Exploring initial correlations in a Gibbs state by application of external field

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We study the time evolution of trace distance between the quantum states which evolve from two kinds of initial states, a correlated Gibbs state and its uncorrelated marginal state. We consider the case where a two-level system linearly and adiabatically interacts with an infinite number of bosons. We find that the trace distance increases above its initial value for any and all parameter settings when a weak external field is applied to the two-level system. This indicates that we can explore the existence of initial system-environment correlations with the breakdown of the contractivity induced by the external field.

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I. INTRODUCTION

Reduced dynamics of open systems plays an important role in various fields, such as condensed-matter physics, quantum optics, and quantum electrodynamics [1]. In order to describe the reduced dynamics, an initial condition is often used where the relevant system is statistically independent from its environment. The uncorrelated condition was originally used to describe NMR phenomena for systems with weak system-environment interactions [2,3]. Besides the initial condition, the Markovian approximation has been used to describe the relaxation phenomena, which gives the so-called Bloch equation [4]. Completely positive dynamical semigroups have been devised to ascertain decay properties in the Markovian process [5], imparting contractivity to the open system dynamics. While dynamical semigroups became the starting point for designing quantum information processing [6], as experimental techniques developed to observe non-Markovian dynamics [7], theoretical treatments for such behavior [1] attracted much attention. The projection operator methods have provided formulations for initially correlated systems [8–10], but the initial correlations have been frequently ignored [11].

The effects of initial system-environment correlations have been discussed regarding quantum measurements [12–24], complete positivity of dynamical maps [25–31], non-Markovian dynamics [32–34], linear-response absorption line shapes [35–37], and general theoretical treatments using the projection operator methods [38]. Especially relevant has been the finding that initial system-environment correlations require extension of the conceptual framework of open system reduced dynamics to include generalization of contractivity [39].

The breakdown of contractivity by including initial correlations has been examined by evaluating the time evolution of the trace distance [39,40] for a finite size of environments, which shows an increase over the initial values. Dajka *et al.* compared four different distance measures, i.e., the trace (which is equivalent to Hilbert-Schmidt distance for the two-dimensional case), Bures, Hellinger, and quantum Jensen-Shannon distances, and found that only the trace distance reveals the distance increase for a system that initially correlates with an infinite size of the environment [41]. However, the correlated state in [41] requires elaborate preparation by quantum engineering. As a correlated state, we can consider a Gibbs state which includes an infinite size of the environment. When we analyze NMR and ESR experiments or design quantum information processing for condensed matter, it might be necessary to find the system-environment correlations that are inherent to the matter. A recent study on the transient linear response of a matter system to a suddenly applied weak external field showed dependence on the initial conditions, correlated Gibbs state, and a conventional factorized state where the relevant system and the environment stay in their equilibrium states [42]. Conversely, one might say that the transient linear response of a matter system is useful to detect the initial correlations. However, in [42], it was difficult to distinguish the effect of initial correlations except for strong system-environment interactions and for intermediate temperatures. In addition, within this approach, we cannot discuss the generalization of contractivity.

In this paper, we evaluate the time evolution of trace distance for a matter system where a two-level system interacts with an infinite number of bosons as the environmental system. We consider that the two-level system linearly and adiabatically interacts with the environment and that a weak external field is suddenly applied to the two-level system. We show that the trace distance between the time evolution from the Gibbs state ρ_{SE} and from an uncorrelated initial condition clearly exhibits effects of initial correlations as an increase in the short-time region. As opposed to [42], as an uncorrelated initial condition, we consider the marginal state of the Gibbs state, namely, $\rho_S \otimes \rho_E$ with $\rho_S = \mathrm{Tr}_{\mathrm{E}}\rho_{SE}$ and $\rho_E = \mathrm{Tr}_{\mathrm{S}}\rho_{SE}$, where Tr_{S} (Tr_{E}) is a partial trace over the relevant (environmental) system. By describing an uncorrelated state as the marginal state, the relevant system interacts with the same environmental features as an average. We find that the trace distance increases in the short-time region even when the system-environment interaction is weak. This indicates that the application of a weak external field can cause information inaccessible at an initial time to flow into the relevant system from its infinitely sized environment included in the Gibbs state, and we can monitor it with the trace distance effectively.

This paper is organized as follows. In Sec. II, we provide our formulation to obtain the time evolution of the two-level system which linearly and adiabatically interacts with an infinite number of bosons under a suddenly applied weak external field. We evaluate the induced dipole moment of the two-level system in Sec. III and the trace distance time evolution from the two different initial conditions in Sec. IV. We state our conclusions in Sec. V.

II. FORMULATION

We consider a matter system which consists of a two-level system and its environment of an infinite number of bosons. We assume that the system-environment interaction is linear and adiabatic, which causes the pure dephasing phenomena to the two-level system. The Hamiltonian of the matter system is written as

$$\mathcal{H} = \mathcal{H}_S + \mathcal{H}_E + \mathcal{H}_{SE}, \tag{1}$$

with

$$\begin{aligned} \mathcal{H}_S &= E_1|1\rangle\langle 1| + E_0|0\rangle\langle 0|, \\ \mathcal{H}_E &= \sum_k \hbar\omega_k b_k^\dagger b_k, \\ \mathcal{H}_{SE} &= \sum_k \hbar g_k (b_k^\dagger + b_k)|1\rangle\langle 1|, \end{aligned} \tag{2}$$

where E_0 and E_1 are the energy of the lower and upper states, respectively, of the relevant system, ω_k is the frequency of the bosonic bath mode, $b_k^\dagger$ and b_k are its creation and annihilation operators, and g_k is the coupling strength. We study the transient behavior of a two-level system after a sudden application of an external field. The matter-field-interaction Hamiltonian is given by

$$\mathcal{H}_P(t) = -\frac{1}{2}\vec{\mu}\cdot\vec{H}\,\theta(t)|1\rangle\langle 0|e^{-i\omega_p t} - \frac{1}{2}\vec{\mu}^*\cdot\vec{H}\,\theta(t)|0\rangle\langle 1|e^{i\omega_p t}, \tag{3}$$

where $\vec{\mu}$ is the transition dipole moment, $\vec{H}$ is the amplitude of the external field, ω_p is the frequency of the external field, and $\theta(t)$ is the step function.

In order to evaluate the time evolution of the induced dipole moment, we use a canonical transformation in terms of $S \equiv \exp[B|1\rangle\langle 1|]$, with $B \equiv \sum_k (g_k/\omega_k)(b_k - b_k^\dagger)$, since we can eliminate the system-bath interaction from the matter Hamiltonian in the form

$$\mathcal{H}' = S^\dagger \mathcal{H} S = \mathcal{H}'_S + \mathcal{H}_E, \tag{4}$$

$$\mathcal{H}'_S = E'_1|1\rangle\langle 1| + E_0|0\rangle\langle 0|, \tag{5}$$

with $E'_1 \equiv E_1 - \hbar\sum_k (g_k^2/\omega_k)$. The matter-field interaction is transformed as

$$\begin{aligned} \mathcal{H}'_P(t) &= S^\dagger \mathcal{H}_P(t) S \\ &= -\frac{1}{2}\vec{\mu}\cdot\vec{H}\theta(t)|1\rangle\langle 0|e^{-i\omega_p t}G(0) + \text{H.c.}, \end{aligned} \tag{6}$$

with $G(0) \equiv e^{B^\dagger}$. From Eqs. (4)–(6), the time evolution of the matter system takes the form

$$\rho(t) = U(t)\rho(0)U^\dagger(t) = Se^{-\frac{i}{\hbar}\mathcal{H}'t}V(t)\rho'(0)V^\dagger(t)e^{\frac{i}{\hbar}\mathcal{H}'t}S^\dagger, \tag{7}$$

where we define the transformed initial condition as $\rho'(0)$ and

$$U(t) \equiv T_+ \exp\left[-\left(\frac{i}{\hbar}\mathcal{H}t + \int_0^t dt'\frac{i}{\hbar}\mathcal{H}_P(t')\right)\right], \tag{8}$$

$$V(t) \equiv T_+ \exp\left[-\frac{i}{\hbar}\int_0^t \hat{\mathcal{H}}'_P(t')dt'\right], \tag{9}$$

with $\hat{\mathcal{H}}'_P(t) \equiv e^{\frac{i}{\hbar}\mathcal{H}'t}\mathcal{H}'_P(t)e^{-\frac{i}{\hbar}\mathcal{H}'t}$ [43]. In the following, we consider the application of a weak external field, which enables us to approximate, up to first order, the matter-field Hamiltonian in $V(t)$ as $V(t) \approx 1 - \frac{i}{\hbar}\int_0^t \hat{\mathcal{H}}'_P(t')dt'$.

A. Correlated initial condition: Gibbs state

As a correlated initial condition, we consider that the whole matter system is in a Gibbs state, $\rho(0) = \rho_{SE} \equiv \frac{1}{Z}\exp[-\beta\mathcal{H}]$. Transforming the correlated initial state, we find $\rho'(0) = \frac{1}{Z}\exp[-\beta\mathcal{H}'_S]\exp[-\beta\mathcal{H}_E]$, with $Z = \text{Tr}[\exp(-\beta\mathcal{H})]$, where Tr is the trace operation for the total system, which gives

$$\rho'(0) \equiv S^\dagger\rho(0)S = \rho'_0|0\rangle\langle 0| + \rho'_1|1\rangle\langle 1|, \tag{10}$$

with

$$\rho'_0 = \frac{e^{-\beta E_0}\tilde{\rho}_E}{Z'_S}, \quad \rho'_1 = \frac{e^{-\beta E'_1}\tilde{\rho}_E}{Z'_S}. \tag{11}$$

In Eq. (11), we define $Z'_S = \text{Tr}_S[\exp(-\beta\mathcal{H}'_S)]$ and $\tilde{\rho}_E = \exp[-\beta\mathcal{H}_E]/Z_E$, with $Z_E = \text{Tr}_E\exp[-\beta\mathcal{H}_E]$. Note that the transformation of the correlated state gives a factorized state of the Gibbs state and the bosonic environment and endows the two-level system with renormalized energies.

The transformed initial condition gives the elements of the reduced statistical operator as

$$\begin{aligned} [\text{Tr}_E\{U(t)\rho_{SE}U(t)\}\,]_{11} &= \frac{e^{-\beta E'_1}}{Z'_S}, \\ [\text{Tr}_E\{U(t)\rho_{SE}U(t)\}]_{10} &= \frac{i\vec{\mu}\cdot\vec{H}}{2\hbar}e^{-i\omega_p t}A_{(\text{corr})}(t), \end{aligned} \tag{12}$$

where we define

$$A_{(\text{corr})}(t) = \int_0^t dt' e^{-i\Delta\omega(t-t')} \times \left(\frac{e^{-\beta E_0}}{Z'_S}\Psi_1(t-t') - \frac{e^{-\beta E'_1}}{Z'_S}\Psi^*(t-t')\right), \tag{13}$$

with $\Delta\omega \equiv (E'_1 - E_0)/\hbar - \omega_p$. In Eq. (13), we define

$$\Psi_1(t-t') \equiv \langle G^\dagger(t)G(t')\rangle = \langle G(t')G^\dagger(t)\rangle^*, \tag{14}$$

with $G(t) \equiv e^{\frac{i}{\hbar}\mathcal{H}'t}e^{B^\dagger}e^{-\frac{i}{\hbar}\mathcal{H}'t}$, and $\langle X\rangle \equiv \text{Tr}_E\tilde{\rho}_E X$ for an arbitrary operator X.

B. Uncorrelated initial condition: Marginal state

As an uncorrelated initial state, we take a factorized state, $\rho(0) = \rho_S \otimes \rho_E$, which is the marginal of ρ_{SE} with $\rho_S = \text{Tr}_E\rho_{SE}$ and $\rho_E = \text{Tr}_S\rho_{SE}$. (Note the difference between ρ_E and $\tilde{\rho}_E$). As shown in Appendix A, the transformation of the marginal state gives

$$\rho'_0 = \frac{1}{Z'_S}e^{-\beta E_0}\rho_E, \quad \rho'_1 = \frac{1}{Z'_S}e^{-\beta E'_1}G(0)\rho_E G^\dagger(0), \tag{15}$$

with $\rho_E = \frac{1}{Z'_S}[e^{-\beta E_0}\tilde{\rho}_E + e^{-\beta E'_1} G^\dagger(0)\tilde{\rho}_E G(0)]$. The elements of the reduced statistical operator are obtained as

$$
\begin{aligned}
[\mathrm{Tr_E}\{U(t)\rho_S \otimes \rho_E U(t)\}]_{11} &= \frac{e^{-\beta E'_1}}{Z'_S}, \\
[\mathrm{Tr_E}\{U(t)\rho_S \otimes \rho_E U(t)\}]_{10} &= \frac{i\vec{\mu}\cdot\vec{H}}{2\hbar} e^{-i\omega_p t} A_{(\mathrm{marg})}(t),
\end{aligned} \tag{16}
$$

where we define

$$
\begin{aligned}
A_{(\mathrm{marg})}(t) = &\int_0^{\tilde{t}} d\tilde{t}' e^{-i\Delta\tilde{\omega}(\tilde{t}-\tilde{t}')} \\
&\times \left\{ \frac{e^{-\beta E_0}}{Z'_S}\left[\frac{e^{-\beta E_0}}{Z'_S}\tilde{\Psi}_1(t-t') + \frac{e^{-\beta E'_1}}{Z'_S}\tilde{\Psi}_2(t,t')\right]\right. \\
&\left. - \frac{e^{-\beta E'_1}}{Z'_S}\left[\frac{e^{-\beta E_0}}{Z'_S}\tilde{\Psi}^*_2(t,t') + \frac{e^{-\beta E'_1}}{Z'_S}\tilde{\Psi}^*_1(t-t')\right]\right\},
\end{aligned} \tag{17}
$$

with

$$
\begin{aligned}
\Psi_2(t,t') &\equiv \langle G(0)G^\dagger(t)G(t')G^\dagger(0)\rangle \\
&= \langle G^\dagger(0)G(t')G^\dagger(t)G(0)\rangle^*.
\end{aligned} \tag{18}
$$

III. TIME EVOLUTION OF THE INDUCED DIPOLE MOMENT

We evaluate the induced dipole moment under the application of an external field given by

$$
\vec{\mu}(t) = \mathrm{Tr}[(\vec{\mu}|1\rangle\langle 0| + \vec{\mu}^*|0\rangle\langle 1|)\rho(t)], \tag{19}
$$

using the formulation in the previous section. Apart from a factor of $2\vec{\mu}(\vec{\mu}^*\cdot\vec{H})/\hbar$, we define the induced dipole moment for the correlated initial condition as $\mu_{(\mathrm{corr})}(t)$ and for the uncorrelated marginal initial condition as $\mu_{(\mathrm{marg})}(t)$. We obtain

$$
\mu_{(m)}(t) = |A_{(m)}(t)|\cos[\omega_p t - \phi_{(m)}(t)], \tag{20}
$$

where $A_{(\mathrm{corr})}(t)$ and $A_{(\mathrm{marg})}(t)$ are defined in Eqs. (13) and (17), respectively; $\phi_{(m)}(t)$ is the argument of $A_{(m)}(t)$. Defining the coupling spectral density as $h(\omega) \equiv \sum_k g_k^2\delta(\omega-\omega_k)$, we obtain

$$
\begin{aligned}
\Psi_1(t) = \exp\Bigg(&-\int_0^\infty d\omega \frac{h(\omega)}{\omega^2} \\
&\times \{[1+2n(\omega)][1-\cos(\omega t)] + i\sin(\omega t)\}\Bigg)
\end{aligned} \tag{21}
$$

and

$$
\begin{aligned}
\Psi_2(t,t') = &\Psi_1(t-t') \\
&\times \exp\left\{-\int_0^\infty d\omega \frac{h(\omega)}{\omega^2} 2i[\sin(\omega t') - \sin(\omega t)]\right\}.
\end{aligned} \tag{22}
$$

Let us now set the spectral density to be Ohmic, i.e., $h(\omega) \equiv s\omega e^{-\omega/\omega_c}$, with the coupling strength s and the cutoff frequency ω_c. In this case, the renormalized energy is given by $E'_1 = E_1 - \hbar s\omega_c$, and we find

$$
\begin{aligned}
\Psi_2(t,t') = \Psi_1(t-t')\exp\{&-2is[\arctan(\omega_c t') \\
&- \arctan(\omega_c t)]\}.
\end{aligned} \tag{23}
$$

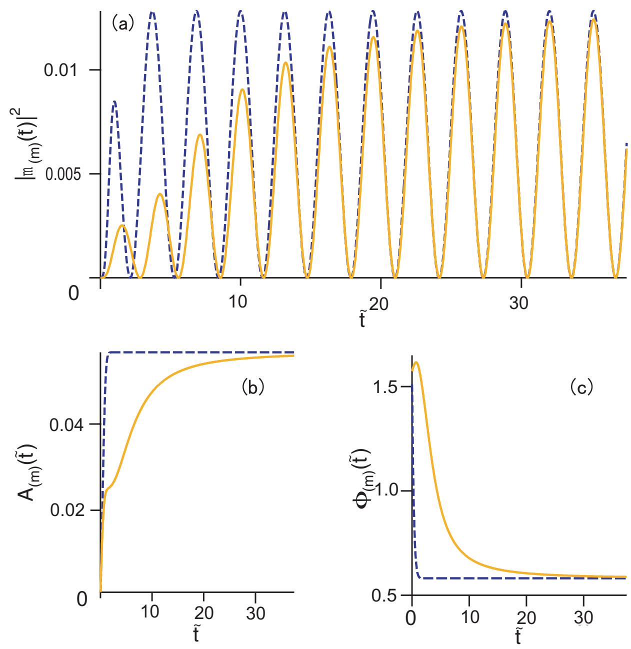

FIG. 1. (Color online) Time evolution of the induced dipole moment for $k_B T = 10\hbar\omega_0$, $s = 1$, $\omega_c = \omega_0/5$, and $\omega_p = \omega_0$. Dashed blue and solid orange lines represent the time evolutions of induced dipole moment for correlated and marginal initial condition, respectively. Time is scaled as $\tilde{t} = \omega_0 t$. (a) Time evolution of intensity of dipole moment $|\mu_{(m)}(\tilde{t})|^2$, (b) the time evolution of amplitude $|A_{(m)}(\tilde{t})|$, and (c) the time evolution of phase $\phi_{(m)}(\tilde{t})$ with $m =$ correlated or marginal.

In Fig. 1, we show the time evolution of the intensity of the dipole moment under the application of an external field for $k_B T = 10\hbar\omega_0$, $s = 1$, $\omega_c = \omega_0/5$, and $\omega_p = \omega_0$. Time is scaled as $\tilde{t} = \omega_0 t$. Dashed blue and solid orange lines represent the time evolutions of the induced dipole moment for the correlated and marginal initial conditions, respectively.

We find that the induced dipole moment for the correlated initial condition approaches a stationary oscillation faster than for the marginal initial condition. As shown in Figs. 1(b) and 1(c), the phase and amplitude of the induced dipole moment for the marginal initial condition approach the same phase and amplitude of the correlated initial condition. We can explain the long-time behavior using the monotonic time dependence of the arctan$(\omega_c t)$ function. The function approaches $\pi/2$ with increasing time, which means that $\Psi_2(t,t')$ approaches $\Psi_1(t-t')$ for large t. Comparison between Eqs. (13) and (17) reveals that the induced dipole moment for the correlated initial condition agrees with that for marginal initial condition at long times.

The time evolutions of the induced dipole moment for lower temperatures of $k_B T = \hbar\omega_0$ and $k_B T = \hbar\omega_0/5$ are shown in Figs. 2 and 3, respectively. As the temperature decreases, we find that the induced dipole moment for the correlated initial condition approaches a stationary oscillation slower,

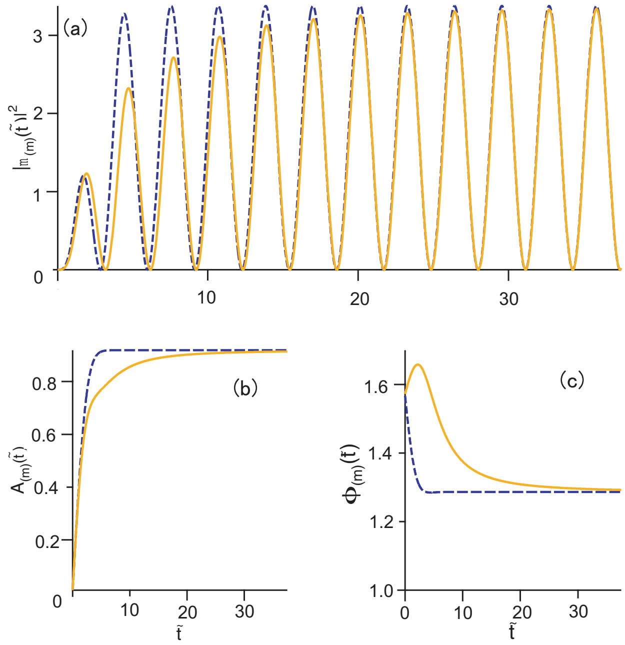

FIG. 2. (Color online) Time evolution of the induced dipole moment for $k_BT = \hbar\omega_0$. Other parameters and evaluated quantities are the same as in Fig. 1.

and we find a decreased dependence of the time evolution on the initial condition. In the above evaluation, the stationary oscillation for the marginal initial condition coincides with that for the correlated initial condition. This coincidence is in contrast to the behavior seen with the factorized initial condition, where the system and environment each stay in equilibrium, as discussed in [42].

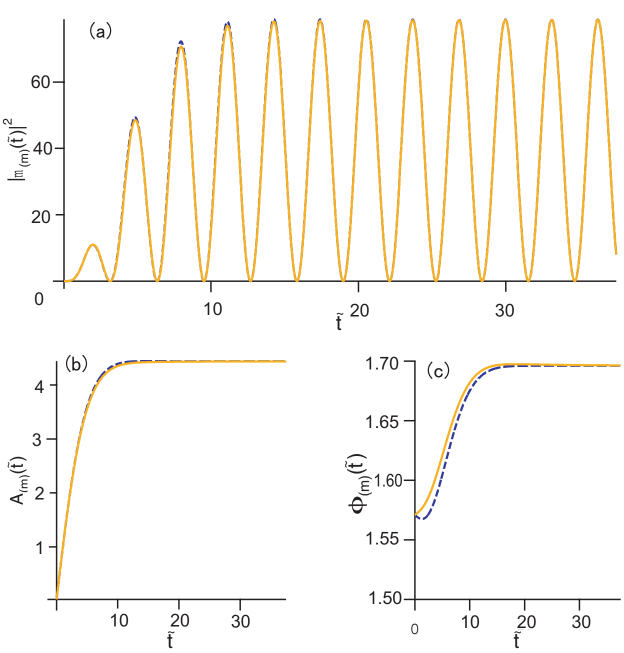

FIG. 3. (Color online) Time evolution of the induced dipole moment for $k_BT = \hbar\omega_0/5$. Other parameters and evaluated quantities are the same as in Fig. 1.

IV. TRACE DISTANCE

We found the difference between the time evolution of the induced dipole moment from the Gibbs state of the whole matter system and its marginal state in the previous section. However, it might be difficult to observe the difference between these time evolutions clearly. Moreover, we cannot discuss contractivity from the evaluations in the previous section.

In order to clarify the effects of initial system-environment correlations on open system dynamics, we need a sensitive and tractable measure. Since the effects of initial correlations appear in time evolutions where non-Markovian features dominate, we can use the trace distance as a measure of non-Markovianity [44,45]. The trace distance for two quantum states expressed with trace class operators ρ^1 and ρ^2 is defined as $D(\rho^1,\rho^2) = \frac{1}{2}\mathrm{Tr}|\rho^1 - \rho^2|$ [6]. Here we want to obtain the distance between the reduced dynamics of the two-level system for two kinds of initial conditions: the Gibbs state $\rho^1(0) = \rho_{SE}$, as a correlated initial condition, and its marginal state $\rho^2(0) = \rho_S \otimes \rho_E$, as an uncorrelated initial condition. When the trace distance increases above an initial value, namely, $D(\rho^1(t),\rho^2(t)) > D(\rho^1(0),\rho^2(0))$, we deduce that information, which is inaccessible at the initial time, flows into the relevant system through system-environment correlations. Moreover, this increase indicates a breakdown of the contractivity [39].

For our model, we obtain the trace distance in the form

$$D(\mathrm{Tr_E}[U(t)\rho_{SE}U^\dagger(t)],\mathrm{Tr_E}[U(t)\rho_S \otimes \rho_E U^\dagger(t)]) = \sqrt{|\mathcal{M}_{11}|^2 + |\mathcal{M}_{10}|^2}, \tag{24}$$

with $\mathcal{M} = \mathrm{Tr_E}[U(t)\rho_{SE}U^\dagger(t)] - \mathrm{Tr_E}[U(t)\rho_S \otimes \rho_E U^\dagger(t)]$.

Using Eqs. (12) and Eq. (16), we obtain the trace distance as

$$D(\mathrm{Tr_E}[U_t\rho_{SE}U_t^\dagger],\mathrm{Tr_E}[U_t\rho_S \otimes \rho_E U_t^\dagger]) = |\mathcal{M}_{10}|. \tag{25}$$

We show the time evolution of the trace distance, apart from the dimensionless quantity $\frac{\vec{\mu}\cdot\vec{H}}{2\hbar\omega_0}$, for Ohmic spectral density in Fig. 4 by setting $\omega_c = \omega_0/5$ and $\omega_p = \omega_0$. Here we scaled the time variable as $\tilde{t}$ ($\equiv \omega_0 t$).

Figures 4(a)–4(c) correspond to the trace distance time evolution at various temperatures: $k_BT = 10\hbar\omega_0$ [Fig. 4(a)], $k_BT = \hbar\omega_0$ [Fig. 4(b)], and $k_BT = \hbar\omega_0/5$ [Fig. 4(c)]. In each plot, we vary s as 1, 0.1, and 0.05. We find in Fig. 4(a) that the trace distance increases in the short-time region, which signifies the breakdown of contractivity for each value of s. In Fig. 4(a), we also find the trace distances approach zero at long times, which arises from the fact that the arctan$(\omega_c t)$ function in Eq. (23) monotonically approaches $\pi/2$ with increasing time and $\Psi_2(t,t')$ approaches $\Psi_1(t-t')$ for large t. For lower-temperature cases, as shown in Figs. 4(b) and 4(c), we find qualitatively similar behavior as in Fig. 4(a). Comparing Figs. 4(a)–4(c), we find that the peak value of the trace distance becomes larger for increasing values of s and at intermediate temperatures.

In the above evaluations, we find that the reduced dynamics of the matter system under a weak external field shows a breakdown of the contractivity for any parameter setting,

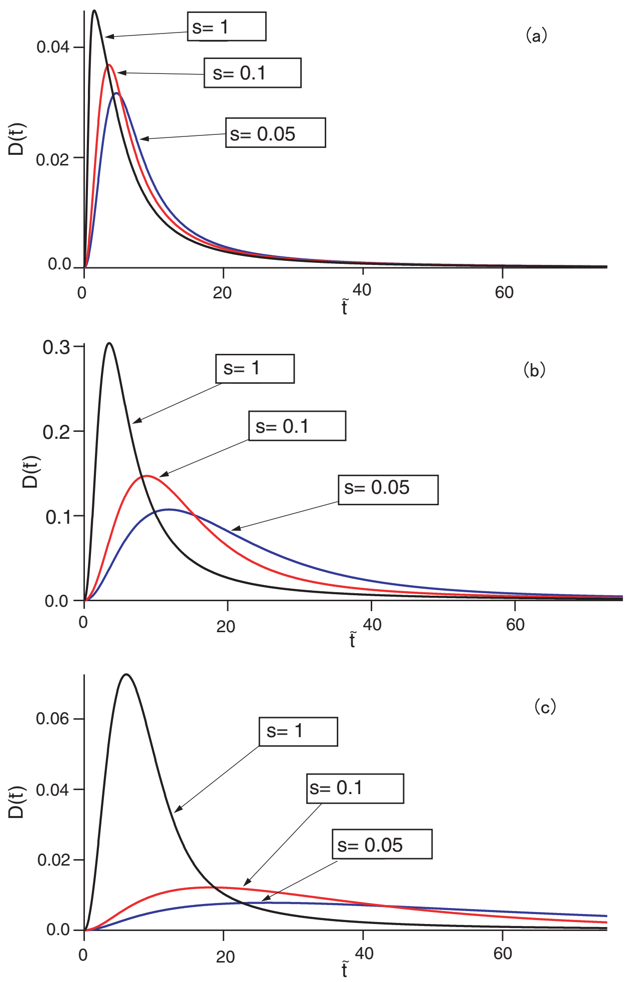

FIG. 4. (Color online) Time evolution of the trace distance for $\omega_c = \omega_0/5$ and $\omega_p = \omega_0$ with changing temperature as (a) $k_B T = 10\hbar\omega_0$, (b) $k_B T = \hbar\omega_0$, and (c) $k_B T = \hbar\omega_0/5$. In each plot, black, red (light gray), and blue (dark gray) lines correspond to the cases of $s = 1$, 0.1, and 0.05, respectively.

which allows us a method to monitor the initial system-environment correlations in the Gibbs state.

V. CONCLUSIONS

We have studied the time evolution of a matter system which is composed of a two-level system and a bosonic environment of infinite size under a suddenly applied weak external field. Assuming the system-environment interaction to be linear and adiabatic, we evaluated the induced dipole moment for two kinds of initial conditions: the correlated Gibbs state and an uncorrelated marginal state. We found that the time evolution of the induced dipole moment depends on the initial condition. However, the difference depends on the parameter settings and, under certain settings, may be difficult to observe. By evaluating the trace distance in the reduced dynamics for these two initial conditions, we found that we can overcome the parameter dependence. The trace distance clearly increases in the short-time region for cases of arbitrary system-environment interactions and temperatures. In order to obtain the increases, application of the weak external field plays an essential role. This can be seen from the fact that the trace distance does not change in the absence of the field, as shown in Appendix B. This indicates that we can effectively monitor the initial system-environment correlations in the Gibbs state with the breakdown of the contractivity induced by the weak external field.

APPENDIX A: DERIVATION OF EQUATION (15)

In this appendix, we explain how to obtain the marginal initial state

$$\rho(0) = \rho_S \otimes \rho_E \tag{A1}$$

with $\rho_S = \mathrm{Tr_E}\rho_{SE}$ and $\rho_E = \mathrm{Tr_S}\rho_{SE}$ for $\rho_{SE}(= \frac{1}{Z}\exp[-\beta\mathcal{H}])$. The correlated state ρ_{SE} is rewritten as

$$\begin{aligned}\rho_{SE} &= \frac{1}{Z}\exp[-\beta\mathcal{H}_0]T_+\exp\left[-\int_0^\beta d\lambda e^{\lambda\mathcal{H}_0}\mathcal{H}_{SE}e^{-\lambda\mathcal{H}_0}\right]\\ &= \frac{1}{Z}\Bigg(e^{-\beta E_0}\exp[-\beta\mathcal{H}_E]|0\rangle\langle 0| + e^{-\beta E_1}\exp[-\beta\mathcal{H}_E]\\ &\quad\times T_+\exp\left[-\sum_k \hbar g_k\int_0^\beta d\lambda(b_k^\dagger e^{\hbar\lambda\omega_k}\right.\\ &\quad\left.+ b_k e^{-\hbar\lambda\omega_k})\right]|1\rangle\langle 1|\Bigg),\end{aligned} \tag{A2}$$

where we define $\mathcal{H}_0 \equiv \mathcal{H}_S + \mathcal{H}_E$ and use the T_+ operator to order λ from right to left. Using the relation as

$$\begin{aligned}&\exp[-\beta\mathcal{H}_E]\ T_+\exp\left[-\sum_k \hbar g_k\int_0^\beta d\lambda(b_k^\dagger e^{\hbar\lambda\omega_k} + b_k e^{-\hbar\lambda\omega_k})\right]\\ &= \exp\left[-\beta\left(\mathcal{H}_E + \sum_k \hbar g_k(b_k^\dagger + b_k)\right)\right]\\ &= G^\dagger(0)\exp\left[-\beta\left(\mathcal{H}_E - \hbar\sum_k\frac{g_k^2}{\omega_k}\right)\right]G(0),\end{aligned} \tag{A3}$$

we obtain

$$\rho_{SE} = \frac{1}{Z_S'}[e^{-\beta E_0}\tilde{\rho}_E|0\rangle\langle 0| + e^{-\beta E_1'}G^\dagger(0)\tilde{\rho}_E G(0)|1\rangle\langle 1|], \tag{A4}$$

which gives the partial trace operation on ρ_{SE} over the system and the environment as

$$\rho_S = \frac{1}{Z_S'}(e^{-\beta E_0}|0\rangle\langle 0| + e^{-\beta E_1'}|1\rangle\langle 1|), \tag{A5}$$

$$\rho_E = \frac{1}{Z_S'}[e^{-\beta E_0}\tilde{\rho}_E + e^{-\beta E_1'}G^\dagger(0)\tilde{\rho}_E G(0)]. \tag{A6}$$

From these, we obtain the marginal state as

$$\rho_S \otimes \rho_E = \frac{1}{Z_S'}(e^{-\beta E_0}\rho_E|0\rangle\langle 0| + e^{-\beta E_1'}\rho_E|1\rangle\langle 1|), \tag{A7}$$

which is transformed into

$$(\rho_S \otimes \rho_E)' = \rho_0'|0\rangle\langle 0| + \rho_1'|1\rangle\langle 1| = \frac{1}{Z_S'}[e^{-\beta E_0}\rho_E|0\rangle\langle 0| + e^{-\beta E_1'}G(0)\rho_E G^\dagger(0)|1\rangle\langle 1|]. \quad (A8)$$

Equation (A8) gives Eq. (15).

APPENDIX B: THE TIME EVOLUTION OF THE TRACE DISTANCE IN THE ABSENCE OF THE EXTERNAL FIELD

In the absence of the external field, the time evolution of the trace distance between the reduced system is given by $D(\mathrm{Tr_E}[W(t)\rho_{SE}W^\dagger(t)],\mathrm{Tr_E}[W(t)\rho_S \otimes \rho_E W^\dagger(t)])$, with $W(t) = \exp[-\frac{i}{\hbar}\mathcal{H}t]$, where $\mathcal{H}$ is the Hamiltonian of the matter system defined in Eq. (1). Since the Gibbs state commutes with the matter Hamiltonian, we have $W(t)\rho_{SE}W^\dagger(t) = \rho_{SE}$. For the marginal state, using Eq. (A8) and the canonical transformation, we obtain

$$\begin{aligned}\mathrm{Tr_E}[W(t)\rho_S \otimes \rho_E W^\dagger(t)] &= \mathrm{Tr_E}[e^{-\frac{i}{\hbar}\mathcal{H}_E t}\rho_0' e^{\frac{i}{\hbar}\mathcal{H}_E t}|0\rangle\langle 0| \\ &\quad + G^\dagger(0)e^{-\frac{i}{\hbar}\mathcal{H}_E t}\rho_1' e^{\frac{i}{\hbar}\mathcal{H}_E t}G(0)|1\rangle\langle 1|] \\ &= \frac{1}{Z_S'}(e^{-\beta E_0}\rho_E|0\rangle\langle 0| + e^{-\beta E_1'}\rho_E|1\rangle\langle 1|) \\ &= \rho_S \otimes \rho_E. \end{aligned} \quad (B1)$$

From these, we find that the trace distance does not evolve in time in the absence of the external field.

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