PHYSICAL REVIEW A **85**, 040306(R) (2012)

Macroscopic quantum computation using Bose-Einstein condensates

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(Received 31 October 2011; published 25 April 2012)

We analyze quantum computation using quantum information stored on two component Bose-Einstein condensates (BECs). We construct a general framework for quantum algorithms to be executed using the collective states of the BECs. The use of BECs allows for an increase of energy scales via bosonic enhancement, resulting in two-qubit gate operations that can be performed at a time reduced by a factor of N, where N is the number of bosons in the BEC. We illustrate the scheme by an application to Grover's algorithm, and discuss possible experimental implementations and decoherence effects.

DOI: 10.1103/PhysRevA.85.040306 PACS number(s): 03.67.Lx, 03.67.Ac, 03.75.Gg, 67.85.Hj

In a recent set of experiments, two-component Bose-Einstein condensates (BECs) were realized on atom chips realizing full coherent control on the Bloch sphere and spin squeezing [1–3]. Currently, the primary application for such two-component BECs is thought to be for quantum metrology [4] and chip-based clocks. In this Rapid Communication we discuss their application toward quantum computation. Although BECs have been considered for quantum computation in the past [5], the results have shown to have been generally unfavorable for these purposes due to enhanced decoherence effects due to the large number of bosons N in the BEC. In this work we consider a different encoding of the quantum information, which to a large extent mitigates this problem. We develop the framework for quantum computation using this encoding, illustrated with several quantum algorithms, and discuss decoherence effects.

Consider a BEC consisting of bosons with two independent degrees of freedom, such as two hyperfine levels in an atomic BEC or spin polarization states of exciton polaritons [4,6]. Denote the bosonic annihilation operators of the two states as a and b, obeying commutation relations $[a,a^\dagger] = [b,b^\dagger] = 1$ [7]. We encode a standard qubit state $\alpha|0\rangle + \beta|1\rangle$ in the BEC using a class of states

$$|\alpha,\beta\rangle\rangle \equiv \frac{1}{\sqrt{N!}}(\alpha a^\dagger + \beta b^\dagger)^N|0\rangle, \tag{1}$$

where α and β are arbitrary complex numbers satisfying $|\alpha|^2 + |\beta|^2 = 1$ (double angle brackets are used to denote the class of states as defined above). For simplicity let us first consider the boson number $N = a^\dagger a + b^\dagger b$ to be a conserved number. The quantum information of the standard qubit state is therefore encoded by N bosonic particles with a collective Hilbert space dimension of $N + 1$.

The state (1) can be visualized by a vector on the Bloch sphere with an angular representation $\alpha = \cos(\theta/2)$, $\beta = \sin(\theta/2)e^{i\phi}$. Points on the Bloch sphere form a set of pseudo-orthogonal states for large N. The overlap between two states can be calculated to be

$$\langle\langle\alpha',\beta'|\alpha,\beta\rangle\rangle = e^{-i(\phi-\phi')N/2}\left[\cos\left(\frac{\theta-\theta'}{2}\right)\cos\left(\frac{\phi-\phi'}{2}\right) + i\cos\left(\frac{\theta+\theta'}{2}\right)\sin\left(\frac{\phi-\phi'}{2}\right)\right]^N.$$

For example, for $\phi = \phi'$ the overlap simplifies to

$$\langle\langle\alpha',\beta'|\alpha,\beta\rangle\rangle = \cos^N\left(\frac{\theta-\theta'}{2}\right) \approx \exp\left(-\frac{N(\theta-\theta')^2}{8}\right).$$

Thus beyond angle differences of the order of $\theta - \theta' \sim 1/\sqrt{N}$, the overlap is exponentially suppressed.

The state $|\alpha,\beta\rangle\rangle$ can be manipulated using Schwinger boson (Stokes operators) operators $S^x = a^\dagger b + b^\dagger a$, $S^y = -ia^\dagger b + ib^\dagger a$, $S^z = a^\dagger a - b^\dagger b$, which satisfy the usual spin commutation relations $[S^i,S^j] = 2i\epsilon_{ijk}S^k$, where ϵ_{ijk} is the Levi-Civita antisymmetric tensor. In the spin language, (1) forms a spin-$N/2$ representation of the SU(2) group (we omit the factor of 2 in our spin definition for convenience). Single-qubit rotations can be performed in a completely analogous fashion to regular qubits. For example, rotations around the z axis of the Bloch sphere can be performed by an evolution

$$\begin{aligned} e^{-i\Omega S^z t}|\alpha,\beta\rangle\rangle &= \frac{1}{\sqrt{N!}}\sum_{k=0}^{N}\binom{N}{k}(\alpha a^\dagger e^{-i\Omega t})^k(\beta b^\dagger e^{i\Omega t})^{N-k}|0\rangle \\ &= |\alpha e^{-i\Omega t},\beta e^{i\Omega t}\rangle\rangle. \end{aligned} \tag{2}$$

Similar rotations may be performed around any axis by an application of

$$H_1 = \hbar\Omega\boldsymbol{n}\cdot\boldsymbol{S} = \hbar\Omega(n_x S^x + n_y S^y + n_z S^z), \tag{3}$$

where $\boldsymbol{n} = (n_x,n_y,n_z)$ is a unit vector. Expectation values of the spin are identical to that of a single spin (up to a factor of N), taking values

$$\begin{aligned} \langle S^x\rangle &= N(\alpha^*\beta + \alpha\beta^*), \\ \langle S^y\rangle &= N(-i\alpha^*\beta + i\alpha\beta^*), \\ \langle S^z\rangle &= N(|\alpha|^2 - |\beta|^2). \end{aligned} \tag{4}$$

Due to the above analogous properties of BEC qubits in relation to standard qubits, a two-component BEC in the class of states (1) hereafter will be called a "BEC qubit," or simply "qubit" for short.

Two-qubit gates can be formed by any product of the Schwinger boson operators of the form $H = \sum_{i,j=x,y,z}\hbar\Omega_{ij}S_1^i S_2^j$, where Ω_{ij} are real symmetric parameters. The combination of single-qubit gates and H_2 may be combined to form an arbitrary Hamiltonian involving

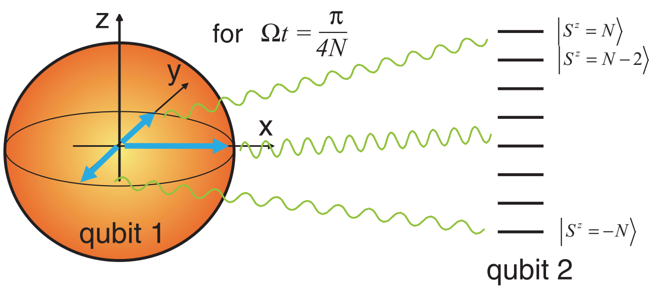

FIG. 1. (Color online) A schematic representation of the entangled state (7).

spin operators according to universality arguments [8]. By successive commutations an arbitrary product of spin Hamiltonians $H \propto \prod_{n=1}^{M} (S_n^j)^{m(n)}$ may be produced, where M is the total number of qubits, $j = x,y,z$, and $m(n) = 0,1$. Although in general higher-order operators may be constructed [e.g., $m(n) \geqslant 2$], our aim here is to perform the analogous operations to a standard qubit system using the BEC qubits. Since for Pauli operators $(\sigma^j)^2 = 1$, such higher-order operators are unnecessary for our purposes.

A key difference between Pauli operators and the Schwinger boson operators are that $\sigma^j \sim O(1)$, whereas $S^j \sim O(N)$. This makes the two-qubit interaction $H_2 \sim O(N^2)$. The consequence of the boosted energy scale of the interaction can be observed by examining explicitly the state evolution of two qubits. For simplicity, let us consider henceforth consider the interaction Hamiltonian

$$H_2 = \hbar\Omega S_1^z S_2^z. \tag{5}$$

This may be done without any loss of generality since an arbitrary two-qubit interaction can be converted to (5) by universality arguments. As a simple illustration, let us perform the analog of the maximally entangling operation

$$e^{-i\sigma_1^z\sigma_2^z\frac{\pi}{4}}(|\uparrow\rangle + |\downarrow\rangle)(|\uparrow\rangle + |\downarrow\rangle) = |+y\rangle|\uparrow\rangle + |-y\rangle|\downarrow\rangle, \tag{6}$$

where $|\pm y\rangle = e^{\mp i\frac{\pi}{4}}|\uparrow\rangle + e^{\pm i\frac{\pi}{4}}|\downarrow\rangle$. Starting from two unentangled qubits, we apply H_2 to obtain

$$e^{-i\Omega S_1^z S_2^z t}\left|\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right\rangle\!\!\right\rangle\left|\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right\rangle\!\!\right\rangle = \frac{1}{\sqrt{2^N}}\sum_{k_2}\sqrt{\binom{N}{k_2}}\left|\frac{e^{i(N-2k_2)\Omega t}}{\sqrt{2}},\frac{e^{-i(N-2k_2)\Omega t}}{\sqrt{2}}\right\rangle\!\!\right\rangle|k_2\rangle, \tag{7}$$

where we have introduced normalized eigenstates of the S^z operator $|k\rangle \equiv \frac{(a^\dagger)^k(b^\dagger)^{N-k}}{\sqrt{k!(N-k)!}}|0\rangle$. For gate times equal to $\Omega t = \pi/4N$ we obtain the analogous state to (6). For example, the maximum z eigenstates $|k_2 = 0,N\rangle$ on qubit 2 are entangled with the states $|\pm y\rangle\rangle \equiv |\frac{e^{\pm i\pi/4}}{\sqrt{2}},\frac{e^{\mp i\pi/4}}{\sqrt{2}}\rangle\rangle$, which is the analog of a Bell state for the bosonic qubits. A visualization of the state (7) is shown in Fig. 1. For each z eigenstate on qubit 2, there is a state $|\frac{e^{i(N-2k_2)\pi/4N}}{\sqrt{2}},\frac{e^{-i(N-2k_2)\pi/4N}}{\sqrt{2}}\rangle\rangle$ on qubit 1 represented on the Bloch sphere entangled with it. The type of entangled state is a continuous version of the original qubit state (6), and has similarities to continuous variable formulations of quantum computing [9], although the class of states that are used here are quite different.

We note here that the analog of the controlled-NOT (CNOT) gate can be produced by further evolving (7) with the Hamiltonian $H_2' = \hbar\Omega(NS_1^z - NS_2^z + N^2)$ for a time $\Omega t = \pi/4N$. For example, for initial states where qubit 1 (2) is in an x (z) eigenstate, we obtain

$$\begin{aligned} U_{\text{CNOT}}\left|\frac{1}{\sqrt{2}},\pm\frac{1}{\sqrt{2}}\right\rangle\!\!\right\rangle|0,1\rangle\rangle &= \left|\frac{1}{\sqrt{2}},\pm\frac{1}{\sqrt{2}}\right\rangle\!\!\right\rangle|0,1\rangle\rangle, \\ U_{\text{CNOT}}\left|\frac{1}{\sqrt{2}},\pm\frac{1}{\sqrt{2}}\right\rangle\!\!\right\rangle|1,0\rangle\rangle &= \left|\frac{1}{\sqrt{2}},\mp\frac{1}{\sqrt{2}}\right\rangle\!\!\right\rangle|1,0\rangle\rangle, \end{aligned} \tag{8}$$

which is exactly the same result as for standard qubits, where $U_{\text{CNOT}} = e^{-i(H_2+H_2')\pi/4N\Omega}$. However, due to the property of BEC qubits that $|\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\rangle\rangle \neq [|1,0\rangle\rangle + |0,1\rangle\rangle]/\sqrt{2}$, we cannot simply superpose (8) to obtain the intermediate cases. Nevertheless, the evolved states have similar properties to the standard qubit case. For example,

$$U_{\text{CNOT}}\left|\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right\rangle\!\!\right\rangle\left|\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right\rangle\!\!\right\rangle = \frac{1}{\sqrt{2^N}}\sum_{k_2}\sqrt{\binom{N}{k_2}}\left|\frac{e^{-i\pi k_2/N}}{\sqrt{2}},\frac{1}{\sqrt{2}}\right\rangle\!\!\right\rangle|k_2\rangle. \tag{9}$$

The correspondence to a CNOT operation may be seen by looking at the extremal states $|k_2 = 0,N\rangle$ states on qubit 2. These states are entangled with the states $|\pm\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\rangle\rangle$ on qubit 1 which is the same result as for a standard qubit CNOT operation with the target qubit in the x basis for qubit 1. As was the case for Fig. 1, there is a quasicontinuum of intermediate states between these extremal states. For BEC qubits it is the collective set of all the intermediate states that constitutes the entanglement between the two qubits.

The effect of the boosted energy scale of (5) is that a gate time of $\Omega t = \pi/4N$ was required to produce this entangled state, in comparison to the standard qubit case of $\Omega t = \pi/4$. The origin of the reduced gate time is due to the bosonic enhancement of the interaction Hamiltonian, originating from the boosted energy scale via bosonic enhancement of many particles occupying the same quantum state in the BEC. This allows for the possibility of fast two-qubit gates using such BEC qubits.

Given a qubit algorithm intended for standard two-level qubits, how does this translate in the bosonic system? For many applications, the procedure amounts to the following: (i) finding the sequence of Hamiltonians required for the algorithm, (ii) making the replacement $\sigma_n^j \rightarrow NS_n^j$, $\sigma_n^i\sigma_m^j \rightarrow S_n^i S_m^j$, and (iii) evolving the same sequence of Hamiltonians for a reduced time $t \rightarrow t/N$. This approach is reasonable from the point of view that we are performing the same algorithm except that a higher representation of SU(2) is being used. Let us illustrate this procedure by an implementation of Grover's algorithm on BEC qubits.

We use the continuous time formulation of the Grover search algorithm (see Sec. 6.2 of Ref. [10]). For the standard qubit case ($N = 1$), a Hamiltonian $H_G = |X\rangle\langle X| + |\text{ANS}\rangle\langle\text{ANS}|$ is applied to an initial state $|X\rangle$. Here, $|X\rangle$ is the $\sigma_n^x = 1$ eigenstate of all the qubits and $|\text{ANS}\rangle$ is the solution state. Under this Hamiltonian evolution, the system executes Rabi oscillations between $|X\rangle$ and $|\mathrm{ANS}\rangle$ with a period of $t = \pi\sqrt{2^M}$, where M is the number of qubits. The bosonic version of the Hamiltonian can be constructed by mapping the projection operators according to $|j\rangle\langle j| = (1+\sigma_n^j)/2 \to (1+S_n^j/N)/2$, where $j = x,z$, giving $H_G = N^2 \prod_{n=1}^M \frac{1}{2}[1 + \frac{S_n^x}{N}] + N^2 \prod_{n=1}^M \frac{1}{2}[1 + \frac{S_n^z}{N}]$. Here we assumed that the solution state is $\{\sigma_n^z = 1\}$ with no loss of generality. The bosonic qubits are prepared in the state $|X\rangle = \prod_{n=1}^M |\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\rangle\rangle_n$ and evolved in time by applying H. The system then executes Rabi oscillations between the initial state $|X\rangle$ and the solution state $|\mathrm{ANS}\rangle$.

The time required for reaching the solution state can be estimated from the period of the Rabi oscillations. For a system undergoing Rabi oscillations of a form $\langle S^z\rangle/N = \sin^2 \omega t$, the frequency can be estimated by $\omega = \sqrt{\frac{d^2\langle S^z(t=0)\rangle}{dt^2}/2N}$. Evaluating the second derivative in the Heisenberg picture gives $\frac{d^2\langle S_n^z/N\rangle}{dt^2} = -\langle[H_G,[H_G,S_n^z/N]]\rangle = 2N^2/2^M$, corresponding to a evolution time of $t \sim \sqrt{2^M}/N$. This has the same square root scaling with the number of sites, but with a further speedup of N, resulting from the fast gates made possible by the use of bosonic qubits. A numerical calculation for a simple three site case is shown in Fig. 2(b), which clearly shows a factor of N improvement in speed of the Grover algorithm. Even for small $N \sim 8$ the results converge quickly, giving almost indistinguishable results to the mean-field curve for the first Rabi oscillation.

The framework for quantum computation using BECs may be applied in principle to a variety of different systems, such as atomic or exciton-polariton BECs. For concreteness, we give the example of a specific configuration of using BECs on atom chips, following the experimental configuration given in Refs. [1–3]. Single-qubit rotations, initialization, and readout may be performed according to existing methods using microwave pulses as discussed in Refs. [1–3]. In these works, the $|F=1,m_F=-1\rangle$ and $|F=2,m_F=1\rangle$ hyperfine levels of the $5S_{1/2}$ ground state of ^{87}Rb are used as the qubit states. In terms of Fig. 3, we make the association for the operator $a^\dagger$ ($b^\dagger$) as creating an atom in the state $|F=1,m_F=-1\rangle$ ($|F=2,m_F=1\rangle$). Since the BEC contains a large number of atoms, there can be more than one atom present in a particular level, as illustrated in Fig. 3. Level c in Fig. 3 corresponds to a suitable higher-energy level satisfying optical selection rules determined by the polarization of the laser fields. Taking the $a \leftrightarrow c$ transitions to be σ^+ circularly polarized light, we make the association that the $c^\dagger$ operator creates an atom in the state $|F'=2,m_F'=0\rangle$ of the $5P_{3/2}$ state. The $b \leftrightarrow c$ transitions are then required to be σ^- polarized light, which connect $|F'=2,m_F'=0\rangle \leftrightarrow |F=2,m_F=1\rangle$.

Two-qubit gates may be implemented by using a quantum bus [11], which is implemented by connecting two BEC qubits via optical cavity quantum electrodynamics (QED), as shown in Fig. 3. Recent experimental advances have allowed for the possibility of incorporating cavity QED with atom chips [12]. For future implementations that may use microwave cavity QED [13], the same scheme can be used except with level c being the state $|F=2,m_F=0\rangle$. We closely follow the methods in Ref. [14] and generalize to the bosonic case. The

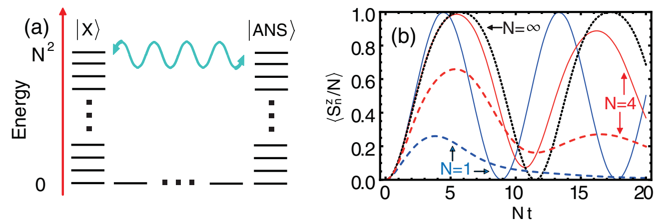

FIG. 2. (Color online) (a) Schematic energy-level structure of the Grover Hamiltonian. Rabi oscillations take place between the initial x eigenstate $|X\rangle$ and the solution state $|\mathrm{ANS}\rangle$. (b) Rabi oscillations executed by the Grover Hamiltonian for $M=3$ for various boson numbers as shown. The dotted line shows the mean-field result corresponding to the $N\to\infty$ limit, calculated by evolving the Heisenberg equations of motion for products of spin operators up to linear in S_n^i. The dashed lines show the Grover evolution including decoherence of the form given by (12) with $\Gamma_z = 0.2$. The solid lines show results with $\Gamma_z = 0$.

cavity QED Hamiltonian for the bosonic case is

$$H_{\mathrm{bus}} = \frac{\hbar\omega_0}{2}\sum_{n=1,2} F_n^z + \hbar\omega p^\dagger p + G\sum_{n=1,2}[F_n^- p^\dagger + F_n^+ p],$$

where $F^z = c^\dagger c - b^\dagger b$, $F^+ = c^\dagger b$, ω_0 is the transition frequency, and p is the photon annihilation operator. Assuming a large detuning $\Delta = \hbar\omega_0 - \hbar\omega \gg G$, we may adiabatically eliminate the photons from the bus and we obtain an effective Hamiltonian $H_{\mathrm{bus}} \approx \frac{G^2}{\Delta}(F_1^+ + F_2^+)(F_1^- + F_2^-) + \mathrm{H.c.}$ Now consider a further detuned single-qubit transition according to $H_{ac} = g\sum_{n=1,2}[c_n^\dagger a_n + \mathrm{H.c.}]$. After adiabatic elimination of level c, we obtain

$$H_{\mathrm{bus}}^{\mathrm{eff}} \approx \hbar\Omega_2^{\mathrm{eff}}(S_1^+ + S_2^+)(S_1^- + S_2^-) + \mathrm{H.c.}, \quad (10)$$

where $\hbar\Omega_2^{\mathrm{eff}} = \frac{G^2g^2}{\Delta^3}$. Although this interaction involves undesired single-qubit interaction terms $S^+S^- + S^-S^+ = -(S^z)^2/2 + \mathrm{const}$, these may be eliminated and converted to the form (5) by combining with single-qubit gates using universality arguments [8].

Finally, we consider decoherence effects due to the use of BEC qubits. Let us first consider the general effects of dephasing and particle loss which are most likely common to any experimental implementation. Consider the simplest case

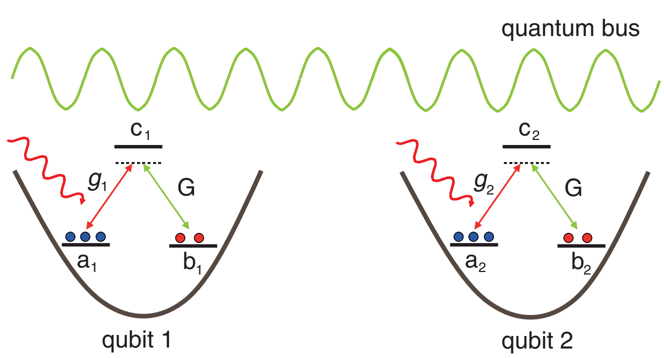

FIG. 3. (Color online) Two bosonic qubits mediated by a quantum bus. The quantum bus couples transitions between levels b and c with energy G. Individual pulses coupling levels a and c with energy g create an adiabatic passage between levels a and b.

when a quantum state is stored in the system of qubits and no gates are applied, i.e., when the BEC qubits are used to simply store a state. The main channels of decoherence in this case are dephasing and particle loss. Considering dephasing first, we model this via the master equation

$$\frac{d\rho}{dt} = -\frac{\Gamma_z}{2}\sum_{n=1}^{M}\left[\left(S_n^z\right)^2\rho - 2S_n^z\rho S_n^z + \rho(S_n^z)^2\right], \qquad (11)$$

where Γ_z is the dephasing rate. For a standard qubit register, the information in a general quantum state can be reconstructed by $4^M - 1$ expectation values of $(I_1, S_1^x, S_1^y, S_1^z) \otimes (I_2, S_2^x, S_2^y, S_2^z) \cdots \otimes (I_M, S_M^x, S_M^y, S_M^z)$ [15]. For the bosonic system, there are in general higher-order correlations involving powers of operators beyond order one, but these are unnecessary for our purposes, as previously discussed. Examining the dephasing of the general correlation $\langle \prod_n S_n^{j(n)} \rangle$ where $j(n) = I, x, y, z$, we obtain the evolution equation $d\langle \prod_n S_n^{j(n)} \rangle / dt = -2\Gamma_z K_z \langle \prod_n S_n^{j(n)} \rangle$, which can be solved to give

$$\left\langle \prod_n S_n^{j(n)} \right\rangle \propto \exp[-2\Gamma_z K_z t]. \qquad (12)$$

Here K_z is the number of noncommuting $S_n^{j(n)}$ operators with S_n^z [i.e., $j(n) = x, y$], which is independent of N and is at most equal to M. The crucial aspect to note here is that the above equation does not have any N dependence. In fact, the equation is identical to that for the standard qubit case ($N = 1$). Physically this difference is due to the statistical independence of the dephasing processes among the bosons. Similar arguments can be made for particle loss, where we find the same N-independent exponential decay as (12). The general result of (12) shows that decoherence is not enhanced by the use of BEC qubits when they are used to store a quantum state. For an implementation using atom chip BECs, the dephasing time h/Γ_z has been estimated to be on the order of seconds [3], a comparable time with other systems proposed for quantum computation [16].

In Fig. 2(b) we show results for evolution of the Grover Hamiltonian including decoherence. We see that the fidelity improves with increasing N, which is a result of the faster evolution times due to bosonic enhancement, while the decoherence rate remains the same, as suggested by (12). All results therefore approach the mean-field curve in the limit $N \to \infty$. Number fluctuations can also degrade the fidelity of the algorithm, although only linearly with the amount of fluctuations, and therefore should be tolerable for reasonable experimental parameters [17].

We have found that two-component BECs can form viable qubits that may be used for quantum computing. Unlike previous proposals [5] where two quantum states of the BEC are used to encode the quantum information, here we use the collective states (1) of the $(N+1)$-dimensional Hilbert space. The theory discussed here is similar to a continuous variables [9] formulation of qubit computing applied to BEC qubits, although the specific details of the encoding are rather different. Our scheme differs from standard continuous variables approaches [18] in that no spin squeezing is necessary to encode quantum information on the qubits. One aspect which we have not discussed is the bosonic mapping procedure for applications that use nonunitary operations such as measurements as part of the algorithm, such as quantum teleportation. We leave such topics as future work.

T.B. thanks R. Schmied, P. van Loock, and P. Turner for discussions. This work is supported by Navy/SPAWAR Grant No. N66001-09-1-2024, and the JSPS through its FIRST program.

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