Short-time quantum detection: Probing quantum fluctuations

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We study the information provided by a detector click about the state of an initially excited two-level system. By computing the time evolution of the corresponding conditioned probability beyond the rotating-wave approximation, we show that a click in the detector is related to the decay of the source only for long interaction times. For short times, non-rotating-wave approximation effects such as self-excitations of the detector forbid a naive interpretation of the detector readings. These effects might appear in cQED experiments.

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Introduction. Quantum detection theory was created to study the behavior of detectors in the presence of radiation [1]. Highly satisfactory up to date, it relies on the conspicuous rotating-wave approximation (RWA), which neglects the so-called counterrotating terms. These terms give important contributions to strong atom-field couplings and very short times as compared to the system time scale, meaning that for any effect beyond RWA (bRWA) to be directly acknowledged, our measurements must be very precise and fast. This is problematic for quantum optics experiments, due to the very small matter-radiation coupling and the fact that observation times must be at the femtosecond scale for most cases (nanosecond for hyperfine qubits), which is too small for current experiments ($\sim\mu$s for trapped ions [2]).

However, cavity quantum electrodynamics (cQED) [3] provides a framework in which those phenomena are accessible to study. By using superconducting qubits coupled to a transmission line, the setup behaves analogously to a one-dimensional radiation-matter interaction model at the microwave frequency range [4]. Moreover, parameters can be easily tuned and the qubit-line coupling modulated up to ultrastrong levels [5,6]. Fast qubit state readout ($\sim$ns) is also possible using a pulsed DC SQUID scheme [7]. Thus, bRWA phenomena have already been reported [8,9], Glauber's theory is no longer valid, and quantum detectors should be described by a non-RWA model like that of [10].

A direct consequence of the breakdown of the RWA is that a detector in its ground state interacting with the vacuum of the field has a certain probability of getting excited and emitting a photon. However, there is not a widespread consensus on the physical reality of this effect. As a matter of fact there had been attempts to design effective detector models in a way which prevents this phenomenon from happening [11]. We should, however, recall here that these peculiar effects should not be that discomforting, as the initial state considered is not an eigenstate of the full Hamiltonian bRWA.

To describe those processes we will neither impose any additional constraints nor question their real existence. We will study the following setup: a source S initially excited, a detector D initially in the ground state, and both interacting with the electromagnetic field in its vacuum state. If the detector clicks at a given time, does it mean that the source is now in the ground state? This problem amounts to computing the probability of decay for the source, conditioned to the excitation of the detector. We will show that unlike Glauber's RWA detector, in which this conditioned probability would be equal to 1 at any time, this cQED detector only achieves this value at long times due to the impact of non-RWA effects.

Mathematical description of the model. We consider a model consisting of two superconducting qubits, S and D, with two levels g and e and separated a distance r. Let us consider that at $t=0$, S is excited, D is in its ground state, and there are no excitations in the transmission line, which will be open, enabling a continuum of modes. Representing the states in terms of qubits and field (F) free eigenstates with the notation $|\psi\rangle = |SDF\rangle$, the initial state would be $|i\rangle_{t=0} = |eg0\rangle$. We intend to study the relevance of bRWA processes by quantifying what information about the state of S can be extracted by knowing the qubit D state after a certain time t. For that we will compute the probability $\mathcal{P}_{S_g|D_e}(t)$ of S to have decayed at a certain instant t based on the condition that we have measured D excited at that moment:

$$\mathcal{P}_{S_g|D_e}(t) = \frac{\mathcal{P}_{[ge*]}}{\mathcal{P}_{[*e*]}} = \frac{\sum_F |\langle geF|e^{-iHt/\hbar}|eg0\rangle|^2}{\sum_{n,F} |\langle neF|e^{-iHt/\hbar}|eg0\rangle|^2}, \qquad (1)$$

$\mathcal{P}_{[ge*]}$ being the probability of having S in the ground state and D excited, and $\mathcal{P}_{[*e*]}$ the total probability of excitation of D.

Naively we would expect that $\mathcal{P}_{[*e*]}(t) = 0 \ \forall\, t \leqslant R/c$ and that $\mathcal{P}_{[*e*]}(t) = \mathcal{P}_{[ge*]}(t) \neq 0 \ \forall\, t \geqslant R/c$, so $\mathcal{P}_{S_g|D_e} = 1$. However, in Ref. [12] it is shown that $\mathcal{P}_{D_e}(t) \geqslant 0 \ \forall\, t \leqslant R/c$. As explained in Refs. [13,14], we can split the probability of detector excitation as

$$\mathcal{P}_{[*e*]}(t) = \mathcal{P}^{(0)}_{[*e*]}(t) + \mathcal{P}^{(R)}_{[*e*]}(t). \qquad (2)$$

The first term, independent of R (and so of S), is the self-excitation term, and so $\mathcal{P}^{(0)}_{[*e*]}(t) \geqslant 0 \ \forall\, t \leqslant R/c$. The second, dependent on R, refers to excitations due to exchange processes, and behaves causally $\mathcal{P}^{(R)}_{[*e*]}(t) = 0 \ \forall\, t \leqslant R/c$.

One might expect that the effects of $\mathcal{P}^{(0)}_{[*e*]}(t)$ could be accounted for by including a sort of "dark current" due to *self-excitations* as compared to the *exchange processes* that would be the only ones to appear if we were thinking naively. In that sense, the analysis here presented might look somehow contrived; however, the very notion of dark current is not valid for short times. The concept itself comes from Fermi's Golden Rule, which predicts a linear dependence of the excitation probability with time. In a bRWA analysis at very short times, akin to that of the Zeno effect or ours, the probability of excitation will result proportional to t^2, and no such thing as a constant stable rate of dark counts can be defined. Since there is no way to experimentally distinguish between the two sorts of processes that lead to a detector excitation, the analysis through a conditioned probability seems a reasonable one.

We will consider the following Hamiltonian [15,16]:

$$H = H_0 + H_I,$$
$$H_0 = \sum_{A=\{S,D\}} \frac{\hbar\Omega_A}{2}\sigma_z^A + \int_{-\infty}^{\infty} dk\,\hbar\,\omega_k\, a_k^\dagger a_k, \tag{3}$$
$$H_I = -\sum_{A=\{S,D\}} d_A V(x_A)\sigma_x^A.$$

Here x_A corresponds to the position of the qubit A, $\hbar\Omega_A$ is the gap between levels for qubit A, and V refers to the one-dimensional field which expands as

$$V(x) = i\int_{-\infty}^{\infty} dk\sqrt{N\omega_k}e^{ikx}a_k + \text{H.c.} \tag{4}$$

This field has a continuum of Fock operators $[a_k, a_{k'}^\dagger] = \delta(k-k')$, and a linear spectrum, $\omega_k = v|k|$, where v is the propagation velocity of the field. The normalization and the speed of photons, $v = (cl)^{-1/2}$, depend on the microscopic details such as the capacitance and inductance per unit length, c and l. Note that this model resembles that of an Unruh-DeWitt detector [10]. For our calculations, we will make use of the interaction picture, so we let the initial state $|eg0\rangle$ evolve for a lapse of time t as

$$\begin{aligned}|\psi(t)\rangle &= U_I(t)|eg0\rangle = \mathcal{T}\{e^{-i\int_0^t dt' H_I(t')/\hbar}\}|eg0\rangle \\ &= I|eg0\rangle + X|ge0\rangle + \sum_k A_{1,k}\,|gg1_k\rangle + \sum_k B_{1,k}\,|ee1_k\rangle \\ &\quad + \sum_{kk'} A_{2,kk'}\,|eg2_{kk'}\rangle + \sum_{kk'} B_{2,kk'}\,|ge2_{kk'}\rangle + \cdots.\end{aligned} \tag{5}$$

Note that all terms but the first three are zero when working in the RWA. For A's coefficients D end in the ground state, for B's, in the excited one. Here and in the following we will only make explicit the terms that contain contributions for the probabilities up to d_A^4. For example, terms with three or more photons in the amplitude will be excluded, as they give contributions of $O(d_A^6)$. We will underline the ones relevant to our analysis.

Probability calculations. Let us define $\mathcal{M}(t;nF) = \langle neF|\psi(t)\rangle$. Thus, the first of the probabilities needed for the computation of $\mathcal{P}_{S_g|D_e}(t)$ [Eq. (1)] can be written down using Eq. (5) as

$$\begin{aligned}\mathcal{P}_{[ge*]} &= \sum_F |\langle geF|U_I(t)|eg0\rangle|^2 = \sum_F |\mathcal{M}(t;gF)|^2 \\ &= \underline{|X|^2 + \sum |B_2|^2} + \sum |B_4|^2 + \cdots.\end{aligned}$$

The first building block needed is $|X|^2$. Note that

$$\mathcal{P}_{[ge0]} = |\langle ge0|U_I(t)|eg0\rangle|^2 = |\mathcal{M}(t;g0)|^2 = |X|^2. \tag{6}$$

To evaluate $|X|^2$ up to fourth order in perturbation theory, one must consider that X has no contributions for orders 0 or 1, so the calculation must be performed for orders 2 and above. As a matter of fact, order 2 alone is sufficient. This calculation has been already performed in the appendix of [17], where the perturbative parameter d_A is included in the dimensionless coupling strength $K_A = \frac{4d_A^2 N}{\hbar^2 v} = 2(\frac{g_A}{\Omega_A})^2$, with $A = \{S,D\}$, g_A being the qubit-line coupling. We must restrict our calculations to times when $K_A\Omega_A t \ll 1$, where our perturbative approach is valid.

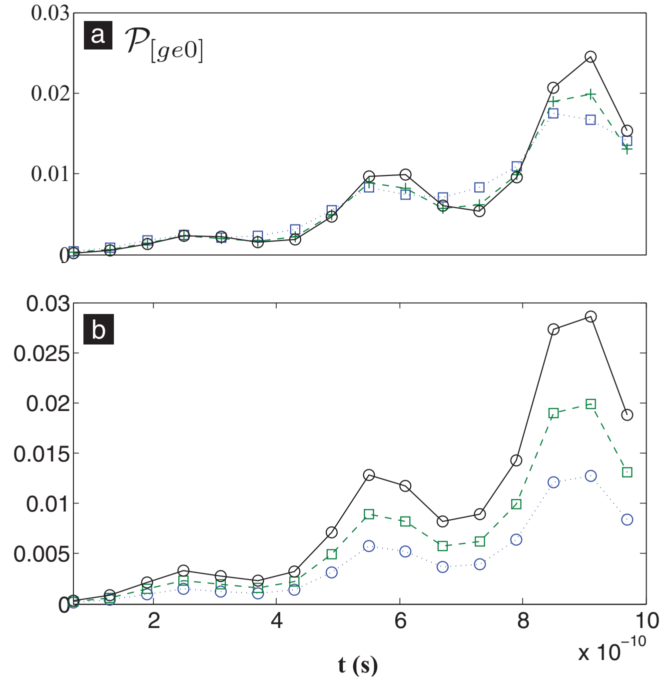

FIG. 1. (Color online) (a) $\mathcal{P}_{[ge0]}$ in front of t for three different values of the distance between qubits $2\pi\frac{r}{\lambda} = 0.1$ (dotted, squares, blue), 0.3 (dashed, crosses, green), and 0.5 (solid, circles, black). For all cases $K_S = K_D = 7.5\times10^{-3}$ and $\Omega/(2\pi) = 1$ GHz. (b) $P_{[ge0]}$ in front of t for three different values of the coupling strength $K = K_S = K_D = 6\times10^{-3}$ (dotted, squares, blue), 7.5×10^{-3} (dashed, crosses, green), and 9×10^{-3} (solid, circles, black). For all cases $2\pi\frac{r}{\lambda} = 0.3$ and $\Omega/(2\pi) = 1$ GHz.

In Fig. 1 we sketch the evolution of the probability $\mathcal{P}_{[ge0]}$ with time, and its dependence with the coupling and the distance between qubits. Typical values for couplings and distances for a setup in cQED are considered from here on. At these early stages $\mathcal{P}_{[ge0]}$ is highly oscillatory in time. For a given time, the probability always grows with the coupling strength but depends on the distance in different ways.

To proceed with the calculation of $\mathcal{P}_{[ge*]}$, the terms $B_{2,kk'} = \langle eg0|U_I(t)|ge2_{kk'}\rangle$ must be evaluated. Because the final associated bare state has two photons, this implies automatically that orders 0 and 1 are discarded. Once again, order 2 alone fits. The final calculation gives a term symmetric respect to a $k \leftrightarrow k'$ exchange:

$$B_{2,kk'} = f_{kk'} + f_{k'k} = f_{kk'} + \{k \leftrightarrow k'\}. \tag{7}$$

After that, they must be squared and summed as in $\sum |B_2|^2 = \frac{1}{2!}\sum_{kk'} B_{2,kk'}B_{2,kk'}^*$ splitting into two terms: $\sum |B_2|^2 = \sum_{kk'} f_{kk'}f_{kk'}^* + \sum_{kk'} f_{kk'}f_{k'k}^*$, a "direct" one, just the product of the square of the emission amplitudes (explicitly computed in Ref. [17]), and a "crossed" one which looks like a photon exchange and is a one-dimensional version of the crossed term computed in Ref. [18]. The summation of the direct terms implies the appearance of expected divergences which can

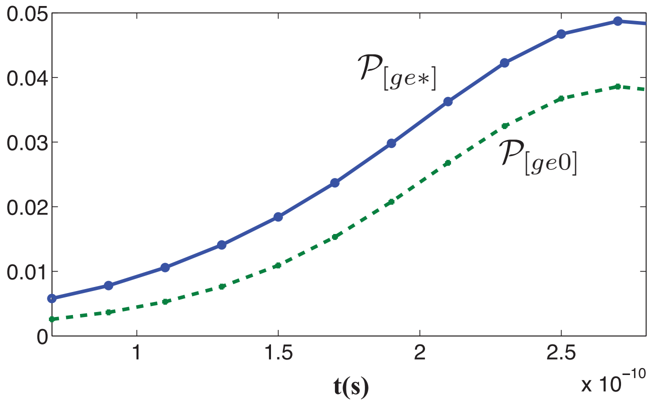

FIG. 2. (Color online) $\mathcal{P}_{[ge*]}$ (solid, blue, circles) and $\mathcal{P}_{[ge0]}$ (dashed, green, crosses) in front of t in s with a distance $2\pi\frac{r}{\lambda} = 0.5$, a coupling strength of $K = K_S = K_D = 3 \times 10^{-2}$, and $\Omega/(2\pi) = 1$ GHz ($\Omega = \Omega_S = \Omega_D$). The difference between the two graphs is the non-RWA term $\sum |B_2|^2$.

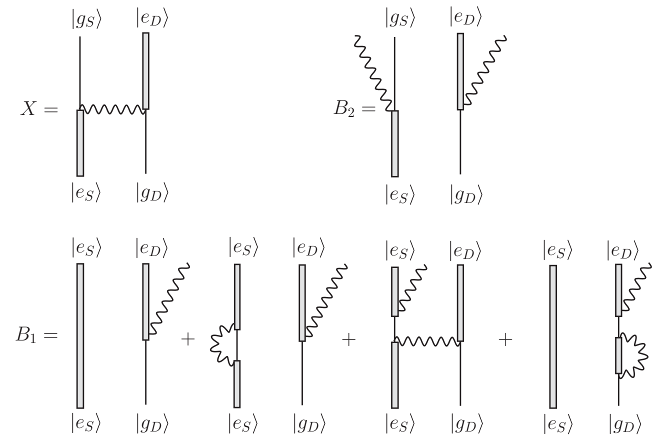

FIG. 3. Diagrams contributing to X, B_1, and B_2. X represents the amplitude for photon exchange between source and detector, while B_2 is just the amplitude for two single-photon emissions, one at each qubit. The leading-order contribution to B_1 is the amplitude for a single-photon emission at the detector qubit (first diagram), but third-order one-loop corrections (second and fourth) and a photon exchange accompanied by an emission at the source (third) have to be also taken into account. B_1 and B_2 are completely non-RWA diagrams.

be resolved using a regularization procedure analogous to the one sketched in the appendix of Ref. [18]. This procedure requires the times of analysis to be larger than a certain cutoff time t_0, which in this case is related to the typical size of a superconducting qubit $d \simeq 10^{-6}$ m [19] and the propagation velocity of the field quanta: $v \simeq 10^8$ m/s. Thus, $t_0 = d/c \simeq 1 \times 10^{-14}$ s, far below the times considered in this work.

Notice that B_2 is only nonzero beyond the RWA. In Fig. 2 we compare $\mathcal{P}_{[ge*]}$ with $\mathcal{P}_{[ge0]}$. The impact of this non-RWA contribution is seen in the subnanosecond regime for a large coupling strength. At larger times, the impact diminishes, $\mathcal{P}_{[ge*]} \simeq \mathcal{P}_{[ge0]}$, and the RWA applies.

The last probability of interest $\mathcal{P}_{[*e*]}$ can be written as

$$\mathcal{P}_{[*e*]} = \sum_{n,F} |\langle neF|U_I(t)|eg0\rangle|^2 = \sum_{n,F} |\mathcal{M}(t;nF)|^2$$
$$= \underline{|X|^2 + \sum |B_1|^2 + \sum |B_2|^2} + \sum |B_3|^2 + \cdots. \tag{8}$$

And so we must obtain $\sum |B_1|^2$, which is again a completely non-RWA contribution. For that case the situation gets more complicated, as there are interfering processes of orders 1 and 3 leading to that final state. The four diagrams contributing to $\sum |B_1|^2$ up to fourth order in perturbation theory can be seen in Fig. 3. The leading contribution is just the probability of self-excitation of the detector (first diagram for B_1 in Fig. 3) and the other contributions come from the interference of this diagram with the other three. In particular, interference with the third diagram of B_1 is crucial for causality [14]. More details on this computation can be found in Ref. [20].

Conditioned detection probability. With the previous probabilities computed we can finally address the conditioned probability $\mathcal{P}_{S_g|D_e}(t)$, which can be calculated as Eq. (1). Note that in the RWA, $\mathcal{P}_{S_g|D_e}(t) = 1$ at any time, since $\mathcal{P}_{[*e*]} = \mathcal{P}_{[ge*]} = \mathcal{P}_{[ge0]}$. The effect of non-RWA contributions to the evolution of $\mathcal{P}_{S_g|D_e}(t)$ can be seen in Figs. 4 and 5, where the dependence with the coupling and the distance between qubits is considered.

The first thing we notice in Fig. 4 is that for short times the information provided by the detector is not very much related to the state of the source, that is, self-excitations and other non-RWA phenomena dominate over the photon exchange between source and detector. For the cases considered, only at interaction times $t \gtrsim 1$ ns $\simeq 1/\Omega$ the conditioned probability converges to the RWA prediction and the excitation of the detector is a reliable way to detect the decay of the source. Since the non-RWA contributions are more relevant for large couplings and short distances, the convergence is faster as the distance grows and the couplings diminish, as can be seen in Figs. 4 and 5. Notice that the ripple frequency we see, for

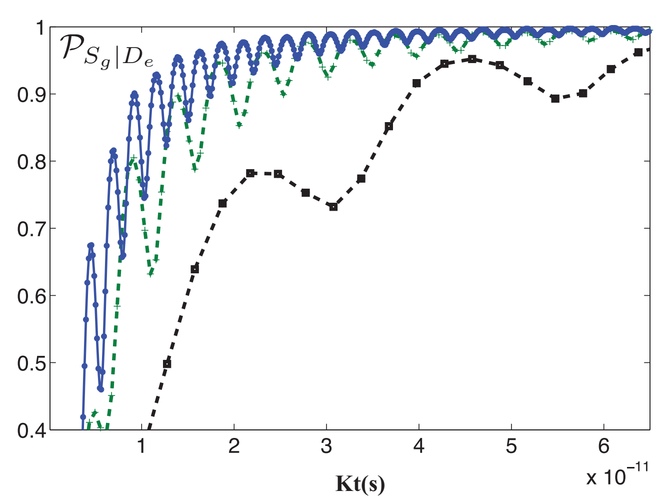

FIG. 4. (Color online) $\mathcal{P}_{S_g|D_e}(t)$ [Eq. (1)] in front of Kt for three different values of $K = K_S = K_D = 7.5 \times 10^{-3}$ (solid, blue, circles), 1.5×10^{-2} (dashed, green, crosses), 7.5×10^{-2} (dashed, black, squares). In the three cases $2\pi\frac{r}{\lambda} = 1$ and $\Omega/(2\pi) = 1$ GHz ($\Omega = \Omega_S = \Omega_D$).

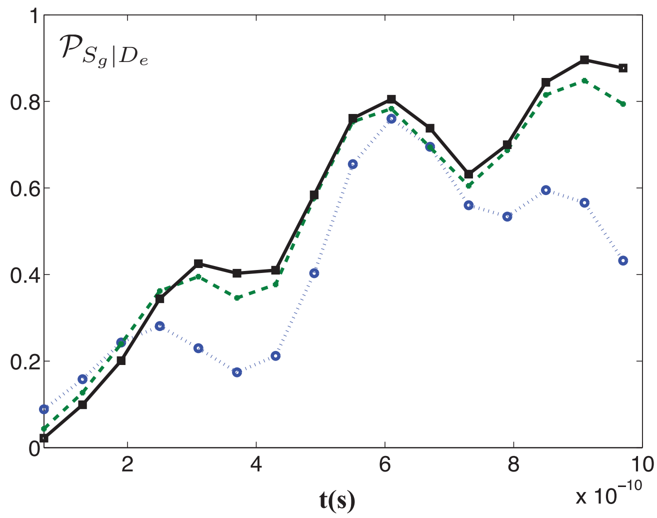

FIG. 5. (Color online) $\mathcal{P}_{S_g|D_e}(t)$ [Eq. (1)] in front of t in s for three different values of the distance $2\pi\frac{r}{\lambda} = 0.5$ (dotted, blue, circles), 0.75 (dashed, green, crosses), 1 (solid, black, squares). In the three cases, $K = K_S = K_D = 1.5 \times 10^{-2}$ and $\Omega/(2\pi) = 1$ GHz ($\Omega = \Omega_S = \Omega_D$).

instance, in Fig. 4 comes from higher harmonics of the qubit frequency Ω. It can be thought as a process similar to that of a Rabi oscillation, where the qubits would be absorbing in cycles the photons previously emitted in self-excitations.

Prospects and conclusions. These theoretical results could have an impact in real experiments of cQED. In particular, a typical setup to measure the internal state of a flux qubit coupled to a transmission line consists of a SQUID surrounding the qubit. Although the total measurement process could take up to tens of nanoseconds, most of the time the coupling SQUID qubit is much stronger than K [7] and the dynamics qubit-transmission line is effectively frozen. Thus this dynamics is only important during the activation of the SQUID, a process that may be in the nanosecond regime. For those measurement times, as we have proved, self-excitation effects cannot be disregarded and should manifest themselves.

Besides, it should, in principle, be possible to prepare experiments in the near future to test our predictions directly. We do not intend to present here more than just a rough sketch. Such experiments would involve the preparation of the system at $t = 0$ in the initial state $|eg0\rangle$, the switching of the interaction for a certain time t (in the line of previous proposals, as [14,21–23]) and then the SQUID measurement of both qubits S and D. By repeating the experiment several times, the resulting frequencies should match our theoretical predictions.

To conclude, we have shown that for typical cQED parameters, a significant amount of time is needed to start trusting the state of a detector as being informative regarding an initially excited source. This is due to the breakdown of the RWA in cQED. By neglecting the counterrotating terms, a total reliability on the information coming out of the detector would be wrongly derived for all time scales. Our result applies to other setups and quantum detectors, although it is in the case of cQED where it might affect the interpretation of coming experimental results.

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