Polarization switching and nonreciprocity in symmetric and asymmetric magnetophotonic multilayers with nonlinear defect

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A one-dimensional magnetophotonic crystal with a nonlinear defect placed either symmetrically or asymmetrically inside the structure is considered. Simultaneous effects of time-reversal nonreciprocity and nonlinear spatial asymmetry in the structure are studied. Bistable response is demonstrated in a such system, accompanied by abrupt polarization switching between two circular or elliptical polarizations for transmitted and reflected waves. The effect is explained in terms of field localization at defect-mode spectral resonances and can be used in the design of thin-film optical isolators and polarization transformation devices.

viewpoint [7].

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to time-reversal symmetry breaking, which is inherent to magnetic media and can be explained from the symmetry

In multilayer structures, material nonlinearity can also

result in optical nonreciprocity. One example is a layered

medium where frequency changing or self-focusing is spatially

I. INTRODUCTION

Magnetophotonic crystals (MPCs) are periodic structures that contain magnetic materials and have a period comparable to the wavelength of electromagnetic radiation [1–6]. The simplest example of such a periodic structure is a multilayer having one-dimensional (1D) periodicity. The main advantage of MPCs in contrast to conventional nonmagnetic photonic crystals (PCs) is their possibility to tune the band edge position in the spectrum of the electromagnetic radiation by means of an external static magnetic field.

Another promising feature of MPCs is a strong enhancement of magneto-optical effects in comparison to bulk magnetic materials. Among these effects, the Faraday effect is of great interest. It manifests itself as rotation of the polarization ellipse of light as it propagates collinearly with an externally applied static magnetic field. It can be seen as a lifting of degeneracy for the left-circular-polarization (LCP) and right-circular-polarization (RCP) states, causing the LCP and RCP components to propagate in the magnetic medium with different phase velocities. The enhancement of Faraday rotation in MPCs originates from localization of light provided by the multiple interference [3,5]. The enhancement becomes even greater in MPCs with microcavity structure where a magnetic defect is introduced into the periodic system [1]. In any geometry, the amount of rotation is linear with respect to the static magnetic field strength and is sensitive to the direction of the applied magnetic field. Since the definitions of LCP vs RCP are tied to the direction of wave propagation, a forward-travelling LCP wave has the same properties as the backward-travelling RCP wave, and vice versa. Hence lifting the degeneracy between LCP and RCP results in optical nonreciprocity; that is, a difference in the properties of a medium for electromagnetic waves propagating in opposite directions. It is well known that this nonreciprocity is related

asymmetric in the presence of nonuniform dichroism [8]. The order in which the nonlinear and dichroic layers are encountered by the incident light significantly influences the

balance between nonlinear effects and absorption, making the optical properties of the structure dependent on the direction of incidence. A similar effect is achieved if a PC contains an asymmetrically placed defect (i.e., microcavity) with a nonlinear (e.g., Kerr-type) material. In such a system, strong field localization inside the defect can makes the internal field intensity sufficient to change its optical characteristics through the Kerr effect. Since the spatial field distribution is different for the waves incident on a spatially asymmetric structure from opposite sides, nonreciprocal response appears [9,10]. These nonlinearity-induced directional sensitivity can be called *reversible nonreciprocity*, to mark that no time-reversal symmetry breaking takes place here.

It is important that such nonlinear reversible nonreciprocity is often accompanied by optical bistability. The dynamical shift of resonant frequencies of the structure (photonic band edges or microcavity modes) due to strong field localization gives rise to two stable transmission and reflection states for the same input intensity. The input-output characteristic of such a system typically contains a hysteresis loop [11,12]. In asymmetric structures, this loop has directional sensitivity, so the intensity level of input light sufficient to achieve bistable switching is different for the waves impinging on the multilayer from opposite sides. This effect can produce such different transmittances for forward- vs backward-propagating waves in the structure that it can be used in the design of a nonlinear electromagnetic diode [11–14]. In analogy with an electronic diode that transmits electric current in only one direction due to its nonlinear current-voltage characteristics, the nonlinear optical diode features unidirectional transmission of the incoming light.

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If a nonlinear multilayer has any kind of optical anisotropy, its bistable response necessarily becomes polarization dependent. This was shown in birefringent nonlinear systems where linear polarization of the field can abruptly change to rightor left-handed elliptical polarization when the light intensity or structure parameters reach a certain threshold level [15,16]. This nonlinear polarization switching, which can appear in any kind of anisotropic nonlinear systems (including birefringent, form anisotropic, chiral media, etc.), is associated with the effects of polarization instability, bistability, multistability, and polarization chaos [17,18].

Of all kinds of anisotropic systems, nonlinear MPCs are distinguished by the possibility to realize switching between two distinct orthogonal polarization states and to make this switching tunable by means of an external static magnetic field. Furthermore, by introducing nonlinearity into the MPC, unidirectional transmission can be achieved for one circular polarization while remaining transparent for the polarization of opposite handedness. An abrupt switching between two opposite circular polarization states can then be realized. Hence it is of special interest to study simultaneous effects of time-reversal nonreciprocity and nonlinear spatial asymmetry on the optical properties of PCs.

In this paper, we consider a MPC with a nonlinear defect placed either symmetrically or asymmetrically inside the periodic structure. An important feature of the studied system is the fact that the asymmetric bistable transmission is accompanied by the polarization conversion [19–21]. The main objective of our study is focused on achieving the bistability-induced abrupt switching between two distinct polarization states. This can be important for thin-film polarization optics devices and polarization-sensitive integrated optics.

The rest of the paper is organized as follows: In Sec. II, we formulate the problem under study and introduce its solution based on the transfer matrix method of multilayer optics. Sections III and IV follow with the results for a nonlinear defect placed symmetrically and asymmetrically into a MPC, respectively. Finally, Sec. V summarizes the paper.

II. PROBLEM FORMULATION AND SOLUTION

We consider a planar multilayer stack of infinite transverse extent (Fig. 1). Each unit cell is composed of a bilayer which consists of magnetic (with constitutive parameters ε_1 , $\hat{\mu}_1$) and nonmagnetic (with parameters ε_2 , μ_2) layers. The magnetic layers are magnetized up to saturation by an external static magnetic field \vec{M} directed along the z axis (Faraday geometry). A defect is created by introducing into the structure a layer with constitutive parameters ε_d and μ_d . We assume that this layer is a Kerr nonlinear dielectric, with a permittivity ε_d that depends linearly on the intensity $|E|^2$ of the electric field. The defect can be settled either symmetrically or asymmetrically in the middle of the structure. The parameters m and n describe the number of bilayers placed before and after the defect layer. In any case the bilayers are arranged symmetrically with respect to the defect layer, (i.e., the structure begins and ends with layers of the same type). We suppose that all layers have the same thickness D. The outer half spaces $z \leq 0$ and $z \geq [2(m + n) +$ 1]D are homogeneous, isotropic, and have parameters ε_0 , μ_0 . Assume that the normally incident field is a linearly polarized



FIG. 1. (Color online) Magnetophotonic structure with nonlinear defect.

plane monochromatic wave of frequency ω and amplitude A. For the sake of definiteness, we also suppose that the vector \vec{E} of the incident wave is directed along the x axis.

As an convenient material for magnetic layers, the family of impurity-doped yttrium-iron garnet (YIG) Y₃Fe₅O₁₂ films can be proposed. These magnetic oxides are well studied and widely used in integrated magneto-optics because they are transparent in the near infrared region [3,5]. As an example, a few types of multilayered films composed of magnetic Bi-substituted YIG (Bi:YIG) and dielectric SiO₂ or glass FR-5 layers were investigated. [2,7] MPCs based on the other materials are also known. Thus, a new class of semiconductor-magnetic hybrid nanostructures consists of GaAs with MnAs nanoclasters (GaAs:MnAs), which are paired with GaAs/AlAs superlattices and have been recently investigated experimentally in the range 900-1100 nm [22]. Also in the nonlinear regime the structure based on the semimagnetic semiconductors such as $Cd_{1-x}Mn_xTe$ with the defect being a quantum well with prescribed spectral characteristics was reported [23,24]. From these papers it may be deduced that the magnetic materials manifest their nonlinear properties at the light intensity about 1 GW/cm². In our present paper we consider the nonlinear defect, which is made of nonmagnetic material due to its greater availability. As an example, AsGa or InSb can be selected for this purpose. We prefer such a structure configuration because these materials require much lower intensities of the incident light to enable the nonlinear effects. From the literature [25] it can be deduced that the nonlinear response in the semiconductor materials can be achieved at the light intensity about 1 kW/cm^2 . Although a defect is made of nonmagnetic material, the studied structure that consists of magnetic layers and such nonlinear defect exhibits a number of very interesting and unique properties that we consider.

Note that, in nonlinear materials, there is always a possibility of three- or four-wave-mixing processes, resulting in second- and third-harmonic generation. In multilayered structures, field enhancement associated with transmission resonances is also known to enhance harmonic generation processes. However, for these processes to be efficient, the fundamental and higher-harmonic waves need to be quasi-phase-matched. This generally requires that *both* the fundamental *and* the higher-harmonic frequency correspond to a resonant mode with good spatial overlap between them [26–28]. Usually, such conditions have to be specially engineered (in particular, to counter material dispersion), and it is easier to meet them using band-edge states [28] than defect states as in this work. Outside the phase matching, it can be shown [29] that second-harmonic transmission and reflection in one-dimensional nonlinear multilayer are about three orders of magnitude weaker that the fundamental transmission and reflection. Hence we will neglect the higher-harmonic generation by the nonlinear layer as is conventional in the theory of bistable Fabry-Perot resonators [30], assuming that any inadvertent phase matching condition can be very easily countered by slightly changing the structure design.

Our solution is based on the transfer matrix formalism [31] which is used to calculate the field distribution inside the structure and the reflection and transmission coefficients of the MPC. In the Faraday geometry, when an external static magnetic field is biased parallel to the direction of wave propagation ($\vec{k} \parallel \vec{M}$), the magnetic permeability $\hat{\mu}_1$ is a tensor quantity with nonzero off-diagonal components:

$$\hat{\mu}_1 = \begin{pmatrix} \mu_1^T & i\alpha & 0\\ -i\alpha & \mu_1^T & 0\\ 0 & 0 & \mu_1^L \end{pmatrix}.$$
 (1)

For the description of electromagnetic waves in this case it is necessary to use a 4×4 transfer matrix formulation [32]. Thus, at the first stage, in the linear case, the equation which defines the coupling of the tangential field components at the input and output of the structure is written in the form [33,34]

$$\dot{\Psi}(0) = \mathfrak{M}\Psi(\Lambda) = \{ (\mathbf{M}_1 \mathbf{M}_2)^m \mathbf{M}_d (\mathbf{M}_2 \mathbf{M}_1)^n \} \Psi(\Lambda), \quad (2)$$

where $\vec{\Psi} = \{E_x, E_y, H_x, H_y\}^T$ is the vector containing the tangential field components at the structure input and output, the upper index *T* is the matrix transpose operator, Λ is the total length of the structure, $\Lambda = [2(m + n) + 1]D$, *m* and *n* are the numbers of periods placed before and after the defect element, and \mathbf{M}_1 , \mathbf{M}_2 , and \mathbf{M}_d are the transfer matrices of the rank four of the first, second, and defect layers, respectively. The elements of the transfer matrices in (2) are determined from the solution of the Cauchy problem and are given in [32].

As the solution of the linear problem (2) is obtained, the intensity of the reflected and transmitted fields and the distribution of the field $\vec{E}_{in}(z)$ inside the MPC can be calculated. Generally, when the defect layer consists of a Kerr nonlinear dielectric, the permittivity ε_d is inhomogeneous and depends on the intensity of the electric field at each point of this layer as follows:

$$\varepsilon_{\rm d}(z) = \varepsilon_{\rm d}^{\rm l} + \varepsilon_{\rm d}^{\rm nl} |E_{\rm in}(z)|^2 \quad [2mD \leqslant z \leqslant (2m+1)D]. \quad (3)$$

Knowing the field intensity in the defect layer, both the actual value of permittivity ε_d and, consequently, the actual value of the transfer matrix \mathfrak{M} can be calculated. Thus we deal with an equation on the unknown function of field intensity distribution inside the defect layer. A magnitude of the incident field *A* is an independent parameter for this equation. Since the parameter ε_d^{nl} is small and the nonlinear contribution to ε_d varies with the longitudinal distance on the scale of one-half wavelength,

we provide an approach which regards ε_d as independent of z and treats the dependence of ε_d on the average intensity of the electric field $|E_{in}|^2$ inside the defect layer. Quantitative reasoning for this approach is presented in [34]. On the basis of this approximation, we suppose that the permittivity of the medium depends on the average intensity of the electric field as $\varepsilon_d = \varepsilon_d^1 + \varepsilon_d^{nl} |\overline{E_{in}}|^2$.

As a result, at the second stage, the nonlinear equation related to the average field intensity distribution in the defect is obtained. The numerical solution of this equation yields us the final field distribution in the MPC and the values of the reflection R and transmission T coefficients, for which expressions can be found in [32].

III. SYMMETRIC MULTILAYERS: POLARIZATION BISTABILITY

Our objective here is to study the main features of optical response for a MPC with a nonlinear defect placed symmetrically inside it. For this reason we consider a MPC consisting of two sections with the same number of bilayers in them (m = n). The sections are located symmetrically on each side of the defect layer. The main idea of such an arrangement is to obtain a significant field localization inside the defect layer, which is achieved by an appropriate choice of the number of periods *m* and the material parameters of layers.

The basic optical properties of the studied MPC are inherited from the characteristics of perfectly periodic structures with nonmagnetic layers. Recall that all periodic structures with layer thicknesses comparable to the wavelength possess forbidden frequency gaps (stopbands or band gaps) as a direct consequence of the Floquet-Bloch theorem [35]. These gaps are determined by the modulation period and the average refractive index. Propagation of waves with frequencies in the stopbands of an idealized infinite structure is completely inhibited, and the band gaps are in this sense perfect. For finite structures these gaps appear as frequency regions with low transmittance and high reflectance, located between hightransmittance passbands. If any distortion (a "defect") is introduced inside a periodic structure, transmission resonances can appear in the stopbands, with the field strongly localized at the defect. The existence of such "localization resonances" is explained by the fact that the defect forms a resonant Fabry-Perot cavity enclosed between two Bragg mirrors.

The main distinctive feature of a MPC in contrast to the nonmagnetic PC is the appearance of circular polarization eigenstates. Such circular polarization eigenstates are also inherent in PCs with chiral isotropic layers [33,34,36] but in the case of MPCs they are controlled with an external static magnetic field. Thus the MPC reacts differently to circularly polarized waves with opposite handedness, with distinct optical spectra for each of them [see Figs. 2(a) and 3(a)]. This way, in the Faraday configuration, both the edges of the forbidden bands and the frequencies of the localized defect modes become different for LCP vs RCP incident waves. As a result, the defect resonances split into doublets [see Figs. 2(a)-3 known as the longitudinal Zeeman-like doublets [20]. These doublets originate from lifting of the degeneracy between resonant conditions for the LCP and RCP waves in the underlying MPC by the external magnetic field. It can be



FIG. 2. (Color online) Frequency dependencies ($\alpha = D/\lambda$) of the transmission coefficient (*T*) of the LCP (–) and RCP (+) waves in the (a) linear (b) nonlinear case for m = n = 5, $\varepsilon_1 = 10$, $\mu_1^T = \mu_1^L = 1$, $\alpha = 0.05$, $\varepsilon_2 = \mu_2 = \mu_d = 1$, $\varepsilon_d^1 = 4$. For the nonlinear case, $\tilde{\varepsilon}_d^{nl} = \varepsilon_d^{nl} I_0 = 1.5 \times 10^{-4}$, which corresponds to the incident light intensity $I_0 = 15 \text{ kW/cm}^2$ for $\varepsilon_d^{nl} \simeq 1.0 \times 10^{-5} \text{ cm}^2/\text{kW}$.

seen that there are two closely spaced resonant modes in the stopband, one of which is a RCP eigenmode and the other is a LCP eigenmode.

In the insets of Figs. 2 and 3 the frequency band where the doublet exists is given on a larger scale. Throughout the paper we suppose that the working frequency is far from the frequency of the ferromagnetic resonance of magnetic layers and their losses are negligibly small. Under this assumption, at the resonant frequencies, the magnitude of the transmission coefficient of the corresponding circularly polarized mode reaches unity, and the structure becomes completely transparent for the LCP wave when $a^- \approx 0.098$ and for the RCP wave when $a^+ \approx 0.0995$. Obviously, the magnitude of splitting (the frequency difference between the peaks $\Delta a = a^+ - a^-$) can be easily tuned by changing the strength of the external static magnetic field.

Now we consider the case when the MPC contains a Kerr-type nonlinear defect. It is known that the introduction of such a defect into an otherwise linear structure can induce bistable behavior in the system. The nature of this bistability is studied in the theory of nonlinear Fabry-Perot resonators quite well [30]. The resonant frequencies a^{\pm} are sensitive to



FIG. 3. (Color online) Same as Fig. 2 but for the reflection coefficient (R).

the refractive index of the material within the cavity. Thus, when the frequency of the incident wave is tuned near a resonant frequency, the field localization induces growth of the light intensity inside the cavity, which, by means of the Kerr effect, eventually alters the refractive index enough to shift the resonant frequency. When this shift brings the resonant condition closer to match the frequency of the incident field, even more energy gets localized in the cavity. This further enhances the shift of the resonance, creating positive feedback that leads to formation of a hysteresis loop in the spectra with respect to the incident field intensity. As a result, for a fixed input field intensity, the frequency dependencies for any resonant mode have a typical shape of "bent resonances." In the spectra of a nonlinear MPC this bending can be seen for both resonant modes in the split doublets [Figs. 2(b)-3(b)].

Now consider a linearly polarized wave incident on a MPC with defect. One can represent it as a superposition of LCP and RCP waves. As a result, the corresponding optical spectra will contain both resonances. This is demonstrated in Fig. 4 for individual polarization components of reflected and transmitted light, as measured in typical experiments. Since the whole system possesses axial symmetry in the considered case of normal incidence and Faraday geometry, we can only distinguish between copolarized (e.g., ss or pp, denoted co) and cross-polarized (sp or ps, denoted cr) components. Since LCP and RCP components are present in



FIG. 4. (Color online) Frequency dependencies ($\alpha = D/\lambda$) of the magnitudes of the copolarized (co) and cross-polarized (cr) components of the transmission (a) and reflection (b) coefficients of linearly polarized waves. The input intensity I_0 in the nonlinear regime is taken to be 5, 10, and 15 kW/cm². Other parameters are as in Fig. 2.

a linearly polarized wave in equal proportion, the magnitudes of the copolarized and cross-polarized components are equal to each other at the resonant frequencies, $|T^{co}| = |T^{cr}| =$ $|R^{co}| = |R^{cr}| = 0.5$. These conditions are satisfied both in the linear and in the nonlinear regimes. In the nonlinear case, both localization resonances are bent. The "angle" of bending clearly depends on the intensity of the incident field and is almost the same for both resonances in the doublet.

Due to the above-mentioned polarization sensitivity of a magnetophotonic system, a linearly polarized wave will very likely undergo a change in its polarization state during reflection or transmission. This is confirmed in Fig. 5, which shows the corresponding frequency dependencies of the ellipticity angle (η) and the polarization azimuth (θ) for the transmitted (solid, dash-dotted lines) and reflected (dashed, dash-dot-dot lines) fields. According to the definition of the Stokes parameters, we introduce the ellipticity η so that the field is linearly polarized when $\eta = 0$, and $\eta = -\pi/4$ for LCP and $+\pi/4$ for RCP (note that, in the latter cases, the preferential azimuthal angle of the polarization ellipse θ becomes undefined). In all other cases ($0 < |\eta| < \pi/4$), the field is elliptically polarized. In the considered frequency



FIG. 5. (Color online) Frequency dependencies ($\alpha = D/\lambda$) of (a) the elipticity angle η and (b) the polarization azimuth θ of the transmitted and reflected fields. The incident light is linearly polarized, and structure parameters are as in Fig. 2. The vertical line marks the bistable polarization switching at α_0 .

band and in the linear regime, the transmitted field experiences the rotation of its polarization ellipse and sequentially changes between LCP and RCP through elliptical and linear polarization states (Fig. 4, solid lines). On the contrary, the reflected field is linearly polarized almost in the whole selected band except the frequencies α^- and α^+ where it becomes circularly polarized (Fig. 5, dashed lines). Note that, at these resonant frequencies, the polarization azimuth $\theta_{ref} = \theta_{ref}(\alpha)$ is a discontinuous function.

Such a drastic difference in the polarization states of the transmitted vs reflected fields can be understood from the fact that the operating frequencies lie in the stopband of the MPC where an impinging wave is almost completely reflected from the structure. As the incident field is linearly polarized, so, too, is the reflected field. Due to the finite size of the structure a small fraction of the wave's energy still gets transmitted through the MPC, undergoing a 90° rotation of its polarization ellipse [Fig. 5(b)] for $x^{-} < x < x^{+}$. At the resonant frequencies, it is evident that the matching circularly polarized eigenmode passes through the system while for the orthogonally polarized eigenmode the transmission is still forbidden. Therefore, both transmitted and reflected fields become circularly polarized within the localized modes frequencies. Note that the reflected field has the same polarization state as the transmitted field because the reflected wave propagates in the opposite direction (see Ref. [7] for clarity).

In the nonlinear regime the ellipticity angle and the polarization azimuth become multivalued functions. Therefore, it is possible to use multistability to switch not only between different transmittances and reflectances but also between two (or, generally, more than two) distinct polarization states in the transmitted and/or reflected light.

The most intriguing scenario for such switching is expected when a bent resonance at x^+ spectrally overlaps with the original location of the other resonance at x^- . This overlap is possible as the resonances are spectrally close to each other. For example, let us fix the operating frequency x_0 at x^- . At this frequency the reflected and transmitted fields ought to be LCP. As the intensity of input field rises, the other resonance corresponding to x^+ and associated with RCP undergoes a redshift and eventually reaches x_0 . It becomes possible to couple the incident wave with frequency x_0 with either of the eigenmodes. Since these have opposite circular polarizations (they are associated with converting a linearly polarized incident light into LCP and RCP), it can be expected that switching between these two polarization states can be achieved.

Indeed, Fig. 5 shows that, at a frequency $w_0 \approx w^-$, the bistable switching occurs between RCP and near LCP for the transmitted light and between linear polarization and RCP for the reflected light. This agrees with the above explanation and is seen in the behavior of resonance bending in the Stokes parameter space (Fig. 5). The shape of the nonlinear resonance bending for ellipticity η_{ref} is similar to that of the reflectance |R| [Fig. 4(b)]. For the transmitted light the bent resonances occur in the immediate vicinity of $\eta = \pm \pi/4$, because only circularly polarized waves can fully couple to the MPC eigenmodes to become transmitted through it.

Finally, note that Fig. 4(b) illustrates another peculiarity of the reflection spectra of the structure under study, namely, the formation of closed loops, which appear in the crosspolarized component of the reflected field. In particular, the closed loop appears in the lower-frequency resonance at α^- . The physical mechanism of loop formation is the difference between the values of T_{co} and R_{co} to either side of the resonance. In the linear regime, $|T_{co}(\alpha^- - \delta)| < |T_{co}(\alpha^- + \delta)|$ since transmittance between the resonances should be higher that to the either side of both defects because it is influenced by the Lorentzian tails of both resonances. Consequently,

$$|R_{\rm co}(a^{-} - \delta)| > |R_{\rm co}(a^{-} + \delta)|.$$
(4)

This inequality can also be influenced by nonsymmetric placement of the resonances in the band gap due to the violation of the quarter-wave condition in the structures under study. In the nonlinear regime, the relation in Eq. (4) holds, and the resonance bending in the direction of lower frequencies will cause a loop to form.

IV. ASYMMETRIC CONFIGURATION: POLARIZATION CONVERSION

Nonlinear multilayer structures with spatial asymmetry are commonly considered to obtain directional sensitivity or reversible nonreciprocity in nonmagnetic PCs. As a few examples, random or deterministically aperiodic media, as well as periodic structures with asymmetrically positioned defects, were recently reported to have direction-dependent or unidirectional transmission. [11,12,14,34,37,38] The general result is that interaction between nonlinearity and asymmetry manifests itself in the simultaneous occurrence of bistability (or multistability) and nonreciprocity.

From a mathematical point of view, this all-optical reversible nonreciprocity is a result of noncommutativity of matrix multiplication in Eq. (2) when the transfer matrix of the structure is calculated. In particular, optical properties of a 1D periodic structure with a defect strongly depend on the position of that defect layer inside the sample. Nevertheless, in the linear regime, specific properties of the transfer matrix that stem from time-reversal reciprocity of the Maxwell equations ensure that the transmission through the system remains the same regardless of whether the field is incident from the left or right side of the structure.

The situation changes drastically if an optically sensitive (e.g., Kerr-type nonlinear) material is used for the defect layer. In this case, due to different field localization patterns within the defect layer for the waves impinging from the left and right sides of the structure, the nonlinear response becomes different. This difference manifests itself in the different bending angles of the localization resonances. [12]

Our goal here is to study the simultaneous effect of the spatial asymmetry and the time-reversal nonreciprocity on the behavior of the localization resonances in the MPC. We modify the structure from Sec. III to make the number of bilayers in two subsections before and after the defect element different $(m \neq n)$. We additionally assume that the static magnetic field direction always coincides with the wave propagation direction. This can be assumed without loss of generality because changing the direction of wave propagation without changing the direction of the static magnetic field reverses the handedness of the circularly polarized states (RCP \Rightarrow LCP). Hence by considering the response of the original structure



FIG. 6. (Color online) Frequency dependencies $(\alpha = D/\lambda)$ of the transmission (*T*) and reflection (*R*) coefficients of the LCP (-) and RCP (+) waves of the MPC with asymmetrically placed $(m \neq n)$ nonlinear defect. Here, $\tilde{\varepsilon}_{d}^{nl} = \varepsilon_{d}^{nl} I_{0} = 1.0 \times 10^{-4}$ (i.e., $I_{0} =$ 10 kW/cm² for $\varepsilon_{d}^{nl} \simeq 1.0 \times 10^{-5}$ cm²/kW). Other parameters are as in Fig. 2. Solid and dashed lines correspond to configurations (*m* = 5, *n* = 6) and (*m* = 6, *n* = 5), respectively.



FIG. 7. (Color online) Frequency dependencies ($\alpha = D/\lambda$) of the elipticity angle (a) and the polarization azimuth (b) of the transmitted and reflected fields of the MPC with asymmetrically placed ($m \neq n$) nonlinear defect. Parameters are as in Fig. 6. The vertical line marks the bistable polarization switching at α_0 .

characterized by (m,n) and its mirror-symmetric counterpart (n,m) to LCP and RCP incident wave solves the problem completely.

Comparison of the results presented in Figs. 2(b)-3(b) and in Fig. 6 shows that adding one bilayer at either side of the MPC drastically changes the spectra of the structure. These changes are associated with the already-mentioned different field distribution inside the structure. The stark difference in the angles of the localization resonance bending results from the all-optical reversible nonreciprocity.

The accompanying change of the magnitude for the reflection and transmission coefficients at the bent resonances (so that $|T_{\max}^{\pm}| < 1$ and $|R_{\min}^{\pm}| > 0$) results from a certain conflict in the design principles for resonant multilayers. Namely, to increase the structure's sensitivity to the direction of incidence, one needs to increase its the spatial asymmetry; yet to increase the maximum transmission at a resonant peak, the structure should remain close to symmetric [12,37]. As a consequence, at the frequencies of the localization resonances the transmission are always below unity and the reflected field is always elliptically rather than circularly polarized. Indeed, as seen in Fig. 7, the ellipticity angle $|\eta_{ref}| < \pi/4$ in the whole selected frequency band. The transmitted field is still circularly polarized at the localization resonances. The polarization azimuth $\theta_{ref} = \theta_{ref}(a)$ is now a continuous function. Hence, while the symmetric structure features polarization switching between two circularly polarized states, the asymmetric one



FIG. 8. (Color online) Frequency dependencies ($\alpha = D/\lambda$) of the magnitudes of the copolarized (co) and cross-polarized (cr) components of the transmission (a) and reflection (b) coefficients of the linearly polarized waves of the MPC with asymmetrically placed ($m \neq n$) nonlinear defect. Parameters are as in Fig. 6.

only enables switching between two elliptically polarized states.

However, it can be seen that changing the position of the defect layer within the structure significantly alters the ratio between the reflected and transmitted field, and in particular the relations between copolarized and cross-polarized components in them (Fig. 8). While the magnitudes of the copolarized and cross-polarized transmission components remain equal to each other ($|T^{co}| = |T^{cr}| \leq 0.5$), the relation between the reflection components ($|R^{co}|$ and $|R^{cr}|$) varies in a much wider range. In one structure configuration (m = 5, n = 6), the peak magnitudes of the copolarized and cross-polarized reflection components are $|R^{co}| \approx 0.8$ and $|R^{cr}| \approx 0.2$. In the other configuration (m = 6, n = 5) they are opposite: $|R^{co}| \approx 0.2$ and $|R^{\rm cr}| \approx 0.8$. In the latter case there is an obvious significant polarization transformation in the reflected field so that a 90° polarization rotation of the incident light can be achieved with good conversion efficiency. This can find useful application as thin-film tunable polarization-rotating mirrors. Also, an appropriate choice of the asymmetric structure configuration, material parameters, layer thicknesses, and magnetic field strength would achieve switching between two orthogonal linear polarization states in the reflected field. This can be

important in the design of tunable thin-film polarization splitters and switchers.

V. CONCLUSIONS

In the present paper, we have studied the effects of bistability, nonreciprocity, and polarization transformation in a magnetophotonic crystal with a nonlinear defect placed either symmetrically or asymmetrically inside the structure. The problem is considered in the Faraday geometry (i.e, the external static magnetic field is applied in the direction of the structure periodicity and is collinear with the wave vector of the incident wave).

The reflection and transmission coefficients of the structures, along with the field distribution inside them, are calculated using the transfer matrix approach. The nonlinear problem is solved under the assumption that the nonlinear permittivity of the medium inside the defect layer depends on the average intensity of the electric field inside the defect.

In the case of symmetric structure configuration, it is shown that a bistable response of a nonlinear magnetophotonic system features switching between two circular polarization states within the localization resonances (defect modes) for reflected and transmitted fields. In the case of asymmetric structure configuration, this switching appears between elliptically polarized states in the reflected field and between nearly circularly polarized states in the transmitted field. The asymmetric structure also features strong 90° polarization rotation in the reflected field, with a potential for bistable switching between linear polarizations.

From the specific parameters used in our numerical calculations, it is reasonable to conclude that bistable response and stepwise polarization switching can already be achieved at the incident power densities of $10-100 \text{ kW/cm}^2$ with available materials in the considered structure configuration.

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