

Highly polarized Fermi gases across a narrow Feshbach resonance

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We address the phase of a highly polarized Fermi gases across a *narrow* Feshbach resonance starting from the problem of a single down-spin fermion immersed in a Fermi sea of up spins. Both polaron and pairing states are considered using the variational wave function approach, and we find that the polaron-to-pairing transition will take place on the BCS side of the resonance, strongly in contrast to a wide resonance where the transition is located at the BEC side. For the pairing phase, we find the critical strength of the repulsive interaction between pairs above which the mixture of pairs and fermions will not phase separate. Therefore, nearby a narrow resonance, it is quite likely that magnetism can coexist with *s*-wave BCS superfluidity at large Zeeman fields, which is a remarkable property absent in conventional BCS superconductors (or fermion-pair superfluids).

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Whether an *s*-wave superconductor (or fermion-pair superfluid) can coexist with magnetism is a long-standing issue in condensed matter physics. Back in the 1960s, Chandrasekhar and Clogston independently considered the response of a BCS superconductor to spin polarization due to a Zeeman field [1,2]. They found that an *s*-wave superconductor will remain unpolarized until a critical Zeeman energy $h_c \sim \Delta/\sqrt{2}$, where Δ is the pairing gap, at which point the system undergoes a sharp phase transition to a partially polarized normal state. This critical field is now known as the Chandrasekhar-Clogston (CC) limit of the superconductor. In this scenario, superconductivity cannot coexist with magnetism. Later on, several proposals were made for magnetized *s*-wave superconducting states; the most famous being the Fulde-Ferrell-Larkin-Ovchinnikov state [3] and the Sarma State [4]. However, so far none of them have been firmly observed in an *s*-wave BCS superconductor.

In the last few years, this problem has been revisited by a series of experiments on two-component Fermi gases with population imbalance [5–9]. Experiments have reached a consensus that, in the resonance regime and on the BCS side, there is a CC limit where a direct transition from a fully paired fermion superfluid to a partially polarized normal state has been observed, and no evidence of a magnetized superfluid has been found [5,8–10]. However, all these studies were done across a wide resonance. Recently, several experimental groups have begun to study narrow Feshbach resonances, such as ^6Li at 543.25 G [11] or $^6\text{Li}-^{40}\text{K}$ mixtures at 154.719 G [12], where the effect of a finite resonance width needs to be taken into account. In this Rapid Communication we find that the resonance width indeed has dramatic effects on the physics of highly polarized Fermi gases.

In contrast to a wide resonance, to characterize a Fermi gas nearby a narrow resonance one not only needs $k_F a_s^0$, where a_s^0 is the zero-energy scattering length between fermions, but one also needs to consider the dimensionless parameter $\hbar^2 k_F / (2mW a_{bg})$, where m is the mass of the atom, W is the width of a resonance, and a_{bg} is the background scattering length. If $\hbar^2 k_F / (2mW a_{bg}) \gtrsim 1$ the resonance is considered

to be a narrow one. Another dimensionless quantity is $k_F a_{bb}$ where a_{bb} is the scattering length between closed channel molecules. In this work we focus on the highly polarized limit and show that all three of these parameters play an important role in determining the nature of many-body phases. This work contains two parts:

First, we consider a single down spin immersed in a Fermi sea of up spins. Two different types of states are compared, which are the polaronic state and the pairing state. For the polaronic state, the single down spin is dressed by particle-hole pairs of up spins, and becomes a fermionic quasiparticle [13]. If this state has lower energy, the system will be a normal state of polaron Fermi liquid at sufficiently high polarization. For the pairing state, one of the up spins will form a bound state with the single down spin. If this state has lower energy, each down spin will form a pair, and the system will be a mixture of condensed pairs and majority fermions. For a wide resonance, a polaron-to-pairing transition takes place on the BEC side of the resonance [14–16]. Here we show that, as the width of resonance gets narrower, the transition point will be shifted toward the BCS side. We find how the critical value of $(k_F a_s^0)_c$ changes with the quantity $\hbar^2 k_F / (2mW a_{bg})$.

Second, when the pairing state has lower energy, the mixture of pairs and fermions may phase separate due to the repulsion between pairs and fermions. A sufficiently strong repulsion between pairs is crucial to stabilize a uniform mixture. For a given $\hbar^2 k_F / (2mW a_{bg})$, we find the critical repulsion $(k_F a_{bb})_c$ as a function of $k_F a_s^0$.

Hence, we conclude that when $1/(k_F a_s^0) > 1/(k_F a_s^0)_c$ and $k_F a_{bb} > (k_F a_{bb})_c$, it is energetically favorable for minority fermions to form pairs, and a condensate of fermion pairs can uniformly mix with majority fermions; that is, magnetism can coexist with fermion-pair superfluids in highly polarized Fermi gases. The fact that this can happen on the BCS side and resonance regime represents a significant distinction between narrow and wide resonances. As far as the response to spin polarization is concerned, at resonance, or even on the BCS side, this system behaves similar as on the BEC side of a wide resonance. This picture is also consistent with a recent high-temperature study of narrow resonances [17].

Model. We use the following two-channel model $\hat{H} = \hat{H}_0 + \hat{V}_c + \hat{V}_{bg} + \hat{V}_{bb}$ to describe a narrow resonance:

$$\hat{H}_0 = \sum_{\mathbf{k}} (\epsilon_{\mathbf{k}}^b + \nu_0) b_{\mathbf{k}}^\dagger b_{\mathbf{k}} + \sum_{\mathbf{k}} (\epsilon_{\mathbf{k}}^u u_{\mathbf{k}}^\dagger u_{\mathbf{k}} + \epsilon_{\mathbf{k}}^d d_{\mathbf{k}}^\dagger d_{\mathbf{k}}), \quad (1)$$

$$\hat{V}_c = g_0 \sum_{\mathbf{k}\mathbf{q}} \Lambda_{\mathbf{k}} (b_{\mathbf{q}-\mathbf{k}}^\dagger d_{\mathbf{q}-\mathbf{k}} u_{\mathbf{k}} + u_{\mathbf{k}}^\dagger d_{\mathbf{q}-\mathbf{k}}^\dagger b_{\mathbf{q}}), \quad (2)$$

$$\hat{V}_{bg} = U_0 \sum_{\mathbf{k}\mathbf{k}'\mathbf{q}} \Lambda_{\mathbf{k}} \Lambda_{\mathbf{k}'} u_{\mathbf{k}}^\dagger d_{\mathbf{q}-\mathbf{k}}^\dagger d_{\mathbf{q}-\mathbf{k}'} u_{\mathbf{k}'}, \quad (3)$$

$$\hat{V}_{bb} = \frac{1}{2} U_{bb} \sum_{\mathbf{k}\mathbf{k}'\mathbf{q}} b_{\mathbf{k}}^\dagger b_{\mathbf{q}-\mathbf{k}}^\dagger b_{\mathbf{q}-\mathbf{k}'} b_{\mathbf{k}'}, \quad (4)$$

where u^\dagger and d^\dagger are creation operators for majority up spin and minority down spin, respectively, b^\dagger is the creation operator for bosonic closed channel molecules, $\epsilon_{\mathbf{k}}^b = \hbar^2 \mathbf{k}^2 / [2(m^u + m^d)]$, and $\epsilon_{\mathbf{k}}^{u/d} = \hbar^2 \mathbf{k}^2 / (2m^{u/d})$. $\gamma = m^d / m^u$ is the mass ratio. \hat{V}_c and \hat{V}_{bg} represent the interchannel coupling and the background scattering, respectively. $\Lambda_{\mathbf{k}} = \Theta(\Lambda - |\mathbf{k}|)$ and Λ is the momentum cutoff. In the Hamiltonian, the molecule detuning ν_0 , the interchannel coupling g_0 , and the background interaction parameter U_0 are bare quantities with Λ dependence, which need to be renormalized as follows [18,19]: $\nu_0(\Lambda) = \nu_r - [1 - Z(\Lambda)]g_r^2/U_r$, $g_0(\Lambda) = Z(\Lambda)g_r$, and $U_0(\Lambda) = Z(\Lambda)U_r$, where $Z(\Lambda) = (1 - U_r m_r \Lambda / \pi^2)^{-1}$, $1/m_r = 1/m^u + 1/m^d$, and $U_r = 2\pi a_{bg}/m_r$. The renormalized quantities U_r , g_r , and ν_r are related to a_s as

$$\frac{2\pi a_s(E)}{m_r} = \left[\left(U_0 + \frac{g_0^2}{E - \nu_0} \right)^{-1} + \frac{m_r \Lambda}{\pi^2} \right]^{-1} = U_r + \frac{g_r^2}{E - \nu_r},$$

and the zero-energy scattering length a_s^0 is given by $a_s^0 = m_r(U_r - g_r^2/\nu_r)/(2\pi)$. Denoting $\nu_r = \Delta\mu(B - B_0)$, where B_0 is the location of the resonance and $\Delta\mu$ is the difference of magnetic moment between two channels, and introducing $W = g_r^2/U_r$, we have $a_s^0 = a_{bg}\{1 - W/[\Delta\mu(B - B_0)]\}$, and $a_s(E) = a_{bg}\{1 + W/[E - \Delta\mu(B - B_0)]\}$, where $W a_{bg}$ is always positive. For U_{bb} , since we only consider it to the mean-field order, we will take it as $U_{bb} = 4\pi\hbar^2 a_{bb}/(m^u + m^d)$. Our following results will be presented in terms of physical parameters (W , B_0 , $k_F a_s^0$, $k_F a_{bg}$, and $k_F a_{bb}$).

Polaronic state. We first adopt the following variational wave function which includes one-particle-hole contribution:

$$|\psi^P\rangle = \left[\phi_0 d_0^\dagger + \sum_{\mathbf{k}\mathbf{q}}' \phi_{\mathbf{k}\mathbf{q}} u_{\mathbf{k}}^\dagger d_{\mathbf{q}-\mathbf{k}}^\dagger u_{\mathbf{q}} + \sum_{\mathbf{q}}' \eta_{\mathbf{q}} b_{\mathbf{q}}^\dagger u_{\mathbf{q}} \right] |\text{FS}\rangle. \quad (5)$$

Here and below, all the summations with the prime (') of \mathbf{k} and \mathbf{q} are restricted to $|\mathbf{k}| > k_F$ and $|\mathbf{q}| < k_F$, respectively. After energy minimization we obtain a self-consistent equation for polaron energy:

$$E = \sum_{\mathbf{q}}' \Gamma_2(\mathbf{q}, E + \epsilon_{\mathbf{q}}^u), \quad (6)$$

where $\Gamma_2(\mathbf{q}, E + \epsilon_{\mathbf{q}}^u)$ coincides with a two-particle vertex with total momentum \mathbf{q} and total energy $E + \epsilon_{\mathbf{q}}^u$ within the ladder approximation. This is because the variational wave function $|\psi^P\rangle$ describes the processes whereby an up spin (taken out from an occupied state \mathbf{q}) and the single down spin undergo repeated scattering, as well as coherent conversion between open and closed channels. This physical process is precisely what is captured by the ladder approximation.

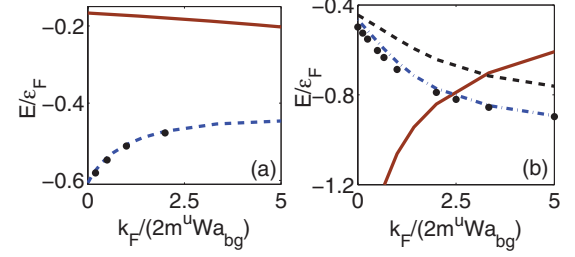


FIG. 1. (Color online) Polaron energy as a function of $\hbar^2 k_F / (2m^u W a_{bg})$. (a) $\gamma = 1$ and different interaction parameters: $1/(k_F a_s^0) = 0$ (dashed line) and $1/(k_F a_s^0) = -2$ (solid line). (b) $1/(k_F a_s^0) = 0$ with different mass ratios: $\gamma = 6/40$ (solid line), $\gamma = 40/6$ (dashed line), and $\gamma = \infty$ (dash-dotted line). All curves result from the one-particle-hole approximation, while solid black dots show results including two-particle-hole pair contributions. $k_F a_{bg}$ is chosen as -0.1 . We also set $\hbar = 1$ in all figures to simplify the presentation.

The explicit form of $\Gamma_2(\mathbf{q}, E + \epsilon_{\mathbf{q}}^u)$ is given as

$$\Gamma_2(\mathbf{q}, E + \epsilon_{\mathbf{q}}^u) = \left[\frac{m_r}{2\pi a_s(E + \epsilon_{\mathbf{q}}^u - \epsilon_{\mathbf{q}}^b)} + I(\mathbf{q}, E + \epsilon_{\mathbf{q}}^u) \right]^{-1}, \quad (7)$$

where $I(\mathbf{q}, E + \epsilon_{\mathbf{q}}^u) = \sum_{\mathbf{k}}' 1/[\epsilon_{\mathbf{k}}^u + \epsilon_{\mathbf{q}-\mathbf{k}}^d - (E + \epsilon_{\mathbf{q}}^u)] - \sum_{\mathbf{k}} 1/(\epsilon_{\mathbf{k}}^u + \epsilon_{\mathbf{k}}^d)$. The difference between Eq. (7) and the previous results for a wide resonance [13,14] is that a constant a_s is replaced by an energy-dependent one $a_s(E + \epsilon_{\mathbf{q}}^u - \epsilon_{\mathbf{q}}^b)$, where $E + \epsilon_{\mathbf{q}}^u - \epsilon_{\mathbf{q}}^b$ represents the energy of the relative motion for two atoms undergoing repeated scatterings.

The polaron energy as a function of $\hbar^2 k_F / (2m^u W a_{bg})$ is plotted in Fig. 1 from solving the self-consistency equation (6). As one can see, when the dimensionless parameter $\hbar^2 k_F / (2m^u W a_{bg})$ increases from zero, (i) for $\gamma = 1$, and nearby resonance $1/(k_F a_s^0) \approx 0$, the polaron energy E will increase, while at the BCS side $1/(k_F a_s^0) \ll 0$, E will decrease; and (ii) at resonance, if γ is greater than a critical value, E will also decrease. We have also checked the energy convergence by considering two-particle-hole contributions [20,21]. The numeric solutions with two-particle-hole contributions are shown as dots in Fig. 1. For $\gamma = 1$, one can see that the corrections from two-particle-hole pairs are always negligibly small, and it becomes even smaller as $|W|$ decreases. While for $\gamma \rightarrow \infty$, the deviation is a little larger [dashed line and the dots in Fig. 1(b)], as already noted for a wide resonance in [20], but it is still within only a few percent. This result justifies the validity of the expansion in terms of the number of particle-hole pairs in computing energy for a narrow resonance.

Pairing state. For the pairing state, we use the variational wave function first introduced in Ref. [14]:

$$|\psi^m\rangle = \left[\eta_0 b_0^\dagger + \sum_{\mathbf{k}}' A_{\mathbf{k}} u_{\mathbf{k}}^\dagger d_{-\mathbf{k}}^\dagger + \sum_{\mathbf{k}\mathbf{q}}' \phi_{\mathbf{k}\mathbf{q}} b_{\mathbf{q}-\mathbf{k}}^\dagger u_{\mathbf{k}}^\dagger u_{\mathbf{q}} + \sum_{\mathbf{k}\mathbf{k}'\mathbf{q}}' \Phi_{\mathbf{k}\mathbf{k}'\mathbf{q}} u_{\mathbf{k}}^\dagger d_{\mathbf{q}-\mathbf{k}}^\dagger u_{\mathbf{k}'}^\dagger u_{\mathbf{q}} \right] |\text{FS}'\rangle, \quad (8)$$

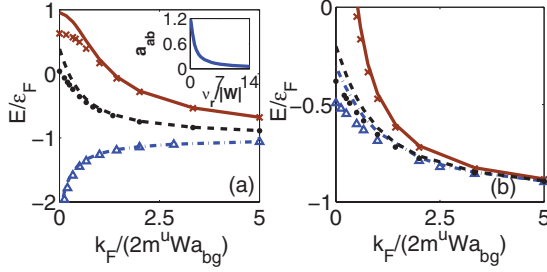


FIG. 2. (Color online) Pairing-state energy as a function of $\hbar^2 k_F / (2m^u W a_{bg})$. (a) Mass ratio $\gamma = 1$ and different interaction parameters: $1/(k_F a_s^0) = 0$ (dashed line), $1/(k_F a_s^0) = -1$ (solid line), and $1/(k_F a_s^0) = 1$ (dash-dotted line). (b) $1/(k_F a_s^0) = 0$ but different mass ratios: $\gamma = 6/40$ (solid line), $\gamma = 40/6$ (dashed line), and $\gamma = \infty$ (dash-dotted line). All the curves are computed from bare pairs, while the dots, crosses, and triangles are results with one-particle-hole contributions. Inset of (a) shows the atom-dimer scattering length a_{ab} (in units of $\sqrt{m^u E_b} / \hbar$ with E_b being the two-body binding energy) as a function of ν_r / W . $k_F a_{bg}$ is chosen as -0.1 .

where $|\text{FS}'\rangle$ refers to the Fermi sea with one spin- \uparrow particle removed from the Fermi surface of $|\text{FS}\rangle$ in the polaron state (5). If we only consider bare pair wave functions without including particle-hole contributions, the pairing-state energy is given by $\Gamma_2^{-1}(0, E + \epsilon_F) = 0$, as shown in the lines of Fig. 2. The interaction between pair and majority up spins can be described by including particle-hole pairs. Up to one particle-hole pair, by minimizing energy, we obtain a closed integral equation [21], and the numeric solution of these equations is also shown in Fig. 2. We find that, as $k_F / (2m^u W a_{bg})$ increases, the pairing-state energy decreases at the BCS side and in the resonance regime, despite different mass ratios; whereas it increases on the BEC side. Another important feature one can find from Fig. 2 is that, in the limit $W \rightarrow 0$, the pairing-state energy always saturates to $-\epsilon_F$. This can be understood as follows: when one down spin is added into the system, an up spin is taken out from the Fermi sea (subtract energy ϵ_F) to form a pair with the down spin, whose energy approaches ν_r in the limit $W \rightarrow 0$. Thus the pairing-state energy should approach $-\epsilon_F + \nu_r$ [22]. Moreover, at any fixed a_s^0 the ratio W / ν_r is fixed, thus $\nu_r \rightarrow 0$ and therefore the pairing-state energy always approaches $-\epsilon_F$ as $W \rightarrow 0$. This also indicates that the interaction between a pair and the residual majority atoms vanishes in the limit $W \rightarrow 0$. We have performed a three-body calculation and find the atom-dimer scattering length a_{ab} from the asymptotic behavior of the three-body wave function [21,23], as shown in the inset of Fig. 2(a), which indeed shows $a_{ab} \rightarrow 0$ as $W \rightarrow 0$.

Polaron-pairing transition. The transition from polaronic state to pairing state can now be determined by comparing their energies. In Figs. 3(a) and 3(b) we consider two concrete samples studied in current experiments: ${}^6\text{Li}$ at 543.25 G and an ${}^6\text{Li}$ - ${}^{40}\text{K}$ mixture at 154.719 G, and the parameters are typical values taken from experimental papers [11,12,24]. We found that, in both cases, the polaron-to-pairing transition is located on the BCS side of the resonance, which is away from resonance with $\Delta\mu(B - B_0)$ on the order of ϵ_F . At the transition points, $1/(k_F a_s^0) = -4.35$ for ${}^6\text{Li}$ and $1/(k_F a_s^0) = -0.55$ for the ${}^6\text{Li}$ - ${}^{40}\text{K}$ mixture, where the systems are very

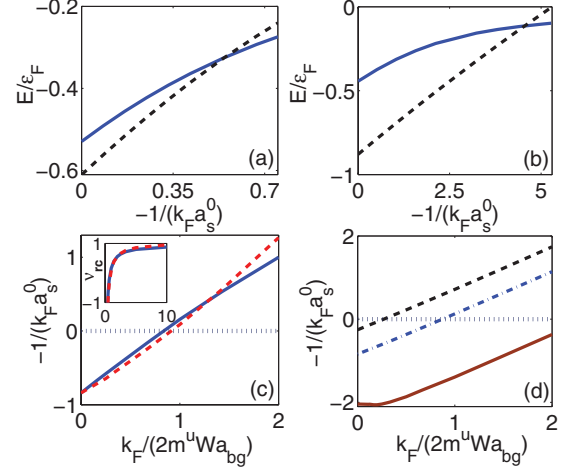


FIG. 3. (Color online) Comparison of polaron energy (solid line) and pairing-state energy (dashed line) as a function of zero-energy scattering length for ${}^6\text{Li}$ - ${}^{40}\text{K}$ mixture (a) and ${}^6\text{Li}$ (b). For (a), we take $k_F a_{bg} = 0.022$ and $W = 54.91\epsilon_F$ [24], and for (b) we take $k_F a_{bg} = 0.016$ and $W = 12.2\epsilon_F$ [11]. In (a), ${}^{40}\text{K}$ is taken as minority component. Panels (c) and (d) show the critical value of the transition $1/(k_F a_s^0)_c$ as a function of $\hbar^2 k_F / (2m^u W a_{bg})$. For (c), $\gamma = 1$ but with different $k_F a_{bg}$: $k_F a_{bg} = -0.1$ (solid line) and $k_F a_{bg} = 0.1$ (dashed line). For (d), $k_F a_{bg} = 0.01$ but with different γ : $\gamma = 6/40$ (solid line), $\gamma = 1$ (dash-dotted line), and $\gamma = 40/6$ (dashed line). Inset of (c) shows the critical value of the transition in term of ν_r^c (in unit of ϵ_F) as a function of $\hbar^2 k_F / (2m^u W a_{bg})$.

BCS like. This transition has also been observed in a recent experiment on ${}^6\text{Li}$ - ${}^{40}\text{K}$ mixtures and the transition is indeed observed on the BCS side [24].

In Fig. 3(c) and 3(d), we plot the critical value $1/(k_F a_s^0)_c$ for the polaron-to-pairing transition as a function of $k_F / (2m^u W a_{bg})$. One finds that, when $\hbar^2 k_F / (2m^u W a_{bg}) \gtrsim 1$, the transition will be shifted to the BCS side. This condition is equivalent to $|W| / \epsilon_F \lesssim 1 / |k_F a_{bg}|$. Since usually $k_F a_{bg} \ll 1$, it means that the resonance width does not need to be very narrow. One also notes that the transition point is not sensitive to the value of $k_F a_{bg}$ itself [Fig. 3(c)] but is sensitive to the mass ratio [Fig. 3(d)]. The inset of Fig. 3(c) shows that, in the limit $W \rightarrow 0$, the critical point will approach $\nu_r^c \rightarrow \epsilon_F$, which means that the pairing state will be favored once the energy of closed channel molecule is below the Fermi energy.

Stability of mixture. The discussion above concludes that, for a sufficiently narrow resonance, the highly polarized Fermi

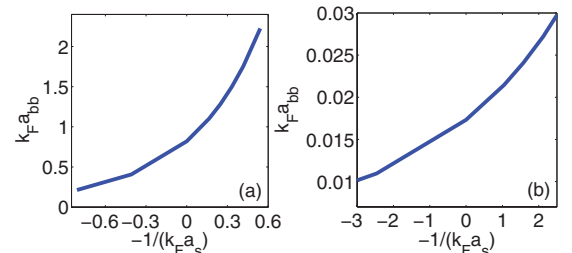


FIG. 4. (Color online) Critical value required for $k_F a_{bb}$ to prevent phase separation as a function of $1/(k_F a_s^0)$ for ${}^6\text{Li}$ - ${}^{40}\text{K}$ mixture (a) and ${}^6\text{Li}$ (b).

gases contain a mixture of majority fermions and bosonic pairs. The next question is whether they will uniformly mix or phase separate. To answer this question, we note that, for a very low density of down spins and a sufficiently narrow resonance, we can expand the equation of state in terms of the density of bosonic pairs n_b up to the second order [25]:

$$\mathcal{E} = \mathcal{E}_F + \mu_b^0 n_b + \frac{1}{2} g n_b^2, \quad (9)$$

where \mathcal{E}_F is the energy density of spin- \uparrow Fermi sea, and $\mu_b^0 = E + \epsilon_F$ where E is the pairing-state energy computed above. The repulsion $g = [4\pi\hbar^2/(m^u + m^d)]a_{bb} + g_{\text{ind}}$ contains the contribution from the bare interaction between closed channel molecules and the induced interaction g_{ind} from the interchannel coupling, which is calculated within Born approximation [21]. From Eq. (9) we obtain

$$\mu_b = \frac{\partial \mathcal{E}}{\partial n_b} = \mu_b^0 + g n_b, \quad (10)$$

$$\mu_{\uparrow} = \frac{\partial \mathcal{E}}{\partial n_{\uparrow}} = \epsilon_F + \frac{\partial \mu_b^0}{\partial n_{\uparrow}} n_b + O(n_b^2), \quad (11)$$

The stability condition against phase separation is given by $\frac{\partial \mu_{\uparrow}}{\partial n_{\uparrow}} \frac{\partial \mu_b}{\partial n_b} - \frac{\partial \mu_{\uparrow}}{\partial n_b} \frac{\partial \mu_b}{\partial n_{\uparrow}} > 0$ [26], from which we can determine the critical value for a_{bb} . The results are plotted in Fig. 4(a) for the ${}^6\text{Li}$ - ${}^{40}\text{K}$ mixture and in Fig. 4(b) for ${}^6\text{Li}$ in the regime where the pairing state is favorable. We can see that it requires $k_F a_{bb} > 0.81$ for ${}^6\text{Li}$ - ${}^{40}\text{K}$ mixture, and $k_F a_{bb} > 0.017$ for ${}^6\text{Li}$ at resonance. Very likely, this condition can be satisfied in ${}^6\text{Li}$ but not in the ${}^6\text{Li}$ - ${}^{40}\text{K}$ mixture.

Note added. Recently, we became aware of the experimental work from Innsbruck group in which the polaron properties and the polaron-to-pairing transition has been observed [24]. We also became aware of two other related theoretical works [27,28].

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