Charge asymmetry in the differential cross section of high-energy e^+e^- photoproduction in the field of a heavy atom

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Quasiclassical correction to the differential cross section of high-energy electron-positron photoproduction in the electric field of a heavy atom is obtained with the exact account of the field. This correction is responsible for the charge asymmetry $\mathcal{A}$ in this process. When the transverse momentum of at least one of the produced particles is much larger than the electron mass m, the charge asymmetry can be as large as tens percent. We also estimate the contribution $\tilde{\mathcal{A}}$ to the charge asymmetry coming from the Compton-type diagram. For heavy nuclei, this contribution is negligible. For light nuclei, $\tilde{\mathcal{A}}$ is noticeable only when the angle between the momenta of the electron and positron is of order of m/ω (ω is the photon energy) while the transverse momenta of both particles are much larger than m.

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I. INTRODUCTION

The production of an electron-positron pair by a photon in an atomic field is one of the most important processes of QED. Because of its importance for various applications [1,2], this process has been investigated in numerous theoretical and experimental papers. The cross section of the process in the Born approximation is known for arbitrary energy ω of the incoming photon [3,4] (we set $\hbar = c = 1$ throughout the paper). For heavy atoms, it is necessary to take into account the higher-order terms of the perturbation theory with respect to the parameter $Z\alpha$ (Coulomb corrections), where Z is the atomic charge number, $\alpha = e^2 \approx 1/137$ is the fine-structure constant, and e is the electron charge. The formal expression for the Coulomb corrections, exact in $Z\alpha$ and ω, was derived in Ref. [5]. However, the numerical computations based on this expression become more and more difficult when ω is increasing, and the numerical results have been obtained so far only for $\omega < 12.5$ MeV [6].

Fortunately, in the high-energy region $\omega \gg m$ (m is the electron mass), a completely different approach, which greatly simplifies the calculations, can be used. As a result of this approach, a simple expression for the Coulomb corrections was obtained in Refs. [7,8] in the leading approximation with respect to m/ω. However, this result has good accuracy only at energies $\omega \gtrsim 100$ MeV. For a long time, the description of the Coulomb corrections for the total cross section at intermediate photon energies (5–100 MeV) was based on the expression obtained in Ref. [9]. This expression is actually an extrapolation of the results obtained at $\omega < 5$ MeV. Recently, corrections of the order of m/ω to the spectrum, as well as to the total cross section, of e^+e^- photoproduction in a strong atomic field were derived in Ref. [10]. The correction to the spectrum was obtained in the region where both produced particles are relativistic. It turns out that this correction is antisymmetric with respect to replacement $\varepsilon_+ \leftrightarrow \varepsilon_-$, where ε_+ and ε_- are the energies of the positron and the electron, respectively, so that the correction to the total cross section comes from the region close to the end of spectrum where $\varepsilon_+ \sim m$ or $\varepsilon_- \sim m$. In Ref. [10], the correction to the total cross section was obtained by means of the dispersion relation for the forward Delbrück scattering amplitude. The account for this correction leads to agreement between the theoretical prediction and available experimental data at intermediate photon energies [11]. The electron (positron) spectrum in the process of e^+e^- photoproduction in a strong Coulomb field in the case $\varepsilon_+ \sim m$ or $\varepsilon_- \sim m$ and $\omega \gg m$ was investigated in Ref. [12]. It was shown that the Coulomb corrections drastically differ from those obtained in the region where $\varepsilon_+ \gg m$ and $\varepsilon_- \gg m$. Integration of the spectrum in Ref. [12] has confirmed the result for the correction to the total cross section obtained in Ref. [11] by means of the dispersion relation.

For many purposes it is very important to know the differential cross section of e^+e^- photoproduction cross section with high accuracy for $\omega \lesssim 100$ MeV, where the accuracy of the results obtained in the leading quasiclassical approximation is not sufficient. In particular, this is very important for the detectors of elementary particles, for correct estimation of efficiency of positron sources. In addition, the e^+e^- photoproduction cross section calculated with high accuracy reveals the charge asymmetry, which is a strong background in the experimental investigation of the charge asymmetry at the decays of elementary particles. The latter is considered as a possible test of the standard model [13]. In the present paper, we calculate, exactly in $\eta = Z\alpha$, the quasiclassical correction to the differential cross section $d\sigma(\boldsymbol{p}, \boldsymbol{q}, \eta)$ of electron-positron pair production by a high-energy photon in a strong atomic field; $\boldsymbol{p}$ and $\boldsymbol{q}$ are the electron and positron momenta, respectively. This correction to the cross section significantly increases the accuracy of the theoretical predictions at $\omega \lesssim 100$ MeV in comparison with that given by the leading term.

The cross section $d\sigma(\boldsymbol{p}, \boldsymbol{q}, \eta)$ can be represented as

$$\begin{aligned} d\sigma(\boldsymbol{p}, \boldsymbol{q}, \eta) &= d\sigma_s(\boldsymbol{p}, \boldsymbol{q}, \eta) + d\sigma_a(\boldsymbol{p}, \boldsymbol{q}, \eta), \\ d\sigma_s(\boldsymbol{p}, \boldsymbol{q}, \eta) &= \frac{d\sigma(\boldsymbol{p}, \boldsymbol{q}, \eta) + d\sigma(\boldsymbol{q}, \boldsymbol{p}, \eta)}{2}, \\ d\sigma_a(\boldsymbol{p}, \boldsymbol{q}, \eta) &= \frac{d\sigma(\boldsymbol{p}, \boldsymbol{q}, \eta) - d\sigma(\boldsymbol{q}, \boldsymbol{p}, \eta)}{2}. \end{aligned} \qquad (1)$$

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Thus, $d\sigma_s(\boldsymbol{p},\boldsymbol{q},\eta)$ is the symmetric quantity with respect to replacement $\boldsymbol{p}\leftrightarrow\boldsymbol{q}$ and $d\sigma_a(\boldsymbol{p},\boldsymbol{q},\eta)$ is the antisymmetric one with respect to this replacement. Using charge conjugation, we have $d\sigma(\boldsymbol{p},\boldsymbol{q},\eta)=d\sigma(\boldsymbol{q},\boldsymbol{p},-\eta)$, so that $d\sigma_s(\boldsymbol{p},\boldsymbol{q},\eta)$ is an even function of η and $d\sigma_a(\boldsymbol{p},\boldsymbol{q},\eta)$ is an odd function of η. The leading quasiclassical term of the cross section obtained in Refs. [7,8] is the even function of η and contributes only to the symmetric part $d\sigma_s(\boldsymbol{p},\boldsymbol{q},\eta)$. The quasiclassical correction obtained in the present paper appears to be the odd function of η and, therefore, contributes only to $d\sigma_a(\boldsymbol{p},\boldsymbol{q},\eta)$, i.e., to the charge asymmetry in the process. Our result is the exact-in-$Z\alpha$ prediction for the charge asymmetry in the differential cross section of e^+e^- photoproduction.

It is known that the atomic screening essentially modifies only the lowest Born contribution (proportional to η^2) to the cross section, while the Coulomb corrections (the higher-order-in-η terms) are not sensitive to it (see, e.g., Ref. [8]). The expansion of $d\sigma_s(\boldsymbol{p},\boldsymbol{q},\eta)$ over η starts from the term proportional to η^2 and, therefore, the symmetric part of the cross section is sensitive to screening. The expansion of $d\sigma_a(\boldsymbol{p},\boldsymbol{q},\eta)$ over η starts from the term proportional to η^3 and $d\sigma_a(\boldsymbol{p},\boldsymbol{q},\eta)$ is not sensitive to screening. So, our result for $d\sigma_a(\boldsymbol{p},\boldsymbol{q},\eta)$ obtained for the case of a pure Coulomb field is also valid for the atomic field.

For the photon energy below the threshold of π-meson photoproduction, we also estimate the contribution of the Compton-type amplitude to the e^+e^- photoproduction cross section. The corresponding term contributes to the charge asymmetry as well.

II. GENERAL DISCUSSION

The cross section of e^+e^- pair production by a photon in an external field reads (see, e.g., Ref. [14])

$$d\sigma(\boldsymbol{p},\boldsymbol{q},\eta)=\frac{\alpha}{(2\pi)^4\omega}\,d\boldsymbol{p}\,d\boldsymbol{q}\,\delta(\omega-\varepsilon_p-\varepsilon_q)\left|M_{\lambda_1\lambda_2}\right|^2, \qquad (2)$$

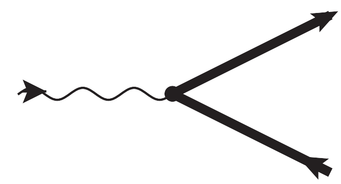

FIG. 1. Diagram for pair production by a photon in a strong Coulomb field. Thick lines correspond to the positive- and negative-energy solutions of the Dirac equation in the Coulomb field.

where $\varepsilon_p=\sqrt{p^2+m^2}$. The matrix element $M_{\lambda_1\lambda_2}$, corresponding to the diagram shown in Fig. 1, has the form

$$M_{\lambda_1\lambda_2}=\int d\boldsymbol{r}\,\bar{u}^{(\text{out})}_{\lambda_1\boldsymbol{p}}(\boldsymbol{r})\,\boldsymbol{\gamma}\cdot\boldsymbol{e}\,v^{(\text{in})}_{\lambda_2\boldsymbol{q}}(\boldsymbol{r})\exp(i\boldsymbol{k}\cdot\boldsymbol{r}). \qquad (3)$$

Here $u^{(\text{out})}_{\lambda_1\boldsymbol{p}}(\boldsymbol{r})$ is a positive-energy solution and $v^{(\text{in})}_{\lambda_2\boldsymbol{q}}(\boldsymbol{r})$ is a negative-energy solution of the Dirac equation in the external field, $\lambda_1=\pm1$ and $\lambda_2=\pm1$ enumerate the independent solutions of the Dirac equation, $\boldsymbol{e}$ is the photon polarization vector, $\boldsymbol{k}$ is the photon momentum, and γ^μ are the Dirac matrices. We remind that the asymptotic forms of $u^{(\text{out})}_{\lambda\boldsymbol{p}}(\boldsymbol{r})$ and $v^{(\text{out})}_{\lambda\boldsymbol{p}}(\boldsymbol{r})$ at large $\boldsymbol{r}$ contain the plane waves and the spherical convergent waves, while the asymptotic forms of $u^{(\text{in})}_{\lambda\boldsymbol{p}}(\boldsymbol{r})$ and $v^{(\text{in})}_{\lambda\boldsymbol{q}}(\boldsymbol{r})$ at large $\boldsymbol{r}$ contain the plane waves and the spherical divergent waves. In order to obtain the wave functions in the leading quasiclassical approximation and the first quasiclassical correction to it, we exploit the convenient integral representation for the exact wave functions in the Coulomb field suggested in Ref. [15]. The derivation of this representation is based on the relations

$$\begin{aligned}
&\lim_{r_1\to\infty} G(\mathbf{r}_2,\mathbf{r}_1|\varepsilon_p)=-\frac{\exp[ipr_1+i\eta_p\ln(2pr_1)]}{4\pi r_1}\sum_{\lambda=1,2}2\varepsilon_p u^{(\text{in})}_{\lambda\mathbf{p}}(\mathbf{r}_2)\bar{u}_{\lambda\mathbf{p}},\quad \boldsymbol{p}=-p\boldsymbol{n}_1,\\
&\lim_{r_1\to\infty} G(\mathbf{r}_2,\mathbf{r}_1|-\varepsilon_p)=\frac{\exp[ipr_1-i\eta_p\ln(2pr_1)]}{4\pi r_1}\sum_{\lambda=1,2}2\varepsilon_p v^{(\text{in})}_{\lambda\mathbf{p}}(\mathbf{r}_2)\bar{v}_{\lambda\mathbf{p}},\quad \boldsymbol{p}=p\boldsymbol{n}_1,\\
&u_{\lambda\mathbf{p}}=\sqrt{\frac{\varepsilon_p+m}{2\varepsilon_p}}\begin{pmatrix}\phi_\lambda\\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{\varepsilon_p+m}\phi_\lambda\end{pmatrix},\quad v_{\lambda\mathbf{p}}=\sqrt{\frac{\varepsilon_p+m}{2\varepsilon_p}}\begin{pmatrix}\frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{\varepsilon_p+m}\chi_\lambda\\ \chi_\lambda\end{pmatrix},\quad \eta_p=Z\alpha\frac{\varepsilon_p}{p},
\end{aligned} \qquad (4)$$

and also

$$\begin{aligned}
&\lim_{r_2\to\infty} G(\mathbf{r}_2,\mathbf{r}_1|\varepsilon_p)=-\frac{\exp[ipr_2+i\eta_p\ln(2pr_2)]}{4\pi r_2}\sum_{\lambda=1,2}2\varepsilon_p u_{\lambda\mathbf{p}}\bar{u}^{(\text{out})}_{\lambda\mathbf{p}}(\mathbf{r}_1),\quad \boldsymbol{p}=p\boldsymbol{n}_2,\\
&\lim_{r_2\to\infty} G(\mathbf{r}_2,\mathbf{r}_1|-\varepsilon_p)=\frac{\exp[ipr_2-i\eta_p\ln(2pr_2)]}{4\pi r_2}\sum_{\lambda=1,2}2\varepsilon_p v_{\lambda\mathbf{p}}\bar{v}^{(\text{out})}_{\lambda\mathbf{p}}(\mathbf{r}_1),\quad \boldsymbol{p}=-p\boldsymbol{n}_2,
\end{aligned} \qquad (5)$$

where $G(\mathbf{r}_2,\mathbf{r}_1|\varepsilon)$ is the Green function of the Dirac equation in the Coulomb field, $\boldsymbol{n}_1=\boldsymbol{r}_1/r_1$, and $\boldsymbol{n}_2=\boldsymbol{r}_2/r_2$. A convenient integral representation for $G(\mathbf{r}_2,\mathbf{r}_1|\varepsilon)$ was obtained in Ref. [16]. Using Eqs. (19)–(22) of that paper, we arrive at the following result for the wave functions $u^{(\mathrm{in})}_{\lambda\mathbf{p}}(\mathbf{r})$ and $v^{(\mathrm{in})}_{\lambda\mathbf{p}}(\mathbf{r})$:

$$
\begin{aligned}
u^{(\mathrm{in})}_{\lambda\mathbf{p}}(\mathbf{r}) &= \frac{\exp(ipr)}{pr}\int_0^\infty dt\, t^{-2i\eta_p-1}\exp(it^2)\left[S_B(-x_{\boldsymbol p},pr)\left(t^2-Z\alpha\frac{m}{p}\gamma^0\right)(1-R_1)+iS_A(-x_{\boldsymbol p},pr)(1+R_1)\right]u_{\lambda\mathbf{p}},\\
v^{(\mathrm{in})}_{\lambda\mathbf{p}}(\mathbf{r}) &= \frac{\exp(ipr)}{pr}\int_0^\infty dt\, t^{2i\eta_p-1}\exp(it^2)\left[S_B(x_{\boldsymbol p},pr)\left(t^2-Z\alpha\frac{m}{p}\gamma^0\right)(1-R_2)+iS_A(x_{\boldsymbol p},pr)(1+R_2)\right]v_{\lambda\mathbf{p}},\\
S_A(x,\rho) &= \sum_{l=1}^{\infty}\mathrm{e}^{-i\pi\nu}J_{2\nu}(2t\sqrt{2\rho})\,l\frac{d}{dx}[P_l(x)+P_{l-1}(x)],\quad S_B(x,\rho)=\sum_{l=1}^{\infty}\mathrm{e}^{-i\pi\nu}J_{2\nu}(2t\sqrt{2\rho})\frac{d}{dx}[P_l(x)-P_{l-1}(x)],\\
x_{\boldsymbol p} &= \frac{\boldsymbol r\cdot\boldsymbol p}{rp},\quad R_{1,2}=(m\pm\gamma^0\varepsilon_p)\frac{\boldsymbol\gamma\cdot\boldsymbol r}{pr}.
\end{aligned}
\tag{6}
$$

Here $P_l(x)$ is the Legendre polynomial, $J_{2\nu}$ is the Bessel function, and $\nu=\sqrt{l^2-(Z\alpha)^2}$. For the wave functions $\bar u^{(\mathrm{out})}_{\lambda\mathbf{p}}(\mathbf{r})$ and $\bar v^{(\mathrm{out})}_{\lambda\mathbf{p}}(\mathbf{r})$ we obtain

$$
\begin{aligned}
\bar u^{(\mathrm{out})}_{\lambda\mathbf{p}}(\mathbf{r}) &= \frac{\exp(ipr)}{pr}\int_0^\infty dt\, t^{-2i\eta_p-1}\exp(it^2)\bar u_{\lambda\mathbf{p}}\left[S_B(x_{\boldsymbol p},pr)(1+R_2)\left(t^2-Z\alpha\frac{m}{p}\gamma^0\right)+iS_A(x_{\boldsymbol p},pr)(1-R_2)\right],\\
\bar v^{(\mathrm{out})}_{\lambda\mathbf{p}}(\mathbf{r}) &= \frac{\exp(ipr)}{pr}\int_0^\infty dt\, t^{2i\eta_p-1}\exp(it^2)\bar v_{\lambda\mathbf{p}}\left[S_B(-x_{\boldsymbol p},pr)(1+R_1)\left(t^2-Z\alpha\frac{m}{p}\gamma^0\right)+iS_A(-x_{\boldsymbol p},pr)(1-R_1)\right].
\end{aligned}
\tag{7}
$$

The integrals over the variable t in Eqs. (6) and (7) can be expressed via the confluent hypergeometric functions. However, the forms (6) and (7) of the wave functions are more convenient for applications than the conventional ones. The results (6) and (7) are in agreement with the well-known solutions of the Dirac equation in the Coulomb field.

III. CALCULATION OF THE MATRIX ELEMENT

Let us introduce the functions

$$
\begin{aligned}
F_A(\boldsymbol r,\boldsymbol p,\eta) &= i\frac{\exp(ipr)}{pr}\int_0^\infty dt\, t^{-2i\eta-1}\exp(it^2)S_A(x_{\boldsymbol p},pr),\quad F_B(\boldsymbol r,\boldsymbol p,\eta)=\frac{\exp(ipr)}{pr}\int_0^\infty dt\, t^{-2i\eta-1}\exp(it^2)S_B(x_{\boldsymbol p},pr),\\
\tilde F_B(\boldsymbol r,\boldsymbol p,\eta) &= \frac{\exp(ipr)}{pr}\int_0^\infty dt\, t^{-2i\eta+1}\exp(it^2)S_B(x_{\boldsymbol p},pr).
\end{aligned}
\tag{8}
$$

In terms of the functions (8), the wave functions $v^{(\mathrm{in})}_{\lambda\mathbf{p}}(\mathbf{r})$ and $\bar u^{(\mathrm{out})}_{\lambda\mathbf{p}}(\mathbf{r})$ have the form

$$
\begin{aligned}
&\bar u^{(\mathrm{out})}_{\lambda\mathbf{p}}(\mathbf{r}) = (\phi^+\mathcal{R}_1^{(+)},-\phi^+_\lambda\mathcal{R}_2^{(+)}),\quad v^{(\mathrm{in})}_{\lambda\mathbf{q}}(\mathbf{r})=\begin{pmatrix}\mathcal{R}_2^{(-)}\chi_\lambda\\ \mathcal{R}_1^{(-)}\chi_\lambda\end{pmatrix},\\
&\mathcal{R}_1^{(+)} = \sqrt{\frac{\varepsilon_p+m}{2\varepsilon_p}}\left[\left(\tilde F_B^{(+)}-\frac{Z\alpha m}{p}F_B^{(+)}\right)(1+\boldsymbol\sigma\cdot\hat{\boldsymbol p}\,\boldsymbol\sigma\cdot\boldsymbol n)+F_A^{(+)}(1-\boldsymbol\sigma\cdot\hat{\boldsymbol p}\,\boldsymbol\sigma\cdot\boldsymbol n)\right],\\
&\mathcal{R}_2^{(+)} = \sqrt{\frac{\varepsilon_p-m}{2\varepsilon_p}}\left[\left(\tilde F_B^{(+)}+\frac{Z\alpha m}{p}F_B^{(+)}\right)(\boldsymbol\sigma\cdot\hat{\boldsymbol p}+\boldsymbol\sigma\cdot\boldsymbol n)+F_A^{(+)}(\boldsymbol\sigma\cdot\hat{\boldsymbol p}-\boldsymbol\sigma\cdot\boldsymbol n)\right],\\
&\mathcal{R}_1^{(-)} = \sqrt{\frac{\varepsilon_q+m}{2\varepsilon_q}}\left[\left(\tilde F_B^{(-)}+\frac{Z\alpha m}{q}F_B^{(-)}\right)(1+\boldsymbol\sigma\cdot\boldsymbol n\,\boldsymbol\sigma\cdot\hat{\boldsymbol q})+F_A^{(-)}(1-\boldsymbol\sigma\cdot\boldsymbol n\,\boldsymbol\sigma\cdot\hat{\boldsymbol q})\right],\\
&\mathcal{R}_2^{(-)} = \sqrt{\frac{\varepsilon_q-m}{2\varepsilon_q}}\left[\left(\tilde F_B^{(-)}-\frac{Z\alpha m}{q}F_B^{(-)}\right)(\boldsymbol\sigma\cdot\hat{\boldsymbol q}+\boldsymbol\sigma\cdot\boldsymbol n)+F_A^{(-)}(\boldsymbol\sigma\cdot\hat{\boldsymbol q}-\boldsymbol\sigma\cdot\boldsymbol n)\right],
\end{aligned}
\tag{9}
$$

where $\boldsymbol n=\boldsymbol r/r$, $\hat{\boldsymbol p}=\boldsymbol p/p$, $\hat{\boldsymbol q}=\boldsymbol q/q$, and

$$
\begin{aligned}
F_A^{(+)} &= F_A(\boldsymbol r,\boldsymbol p,\eta_p),\quad F_B^{(+)}=F_B(\boldsymbol r,\boldsymbol p,\eta_p),\quad \tilde F_B^{(+)}=\tilde F_B(\boldsymbol r,\boldsymbol p,\eta_p),\\
F_A^{(-)} &= F_A(\boldsymbol r,\boldsymbol q,-\eta_q),\quad F_B^{(-)}=F_B(\boldsymbol r,\boldsymbol q,-\eta_q),\quad \tilde F_B^{(-)}=\tilde F_B(\boldsymbol r,\boldsymbol q,-\eta_q).
\end{aligned}
\tag{10}
$$

Then the matrix element $M_{\lambda_1\lambda_2}$, Eq. (3), is

$$
M_{\lambda_1\lambda_2}=\int d\boldsymbol r\,\exp(i\boldsymbol k\cdot\boldsymbol r)\phi^+_{\lambda_1}[\mathcal{R}_1^{(+)}\boldsymbol\sigma\cdot\boldsymbol e\,\mathcal{R}_1^{(-)}+\mathcal{R}_2^{(+)}\boldsymbol\sigma\cdot\boldsymbol e\,\mathcal{R}_2^{(-)}]\chi_{\lambda_2}. \tag{11}
$$

For any vector $\boldsymbol{X}$ we introduce the notation $\boldsymbol{X}_\perp = \boldsymbol{X} - (\boldsymbol{\nu}\cdot\boldsymbol{X})\boldsymbol{\nu}$, $\boldsymbol{\nu} = \boldsymbol{k}/k$, and we write the matrix element $M_{\lambda_1\lambda_2}$ in the form

$$M_{\lambda_1\lambda_2} = \phi^+_{\lambda_1}[(a_0 + a_1) + \boldsymbol{\sigma}\cdot(\boldsymbol{b}_0 + \boldsymbol{b}_1)]\chi_{\lambda_2}, \tag{12}$$

where a_0 and $\boldsymbol{b}_0$ are linear in $\boldsymbol{\theta}_p = \hat{\boldsymbol{p}}_\perp$, $\boldsymbol{\theta}_q = \hat{\boldsymbol{q}}_\perp$, m/ε_p, and m/ε_q, while a_1 and $\boldsymbol{b}_1$ are quadratic in these variables. We have

$$\begin{aligned} &a_0 = 2i[\boldsymbol{\nu}\times\boldsymbol{e}]\cdot\boldsymbol{g}^{(-)}, \quad \boldsymbol{b}_0 = 2\boldsymbol{e}\cdot\boldsymbol{g}^{(+)}\,\boldsymbol{\nu} + 2g\,\boldsymbol{e}, \quad a_1 = i[\boldsymbol{\nu}\times\boldsymbol{e}]\cdot(\boldsymbol{\theta}_p - \boldsymbol{\theta}_q)\,g,\\ &\boldsymbol{b}_1 = \boldsymbol{e}\cdot\boldsymbol{g}^{(+)}(\boldsymbol{\theta}_p + \boldsymbol{\theta}_q) + \boldsymbol{e}\cdot(\boldsymbol{\theta}_p - \boldsymbol{\theta}_q)\,\boldsymbol{g}^{(-)} - (\boldsymbol{\theta}_p - \boldsymbol{\theta}_q)\cdot\boldsymbol{g}^{(-)}\,\boldsymbol{e} - \boldsymbol{e}\cdot(\boldsymbol{\theta}_p + \boldsymbol{\theta}_q)\,g\boldsymbol{\nu}. \end{aligned} \tag{13}$$

Here

$$\begin{aligned} \boldsymbol{g}^{(\pm)} &= \int d\boldsymbol{r}\,\exp(i\boldsymbol{k}\cdot\boldsymbol{r})F_A^{(+)}[(\boldsymbol{n}_\perp + \boldsymbol{\theta}_q)\tilde{F}_B^{(-)} - \boldsymbol{n}_\perp F_A^{(-)}] \pm \int d\boldsymbol{r}\,\exp(i\boldsymbol{k}\cdot\boldsymbol{r})F_A^{(-)}[(\boldsymbol{n}_\perp + \boldsymbol{\theta}_p)\tilde{F}_B^{(+)} - \boldsymbol{n}_\perp F_A^{(+)}],\\ g &= \frac{m\omega}{\varepsilon_p\varepsilon_q}\int d\boldsymbol{r}\,\exp(i\boldsymbol{k}\cdot\boldsymbol{r})F_A^{(+)}F_A^{(-)}. \end{aligned} \tag{14}$$

In Ref. [10], the following expressions for the sum S_A and S_B, which take into account the leading terms and first quasiclassical corrections, have been obtained:

$$S_A(x,\rho) = -\frac{y^2}{8}J_0\left(y\sqrt{\frac{1+x}{2}}\right)\left[1 + \frac{i\pi(Z\alpha)^2}{y}\right], \quad S_B(x,\rho) = -\frac{y}{2\sqrt{2(1+x)}}J_1\left(y\sqrt{\frac{1+x}{2}}\right)\left[1 + \frac{i\pi(Z\alpha)^2}{y}\right], \tag{15}$$

where $y = 2t\sqrt{2\rho}$. These formulas are obtained for $y \gg 1$ and $1 + x \ll 1$. Substituting Eq. (15) into Eq. (8) and taking the integrals over the variable t we find

$$\begin{aligned} F_A(\boldsymbol{r},\,\boldsymbol{p},\,\eta) &= \frac{1}{2}\exp\left(\frac{\pi\eta}{2} - i\,\boldsymbol{p}\cdot\boldsymbol{r}\right)\bigg[\Gamma(1-i\eta)F(i\eta,1,i(pr + \boldsymbol{p}\cdot\boldsymbol{r}))\\ &\quad + \frac{\pi\eta^2\exp\left(i\frac{\pi}{4}\right)}{2\sqrt{2pr}}\Gamma(1/2 - i\eta)F(1/2 + i\eta,1,i(pr + \boldsymbol{p}\cdot\boldsymbol{r}))\bigg],\\ F_B(\boldsymbol{r},\,\boldsymbol{p},\,\eta) &= -\frac{i}{2}\exp\left(\frac{\pi\eta}{2} - i\,\boldsymbol{p}\cdot\boldsymbol{r}\right)\bigg[\Gamma(1-i\eta)F(1+i\eta,2,i(pr + \boldsymbol{p}\cdot\boldsymbol{r}))\\ &\quad + \frac{\pi\eta^2\exp\left(i\frac{\pi}{4}\right)}{2\sqrt{2pr}}\Gamma(1/2 - i\eta)F(3/2 + i\eta,2,i(pr + \boldsymbol{p}\cdot\boldsymbol{r}))\bigg],\\ \tilde{F}_B(\boldsymbol{r},\,\boldsymbol{p},\,\eta) &= \frac{1}{2}\exp\left(\frac{\pi\eta}{2} - i\,\boldsymbol{p}\cdot\boldsymbol{r}\right)\bigg[\Gamma(2-i\eta)F(i\eta,2,i(pr + \boldsymbol{p}\cdot\boldsymbol{r}))\\ &\quad + \frac{\pi\eta^2\exp\left(i\frac{\pi}{4}\right)}{2\sqrt{2pr}}\Gamma(3/2 - i\eta)F(1/2 + i\eta,2,i(pr + \boldsymbol{p}\cdot\boldsymbol{r}))\bigg]. \end{aligned} \tag{16}$$

Here $\Gamma(x)$ is the Euler gamma function, and $F(\alpha,\beta,x)$ is the confluent hypergeometric function. Then we use the approach of Ref. [17] based on the integral taken in Ref. [18],

$$\begin{aligned} &\int\frac{d\boldsymbol{r}}{r}\exp\left[-i\,\boldsymbol{Q}\cdot\boldsymbol{r} - i\,\frac{m^2\omega}{2\varepsilon_p\varepsilon_q}\,\lambda r\right]F(-ia_1,1,i(qr + \boldsymbol{q}\cdot\boldsymbol{r}))\,F(ia_2,1,i(pr + \boldsymbol{p}\cdot\boldsymbol{r}))\\ &\quad = \frac{4\pi}{Q^2}\left(\frac{m^2\omega(1+\xi_p\lambda)}{\varepsilon_p\xi_p Q^2}\right)^{ia_1}\left(\frac{m^2\omega(1+\xi_q\lambda)}{\varepsilon_q\xi_q Q^2}\right)^{-ia_2}F(-ia_1,ia_2,1,z),\\ &z = 1 - \frac{Q^2\xi_p\xi_q(1+\lambda)}{m^2(1+\xi_p\lambda)(1+\xi_q\lambda)}, \quad \xi_p = \frac{1}{1+\delta_p^2}, \quad \xi_q = \frac{1}{1+\delta_q^2}, \quad \boldsymbol{\delta}_p = \frac{\varepsilon_p\boldsymbol{\theta}_p}{m}, \quad \boldsymbol{\delta}_q = \frac{\varepsilon_q\boldsymbol{\theta}_q}{m}, \quad \boldsymbol{Q} = \boldsymbol{p} + \boldsymbol{q} - \boldsymbol{k}. \end{aligned} \tag{17}$$

Here we assume that $|\lambda| \sim 1$. We write $g = g_0 + \delta g$ and $\boldsymbol{g}^{(\pm)} = \boldsymbol{g}_0^{(\pm)} + \delta\boldsymbol{g}^{(\pm)}$, where the leading terms are

$$\begin{aligned} &g_0 = N[(\xi_q - \xi_p)\,i\eta\mathcal{F} + (1 - \xi_p - \xi_q)(1-u)\mathcal{F}'], \quad \boldsymbol{g}_0^{(\pm)} = N\,\frac{(\varepsilon_p \mp \varepsilon_q)}{\omega}[(\xi_p\boldsymbol{\delta}_p + \xi_q\boldsymbol{\delta}_q)\,i\eta\mathcal{F} + (\xi_p\boldsymbol{\delta}_p - \xi_q\boldsymbol{\delta}_q)(1-u)\mathcal{F}'],\\ &N = -i\,\frac{2\pi}{mQ^2}\left(\frac{\varepsilon_q\xi_q}{\varepsilon_p\xi_p}\right)^{i\eta}|\Gamma(1-i\eta)|^2, \quad \mathcal{F} = F(-i\eta,i\eta,1,u), \quad \mathcal{F}' = \frac{\partial\mathcal{F}}{\partial u}, \quad u = 1 - \frac{\boldsymbol{Q}_\perp^2}{m^2}\xi_p\xi_q. \end{aligned} \tag{18}$$

Here $\eta = Z\alpha$ and $F(a,b,c,x)$ is the hypergeometric function.

The first quasiclassical corrections, δg and $\delta\boldsymbol{g}^{(\pm)}$, are given by the integrals

$$
\begin{aligned}
\delta g &= \frac{\pi\eta^2\exp\left(i\frac{\pi}{4}\right)}{8\sqrt{2}}\int\frac{d\boldsymbol{r}}{\sqrt{r}}\exp(-i\boldsymbol{Q}\cdot\boldsymbol{r})\bigg\{\frac{1}{\sqrt{\varepsilon_q}}\Gamma(1-i\eta)\Gamma(1/2+i\eta)\\
&\quad\times F(i\eta,1,i(pr+\boldsymbol{p}\cdot\boldsymbol{r}))F(1/2-i\eta,1,i(qr+\boldsymbol{q}\cdot\boldsymbol{r}))+(\boldsymbol{p}\leftrightarrow\boldsymbol{q},\eta\to-\eta)\bigg\},\\
\delta\boldsymbol{g}^{(\pm)} &= \frac{\pi\eta^2\exp\left(i\frac{\pi}{4}\right)(\varepsilon_p\mp\varepsilon_q)}{8\sqrt{2\varepsilon_p\varepsilon_q}}\int\frac{d\boldsymbol{r}}{\sqrt{r}}\exp(-i\boldsymbol{Q}\cdot\boldsymbol{r})\bigg\{\frac{1}{\sqrt{\varepsilon_p}}\Gamma(1-i\eta)\Gamma(1/2+i\eta)\\
&\quad\times F(i\eta,1,i(pr+\boldsymbol{p}\cdot\boldsymbol{r}))[(\boldsymbol{n}_\perp+\boldsymbol{\theta}_q)(1/2+i\eta)F(1/2-i\eta,2,i(qr+\boldsymbol{q}\cdot\boldsymbol{r}))\\
&\quad-\boldsymbol{n}_\perp F(1/2-i\eta,1,i(qr+\boldsymbol{q}\cdot\boldsymbol{r}))]-(\boldsymbol{p}\leftrightarrow\boldsymbol{q},\eta\to-\eta)\bigg\}.
\end{aligned}
\tag{19}
$$

In order to take the integral over $\boldsymbol{r}$, we use the parametrization

$$
\frac{1}{\sqrt{r}}=\frac{\exp(i\pi/4)}{\sqrt{\pi}}\int_0^\infty\frac{d\lambda}{\sqrt{\lambda}}\exp(-i\lambda r).
\tag{20}
$$

Then, we obtain

$$
\begin{aligned}
\delta g &= \frac{\pi^{3/2}\eta^2}{2mQ}\left(\frac{\varepsilon_q\xi_q}{\varepsilon_p\xi_p}\right)^{i\eta}\int_0^\infty\frac{d\lambda}{\sqrt{\lambda}}\left(\frac{1+\xi_p\lambda}{1+\xi_q\lambda}\right)^{i\eta}\left\{\frac{\sqrt{\xi_p}\Gamma(1-i\eta)\Gamma(1/2+i\eta)}{\varepsilon_q\sqrt{1+\xi_p\lambda}}\left[\left((1/2-i\eta)\frac{\xi_p}{1+\xi_p\lambda}+i\eta\frac{\xi_q}{1+\xi_q\lambda}\right)\mathcal{G}\right.\right.\\
&\quad\left.\left.+\left(\frac{1}{1+\lambda}-\frac{\xi_p}{1+\xi_p\lambda}-\frac{\xi_q}{1+\xi_q\lambda}\right)(1-z)\mathcal{G}'\right]+(\boldsymbol{p}\leftrightarrow\boldsymbol{q},\eta\to-\eta)\right\},\\
\delta\boldsymbol{g}^{(\pm)} &= \frac{\pi^{3/2}\eta^2(\varepsilon_p\mp\varepsilon_q)}{2mQ\omega}\left(\frac{\varepsilon_q\xi_q}{\varepsilon_p\xi_p}\right)^{i\eta}\int_0^\infty\frac{d\lambda}{\sqrt{\lambda}}\left(\frac{1+\xi_p\lambda}{1+\xi_q\lambda}\right)^{i\eta}\left\{\frac{\sqrt{\xi_p}\Gamma(1-i\eta)\Gamma(1/2+i\eta)}{\varepsilon_q\sqrt{1+\xi_p\lambda}}\right.\\
&\quad\left.\times\left[\left(-(1/2-i\eta)\frac{\xi_p\boldsymbol{\delta}_p}{1+\xi_p\lambda}+i\eta\frac{\xi_q\boldsymbol{\delta}_q}{1+\xi_q\lambda}\right)\mathcal{G}+\left(\frac{\xi_p\boldsymbol{\delta}_p}{1+\xi_p\lambda}-\frac{\xi_q\boldsymbol{\delta}_q}{1+\xi_q\lambda}\right)(1-z)\mathcal{G}'\right]-(\boldsymbol{p}\leftrightarrow\boldsymbol{q},\eta\to-\eta)\right\},\\
\mathcal{G} &= F(1/2-i\eta,i\eta,1,z),\quad \mathcal{G}'=\frac{\partial\mathcal{G}}{\partial z}.
\end{aligned}
\tag{21}
$$

Here z is defined in Eq. (17).

IV. CALCULATION OF THE PHOTOPRODUCTION CROSS SECTION

Using the matrix element obtained it is easy to write the cross section with all polarizations taken into account. For the cross section summed over the polarization of the electron and positron, it is necessary to calculate

$$
\sum_{\lambda_{1,2}}\left|M_{\lambda_1\lambda_2}\right|^2=2[|a_0+a_1|^2+|\boldsymbol{b}_0+\boldsymbol{b}_1|^2]=2[|a_0|^2+|\boldsymbol{b}_0|^2+2\,\mathrm{Re}(a_0a_1^*+\boldsymbol{b}_0\cdot\boldsymbol{b}_1^*)],
\tag{22}
$$

where we neglect the terms $|a_1|^2$ and $|\boldsymbol{b}_1|^2$. It follows from Eq. (13) that

$$
\mathrm{Re}(a_0a_1^*+\boldsymbol{b}_0\cdot\boldsymbol{b}_1^*)=0
$$

for any photon polarization. Thus, the terms with a_1 and $\boldsymbol{b}_1$ do not contribute to the next-to-leading correction to the cross section summed over the electron and positron polarizations. For simplicity, we restrict ourselves to the case of unpolarized photon. From Eqs. (13), (22), (18), and (21), we have

$$
d\sigma=\frac{\alpha m^4\,d\varepsilon_p\,d\boldsymbol{\delta}_p\,d\boldsymbol{\delta}_q}{2\pi^4\omega^3}\left[\left(\varepsilon_p^2+\varepsilon_q^2\right)|\boldsymbol{g}^{(-)}|^2+\omega^2|g|^2\right],
\tag{23}
$$

where we used the relation $\boldsymbol{g}^{(+)} = (\varepsilon_p - \varepsilon_q)\boldsymbol{g}^{(-)}/\omega$ [see Eqs. (18) and (21)]. We write $d\sigma = d\sigma_s + d\sigma_a$, where the leading term is

$$d\sigma_s = \frac{2\alpha m^2|\Gamma(1-i\eta)|^4\, d\varepsilon_p\, d\boldsymbol{\delta}_p\, d\boldsymbol{\delta}_q}{\pi^2\omega^3 Q^4}\{[(1-u)(\varepsilon_p^2+\varepsilon_q^2) + 2\varepsilon_p\varepsilon_q(\xi_p-\xi_q)^2]\eta^2\mathcal{F}^2 + [u(\varepsilon_p^2+\varepsilon_q^2) + 2\varepsilon_p\varepsilon_q(1-\xi_p-\xi_q)^2](1-u)^2\mathcal{F}'^2\}. \tag{24}$$

Here u and $\mathcal{F}$ are defined in Eq. (18). The leading term is symmetric with respect to replacement $\boldsymbol{p} \leftrightarrow \boldsymbol{q}$. The correction $d\sigma_a$ has the form

$$d\sigma_a = -\frac{\alpha m^2\eta^2|\Gamma(1-i\eta)|^2\, d\varepsilon_p\, d\boldsymbol{\delta}_p\, d\boldsymbol{\delta}_q}{2\pi^{3/2}\omega^3 Q^3}\,\mathrm{Im}\left\{\int_0^\infty \frac{d\lambda}{\sqrt{\lambda}}\left(\frac{1+\xi_p\lambda}{1+\xi_q\lambda}\right)^{i\eta}\frac{\sqrt{\xi_p}\,\Gamma(1-i\eta)\Gamma(1/2+i\eta)}{\varepsilon_q\sqrt{1+\xi_p\lambda}}\mathcal{M} + (\boldsymbol{p}\leftrightarrow\boldsymbol{q}\,,\eta\to-\eta)\right\},$$

$$\mathcal{M} = [(\xi_p-\xi_q)\,i\eta\mathcal{F} + (1-\xi_p-\xi_q)(1-u)\mathcal{F}'][4\varepsilon_p\varepsilon_q(\xi_p f_1 + \xi_q f_2 + f_3) + (\varepsilon_p^2+\varepsilon_q^2)(f_1+f_2+2f_3)] + (\varepsilon_p^2+\varepsilon_q^2)(1-u)[(f_1-f_2)i\eta\mathcal{F} - u(f_1+f_2)\mathcal{F}'], \tag{25}$$

$$f_1 = \frac{(1/2-i\eta)\mathcal{G} - (1-z)\mathcal{G}'}{1+\xi_p\lambda}, \quad f_2 = \frac{i\eta\mathcal{G} - (1-z)\mathcal{G}'}{1+\xi_q\lambda}, \quad f_3 = \frac{(1-z)\mathcal{G}'}{1+\lambda}.$$

Here z and $\mathcal{G}$ are defined in Eq. (17). As it should be, the correction $d\sigma_a$ is invariant under the replacement $\boldsymbol{p} \leftrightarrow \boldsymbol{q}\,, \eta \to -\eta$. Since i enters this expression only in the combination $i\eta$, it is evident that the correction $d\sigma_a$ is antisymmetric with respect to replacement $\eta \to -\eta$, as well as with respect to replacement $\boldsymbol{p} \leftrightarrow \boldsymbol{q}$.

V. SPECIAL CASES

If $\eta \ll 1$, the leading term $d\sigma_s$ has the form

$$d\sigma_s = \frac{2\alpha m^2\eta^2\, d\varepsilon_p\, d\boldsymbol{\delta}_p\, d\boldsymbol{\delta}_q}{\pi^2\omega^3 Q^4}\left[\frac{Q^2}{m^2}\xi_p\xi_q(\varepsilon_p^2+\varepsilon_q^2) + 2\varepsilon_p\varepsilon_q(\xi_p-\xi_q)^2\right]. \tag{26}$$

The correction Eq. (25) at $\eta \ll 1$ reads

$$d\sigma_a = -\frac{\alpha m^2\eta^3 d\varepsilon_p\, d\boldsymbol{\delta}_p\, d\boldsymbol{\delta}_q}{2\pi\omega^3 Q^3}\left\{(\xi_p-\xi_q)\left[4(\varepsilon_p\xi_p+\varepsilon_q\xi_q) + \frac{\omega(\varepsilon_p^2+\varepsilon_q^2)}{\varepsilon_p\varepsilon_q}\right] + (\varepsilon_p-\varepsilon_q)\frac{(\varepsilon_p^2+\varepsilon_q^2)}{\varepsilon_p\varepsilon_q}\frac{Q^2}{m^2}\xi_p\xi_q\right\}. \tag{27}$$

In the limit $\delta_p \gg 1$ and $\delta_q \gg 1$ this formula reduces to

$$d\sigma_a = -\frac{\alpha\eta^3(\varepsilon_p^2+\varepsilon_q^2)d\varepsilon_p\, d\boldsymbol{\delta}_p\, d\boldsymbol{\delta}_q}{2\pi\varepsilon_p\varepsilon_q\delta_p^2\delta_q^2\omega^3 Q^3}[m^2(\delta_q^2-\delta_p^2)\omega + (\varepsilon_p-\varepsilon_q)Q^2] = -\frac{\alpha\eta^3(\varepsilon_p^2+\varepsilon_q^2)d\varepsilon_p\, d\boldsymbol{\delta}_p\, d\boldsymbol{\delta}_q}{\pi\delta_p^2\delta_q^2\omega^3 Q^3}(\boldsymbol{\theta}_q-\boldsymbol{\theta}_p)\cdot\boldsymbol{Q}. \tag{28}$$

The correction $d\sigma_a$ at $\eta \ll 1$, $\delta_p \gg 1$, and $\delta_q \gg 1$ was also investigated in Ref. [19] in scalar electrodynamics. Our result (28), obtained for fermions, differs from the result of Ref. [19] for scalar particles by the factor $(\varepsilon_p^2+\varepsilon_q^2)/(\varepsilon_p\varepsilon_q)$. This factor is equal to 2 for $|\varepsilon_p-\varepsilon_q| \ll \omega$ in accordance with the statement of Ref. [19].

From the experimental point of view, it may be interesting to consider the case $\delta_p \gg \delta_q \gg 1$ or $\delta_q \gg \delta_p \gg 1$ at $\eta \lesssim 1$. Then the leading symmetric term is

$$d\sigma_s = \frac{2\alpha\eta^2\xi_p\xi_q\,(\varepsilon_p^2+\varepsilon_q^2)\, d\varepsilon_p\, d\boldsymbol{\delta}_p\, d\boldsymbol{\delta}_q}{\pi^2\omega^3 Q^2}, \tag{29}$$

where $Q \approx m|\boldsymbol{\delta}_p+\boldsymbol{\delta}_q|$, $\xi_p \approx 1/\delta_p^2$, and $\xi_q \approx 1/\delta_q^2$. This term is proportional to η^2 for any η. The leading antisymmetric term is

$$d\sigma_a = -\frac{\alpha m^2\eta^2(\varepsilon_p\xi_p-\varepsilon_q\xi_q)(\varepsilon_p^2+\varepsilon_q^2)\, d\varepsilon_p\, d\boldsymbol{\delta}_p\, d\boldsymbol{\delta}_q}{\pi\varepsilon_p\varepsilon_q\omega^3 Q^3}\,\mathrm{Re}\, g(\eta),$$

$$g(\eta) = \eta\frac{\Gamma(1-i\eta)\Gamma(1/2+i\eta)}{\Gamma(1+i\eta)\Gamma(1/2-i\eta)}. \tag{30}$$

It is also important to consider the asymptotics of the charge asymmetry in the region $|\boldsymbol{\delta}_p+\boldsymbol{\delta}_q| \ll |\boldsymbol{\delta}_p-\boldsymbol{\delta}_q|$. In this case, the arguments u and z of the hypergeometric functions $\mathcal{F}$ and $\mathcal{G}$, as well as the factor $[(1+\xi_p\lambda)/(1+\xi_q\lambda)]^{i\eta}$, in Eq. (25) can be replaced by unity. As a result, we find that $d\sigma_s \propto \eta^2$ and $d\sigma_a \propto \eta^3$ for any η, and one can use Eqs. (26) and (27) for this region.

Integration of Eq. (24) over $\boldsymbol{\delta}_p$ gives, for $d\sigma_s$ [17],

$$d\sigma_s = \frac{4\alpha\eta^2\xi_q^2\, d\boldsymbol{\delta}_p\, d\varepsilon_p}{\pi m^2\omega^3}\{(\varepsilon_p^2+\varepsilon_q^2)(L+3/2) + \varepsilon_p\varepsilon_q[1+4\xi_p(1-\xi_p)L]\}, \tag{31}$$

$$L = \ln\left(\frac{2\varepsilon_p\varepsilon_q}{m\omega}\right) - 2 - \mathrm{Re}[\psi(1+i\eta)+C],$$

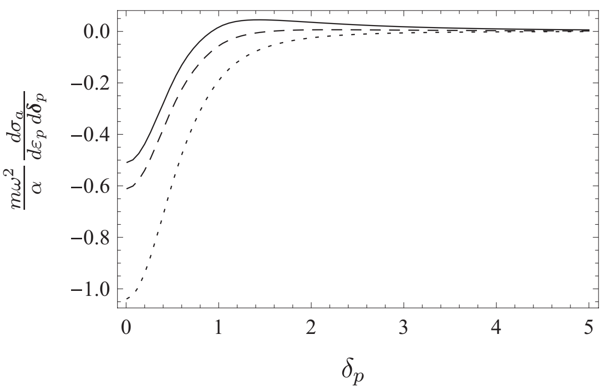

FIG. 2. The dependence of $\frac{d\sigma_a}{d\varepsilon_p\, d\delta_p}$ in units $\frac{\alpha}{m\omega^2}$ on δ_p for a few values of $x = \varepsilon_p/\omega$: $x = 0.25$ (solid curves), $x = 0.5$ (dashed curves), and $x = 0.75$ (dotted curves); $\eta = 0.54$ (tungsten).

where $C = 0.577\ldots$ is the Euler constant. The integration over $\boldsymbol{\delta}_p$ gives the well-known result [8]

$$d\sigma_s = \frac{4\alpha\eta^2}{m^2\omega^3}\left(\varepsilon_p^2 + \varepsilon_q^2 + \frac{2}{3}\varepsilon_p\varepsilon_q\right)\left(L + \frac{3}{2}\right) d\varepsilon_p. \qquad (32)$$

We have performed numerical integration of Eq. (25) over $\boldsymbol{\delta}_q$. Figure 2 shows the result of this integration, $\frac{d\sigma_a}{d\varepsilon_p\, d\boldsymbol{\delta}_p}$ in units $\frac{\alpha}{m\omega^2}$ as a function of δ_p for a few values of $x = \varepsilon_p/\omega$.

The cross section $d\sigma_a$ integrated over both $\boldsymbol{\delta}_p$ and $\boldsymbol{\delta}_q$ was obtained in our previous paper [10] and has the form

$$d\sigma_a = -\frac{\pi^3\alpha\eta^2(\varepsilon_p - \varepsilon_q)(2\omega^2 - 3\varepsilon_p\varepsilon_q)\, d\varepsilon_p}{4m\omega^3\varepsilon_p\varepsilon_q}\,\mathrm{Re}\, g(\eta). \qquad (33)$$

The result of numerical integration of Eq. (25) over $\boldsymbol{\delta}_p$ and $\boldsymbol{\delta}_q$ is in agreement with the above result.

VI. COMPTON-TYPE CONTRIBUTION

In this section we estimate the contribution of the Compton-type amplitude to the photoproduction cross section. Since this amplitude is small in comparison with the leading amplitude found above, it is necessary to take into account only the interference between the Compton-type amplitude and the leading amplitude. The leading amplitude is enhanced at small angles θ_p and θ_q of the final particles. Therefore, it is sufficient to calculate the Compton-type amplitude also at $\theta_p, \theta_q \ll 1$. For real initial and final photons with $\omega \ll m_A$ (m_A is the nuclear mass), the nuclear Compton scattering amplitude, corresponding to the left-hand diagram in Fig. 3, in the forward direction reads

$$M_C = T(\omega)\, \boldsymbol{e}_1 \cdot \boldsymbol{e}_2^*, \qquad (34)$$

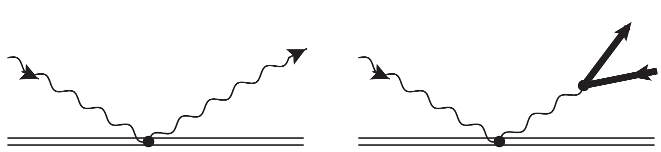

FIG. 3. Real Compton scattering diagram (left) and Compton-type diagram for pair production by a photon in a strong Coulomb field (right). Thick lines correspond to the positive- and negative-energy solutions of the Dirac equation in the Coulomb field. Double line denotes nucleus.

where $T(\omega)$ is the function measured for many nuclei (see Ref. [20]), and $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ are the photon polarization vectors of the initial and final photons, respectively. The function $T(\omega)$ satisfies the relations

$$T(0) = -\frac{Z^2e^2}{m_A}, \quad \mathrm{Im}\, T(\omega) = \frac{\omega}{4\pi}\sigma_{\gamma N}(\omega),$$
$$\mathrm{Re}\,[T(\omega) - T(0)] = \frac{\omega^2}{2\pi^2}\,\mathrm{P}\!\int_0^\infty \frac{\sigma_{\gamma N}(\omega)}{\omega'^2 - \omega^2}\, d\omega', \qquad (35)$$

where $\sigma_{\gamma N}(\omega)$ is the nuclear photoabsorption cross section and P denotes the integration in the principal value sense. Below the pion photoproduction threshold, the cross section $\sigma_{\gamma N}(\omega)$ is conventionally written as a superposition of Lorentzian lines

$$\sigma_{\gamma N}(\omega) = \sum_n \sigma_n \frac{(\omega\Gamma_n)^2}{\left(E_n^2 - \omega^2\right)^2 + (\omega\Gamma_n)^2}, \qquad (36)$$

where the parameters σ_n, E_n, and Γ_n are extracted from the experiment. The corresponding function $T(\omega)$ has the form

$$T(\omega) = -\frac{Z^2e^2}{m_A} + \frac{\omega^2}{4\pi}\sum_n \frac{\sigma_n\Gamma_n}{E_n^2 - \omega^2 - i\omega\Gamma_n}. \qquad (37)$$

Below the pion threshold, but above the resonance region ($\omega \gg E_n$), the function $T(\omega)$ has the form

$$T(\infty) = -\frac{Z^2e^2}{m_A} - \frac{1}{4\pi}\sum_n \sigma_n\Gamma_n. \qquad (38)$$

The last term in this asymptotics is equal to $(1 + \varkappa)NZe^2/m_A$, where $\varkappa$ is the so-called enhancement factor (see Ref. [20]), and N is the number of neutrons, and we obtain

$$T(\infty) = -\frac{Ze^2}{m_p}\left(1 + \frac{N}{A}\varkappa\right), \qquad (39)$$

where m_p is the proton mass, and $A = Z + N$. For heavy nuclei $\varkappa \sim 0.3$–0.4 [21], so that

$$T(\infty)/T(0) \sim 3.$$

Using the function $T(\omega)$, we write the additional contribution to the photoproduction amplitude, corresponding to the right-hand diagram in Fig. 3, as follows:

$$\tilde{M}_{\lambda_1\lambda_2} = -T(\omega)\int \frac{d\boldsymbol{\kappa}}{(2\pi)^3}\frac{4\pi}{\omega^2 - \kappa^2 + i\,0}\left(\boldsymbol{e} - \frac{(\boldsymbol{\kappa}\cdot\boldsymbol{e})}{\omega^2}\boldsymbol{\kappa}\right)$$
$$\cdot \int d\boldsymbol{r}\, \bar{u}^{(\mathrm{out})}_{\lambda_1\boldsymbol{p}}(\boldsymbol{r})\, \boldsymbol{\gamma}\, v^{(\mathrm{in})}_{\lambda_2\boldsymbol{q}}(\boldsymbol{r}) \exp(i\boldsymbol{\kappa}\cdot\boldsymbol{r}). \qquad (40)$$

We assume that $\theta_p \ll 1$ and $\theta_q \ll 1$. Taking the integral over $\boldsymbol{\kappa}$ we obtain

$$\tilde{M}_{\lambda_1\lambda_2} = T(\omega)\int \frac{d\boldsymbol{r}}{r}\, \bar{u}^{(\mathrm{out})}_{\lambda_1\boldsymbol{p}}(\boldsymbol{r})\,[\boldsymbol{e} - (\boldsymbol{n}\cdot\boldsymbol{e})\boldsymbol{n}]$$
$$\cdot\, \boldsymbol{\gamma}\, v^{(\mathrm{in})}_{\lambda_2\boldsymbol{q}}(\boldsymbol{r}) \exp(i\omega r), \qquad (41)$$

where $\boldsymbol{n} = \boldsymbol{r}/r$. The main contribution to the integral over $\boldsymbol{r}$ is given by the region $\boldsymbol{p}\cdot\boldsymbol{r} \sim pr \sim \omega^2/m^2$ and $\boldsymbol{q}\cdot\boldsymbol{r} \sim qr \sim \omega^2/m^2$. In this case

$$F_A(\boldsymbol{r},\, \boldsymbol{p},\, \eta) = \tilde{F}_B(\boldsymbol{r},\, \boldsymbol{p},\, \eta) = \frac{\exp(-i\,\boldsymbol{p}\cdot\boldsymbol{r})}{2(pr + \boldsymbol{p}\cdot\boldsymbol{r})^{i\eta}},$$
$$F_B(\boldsymbol{r},\, \boldsymbol{p},\, \eta) = \frac{\exp(-i\,\boldsymbol{p}\cdot\boldsymbol{r})}{2(pr + \boldsymbol{p}\cdot\boldsymbol{r})^{i\eta+1}}. \qquad (42)$$

Using these asymptotics and taking the integral over $\boldsymbol{r}$ we arrive at the Compton-type correction to the photoproduction amplitude,

$$\begin{aligned}\tilde{M}_{\lambda_1\lambda_2} &= \phi^{+}_{\lambda_1}(\tilde{a}+\boldsymbol{\sigma}\cdot\tilde{\boldsymbol{b}})\chi_{\lambda_2},\\ \tilde{a} &= i\tilde{N}\,[\boldsymbol{v}\times\boldsymbol{e}]\cdot\boldsymbol{\vartheta}\,,\quad \tilde{\boldsymbol{b}}=\tilde{N}\frac{(\varepsilon_p-\varepsilon_q)}{\omega}(\boldsymbol{e}\cdot\boldsymbol{\vartheta})\,\boldsymbol{v},\\ \tilde{N} &= \frac{2\pi\,T(\omega)}{m\omega}\left(\frac{\varepsilon_q}{\varepsilon_p}\right)^{i\eta}\frac{1}{1+\vartheta^2},\\ \boldsymbol{\vartheta} &= \frac{\varepsilon_p\varepsilon_q}{m\omega}(\boldsymbol{\theta}_p-\boldsymbol{\theta}_q)=\frac{\varepsilon_q\boldsymbol{\delta}_p-\varepsilon_p\boldsymbol{\delta}_q}{\omega}.\end{aligned} \tag{43}$$

The corresponding correction to the cross section has the form

$$\begin{aligned}d\tilde{\sigma} &= \frac{4\alpha m^4\,d\varepsilon_p\,d\boldsymbol{\delta}_p\,d\boldsymbol{\delta}_q}{(2\pi)^4\omega}\mathrm{Re}(a_0\tilde{a}^*+\boldsymbol{b}_0\cdot\tilde{\boldsymbol{b}}^*)\\ &= \frac{8\alpha m^4\left(\varepsilon_p^2+\varepsilon_q^2\right)d\varepsilon_p\,d\boldsymbol{\delta}_p\,d\boldsymbol{\delta}_q}{(2\pi)^4\omega^3}\mathrm{Re}[(\boldsymbol{g}_0^{(-)}\cdot\boldsymbol{\vartheta})\tilde{N}^*].\end{aligned} \tag{44}$$

This correction contains both symmetric and antisymmetric parts with respect to replacement $\eta\to-\eta$. The symmetric part is proportional to $\mathrm{Im}\,T(\omega)$ and the antisymmetric part is proportional to $\mathrm{Re}\,T(\omega)$:

$$\begin{aligned}d\tilde{\sigma}_a &= \frac{2\alpha m^2|\Gamma(1-i\eta)|^2\left(\varepsilon_p^2+\varepsilon_q^2\right)d\varepsilon_p\,d\boldsymbol{\delta}_p\,d\boldsymbol{\delta}_q}{\pi^2\omega^4(1+\vartheta^2)Q^2}\,\mathrm{Re}\,T(\omega)\\ &\quad\times(\boldsymbol{\vartheta}\cdot[\cos\mu\,(\xi_p\boldsymbol{\delta}_p+\xi_q\boldsymbol{\delta}_q)\,\eta\mathcal{F}\\ &\quad+\sin\mu\,(\xi_p\boldsymbol{\delta}_p-\xi_q\boldsymbol{\delta}_q)(1-u)\mathcal{F}']),\\ \mu &= \eta\ln\left(\frac{\xi_q}{\xi_p}\right).\end{aligned} \tag{45}$$

VII. DISCUSSION

In quantum electrodynamics, an electron differs from a positron only by its charge. Therefore, the cross section of e^+e^- pair photoproduction satisfies the relation

$$d\sigma(\boldsymbol{p},\boldsymbol{q},\eta)=d\sigma(\boldsymbol{q},\boldsymbol{p},-\eta).$$

We define the charge asymmetry $\mathcal{A}$ as

$$\begin{aligned}\mathcal{A} &= \frac{d\sigma(\boldsymbol{p},\boldsymbol{q},\eta)-d\sigma(\boldsymbol{p},\boldsymbol{q},-\eta)}{d\sigma(\boldsymbol{p},\boldsymbol{q},\eta)+d\sigma(\boldsymbol{p},\boldsymbol{q},-\eta)}\\ &= \frac{d\sigma(\boldsymbol{p},\boldsymbol{q},\eta)-d\sigma(\boldsymbol{q},\boldsymbol{p},\eta)}{d\sigma(\boldsymbol{p},\boldsymbol{q},\eta)+d\sigma(\boldsymbol{q},\boldsymbol{p},\eta)}.\end{aligned} \tag{46}$$

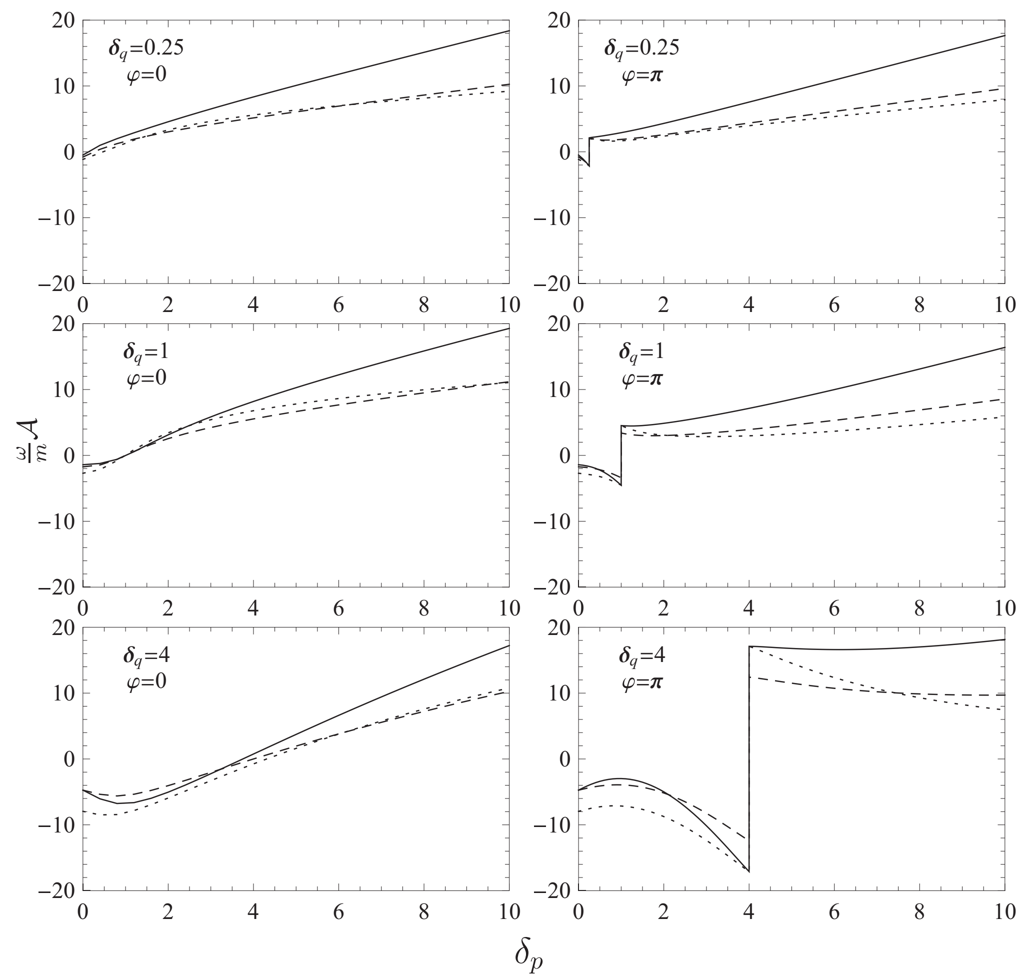

FIG. 4. The dependence of $\mathcal{A}$ on δ_p in units m/ω for a few values of δ_q, φ, and $x=\varepsilon_p/\omega$: $x=0.25$ (solid curves), $x=0.5$ (dashed curves), and $x=0.75$ (dotted curves), $\eta=0.54$ (tungsten).

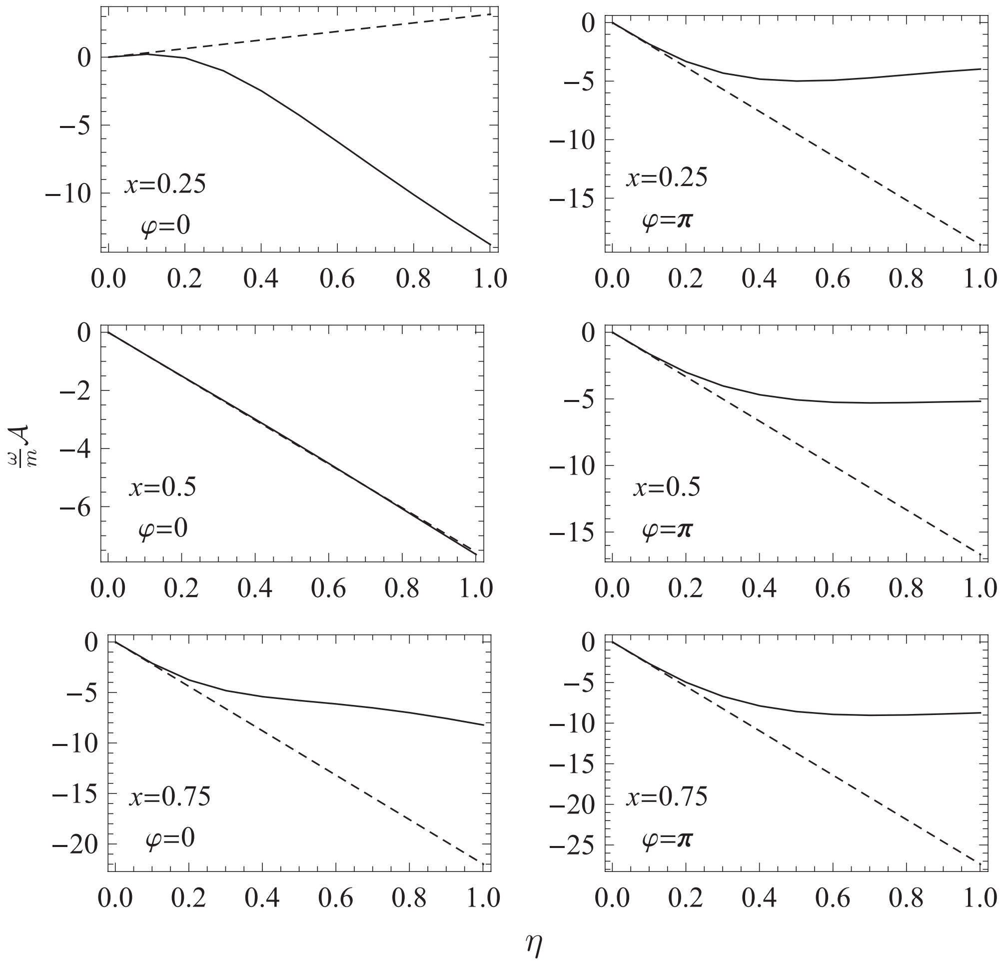

FIG. 5. The dependence of $\mathcal{A}$ on $\eta = Z\alpha$ in units m/ω for $\delta_p = 2$, $\delta_q = 4$, and a few values of x and φ. The solid curve represents the exact-in-η result, and the dashed curve is obtained in the leading-in-η approximation (linear in η).

Let us first neglect $d\tilde{\sigma}_a$ and calculate $\mathcal{R}$ as a ratio of $d\sigma_a$ in Eq. (25) and $d\sigma_s$ in Eq. (24). Outside the very narrow region $Q_\perp \lesssim |Q_\parallel| = |\boldsymbol{v}\cdot\boldsymbol{Q}| \sim m^2/\omega$, we can replace $Q^2 \to \boldsymbol{Q}_\perp^2 = m^2(\boldsymbol{\delta}_p + \boldsymbol{\delta}_q)^2$. Then, at fixed $\boldsymbol{\delta}_p$, $\boldsymbol{\delta}_q$, and $x = \varepsilon_p/\omega$, the asymmetry $\mathcal{A}$ scales as m/ω, as can be seen from Eqs. (25) and (24). Figure 4 shows the dependence of $\mathcal{A}$ on δ_p in units m/ω for tungsten ($\eta = 0.54$) for a few values of δ_q and φ, where φ is the angle between vectors $\boldsymbol{\delta}_p$ and $\boldsymbol{\delta}_q$. It is seen that the charge asymmetry may be rather large ($\mathcal{A} \sim$ 20–30% for $\omega/m = 50$). The asymmetry is large when δ_p and/or δ_q are much larger than unity. Note that this statement is also valid in the region $|\boldsymbol{\delta}_p + \boldsymbol{\delta}_q| \ll |\boldsymbol{\delta}_p - \boldsymbol{\delta}_q|$ (but $\delta_p \gg 1$ and $\delta_q \gg 1$).

For $\varphi = \pi$, one can see a jump in $\mathcal{A}$ at $\delta_p = \delta_q$ where $|\boldsymbol{Q}_\perp| = 0$. At this point $\mathcal{A}$ changes its sign. The origin of the jump is the following. As it was shown in Sec. V, at $Q_\perp/m \ll |\boldsymbol{\delta}_p - \boldsymbol{\delta}_q|$ one can use Eqs. (26) and (27) for any η; see discussion following Eq. (30). In this region, for $Q_\perp \gg m^2/\omega$, we have $d\sigma_s \propto 1/Q^2$ and $d\sigma_a \propto 1/Q^2$ so that $d\sigma_a/d\sigma_s$ is finite. However, for $\varphi = 0$ the ratio $d\sigma_a/d\sigma_s$ has opposite sign for $\delta_p > \delta_q$ and $\delta_p < \delta_q$. Therefore, the dependence $\mathcal{A}$ on δ_p at $\varphi = \pi$ looks like a jump though it is a rapidly varying continuous function in the very narrow region $Q_\perp \sim m^2/\omega$, which is not considered in this paper.

Note that screening should be taken into account only in the very narrow region $|\boldsymbol{Q}_\perp| \lesssim r_{\rm scr}^{-1} \sim m\alpha Z^{1/3} \ll m$, where $r_{\rm scr}$ is the atomic screening radius. Outside of this region, at $(\delta_p + \delta_q)|\varphi - \pi| \gg \alpha Z^{1/3}$ or $|\delta_p - \delta_q| \gg \alpha Z^{1/3}$, screening is unimportant.

It is interesting to understand the importance of high-order-in-η terms in the charge asymmetry. Figure 5 shows the dependence of $\mathcal{A}$ on $\eta = Z\alpha$ in units m/ω for $\delta_p = 2$, $\delta_q = 4$, and a few values of $x = \varepsilon_p/\omega$ and φ. The dashed curve in this figure is obtained in the leading-in-η approximation (linear

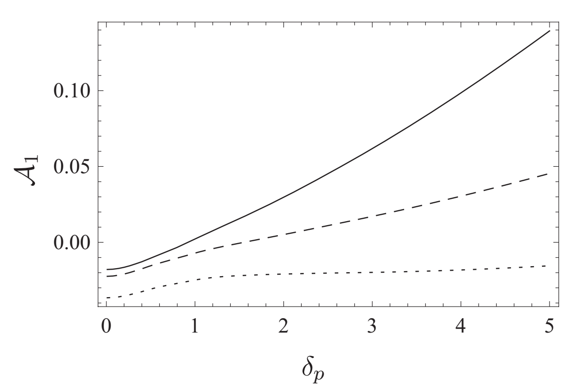

FIG. 6. The dependence of $\mathcal{A}_1$ on δ_p for $x = 0.25$ (solid curves), $x = 0.5$ (dashed curves), and $x = 0.75$ (dotted curves); $\eta = 0.54$ (tungsten), $\omega/m = 50$.

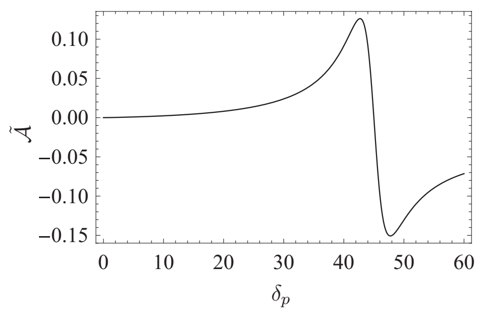

FIG. 7. Contribution $\tilde{\mathcal{A}}$ as a function of δ_p for $\omega/m = 200$, $Z = 1$ (proton), $x = \varepsilon_p/\omega = 0.6$, $\delta_q = 30$, $\varphi = 0$.

in η). It is seen that η dependence is very strong even for intermediate values of η. Let us also introduce the charge asymmetry $\mathcal{A}_1$ for the cross section integrated over the angles of one of the particles,

$$\mathcal{A}_1 = \frac{d\sigma(\boldsymbol{p},\eta) - d\sigma(\boldsymbol{p},-\eta)}{d\sigma(\boldsymbol{p},\eta) + d\sigma(\boldsymbol{p},-\eta)}. \quad (47)$$

We calculate the nominator in $\mathcal{A}_1$ integrating $d\sigma_a$ in Eq. (25) over $\boldsymbol{\delta}_q$ and the denominator using Eq. (31). Figure 6 shows the dependence of $\mathcal{A}_1$ on δ_p for $\eta = 0.54$ (tungsten), $\omega/m = 50$, and a few values of x. Note that $\mathcal{A}_1$, in contrast to $\mathcal{A}$, does not scale as m/ω due to logarithmic dependence of $d\sigma_s(\boldsymbol{p},\eta)$ on ω; see Eq. (31). It is seen that $\mathcal{A}_1$ is noticeable though it is smaller than $\mathcal{A}$. The charge asymmetry corresponding to the cross section integrated over the angles of both particles (over $\boldsymbol{\delta}_p$ and $\boldsymbol{\delta}_q$) is very small (see Ref. [10]).

Let us discuss now the contribution $\tilde{\mathcal{A}}$ to the charge asymmetry,

$$\tilde{\mathcal{A}} = \frac{d\tilde{\sigma}_a(\boldsymbol{p},\boldsymbol{q},\eta)}{d\sigma_s(\boldsymbol{p},\boldsymbol{q},\eta)}, \quad (48)$$

where $d\tilde{\sigma}_a(\boldsymbol{p},\boldsymbol{q},\eta)$ is given by Eq. (45) and $d\sigma_s(\boldsymbol{p},\boldsymbol{q},\eta)$ is given by Eq. (24).

In the region $\delta_p \sim \delta_q \sim 1$, we have $d\tilde{\sigma}_a/d\sigma_a \sim m/(\eta m_p)$ and $d\sigma_a/d\sigma_s \sim \eta m/\omega$. In this region, $d\tilde{\sigma}_a$ may be comparable with $d\sigma_a$ only for light nuclei ($\eta \ll 1$), where the asymmetry is very small.

In the region $\delta_p \sim 1$, $\delta_q \gg 1$, we have $d\tilde{\sigma}_a/d\sigma_a \sim m/(\eta m_p)$ and $d\sigma_a/d\sigma_s \sim \eta\theta_q$. Again, $d\tilde{\sigma}_a$ may be comparable with $d\sigma_a$ only for light nuclei where the asymmetry is very small.

The only region where $d\tilde{\sigma}_a \gtrsim d\sigma_a$ and $\tilde{\mathcal{A}}$ is not too small is $\eta \ll 1$, $\delta_p \gg 1$, $\delta_q \gg 1$, but $\vartheta = \frac{\varepsilon_p\varepsilon_q}{m\omega}|\boldsymbol{\theta}_p - \boldsymbol{\theta}_q| \sim 1$. In this region the ratio $d\tilde{\sigma}_a/d\sigma_a$ is

$$\frac{d\tilde{\sigma}_a}{d\sigma_a} = -\frac{2\left(1 + \frac{N}{A}\varkappa\right)\varepsilon_p^2\varepsilon_q^2\theta_p^3}{\pi(1+\vartheta^2)\,\eta\,\omega m^2 m_p}, \quad (49)$$

and may be larger than unity. Here we took into account that $\theta_p \approx \theta_q \gg m/\omega$ but $|\boldsymbol{\theta}_p - \boldsymbol{\theta}_q| \sim m/\omega$. For the contribution $\tilde{\mathcal{A}}$ to the charge asymmetry we have

$$\tilde{\mathcal{A}} = \frac{d\tilde{\sigma}_a}{d\sigma_s} = -\frac{\left(1 + \frac{N}{A}\varkappa\right)\varepsilon_p\varepsilon_q\theta_p^2(\boldsymbol{\theta}_p \cdot \boldsymbol{\vartheta})}{(1+\vartheta^2)mm_p}, \quad (50)$$

where $\boldsymbol{\vartheta} = (\boldsymbol{\theta}_p - \boldsymbol{\theta}_q)\varepsilon_p\varepsilon_q/(m\omega)$, so that $\tilde{\mathcal{A}}$ may reach about 10% at large transverse momenta compared to the electron mass (see Fig. 7). In this figure, $\tilde{\mathcal{A}} = 0$ in the point corresponding to the condition $\vartheta = 0$, which is equivalent to $\delta_p = \delta_q x/(1-x)$.

VIII. CONCLUSION

We have derived exactly in the parameter $\eta = Z\alpha$ the charge asymmetry $\mathcal{A}$, Eq. (46), in the process of e^+e^- photoproduction in a Coulomb field at photon energy $\omega \gg m$, $\varepsilon_p \gg m$, and $\varepsilon_q \gg m$. This asymmetry is related to the first quasiclassical correction to the differential cross section of the process, Eq. (25). When $p_\perp$ and/or $q_\perp$ are much larger than the electron mass m, the charge asymmetry can be as large as tens percent. The charge asymmetry $\mathcal{A}_1$, Eq. (47), in the cross section integrated over the transverse momenta of one of the particles is several times smaller than the asymmetry $\mathcal{A}$ in the cross-section differential with respect to the transverse momenta of both particles. We have also estimated the contribution $\tilde{\mathcal{A}}$, Eqs. (45) and (48), to the charge asymmetry of the Compton-type diagram. For $\eta \sim 1$, this contribution is negligible. The only region where $\tilde{\mathcal{A}}$ can be important is $\eta \ll 1$ (light nucleus); $\theta_p \approx \theta_q \gg m/\omega$ but $|\boldsymbol{\theta}_p - \boldsymbol{\theta}_q| \sim m/\omega$. Though we have performed our calculation for the pure Coulomb field, our results are also applicable for photoproduction in the electric field of atoms except the very narrow region $Q \lesssim r_{\rm scr}^{-1} \sim m\alpha Z^{1/3} \ll m$. Note that the effect of the finite nuclear size, coming from the difference between the electric field inside the nucleus and the Coulomb field, is not important for the process under consideration because the characteristic distances for our process are of the order of the Compton wavelength, which is much larger than the nuclear radius for all Z. Our results clearly demonstrate that experimental observation of the charge asymmetry in the process of e^+e^- photoproduction in a Coulomb field is a realistic task.

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