_{\text{eff}}\delta_{j',j-1} + (m\omega + E_0)\delta_{j',j}, \tag{5}$$

where $E_0 = \langle W_j|(p^2/2m + V)|W_j\rangle$ and the effective tunneling amplitude

$$J_{\text{eff}} = J\sum_{k=-\infty}^{\infty}\mathcal{J}_{2k}(s_1)\mathcal{J}_k(s_2)e^{ik\varphi}, \tag{6}$$

where

$$s_i = \frac{aK_i}{\omega} \tag{7}$$

are the dimensionless strengths of the first ($i = 1$) and the second ($i = 2$) harmonic, the bare tunneling amplitude is $J = -\langle W_{j+1}|(p^2/2m + V)|W_j\rangle$, and $\mathcal{J}_n$ is the ordinary Bessel function. If $\hbar\omega \gg J$ the description of a single-particle system may be restricted to a single diagonal block

$$H_{\text{eff}} = \langle\langle\phi_{j',0}|\mathcal{H}|\phi_{j,0}\rangle\rangle = -J_{\text{eff}}\delta_{j',j+1} - J^*_{\text{eff}}\delta_{j',j-1}, \tag{8}$$

where the constant term E_0 has been omitted.

If there is only one harmonic present in Eq. (1) or the phase $\varphi = 0$, the Floquet Hamiltonian is time-reversal invariant and $\mathcal{H}$ is represented by a real symmetric matrix in a generic basis [21]. Then the effective tunneling amplitude (6) is real. Single-harmonic driving has been used to change the interaction from ferromagnetic (positive J_{eff}) to antiferromagnetic (negative J_{eff}) and to realize frustrated magnetic phases [11,12]. By breaking TRI we are able to realize nearly arbitrary complex values of the tunneling amplitude $J_{\text{eff}} = |J_{\text{eff}}|e^{i\varphi_J}$. In Fig. 1 we present the absolute value $|J_{\text{eff}}|$ and the phase φ_J as functions of the parameter s_1.

The eigenstates of the Hamiltonian (8) are the Bloch waves $\psi_j = e^{ikx_j}/\sqrt{N_s}$, where N_s is the number of lattice sites, with the dispersion relation $E(k) = -2|J_{\text{eff}}|\cos(ka - \varphi_J)$. Single-harmonic driving allows for $\varphi_J = 0$ or π and thus for the ground state with $k = 0$ or with k at the edge of the first Brillouin zone. The ground state of the system with broken TRI may correspond to any value of k.

We have concentrated on a 1D problem. However, a similar control of phases of the tunneling amplitudes can also be realized in higher dimensions. Indeed, modulations applied to the lattice along orthogonal axes enable us to produce arbitrary tunnelings along the corresponding directions [11]. In the following we will focus on a 2D triangular lattice.

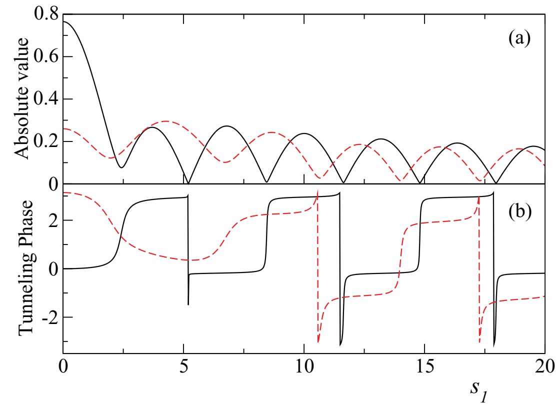

FIG. 1. (Color online) The absolute value A (top) and the complex phase φ_J (bottom) of the effective tunneling amplitude $J_{\text{eff}}/J = A\exp(i\varphi_J)$, Eq. (6), for a double-harmonic modulation of the optical lattice potential as a function of the dimensionless strength s_1 of the ω component for $s_2 = 1$ [see Eq. (7)], $\varphi = 0.2$ (black solid lines) and $s_2 = 3$, $\varphi = 0.5$ (red dashed lines).

B. Two-dimensional triangular optical lattice

The triangular optical lattice can be realized experimentally by means of three laser beams. Single-harmonic modulations of the lattice along the two orthogonal directions allow one to control the sign of the tunneling amplitudes of particles loaded in the lattice. This setup was used in the experiments that demonstrated frustrated classical magnetism [12]. With the help of double-harmonic modulations we are able to realize any phase of the tunneling amplitudes

$$J_\alpha = |J_\alpha|e^{i\varphi_\alpha}, \quad J_\beta = |J_\beta|e^{i\varphi_\beta}; \tag{9}$$

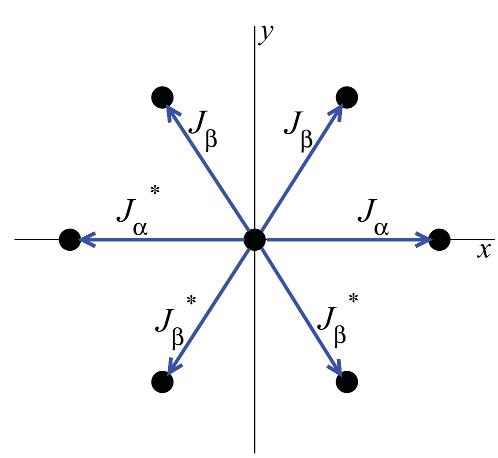

FIG. 2. (Color online) Triangular Bravais lattice points (black circles) and amplitudes $J_{\alpha,\beta}$ corresponding to tunneling from a lattice point to its nearest neighbors.

see Fig. 2. Eigenstates of a single particle in such a lattice are Bloch waves with the dispersion relation

$$E(\mathbf{k}) = -2|J_\alpha|\cos(k_x a - \varphi_\alpha) - 2|J_\beta|\left\{\cos\left[(\sqrt{3}k_y + k_x)\frac{a}{2} - \varphi_\beta\right] + \cos\left[(\sqrt{3}k_y - k_x)\frac{a}{2} - \varphi_\beta\right]\right\}. \quad (10)$$

We induce a shift of the dispersion relation along the k_y direction in reciprocal space by changing the value of φ_β (with the other parameters fixed). The modification of φ_α alters the structure of the dispersion relation. It can reveal a doubly degenerate ground state for $\varphi_\alpha = \pi$. The presence of such a degeneracy has been observed experimentally in a Bose system [12]. For example, for $J_\alpha = J_\beta = -|J_\beta|$ the system in most experimental realizations chooses spontaneously one of two ground states. With the double-harmonic modulation breaking TRI, the two degenerate minima for $\varphi_\alpha = \pi$ can be moved arbitrarily along the k_y direction with a change of φ_β; see Fig. 3.

In the experiment [12] a Bose-Einstein condensate (BEC) was prepared in a triangular lattice. Although in that case particle interactions are present, the ground state is still determined by the single-particle dispersion relation (10). Indeed, assuming a homogeneous system (which is a good approximation of the experimental situation), the solution of the Gross-Pitaevskii equation has the chemical potential given by $\mu_B = E(\mathbf{k}) + n_B U_B$, where U_B characterizes the on-site particle interactions and n_B is the average number of bosons per a lattice site. We would like to stress that in the presence of interactions, the restriction to a single block of the Floquet Hamiltonian as in Eq. (8) is valid provided $\hbar\omega \gg U_B$ [8]. On the other hand, $\hbar\omega$ must be much smaller than the energy separation between bands of the periodic lattice problem for the description limited to the lowest band to be valid.

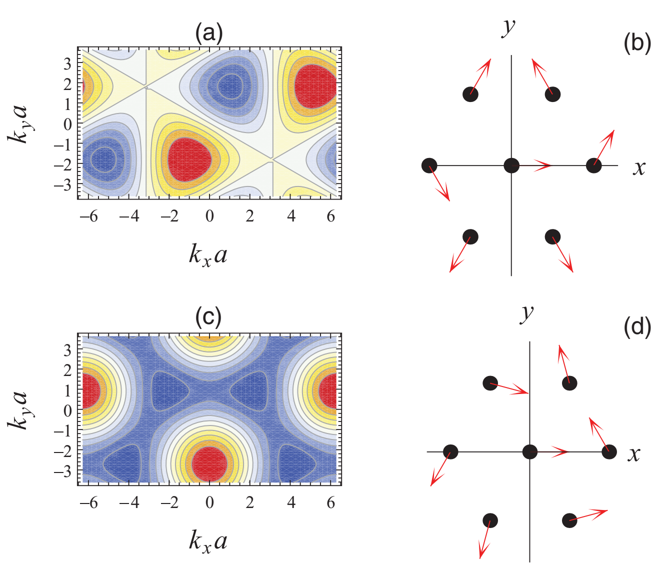

FIG. 3. (Color online) Contour plots of the dispersion relation Eq. (10) for $|J_\alpha| = |J_\beta|$ and $\varphi_\alpha = \varphi_\beta = \pi/2$ (a) and $\varphi_\alpha = \pi$ and $\varphi_\beta = \pi/4$ (c); cool colors indicate regions around energy minima. In the right panels the directions of the arrows indicate the phases $e^{i\mathbf{k}\cdot\mathbf{r}_i}$, where $\mathbf{k}$ corresponds to a minimum of the dispersion relation. Specifically $\mathbf{k}a = (\pi/3, \pi/\sqrt{3})$ for the minimum in (a) and $\mathbf{k}a = (+2\pi/3, \pi/2\sqrt{3})$ for one of the two degenerate, nonequivalent minima in (c). The arrows in (b) and (d) relate to (a) and (c), respectively.

III. FERMIONS IN A TRIANGULAR LATTICE

Frustrated classical magnetism in a triangular optical lattice has been demonstrated experimentally in a Bose system [12]. Within the mean-field approximation the Bose-Einstein-condensate wave function is a Bloch wave with wave vector corresponding to the minimum of the dispersion relation (10). For antiferromagnetic interactions the system experiences frustration, because the tendency of the wave function to change phase by π, when we jump between neighboring sites, cannot be reconciled with the triangular lattice geometry.

Consider now a mixture of fermions in different internal states (say spin-up ↑ and -down ↓ states) with attractive contact interactions in a 2D triangular optical lattice. We assume that the double-harmonic modulation of the lattice allows us to adjust any phase of the complex tunneling amplitudes (9). In the tight-binding approximation the Hamiltonian of the Fermi system reads

$$\hat{H}_F = -\sum_{\langle ij\rangle} J_{ij}(\hat{a}^\dagger_{i\uparrow}\hat{a}_{j\uparrow} + \hat{a}^\dagger_{i\downarrow}\hat{a}_{j\downarrow}) - \mu\sum_i(\hat{n}_{i\uparrow} + \hat{n}_{i\downarrow}) - U\sum_i \hat{a}^\dagger_{i\downarrow}\hat{a}^\dagger_{i\uparrow}\hat{a}_{i\uparrow}\hat{a}_{i\downarrow}, \quad (11)$$

where the operator $\hat{a}_{i\uparrow}$ annihilates a spin-up fermion at site i, $\hat{n}_{i,\uparrow} = \hat{a}^\dagger_{i\uparrow}\hat{a}_{i\uparrow}$, and similarly for spin-down fermions. The tunneling amplitude $J_{ij} = J^*_{ji}$ and it is equal to J_α or J_β, Eqs. (9), depending on the direction of the tunneling in the triangular lattice; see Fig. 2. The parameter $U > 0$ characterizes the interspecies, on-site, attractive interactions and μ stands for the chemical potential of the Fermi system.

The standard BCS approach [22] leads to the effective Hamiltonian

$$H_{F,\mathrm{eff}} = -\sum_{\langle ij\rangle} J_{ij}(\hat{a}^\dagger_{i\uparrow}\hat{a}_{j\uparrow} + \hat{a}^\dagger_{i\downarrow}\hat{a}_{j\downarrow}) - \mu\sum_i(\hat{n}_{i\uparrow} + \hat{n}_{i\downarrow}) + \sum_i(\Delta_i\,\hat{a}^\dagger_{i\uparrow}\hat{a}^\dagger_{i\downarrow} + \Delta^*_i\,\hat{a}_{i\downarrow}\hat{a}_{i\uparrow}), \quad (12)$$

where the pairing function

$$\Delta_i = U\langle\hat{a}_{i,\uparrow}\hat{a}_{i,\downarrow}\rangle. \quad (13)$$

If the phases of the tunneling amplitudes (9) are zero the ground state of the system corresponds to a constant pairing function $\Delta_i = \mathrm{const}$. However, the pairing function can acquire a nontrivial phase when the tunneling amplitudes become complex. In order to find the ground state of the system let us look for the solutions of the Bogoliubov–de Gennes equations in the form

$$\begin{bmatrix} u_{\mathbf{k}}(\mathbf{r}_i) \\ v_{\mathbf{k}}(\mathbf{r}_i)\end{bmatrix} = \frac{e^{i\mathbf{k}\cdot\mathbf{r}_i}}{\sqrt{N_s}}\begin{bmatrix} U_{\mathbf{k}}\, e^{i\mathbf{k}_0\cdot\mathbf{r}_i} \\ V_{\mathbf{k}}\, e^{-i\mathbf{k}_0\cdot\mathbf{r}_i}\end{bmatrix}, \quad (14)$$

where $U_{\mathbf{k}}$ and $V_{\mathbf{k}}$ satisfy

$$\begin{bmatrix} E(\mathbf{k}+\mathbf{k}_0) - \mu & \bar{\Delta} \\ \bar{\Delta}^* & -\tilde{E}(\mathbf{k}-\mathbf{k}_0) + \mu \end{bmatrix}\begin{bmatrix} U_{\mathbf{k}} \\ V_{\mathbf{k}}\end{bmatrix} = \varepsilon_{\mathbf{k}}\begin{bmatrix} U_{\mathbf{k}} \\ V_{\mathbf{k}}\end{bmatrix}, \quad (15)$$

and $|U_{\mathbf{k}}|^2 + |V_{\mathbf{k}}|^2 = 1$. In Eqs. (15), $E(\mathbf{k})$ is the dispersion relation (10) while $\tilde{E}(\mathbf{k}) = E(\mathbf{k}; \varphi_\alpha \to -\varphi_\alpha, \varphi_\beta \to -\varphi_\beta)$. Solving (15), we obtain the eigenvalues

$$\varepsilon_{\mathbf{k},\pm} = \frac{E(\mathbf{k}+\mathbf{k}_0) - \tilde{E}(\mathbf{k}-\mathbf{k}_0)}{2} \pm \delta\varepsilon_{\mathbf{k}}, \tag{16}$$

where

$$\delta\varepsilon_{\mathbf{k}} = \sqrt{\frac{[E(\mathbf{k}+\mathbf{k}_0) + \tilde{E}(\mathbf{k}-\mathbf{k}_0) - 2\mu]^2}{4} + |\bar{\Delta}|^2}. \tag{17}$$

The excitation spectrum of the system, i.e., the upper branch $\varepsilon_{\mathbf{k},+}$, may become negative for some $\mathbf{k}$. In such a case, the corresponding quasiparticles are present even at zero temperature. Therefore, at $T = 0$, the proper equation for $\bar{\Delta}$ reads

$$\bar{\Delta} = \frac{U}{N_s} \sum_{\mathbf{k}} \frac{\bar{\Delta}}{2\,\delta\varepsilon_{\mathbf{k}}} [1 - 2\theta(-\varepsilon_{\mathbf{k},+})], \tag{18}$$

where the Heaviside function $\theta(-\varepsilon_{\mathbf{k},+})$ ensures that quasiparticles corresponding to the negative-energy spectrum are also included [22]. Finally the desired pairing function becomes

$$\Delta_i = U \sum_{\mathbf{k}} u_{\mathbf{k},+}(\mathbf{r}_i) v^*_{\mathbf{k},+}(\mathbf{r}_i)[1 - 2\theta(-\varepsilon_{\mathbf{k},+})] = e^{i2\mathbf{k}_0\cdot\mathbf{r}_i}\,\bar{\Delta}. \tag{19}$$

When we switch from $\varphi_\alpha = \varphi_\beta = 0$ to $\varphi_\alpha = 0$ and $\varphi_\beta \neq 0$, the minimum of the dispersion relation (10) is shifted from $\mathbf{k} = \mathbf{0}$ to $\mathbf{k} = \mathbf{k}_0 = (0, \frac{2\varphi_\beta}{a\sqrt{3}})$. In the ground state of the system fermions occupy energy levels starting from the new minimum up to the Fermi level. Thus, all fermions acquire quasimomentum $\mathbf{k}_0$ and consequently the pairing function gets the quasimomentum $2\mathbf{k}_0$; see (19).

For $|J_\alpha| = |J_\beta|$ and $\varphi_\alpha = \pi$, there are two nonequivalent, degenerate minima of $E(\mathbf{k})$. For example, for $\varphi_\beta = \pi/4$ they are located at $\mathbf{k} = (\pm\frac{2\pi}{3a}, \frac{\pi}{2a\sqrt{3}})$; see Fig. 3(c). In the ground state, fermions occupy energy levels with quasimomenta around both of the minima. A nonzero pairing function exists for different values of $\mathbf{k}_0$. However, for $\mathbf{k}_0 = (0, \frac{\pi}{2a\sqrt{3}})$ we obtain the lowest energy of the Hamiltonian (12). A slight change of $\mathbf{k}_0$ causes a rapid decrease of the energy gap in the excitation spectrum (16). In Fig. 4(a) we present the Fourier transform $|\Delta_{\mathbf{k}}|^2$ of the pairing function, where $\Delta_{\mathbf{k}} = \sum_i \Delta_i e^{-i\mathbf{k}\cdot\mathbf{r}_i}/\sqrt{N_s}$, obtained numerically for a finite system. For the chemical potential $\mu = 0$, which corresponds to the half-filling regime for noninteracting particles, $|J_\beta|/|J_\alpha| = 1$, $U/|J_\alpha| = 2$, we obtain $|\Delta_i|/|J_\alpha| = 0.111$ at the center of the lattice, which agrees with the analytical solution $\bar{\Delta}/|J_\alpha| = 0.109$ for an infinite lattice. In Fig. 4(a) we see that even for the lattice of 60×60 sites there exists a clearly resolved peak at $\mathbf{k} \approx 2\mathbf{k}_0 = (0, \frac{\pi}{a\sqrt{3}})$.

As was discussed in Ref. [12], Bose systems can simulate frustrated classical magnetism. We show that a similar phenomenon can be simulated in the Fermi system. Indeed, in a triangular lattice with complex tunnelings, the phase of the complex pairing function is the one that is mapped onto the orientation of the classical spins.

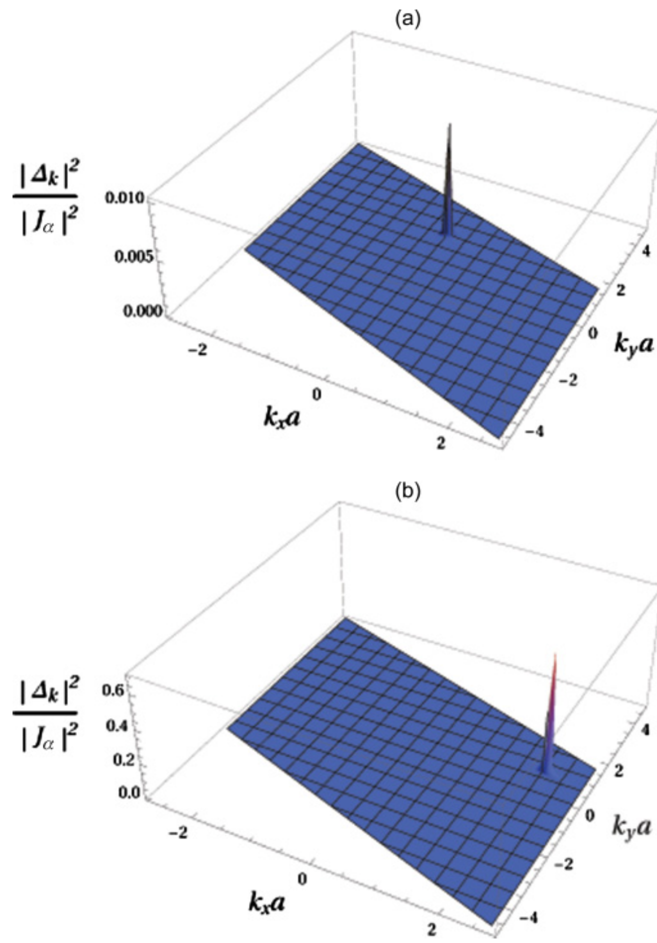

FIG. 4. (Color online) Modulus squared of the Fourier transform of the BCS pairing function $|\Delta_{\mathbf{k}}|^2$, where $\Delta_{\mathbf{k}} = \sum_i \Delta_i e^{-i\mathbf{k}\cdot\mathbf{r}_i}/\sqrt{N_s}$, obtained numerically for a finite system of 60×60 lattice sites for $|J_\alpha| = |J_\beta|$, $\varphi_\alpha = \pi$, $\varphi_\beta = \pi/4$, $U/|J_\alpha| = 2$, and $\mu = 0$. (a) shows $|\Delta_{\mathbf{k}}|^2$ corresponding to the ground state of the isolated Fermi system. (b) presents similar results but for the Fermi system coupled to a Bose-Einstein condensate, with wave function $\psi_i = \sqrt{n_B}e^{i\mathbf{q}_0\cdot\mathbf{r}_i}$, where $\gamma\sqrt{n_B}/|J_\alpha| = 2.3$. Note that the peak in (a) is located at $\mathbf{k}a = (0.00, 1.78) \approx (0, \pi/\sqrt{3})$ while in (b) it is at $\mathbf{k}a = (2.09, 0.89) \approx \mathbf{q}_0 a = (+2\pi/3, \pi/2\sqrt{3})$.

IV. BOSE-FERMI MIXTURE IN A TRIANGULAR LATTICE

In this section we consider a situation when fermions coexist with molecular dimers—pairs of spin-up and spin-down fermions. The dimers form a Bose-Einstein condensate. Such a mixture can be prepared by sweeping the system over a Feshbach resonance that creates a molecular BEC and leaves some fraction of unbound, repulsively interacting fermions. Then by crossing a second Feshbach resonance one is able to change the interactions between fermions from repulsive to attractive, turning unbound fermions into BCS pairs [17]. The process does not affect molecular BEC at the same time. For this purpose the Feshbach resonances at 202 and 224 G for ^{40}K atoms [23] seem to be quite suitable. We also assume the presence of a weak coupling that transforms dimers into unbound fermions and vice versa. It can be realized via photodissociation and photoassociation. For a large molecular BEC the weak coupling does not significantly influence the condensate wave function and therefore we neglect the dynamics of the BEC. The system under our consideration can be reduced to the following Hamiltonian:

$$\hat{H} = \hat{H}_F + \hat{H}_{BF}, \tag{20}$$

with

$$\hat{H}_{BF} = \gamma \sum_i (\psi_i^* \, \hat{a}_{i\downarrow}\hat{a}_{i\uparrow} + \psi_i \, \hat{a}_{i\uparrow}^\dagger \hat{a}_{i\downarrow}^\dagger), \tag{21}$$

where the BEC wave function $\psi_i = \sqrt{n_B} e^{i\mathbf{q}_0\cdot\mathbf{r}_i}$ is the ground-state solution for bosons in a triangular lattice, i.e., $\mathbf{q}_0$ corresponds to the minimum of the dispersion relation (10). For reasons of simplicity but without loss of generality we choose the same dispersion relation for molecules and for fermions. In the system under consideration, the tunneling amplitudes for molecules in a shaken optical lattice depend on a molecular state populated in the photoassociation process. The details of this process are not considered in the present paper. The coupling constant γ characterizes transfer of dimers into unbound fermions and vice versa. We consider real $\gamma \geqslant 0$.

In the presence of the condensate of dimers the BCS effective Hamiltonian (12) has to be supplemented with (21), that is,

$$\hat{H}_{\text{eff}} = \hat{H}_{F,\text{eff}} + \hat{H}_{BF}. \tag{22}$$

In the presence of bosons, if $\mathbf{k}_0 = \mathbf{q}_0/2$, a simple analytical solution (14) of the corresponding Bogoliubov–de Gennes equations exists. This solution need not correspond to the ground state of the system. However, we will see that for sufficiently strong coupling between bosons and fermions this becomes the ground-state solution. Employing (14) with $\mathbf{k}_0 = \mathbf{q}_0/2$, we obtain the following equation for $\bar{\Delta}$:

$$\bar{\Delta} = \frac{U}{N_s} \sum_{\mathbf{k}} \frac{\bar{\Delta} + \gamma\sqrt{n_B}}{2\delta\varepsilon_{\mathbf{k}}} [1 - 2\theta(-\varepsilon_{\mathbf{k},+})], \tag{23}$$

where, in the present case, the excitation spectrum is

$$\varepsilon_{\mathbf{k},+} = \frac{E(\mathbf{k}+\mathbf{q}_0/2) - \tilde{E}(\mathbf{k}-\mathbf{q}_0/2)}{2} + \delta\varepsilon_{\mathbf{k}}, \tag{24}$$

with

$$\delta\varepsilon_{\mathbf{k}} = \left[\frac{[E(\mathbf{k}+\mathbf{q}_0/2) + \tilde{E}(\mathbf{k}-\mathbf{q}_0/2) - 2\mu]^2}{4} + |\bar{\Delta} + \gamma\sqrt{n_B}|^2 \right]^{1/2}, \tag{25}$$

and the resulting pairing function

$$\Delta_i = e^{i\mathbf{q}_0\cdot\mathbf{r}_i} \bar{\Delta}. \tag{26}$$

Let us concentrate on the triangular lattice with $|J_\beta|/|J_\alpha| = 1$, $\varphi_\alpha = \pi$, and $\varphi_\beta = \pi/4$ that corresponds to the dispersion relation plotted in Fig. 3(c). The dispersion relation reveals two nonequivalent minima, but the solution of the Gross-Pitaevskii equation for bosons chooses the Bloch wave with the quasimomentum corresponding to one of the minima. The signatures of such a spontaneous symmetry breaking have been observed experimentally [12]. We assume that the Bose system chooses $\mathbf{q}_0 = (+\frac{2\pi}{3a}, \frac{\pi}{2a\sqrt{3}})$ and analyze its influence on the Fermi system.

We consider the system with $\mu = 0$. If $\gamma = 0$ Cooper pairs with the quasimomentum $\mathbf{q}_0$ do not exist, i.e., $\bar{\Delta} = 0$ is the only solution of (23). If coupling between bosons and fermions is present, but $\gamma\sqrt{n_B}/|J_\alpha| < 2.112$, the system reveals gapless superfluidity [22]. Cooper pairs with the quasimomentum $\mathbf{q}_0$ appear ($\bar{\Delta} \neq 0$), but there is no energy gap in the excitation spectrum. The system possesses quasimomenta $\mathbf{k}$ for which the excitation energies $\varepsilon_{\mathbf{k},+} < 0$ and consequently the corresponding quasiparticles are present even at $T = 0$. Concerning the ground state of the system, numerical solutions of the Bogoliubov–de Gennes equations are analyzed. It is found that an increase in the parameter γ causes a gradual enlargement of the peak at $\mathbf{k} = \mathbf{q}_0$ in the Fourier transform of the pairing function, together with a reduction of the peak at $\mathbf{k} = (0, \frac{\pi}{a\sqrt{3}})$ (the solution in the absence of bosons considered in the previous section). For $\gamma\sqrt{n_B}/|J_\alpha| \approx 0.3$ we observe a crossover: the peak at $\mathbf{k} = (0, \frac{\pi}{a\sqrt{3}})$ becomes hardly visible and the ground state starts to be well reproduced by the pairing function (26).

For $\gamma\sqrt{n_B}/|J_\alpha| \geqslant 2.112$ an energy gap shows up, $\varepsilon_{\mathbf{k},+} > 0$. There is no quasiparticle at zero temperature and the pairing function (26) is related to the ground state of the system. In Fig. 4(b) we show the Fourier transform of the pairing function obtained numerically for a triangular lattice of 60×60 sites where the strong peak at $\mathbf{k} \approx \mathbf{q}_0$ is clearly visible. The pairing function at the center of the lattice is $|\Delta_i|/|J_\alpha| = 0.891$ and the energy gap in the excitation spectrum is $0.187|J_\alpha|$. Those numbers agree with the solutions for an infinite system, i.e., $\bar{\Delta}/|J_\alpha| = 0.891$ and $\min(\varepsilon_{\mathbf{k},+}) = 0.194|J_\alpha|$.

Thus, we can describe the behavior of the system in the following way. In a triangular optical lattice with complex tunnelings, we are able to realize a BEC in the ground state with wave vector located at an arbitrary position in reciprocal space. If superfluid fermions are also present in the lattice and there is sufficiently strong coupling between fermions and bosons, the phase of the BCS pairing function reflects the phase of the BEC wave function.

V. CONCLUSIONS

In summary, we have shown that time-reversal symmetry breaking in an optical lattice potential allows us to realize complex tunneling amplitudes in the corresponding tight-binding model. We have considered a simple scheme of symmetry breaking by means of two harmonic modulations of the lattice, but the generalization to more complicated modulations is straightforward.

We have studied a fermionic system as well as a Bose-Fermi mixture in a triangular lattice potential with complex tunnelings. In such a lattice the Bose system can simulate frustrated classical magnetism [12]. We have shown that this behavior is similar for fermions where the pairing function acquires a complex phase. Assuming the presence of a coupling mechanism—an exchange of unbound fermions and bosonic molecules—we have shown that the complex phase of the Bose wave function is mapped to the fermions as reflected in the Fermi pairing function.

We became aware very recently of Ref. [24], where the authors consider a similar idea for realization of complex tunneling amplitudes in Bose systems both theoretically and experimentally.

ACKNOWLEDGMENTS

The work of K.S. and K.T. was financed from Polish National Center for Science funds received through Grants No. DEC-2011/01/B/ST3/00512 and No. DEC-2011/01/N/ST2/00424, respectively. J.Z. acknowledges support from Polish Government scientific funds for 2009-2012 as a research project.

[1] D. Jaksch and P. Zoller, Ann. Phys. (NY) **315**, 52 (2005).
[2] M. Lewenstein, A. Sanpera, V. Ahufinger, B. Damski, A. Sen, and U. Sen, Adv. Phys. **56**, 243 (2007).
[3] I. Bloch, J. Dalibard, and W. Zwerger, Rev. Mod. Phys. **80**, 885 (2008).
[4] C. Chin, R. Grimm, P. Julienne, and E. Tiesinga, Rev. Mod. Phys. **82**, 1225 (2010).
[5] T. M. Hanna, E. Tiesinga, and P. S. Julienne, New J. Phys. **12**, 083031 (2010).
[6] D. Jaksch, C. Bruder, J. I. Cirac, C. W. Gardiner, and P. Zoller, Phys. Rev. Lett. **81**, 3108 (1998).
[7] M. Greiner, O. Mandel, T. Esslinger, T. W. Hansch, and I. Bloch, Nature (London) **415**, 39 (2002).
[8] A. Eckardt, C. Weiss, and M. Holthaus, Phys. Rev. Lett. **95**, 260404 (2005).
[9] H. Lignier, C. Sias, D. Ciampini, Y. Singh, A. Zenesini, O. Morsch, and E. Arimondo, Phys. Rev. Lett. **99**, 220403 (2007).
[10] A. Zenesini, H. Lignier, D. Ciampini, O. Morsch, and E. Arimondo, Phys. Rev. Lett. **102**, 100403 (2009).
[11] A. Eckardt, P. Hauke, P. Soltan-Panahi, C. Becker, K. Sengstock, and M. Lewenstein, Europhys. Lett. **89**, 10010 (2010).
[12] J. Struck, C. Olschlager, R. Le Targat, P. Soltan-Panahi, A. Eckardt, M. Lewenstein, P. Windpassinger, and K. Sengstock, Science **333**, 996 (2011).
[13] K. Sacha, J. Zakrzewski, and D. Delande, Phys. Rev. Lett. **83**, 2922 (1999); **84**, 200(E) (2000).
[14] K. Sacha, J. Zakrzewski, and D. Delande, Ann. Phys. (NY) **283**, 94 (2000).
[15] K. Sacha and J. Zakrzewski, Phys. Rev. Lett. **86**, 2269 (2001).
[16] N. Bray-Ali, L. Ding, and S. Haas, Phys. Rev. B **80**, 180504(R) (2009).
[17] A. Niederberger, J. Wehr, M. Lewenstein, and K. Sacha, Europhys. Lett. **86**, 26004 (2009).
[18] G. Floquet, Ann. École Norm. Sup. **12**, 47 (1883).
[19] J. H. Shirley, Phys. Rev. **138**, B979 (1965).
[20] Y. A. Zel'dovich, Zh. Eksp. Teor. Fiz. **51**, 1492 (1966) [Sov. Phys. JETP **24**, 1006 (1967)].
[21] F. Haake, *Quantum Signatures of Chaos*, 2nd ed. (Springer-Verlag, Berlin, 2001).
[22] A. S. Alexandrov, *Theory of Superconductivity, From Weak to Strong Coupling* (Institute of Physics Publishing, Bristol, 2003).
[23] C. A. Regal and D. S. Jin, Phys. Rev. Lett. **90**, 230404 (2003); C. A. Regal, M. Greiner, and D. S. Jin, *ibid*. **92**, 083201 (2004).
[24] J. Struck, C. Ölschläger, M. Weinberg, P. Hauke, J. Simonet, A. Eckardt, M. Lewenstein, K. Sengstock, and P. Windpassinger, e-print arXiv:1203.0049.