

# Dynamics of nonclassical light in integrated nonlinear waveguide arrays and generation of robust continuous-variable entanglement

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We study a class of nonlinear waveguide arrays in which the waveguides are endowed with  $\chi^{(2)}$  nonlinearity and coupled through the evanescent overlap of the guided modes. We investigate the viability of such an array as a platform for generating both bipartite and tripartite continuous-variable entanglement. We explicitly address the effect of realistic losses on the entanglement produced, discuss the possible types of nonlinear materials that could be used, and suggest solutions for the possible phase matching issues in the waveguides. The simultaneous generation and manipulation of the light on a single waveguide chip circumvents the usual bandwidth problems associated with the use of external bulky optical elements and makes this avenue promising for further investigation.

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## I. INTRODUCTION

The generation and manipulation of light are at the heart of optical physics and photonics technologies. In particular, waveguides offer highly flexible tools for manipulating and processing light over short distances and have thus found applications in diverse areas of research [1]. For example, in optics they form the building blocks of a more complex structure commonly known as waveguide arrays [2]. The possibility of manipulating various interactions by design makes these arrays an experimentally accessible tool for studying a variety of effects from a large number of fields of physics. Another salient feature of this system is the possibility of controlling the exact initial conditions for the light propagating inside the array. Moreover, the decoherence rate in this system is very low, even for longer propagation distances [3].

Relevant studies so far include realization of condensed-matter-like effects [4–11], quantum random walks [3,12], and the quantum Zeno effect [13]. Quite recently, it has also been shown that one can tailor the dispersion relation for the waveguides to study the famous Bose-Hubbard Hamiltonian in a classical setup [14]. Furthermore, waveguides along with single photons have been shown to form basic units for quantum network architectures [15].

The above works have shown that the waveguide arrays are quite suitable for investigating a variety of physical effects and can also be used to effectively manipulate various quantum states of light. The question naturally arises as to whether such structures can also be utilized as a flexible tool to both generate and manipulate the quantum states of light at the same time. Here we explore the issue by studying an experimentally accessible model for the waveguide arrays where the waveguides are endowed with  $\chi^{(2)}$  nonlinearity and coupled through the evanescent overlap of the guided modes.

As a relevant application of our study, we choose to study the generation of continuous-variable entanglement using the waveguide structure in an integrated manner on a single waveguide chip [16]. We note that a detailed discussion about the entanglement in continuous-variable systems can be found in Ref. [17]. Conventionally, the continuous wave entangled

light has been generated from optical parametric process inside an optical cavity. However, the use of optical cavities severely limits the bandwidth of the entangled beam generated in such processes. It would thus be highly desirable to avoid the use of optical cavities. The use of optical waveguides, where the transverse field confinement results in an increase of the nonlinear efficiency, can make up for the buildup of a reasonably high finesse cavity.

## II. SYSTEM DESCRIPTION

We consider a waveguide system which contains an array of  $N$  identical waveguides endowed with  $\chi^{(2)}$  nonlinearity. We assume that each of these waveguides is pumped with coherent light (as shown in Fig. 1). The waveguide arrays studied here can be implemented, for example, by using the periodically poled lithium niobate waveguides employed in the work of Yoshino *et al.* [18]. A recent manuscript has also investigated the intriguing quantum correlations generated in a system consisting of quadratic waveguide arrays [19]. However, the emphasis in that manuscript is on very different features. Specifically, that manuscript investigates the generation and manipulation of light at the single-photon level. In our work we study the possibility of generating broadband continuous-variable entanglement in the waveguide structure and discuss the possible applications of the system in the field of continuous-variable information processing [16]. The coupling between the waveguide is achieved by the evanescent overlap of the guided modes [20]. We assume that the pump field is in a strong coherent classical field, which is strong enough to remain undepleted of photons over the entire length of the waveguide. The field operators evolve according to the Heisenberg equations given by

$$\dot{a}_j = -2ig_j a_j^\dagger + iJ_j(a_{j-1} + a_{j+1}), \quad (1)$$

where  $a_j(a_j^\dagger)$  refers to the bosonic creation (annihilation) operator for the field in the  $j$ th waveguide. The nonlinear coupling parameter  $g_j$  depends on strength of the pump laser and the nonlinear susceptibility of the waveguide medium. The linear coupling parameter  $J_j$  represents the rate at which

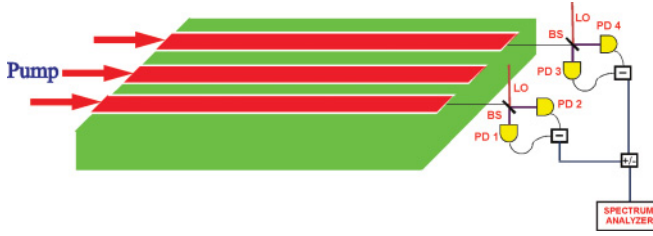


FIG. 1. (Color online) Schematic of a waveguide array with  $\chi^{(2)}$  nonlinearity. The correlation function  $M(j,k)$  can be measured by feeding the waveguide modes into the homodyne detectors. The signals from the homodyne detectors are subtracted or summed electronically and the variance is measured by a spectrum analyzer. The resulting values of the variances can be used to calculate the correlation function  $M(j,k)$ . The elements are: symmetric beam splitters BS, local oscillator LO, photodiode PD.

the photons are transferred to the neighboring waveguides. Note that we have ignored the evanescent coupling between the pump beams because the coupling coefficient for the pump beam would generally be much smaller compared to the signal ones, due to the weaker overlap between the waveguide modes at higher frequencies [20].

The evolution of the photon annihilation operators  $a_j$  of output modes can be written as

$$a_j(t) = \sum_k a_k(0) A_{j,k} + \sum_k a_k^\dagger(0) B_{j,k}, \quad (2)$$

where the summation is over all possible input modes. Moreover,  $A_{j,k}$  and  $B_{j,k}$  denote the complex matrix element, which depends on the coupling parameters  $g$  and  $J_j$ . In addition, the functions  $A_{j,k}$  and  $B_{j,k}$  depend on the propagation distance  $z$  over which the light distribution evolves. We note that Eq. (2) signifies the simultaneous generation and manipulation of light in waveguide arrays. Pump photons are converted to the signal photons within the individual waveguides, whereas the evanescent coupling between the signal modes leads to the linear spreading of the light across the waveguide array. Although the case of two coupled nonlinear waveguides which constitute the nonlinear optical couplers has been studied earlier [21], we emphasize here that our quantum formalism also applies to the case of a nonlinear waveguide array of a large number of waveguides, as is now feasible according to the experimental work of Ref. [20].

### III. DYNAMICS

Before moving to the main part, we first analyze the mean intensities behavior at the output which is an obvious measurable quantity in any similar experiment. The input to the waveguide system can either be in the form of a separable or entangled state. For the case when the signal modes are initially in the vacuum state, the intensity will be given by  $I_j(t) = \sum_{l=1}^N |B_{j,l}|^2$ . In this case, the pump mode is spontaneously converted to the signal modes within the individual waveguides. Further, because of the linear coupling between the inter-waveguide modes, the light spreads across the array. We note that in this case the light generated at the output of the waveguide arises solely because of the vacuum fluctuations. For the obvious, most interesting case of

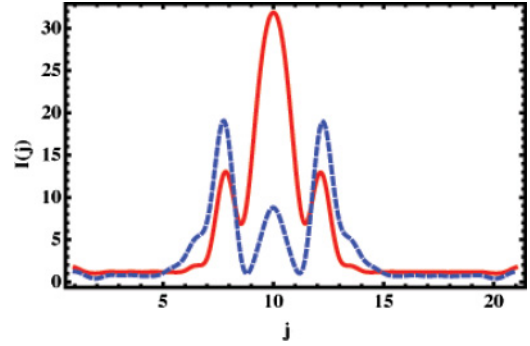


FIG. 2. (Color online) The intensity  $I_j(t)$  as a function of waveguide index  $j$  for the case of 21 waveguides. The coupling parameters are chosen such as  $g/J = 1/2$  (red solid line) and  $g/J = 1/3$  (blue dashed line). The input to the waveguide is in  $j = 10$  waveguide. The input is a coherent state with amplitude  $\alpha = 5$ . We are looking at the output intensity at  $t = 2.5$ .

a coherent light  $|\alpha\rangle$  fed into the  $m$ th waveguide, the intensity evolution among the waveguide sites can be written as

$$I_j(t) = (|\alpha|^2)(|A_{j,m}|^2 + |B_{j,m}|^2) + (\alpha^2 A_{j,m} B_{j,m}^* + \text{H.c.}) + \sum_{l=1}^N |B_{j,l}|^2. \quad (3)$$

It is interesting here to briefly compare the quantum walk of coherent light in linear waveguide arrays as analyzed, for example, in Ref. [3], with the arrays discussed here. In the linear case, we just have the contribution from the first term for  $I(t)$  and we see the ballistic propagation of the coherent light across the waveguide arrays. The quantum walk of coherent light in our system shows a very different behavior depending on the ratio of  $g/J$ . For a higher value of  $g/J$ , we found that the input light is amplified and remains localized in the input waveguide. For a lower value of  $g/J$ , the input light spreads across the array. This is schematically shown in Fig. 2. We note here the difference from all previous works, except Ref. [19], which essentially correspond to the transfer of the quantum light while the input light itself is generated outside the waveguide system by using bulk optical elements [12,22]. Such schemes involving bulk optical elements suffer from severe limitations as far as stability and physical size are concerned and may also introduce quantum decoherence.

### IV. GENERATING CONTINUOUS-VARIABLE ENTANGLEMENT

As a relevant application to our system, we investigate the possibility of generating continuous-variable entanglement in an integrated manner. We define the quadrature operators for the  $j$ th waveguide given by  $q_j \equiv (a_j e^{-i\phi} + a_j^\dagger e^{i\phi})/\sqrt{2}$  and  $p_j \equiv (a_j e^{-i\phi} - a_j^\dagger e^{i\phi})/\sqrt{2}i$ . We use the criterion of Ref. [23] for studying the entanglement between the waveguide modes and define the correlation between two waveguide modes as  $M(j,k) = \langle a_j^\dagger a_j \rangle + \langle a_k^\dagger a_k \rangle + e^{-2i\phi} \langle a_j a_k \rangle + e^{2i\phi} \langle a_j^\dagger a_k^\dagger \rangle$ . The correlation function  $M(j,k)$ , our entanglement measure, can be probed by using homodyne detectors scanning across the waveguide array output as shown in Fig. 1. We note

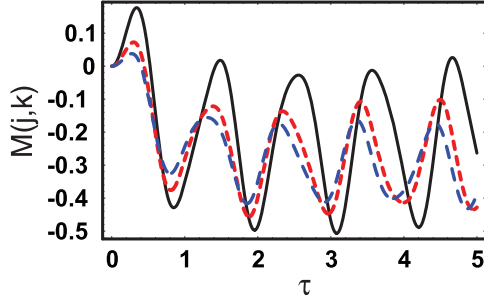


FIG. 3. (Color online) The correlation function  $M(j,k)$  as a function of  $\tau$  ( $\tau \equiv Jt/\pi$ ) for the case of five waveguides. The coupling parameters are chosen such that (a)  $g/J = 1/5$  (black line), (b)  $g/J = 1/7$  (red dashed line), and (c)  $g/J = 1/9$  (blue line).

that a similar correlation measurement has been carried out in the work of Peruzzo *et al.* [22], although we emphasize that the detection process in our case is fundamentally different than the one reported in that work. In the experiment by Peruzzo *et al.*, the output from the waveguide modes is detected with single-photon detectors. On the other hand, in our case the waveguide modes at the output are mixed with a strong local oscillator, as shown in Fig. 1. Since the local oscillator amplifies the waveguide modes, one can use high efficiency detectors that work only with strong signals. Further, note that we have written  $M(j,k)$  in terms of photon annihilation and creation operators of the waveguide modes, but in the actual homodyne measurement one measures the quadrature observable, which can then be used to calculate the correlation function  $M(j,k)$ . The negativity of  $M$  is a sufficient condition for entanglement. We first discuss our analytical result for the bipartite case of two waveguides, which reads

$$M(1,2) = 4 \sin^2(\Omega t) g(2g - J)/\Omega^2, \quad (4)$$

where  $\Omega \equiv (J^2 - 4g^2)^{1/2}$ . Clearly,  $M(1,2) < 0$  for  $J > 2g$ , and thus entanglement occurs. For the case of five waveguides, we show our numerical results in Fig. 3. The negative values of  $M(i,j)$  in Fig. 3 clearly demonstrate the entanglement between the inter-waveguide modes.

The system of waveguides we propose can also act as a compact source of tripartite entangled light. van Loock and Furusawa [24] have shown that for a fully inseparable three-mode system, it is sufficient to measure  $V_{i,j,k} = V(X_i - (X_j + X_k)/\sqrt{2}) + V(Y_i + (Y_j + Y_k)/\sqrt{2}) < 4$ , where the mode indices  $i, j, k$  are all different, to demonstrate the inseparability. The quadrature operators for each mode are defined as follows:  $X_i = a_i e^{-i\phi} + a_i^\dagger e^{i\phi}$ ,  $Y_i = -i(a_i e^{-i\phi} - a_i^\dagger e^{i\phi})$ . In our case, we choose  $\phi = \pi/2$  and study the evolution of  $V_{2,1,3}$  for  $g/J = 2/3$  for the case of three waveguides in Fig. 4 and show that the conditions  $V_{2,1,3} < 4$  are clearly satisfied, meaning the three waveguide modes are entangled.

## V. LOSSES AND EXPERIMENTAL FEASIBILITY

Loss is the greatest challenge facing the implementation of integrated photonic technologies, and it is inevitable in real-world systems. Thus an immediate question of interest arises: How does this loss affect the entanglement in the

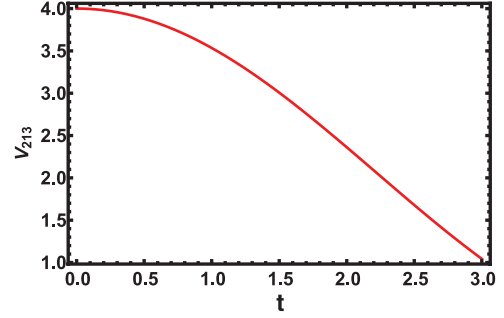


FIG. 4. (Color online) The correlation function  $V_{2,1,3}$  as a function of  $t$  for the case of three waveguides. The coupling parameters are chosen such that  $g/J = 2/3$ .

waveguide modes? It is well known that entanglement is quite susceptible to decoherence [25]; thus the previous question is quite relevant in context to quantum information processing using waveguides. Since the two waveguides are identical, we have taken the loss rate of both modes to be the same. We can model the loss in waveguides in the framework of system-reservoir interaction well known in quantum optics, and it is given by

$$\begin{aligned} \mathcal{L}\rho = & -\frac{\gamma}{2}(\hat{a}^\dagger \hat{a} \rho - 2\hat{a} \rho \hat{a}^\dagger + \rho \hat{a}^\dagger \hat{a}) \\ & -\frac{\gamma}{2}(\hat{b}^\dagger \hat{b} \rho - 2\hat{b} \rho \hat{b}^\dagger + \rho \hat{b}^\dagger \hat{b}), \end{aligned} \quad (5)$$

where  $\rho$  is the density operator corresponding to the system consisting of fields in the modes  $a$  and  $b$ . The dynamical evolution of any measurable  $\langle O \rangle$  in the coupled waveguide system is then governed by the quantum-Liouville equation of motion given by  $\dot{\rho} = -\frac{i}{\hbar}[H, \rho] + \mathcal{L}\rho$ , where  $\langle \dot{O} \rangle = \text{Tr}\{O\dot{\rho}\}$ , the commutator gives the unitary time evolution of the system under the influence of coupling, and the last term accounts for the loss.

We use the above equation for the density operator to evaluate the correlation  $M(1,2)$  given in the presence of loss. We also set  $\phi = 0$  in the equation for  $M(j,k)$ . The results are shown in Fig. 5. We see there that even for reasonable large values of the loss to coupling ratio the waveguides will still be entangled.

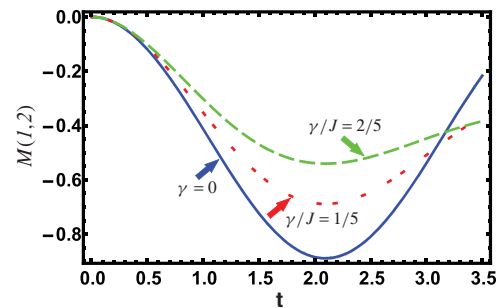


FIG. 5. (Color online) The correlation function  $M(1,2)$  as a function of  $t$  for the case of two waveguides in the presence of loss. The coupling parameters are chosen such that  $g/J = 2/5$ . The loss  $\gamma$  is taken to be  $\gamma = 0$  (solid line),  $\gamma/J = 1/5$  (dotted line), and  $\gamma/J = 2/5$  (dashed line).

We emphasize that such an integrated approach for the simultaneous generation and manipulation of nonclassical light seems feasible. In particular, the experiments by Iwanow *et al.* [20] investigate the process of second harmonic generation in the waveguide arrays of periodically poled LiNbO<sub>3</sub>, which feature a quadratic nonlinearity. We note that the second harmonic generation is the reverse of the downconversion process. Low loss waveguides with losses of 0.2 dB/cm for the fundamental wave and 0.4 dB/cm for its second harmonic were fabricated in these experiments using the technique of quasi-phase-matching (QPM) [26]. Because of their lower values for the loss, such arrays are ideally suited for the experimental implementation of our proposal. Moreover, integrated optical circuits based on LiNbO<sub>3</sub> substrates are now very well established, and a great variety of devices based on this technology can be integrated on the single chip. For instance, lithium niobate waveguides have been extensively employed for applications in electro-optic switching and phase modulators.

Except for the losses, for a successful implementation one would also need to ensure the downconversion process to be efficient, in which the usual phase-matching condition must be satisfied [1,27]. The phase-matching in waveguide structures can be achieved using the quasi-phase-matching in combination with ferroelectric materials. For example, in addition to an enhanced nonlinear efficiency due to tight confinement of the interacting waves inside the waveguides, the QPM allows working with the highest nonlinear coefficient of lithium niobate ( $d_{33} \approx 30$  pm/V), which is much larger than that ( $d_{31}$ ) commonly used in birefringent phase-matching in bulk crystals. Other materials which can be used as a nonlinear medium inside optical waveguides include periodically poled potassium titanyl phosphate (PPKTP), quasi-phase-matched LiTaO<sub>3</sub>, and periodically

poled stoichiometric LiTaO<sub>3</sub>. Recent experiments have shown that with the advancement in waveguide fabrication techniques and improvement in the detection schemes, it is possible to achieve better than  $-5$  dB of pulsed traveling-wave squeezing in MgO-doped periodically poled LiNbO<sub>3</sub> (PPLN) waveguides [28,29].

## VI. CONCLUSION

In conclusion, we considered the waveguide arrays with  $\chi^{(2)}$  nonlinearity and studied the generation and manipulation of nonclassical light in such a system. We also investigated the possibility of generating broadband continuous-variable entanglement in the waveguide structure. In our study, we propose an integrated approach toward continuous-variable entanglement based on integrated waveguide quantum circuits, which are compact and relatively more stable. In addition, the use of waveguide structure also eliminates the need for the optical path alignment which is required in the bulk optical systems. The possibility of generating broadband entanglement from waveguides could also help to avoid the use of optical cavities from many key experiments in the area of broadband continuous-variable quantum information processing. We can further investigate the evolution of the Wigner functions in the waveguide arrays [30]. Further, in our study we have assumed the coupling parameter  $J_j$  to be constant, but one can also make the value of  $J_j$  random [31], which is left for a future contribution.

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