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Entanglement creation with negative index metamaterials

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We propose a scheme for creating a maximally entangled state comprising two field quanta. In our scheme, two weak light fields, which are initially prepared in either coherent or polarization states, interact with a composite medium near an interface between a dielectric and a negative index metamaterial. This interaction leads to a large Kerr nonlinearity, reduction of the group velocity of the light, and significant confinement of the light fields, while simultaneously avoiding amplitude losses of the incoming radiation. All these considerations make our scheme efficient.

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Entanglement is an essential feature of quantum theory, which manifests impressive advantages of recently established quantum technologies over their classical counterparts [1,2]. A variety of physical systems, starting from atoms, electrons, and photons [3] and ending with sophisticated molecules and even living organisms [4], can exhibit quantum entanglement. Among these systems, photons, the quanta of the electromagnetic field, have a privileged position because of their exceptional properties. Photons can be easily generated and measured and can carry information over long distances since they are resistant to the detrimental effect of decoherence [5]. At the same time, photons do not directly interact with each other, which makes it difficult to prepare them in entangled states.

A conventional way to create entanglement of light quanta is through their interaction with nonlinear (Kerr) media, which have intensity-dependent refractive indices [6]. In natural media, however, the Kerr nonlinearity is very small. Therefore, to achieve significant entanglement of photons, one has to increase both the intensity of the field pulses and the interaction time with the medium. Such actions may not always be possible in practice, because of diffraction and the finite size of the medium, and are very unlikely from the viewpoint of applications [5,7], where weak light fields (i.e., of the energy of a single photon) are desired.

In 2000, a pioneering proposal to "design" materials exhibiting strong nonlinear interaction at the single-photon level was made by Lukin and Imamoğlu [8]. This idea stimulated a number of theoretical investigations [9–11] as well as experiments [12–14], to name just a few. However, despite these remarkable results in achieving large Kerr nonlinearity, efficient creation of entanglement at the low-energy limit remains challenging in many respects [15,16].

In this work we suggest a scheme in which initially uncorrelated states of the light field become entangled due to their interaction with a medium near an interface between a dielectric and a negative index metamaterial. The medium of interest consists of a dielectric (which has a layer of thickness z_0 doped with six-level atoms [9]) and a metamaterial placed together, as shown in Fig. 1. Due to interaction with the medium, an incident light beam creates a spatially confined surface polariton [17] which propagates along the interface with a substantially reduced group velocity $\upsilon_g < c$. Although in natural media a surface polariton undergoes large amplitude loss, specific design of the medium makes it possible to suppress losses significantly in a narrow frequency bandwidth of the incoming light [18]. Placing the layer of six-level atoms near the interface allows us to establish double electromagnetically induced transparency [19] (i.e., for two incoming pulses simultaneously) and, at the same time, create large Kerr nonlinearity [9]. All the mentioned factors contribute to an efficient nonlinear interaction between the two surface polaritons in the medium. Such interaction makes possible a mutual π phase shift between the polaritons, which leads to entanglement of the light fields.

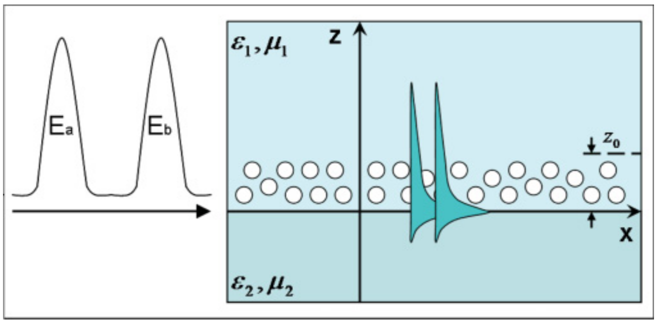

FIG. 1. (Color online) Two weak light pulses create two surface polaritons near the interface between the dielectric $z > 0$ and the metamaterial $z < 0$. When electromagnetically induced transparency is established for both polaritons simultaneously, they will propagate along the interface with small group velocities $\upsilon_g < c$ and interact nonlinearly with each other.

Ignoring the presence of the atomic layer near the interface, the process of interaction between the light fields and the medium can be considered from the viewpoint of classical electrodynamics. Macroscopic properties of the material can be characterized with the electric permittivity ε and magnetic permeability μ. For a dielectric, these parameters are strictly positive, while both of them may be simultaneously negative for a metamaterial [20]. Let us assume that the dielectric has constant homogeneous parameters ε_1 and μ_1, while the parameters ε_2 and μ_2 are frequency dependent for the

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metamaterial and are given by [18]

$$\varepsilon_2(\omega) = \varepsilon_\infty - \frac{\omega_e^2}{\omega(\omega + i\gamma_e)},$$

$$\mu_2(\omega) = \mu_\infty - \frac{\omega_m^2}{\omega(\omega + i\gamma_m)}, \qquad (1)$$

where ω_e and ω_m are the electric and magnetic plasma frequencies, γ_e and γ_m are corresponding (empirical) decay rates, and ε_∞ and μ_∞ are background constants [17]. Here we have chosen the simplest (Drude-like) model for the magnetic permeability $\mu_2(\omega)$. This model is known to be adequate in the optical region [17,21], although more sophisticated models can be taken into consideration [22].

The electromagnetic field of the surface polaritons can be found from the Maxwell equations with boundary conditions for ε_i and μ_i ($i = 1,2$). Since the permittivity and the permeability (1) may be simultaneously negative, both transverse magnetic (TM) and transverse electric (TE) polarizations of the electromagnetic field may exist in the medium. Natural media, in contrast, support only TM polarization [17]. To be specific, we shall later focus on the TM waves. The wave vector of the electromagnetic field in the medium is given by the dispersion relation

$$K(\omega) = \frac{\omega}{c}\sqrt{\varepsilon_1\,\varepsilon_2\frac{\varepsilon_2\mu_1 - \varepsilon_1\mu_2}{\varepsilon_2^2 - \varepsilon_1^2}}. \qquad (2)$$

The real part of this expression gives the dispersion of the field, while its imaginary part stands for the absorption loss.

It has been shown that absorption loss can be completely suppressed in a narrow frequency bandwidth due to destructive interference of the electric and magnetic absorption responses of the medium [18]. For example, taking $\varepsilon_1 = 1.3$ and $\mu_1 = 1$ and $\omega_e = 1.37 \times 10^{16}\ \mathrm{s}^{-1}$, $\gamma_e = 2.73 \times 10^{13}\ \mathrm{s}^{-1}$ (as for Ag), and assuming $\omega_m = 10^{15}\ \mathrm{s}^{-1}$, $\gamma_m = 10^{12}\ \mathrm{s}^{-1}$, $\varepsilon_\infty = 5$, and $\mu_\infty = 5$, one can see that the absorption loss Im$[K(\omega)]$ vanishes for $\omega_0 \approx 4.4 \times 10^{14}\ \mathrm{s}^{-1} = 440$ THz, which corresponds to red light of the visible spectrum. It is important to note that metamaterials with negative refractive index have been observed in the red region of the visible spectrum [20]. More details about the dispersion relation (2) and possible parameters of the medium can be found in Ref. [18] and references therein.

We are now in a position to consider the interaction between the surface polariton light fields and the medium quantum mechanically. The electric field of each of the surface polaritons can be quantized near the surface and written in the plane-wave expansion as [23]

$$\mathbf{E}(\mathbf{r},t) = \int dk[\mathbf{E}_0(k)a(k)e^{ikx-\omega t} + \text{H.c.}]. \qquad (3)$$

Here we introduced $k \equiv \text{Re}[K(\omega)]$, taking into account that the wave vector is approximated by its real part $k(\omega) \approx \text{Re}[K(\omega)]$ in the low-loss frequency range. The amplitude $\mathbf{E}_0(k)$ can be found from the requirement that it should obey the field Hamiltonian $H_F = (1/2)\int d^3r[\tilde{\varepsilon}\langle|\mathbf{E}|^2\rangle + \tilde{\mu}\langle|\mathbf{H}|^2\rangle]$ in a dispersive lossless medium [22]. It is important to note that our quantization procedure is applicable only in a narrow frequency bandwidth where the losses are low. In the general case, the quantization of the electromagnetic field in dispersive and absorptive media is a much more complicated task [24–26].

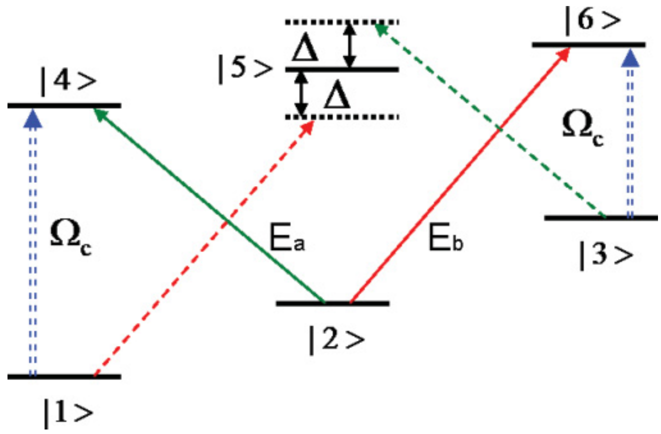

FIG. 2. (Color online) Configuration of a six-level atom for creating double electromagnetically induced transparency for two weak fields E_a and E_b. Here, the fields E_a and E_b are coupled resonantly with the transitions $|2\rangle \rightarrow |4\rangle$ and $|2\rangle \rightarrow |6\rangle$ and off-resonantly with the transitions $|3\rangle \rightarrow |5\rangle$ and $|1\rangle \rightarrow |5\rangle$ with detuning Δ. Two classical control fields drive the transitions $|1\rangle \rightarrow |4\rangle$ and $|3\rangle \rightarrow |6\rangle$ [9].

The quantized polariton fields exhibit a remarkable property of confinement along the z direction. The confinement can be quantified as $\xi(\omega) = 1/\text{Re}[k^\perp(\omega)]$, where $k^\perp(\omega) = \sqrt{K^2(\omega) - \omega^2\varepsilon_1\mu_1/c^2}$ is the normal component of the real part of the wave vector (2). This property of the quantized fields ensures interaction of the polaritons with the six-level atoms embedded into the dielectric and defines the effective thickness of the atomic layer $\xi(\omega) \approx z_0$ in practice. The reason for injection of the six-level atoms near the dielectric-metamaterial interface is that such atomic systems have been shown to cause the effect of double electromagnetically induced transparency, simultaneously exhibiting symmetric nonlinearity for two incoming pulses [9]. The energy levels of the system are shown schematically in Fig. 2. The model of a six-level atom can be practically realized in the rubidium isotope ^{87}Rb [9], for example.

The interaction of the surface polaritons and the six-level atoms can be modeled with the electric dipole Hamiltonian $H_{ED} = -\sum \mathbf{d}_i \cdot \mathbf{E}(\mathbf{r}_i)$, where $\mathbf{E}(\mathbf{r}_i)$ is the electric field (3) of the surface polaritons, $\mathbf{r}_i$ is the position of atom i, and the summation is to be done over all atoms in the interaction volume [22].

Because of the symmetry of the atomic level structure, the two surface polaritons propagate in the medium with equal group velocities $\upsilon_a = \upsilon_b \equiv \upsilon_g$ [9]. The dynamics of the surface polariton field operators can be obtained in the Heisenberg picture by solving the corresponding set of equations

$$\left(\frac{1}{\upsilon_g}\frac{\partial}{\partial t} + \frac{\partial}{\partial x}\right)\mathbf{E}_n(\mathbf{r},t) = i\chi I_m \mathbf{E}_n(\mathbf{r},t), \qquad (4)$$

where the adiabatic approximation has been used to ignore time derivatives of higher order [22]. Here $n,m = a,b$ ($n \neq$

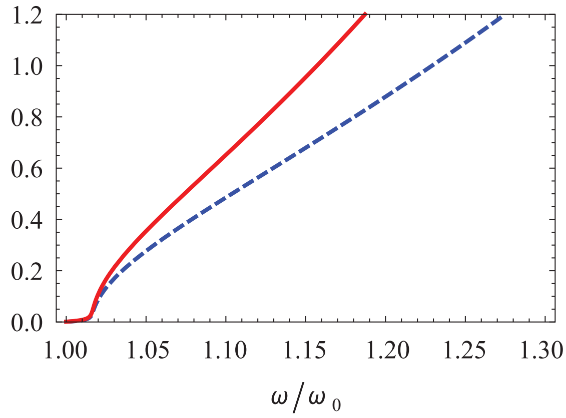

FIG. 3. (Color online) Kerr coefficient χ ($\times 10^4$) (dashed blue) and corresponding mutual phase shift ϕ in units of π (solid red) as functions of the field frequency ω/ω_0.

m), $I_m = |\mathbf{E}_m(\mathbf{r},t)|^2$, and χ is the Kerr coefficient given by

$$\chi = \frac{2\pi n z_0 f[(k_a + k_b - k_c)z_0]}{\hbar^4 \upsilon_{g,0}|\Omega_c|^2\Delta}\langle|\mathbf{d}_{24}\mathbf{E}_a|^2|\mathbf{d}_{26}\mathbf{E}_b|^2\rangle, \quad (5)$$

where n is the atomic density, z_0 is the thickness of the atomic layer, $f[x] \equiv (e^{-x}\sinh x)/x$, k_a and k_b are the real parts of the polariton wave numbers, k_c and Ω_c are the wave number and the Rabi frequency of the driving field, $\upsilon_{g,0}$ is the group velocity of the polaritons, ignoring the atomic layer, Δ stands for the spectral detuning, $\mathbf{d}_{24}$ and $\mathbf{d}_{26}$ give the atomic dipole moments of the corresponding transitions, $\mathbf{E}_a$ and $\mathbf{E}_b$ are the electric field operators of the polaritons, and $\langle\cdots\rangle$ denotes averaging over orientations of the dipole moments. A typical atomic density in a gas is 2×10^{14} cm^{-3}. To establish double electromagnetically induced transparency in ^{87}Rb, the Rabi frequency of the control field is to be $\Omega_c =$ 1 MHz, the transition wavelength is 780 nm, the detuning is $\Delta = 1.4$ MHz, and the dipole moments are about $5ea_0$, where e is the electron charge and a_0 is the Bohr radius. Assuming the thickness of the atomic layer $z_0 = 2$ μm, we obtain the Kerr nonlinearity as displayed in Fig. 3.

Although the Kerr nonlinearity χ is of the order of 10^{-4}, a significant mutual phase shift of the order of unity can be achieved between the surface polaritons. The mutual phase shift is given by $\phi = \chi\omega L/\upsilon_g$, where ω is the light frequency, L is the length of interaction in the medium, and υ_g is the group velocity of the light in the medium, ignoring the layer of five-level atoms. For the chosen parameters of the medium $\upsilon_g \approx 0.4c$ and assuming $L = 1$ mm, the mutual phase shift is shown in Fig. 3. The surface polaritons receive a mutual phase shift of the order of π at the frequency $\omega_\pi \approx 1.24\,\omega_0 =$ 545 THz (green light), which is close to the no-loss frequency ω_0.

Here we would like to point out that, because of the symmetry of the levels of the six-level atom, the refractive index of the medium is exactly the same for the two surface polaritons. That is why the polaritons propagate in the medium with equal group velocities and experience identical nonlinearity. Alternatively, five-level atoms [10] can be placed near the dielectric-metamaterial interface [27]. In this case, the two surface polaritons propagate in the medium with different group velocities and experience different nonlinearity in the double electromagnetically induced transparency regime. The latter scheme is best suited to achieving a uniform cross-phase-modulation [28].

The mutual (symmetric) phase shift can be used to create entanglement between initially uncorrelated field modes. For single-mode incident fields, the interaction of the surface polaritons in the (Kerr) medium can be described with the help of an effective Hamiltonian

$$H_{\text{eff}} = \hbar\chi a^\dagger a b^\dagger b, \quad (6)$$

where $a^\dagger, b^\dagger$ and a,b are creation and annihilation operators of the field modes of the two polaritons. The time evolution of these operators is initiated by the unitary transformation $U(t) = \exp(-i\phi a^\dagger a b^\dagger b)$ and is given by

$$a(t) = e^{-i\phi b^\dagger b}a(0), \quad b(t) = e^{-i\phi a^\dagger a}b(0). \quad (7)$$

If the initial states of the incident fields are uncorrelated single-mode coherent states $|\alpha\rangle$ and $|\beta\rangle$ [23], the final state $|\psi(t)\rangle_{ab}$ after the interaction can be written in the Fock basis as [29]

$$|\psi(t)\rangle_{ab} = e^{-|\beta|^2/2}\sum_{n=0}^{\infty}\frac{\beta^n}{\sqrt{n!}}|\alpha e^{-i\phi n}\rangle_a \otimes |n\rangle_b, \quad (8)$$

where the dynamics of the creation and annihilation operators (7) has been taken into account. Assuming $\phi = \pi$ and decomposing the sum in Eq. (8) over odd and even values of the index n, we obtain the following form of the final state:

$$|\psi\rangle^f_{ab} = \frac{1}{2}[|\alpha\rangle_a(|\beta\rangle_b + |-\beta\rangle_b) + |-\alpha\rangle_a(|\beta\rangle_b - |-\beta\rangle_b)].$$

This state is the local unitary equivalent to the entangled state $(|\alpha\rangle_a|\beta\rangle_b + |-\alpha\rangle_a|-\beta\rangle_b)/\sqrt{N}$ [29], where $N = 2 - 2\exp(-2|\alpha|^2 - 2|\beta|^2)$, which is known to preserve exactly one (entangled) bit of quantum information [30].

In contrast to our assumption above, the initial states $|\psi\rangle_a$ and $|\psi\rangle_b$ of the incident field can also be assumed to be polarization states of the photons. In this case, entanglement of the field modes can be achieved, for example, with the help of the Nemoto-Munro protocol [31]. To understand this protocol better, let us assume that the incident field a is prepared in a superposition of vacuum and single-photon states and is given by $|\psi\rangle_a = c_0|0\rangle + c_1|1\rangle$ in the Fock basis. The second incident field is prepared in the coherent state $|\alpha\rangle_b$. When the two fields interact with the medium, the resulting state of the fields is given by

$$U(t)\,|\psi\rangle_a|\alpha\rangle_b = c_0|0\rangle|\alpha\rangle_b + c_1|1\rangle|\alpha\, e^{i\phi}\rangle_b. \quad (9)$$

The state of the field a is unaffected by the interaction, while the state of the field b receives a phase shift, which is proportional to the number of photons in the state $|\psi\rangle_a$.

Assume that we have two polarization qubits to become entangled. The qubits are initially prepared in single-photon superposition states $|\psi\rangle_a = c_0|H\rangle + c_1|V\rangle$ and $|\psi\rangle_b = d_0|H\rangle + d_1|H\rangle$, where $|V\rangle$ and $|H\rangle$ are polarization degrees of freedom. These qubits are split individually on polarizing beam splitters into spatial modes and interact with an additional probe beam (which is in a coherent state $|\alpha\rangle_p$) in the Kerr medium. The resulting state of the three beams is given by

$$|\psi\rangle_{abc} = (c_0d_0|HH\rangle + c_1d_1|VV\rangle)|\alpha\rangle_p + c_0d_1|HV\rangle|\alpha e^{i\phi}\rangle_p + c_1d_0|VH\rangle|\alpha e^{-i\phi}\rangle. \quad (10)$$

The first term in this expression does not receive any phase shift, while the the second and third terms receive opposite-sign shifts. This makes it possible to transform the three-party state into entangled (Bell) bipartite states by performing a homodyne measurement on the probe. The measurement results in either the $c_0d_0|HH\rangle + c_1d_1|VV\rangle$ or the $c_0d_1|HV\rangle + c_1d_0|VH\rangle$ state, which are both maximally entangled states of qubits for $c_0 = c_1 = d_0 = d_1 = 1/\sqrt{2}$. It is also important to note that the Nemoto-Munro protocol described above allows us to construct entangling controlled-NOT gates [7] with large Kerr nonlinearity, opening a prominent possibility to use metamaterials in quantum computing.

The presented scheme for entanglement creation with negative index metamaterials may also find applications in quantum communication and quantum teleportation with both coherent [5,30] and polarization states [7]. Moreover, the Kerr nonlinearity created with the described medium can be used to generate multimode entangled coherent states [32] and multiphoton Greenberger-Horne-Zeilinger states [33].

We also would like to comment that in the present discussion we restricted ourselves to TM polarization of the surface polaritons. As we mentioned before, both TM and TE polarizations may exist at the dielectric-metamaterial interface. These polarizations may be used for information encoding on a par with encoding in quantum states of the field quanta. Another attractive idea is to use a trade-off between confinement and losses of the surface polaritons [22,27]. This trade-off may be used to establish two regimes, corresponding to "manipulation" and "low-loss transmission,' which are highly desired in quantum computation [34]. Both the possibilities mentioned will be the subject of further investigations.

In conclusion, we presented a scheme for entanglement creation with a composite medium consisting of a dielectric and a negative index metamaterial. Surface polaritons, which are created by the incident light in the medium, propagate along the dielectric-metamaterial interface with substantially reduced group velocity, exhibiting spatial confinement and with suppressed amplitude losses. Placing a layer of six-level atoms near the interface allowed us to establish a symmetric nonlinear interaction between the surface polaritons, which can be utilized to create entanglement between initially uncorrelated coherent or polarization states of light.

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