Impact of dispersion and non-Kerr nonlinearity on the modulational instability of the higher-order nonlinear Schrödinger equation

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(Received 24 January 2012; published 19 March 2012)

We have studied the modulational instability (MI) of the higher-order nonlinear Schrödinger (HNLS) equation with non-Kerr nonlinearities in an optical context and presented an analytical expression for MI gain to show the effects of non-Kerr nonlinearities and higher-order dispersions on MI gain spectra. In our study, we demonstrate that MI can exist not only for the anomalous group-velocity dispersion (GVD) regime, but also in the normal GVD regime in the HNLS equation in the presence of non-Kerr quintic nonlinearities. The non-Kerr quintic nonlinear effect reduces the maximum value of the MI gain and bandwidth and plays a sensitive role over the Kerr nonlinearity, which leads to continuous wave breaking into a number of stable wave trains of ultrashort optical pulses that can be used to generate the stable supercontinuum white-light coherent sources.

DOI: 10.1103/PhysRevA.85.033820 PACS number(s): 42.65.Tg, 05.45.Yv, 42.79.Sz, 42.81.Dp

I. INTRODUCTION

The interplay between dispersion and nonlinearity gives rise to several important phenomena in optical fibers, including parametric amplification, wavelength conversion, modulational instability (MI), soliton propagation, and supercontinuum generation [1]. MI is a fundamental and ubiquitous nonlinear phenomenon that pertains to a large variety of subfields of physics, such as fluid dynamics [2], plasma physics [3], atomic physics [atomic Bose-Einstein condensates (BECs)], and nonlinear optics [4]. MI is a process in which the amplitude and phase modulations of a wave grow as a result of an interplay between the nonlinearity and dispersion. It refers to the exponential growth of weak perturbations through the amplification of sideband frequencies and is closely associated with the concept of self-localized waves, or solitons. In all of the above cited fields, MI is one of the principal mechanisms which leads to the emergence of localized coherent nonlinear structures and the formation of a train of ultrashort pulses. MI was introduced in an optical context in 1980 [5] and experimentally verified by Tai *et al.* [6] in 1986, which showed that MI only occurs in the anomalous GVD regime for positive cubic nonlinearity. In optical fibers, MI occurs due to an interplay between nonlinear and dispersive effects, and manifests itself as the breakup of continuous wave (cw) or quasi-cw into a train of ultrashort pulses. Although the phenomena of MI has some detrimental effect on beam propagation, it has been shown that MI in an optical fiber can be exploited in an optical pulse source with high repetition rates that are not achievable by active mode locking. In other words, MI can be exploited as a passive mode-locked mechanism in a fiber laser [7]. MI has been well analyzed in an optical context with a view to applications in the generation of train of ultrashort pulses with Terahertz repetition rates. In this context, Pitois *et al.* [8] experimentally investigated the influence of fourth-order dispersion (FOD) in the MI study in a single-mode optical fiber and demonstrated evidence of a new MI spectral window due to the FOD effect in the normal-dispersion regime, and also showed that the FOD-induced MI can be used for broadband wavelength conversion. It also has been shown [9] that MI is able to generate extremely broad spectra. Demircan *et al.* [10] showed that MI is responsible for the generation of ultrabroadband octave-spanning continua for pico- and subpicosecond pulses in the anomalous-dispersion as well as in the normal-dispersion region in the context of the extended nonlinear Schrödinger (NLS) equation with cubic nonlinearities. MI is once again the subject of significant interest as a mechanism to describe the emergence of strongly localized rogue wave structures in hydrodynamics and optics. In this context, Peregrine [11] identified the key role of the modulational instability in the formation of patterns resembling freak waves or a rogue wave. Recently, Dinda and Porsezian [12] analyzed the MI of light waves in glass fibers with a local saturable nonlinear refractive index in the presence of the fourth order of the fiber dispersion [10,13,14].

In this paper, we consider that the higher-order NLS (HNLS) equation with non-Kerr nonlinear terms can be written in terms of a slowly varying complex envelope of the electric field $E(z,t)$ as

$$E_z = i(\beta_2 E_{tt} + \gamma_1 |E|^2 E) + \beta_3 E_{ttt} + i\beta_4 E_{tttt} + \alpha_1(|E|^2 E)_t + \alpha_2 E(|E|^2)_t + i\gamma_2 |E|^4 E + \alpha_3(|E|^4 E)_t + \alpha_4 E(|E|^4)_t. \quad (1)$$

Here, z is the normalized distance along the fiber and t is the normalized time with the frame of the reference moving along the fiber at the group velocity. The subscripts z and t denote the spatial and temporal partial derivatives, respectively. The coefficients β_i ($i = 2, 3, 4$) are the real parameters related to group-velocity dispersion (GVD), third-order dispersion (TOD), and fourth-order dispersion (FOD), respectively. γ_1, α_1, and α_2 are related to self-phase modulation (SPM), self steepening, and self-frequency shift due to stimulated Raman scattering (SRS). The terms related to the coefficients γ_2, α_3, and α_4 in Eq. (1) represent the quintic non-Kerr nonlinearity. The quintic nonlinearities arise from the expansion of the refractive index in powers of intensity I of the light pulse: $n = n_0 + n_2 I + n_4 I^2 + \cdots$. Here, n_0 is the linear refractive index coefficient, and n_2 and n_4 are the nonlinear refractive

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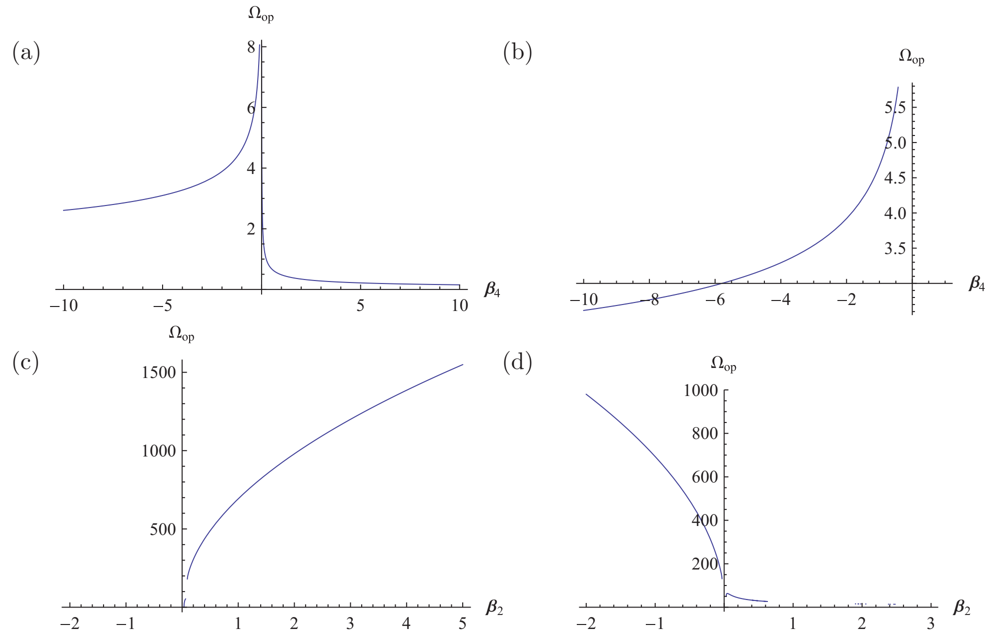

FIG. 1. (Color online) Variation of optimum modulation frequency (Ω_{op}) (a) $\beta_2 = \frac{1}{2}$ ps^2/km > 0, (b) $\beta_2 = -\frac{1}{2}$ ps^2/km < 0 as a function of β_4, and (c) $\beta_4 = \frac{5}{24} \times 10^{-5}$ ps^4/km > 0, (d) $\beta_4 = -\frac{5}{24} \times 10^{-5}$ ps^4/km < 0 as a function of β_2. The other parameter values used here are $P_0 = 15$ W, $\gamma_1 = 1$ W^{-1}/km, $\gamma_2 = 1$ W^{-2}/km, $\alpha_1 = -0.0247$ W^{-1}/[(2π) km THz], $\alpha_2 = 0.03705$ W^{-1} fs/km, $\alpha_3 = -0.0247$ W^{-2}/[(2π) km THz], and $\alpha_4 = 0.030875$ W^{-2} fs/km.

index coefficients, which originate from third- and fifth-order susceptibility. The polarizations induced through these susceptibilities give the cubic and quintic (non-Kerr) terms in the NLS equation, respectively. The nonlinearity arising due to fifth-order susceptibility can be obtained in many optical materials, such as semiconductors, semiconductor doped glasses, polydiacctylene toluene sulfonate (PTS), calcogenide glasses, and some transparent organic materials. When the terms related to β_4, γ_2, α_3, and α_4 of Eq. (1) are ignored, the resulting equation becomes the higher-order nonlinear Schrödinger (HNLS) equation, which includes shock and Raman terms. This equation is well analyzed by many authors from different points of view (e.g., Painlevé property, inverse scattering transform, Hirota direct method, and conservation laws) [15]. Special attention is also given to study the MI of the HNLS equation by many authors [16]. But the non-Kerr nonlinear terms in Eq. (1) become important when one increases the intensity of the incident light power to produce shorter (beyond femtosecond) pulses, in which we can see day-by-day progress in the high repetition rate beyond ultrashort (autosecond) pulse sources based on fiber technology [17]. Equation (1) is very important in order to adapt to the current progress in high-repetition-rate (beyond ultrashort, even autosecond) optical systems. Recently, microstructured photonic crystal fibers (PCFs) have attracted significant attention since they provide extra degrees of freedom for the manipulation of mode propagation. PCFs are made up of a pure silica core surrounded by an array of microscopic air holes running along their entire length. The large refractive-index step between silica and air allows light to be concentrated into a very small area, resulting in enhanced nonlinear effects. To explain the optical pulse communication in highly nonlinear photonic crystal fiber, one has to extend the order of nonlinear terms in the standard nonlinear Schrödinger equation beyond Kerr nonlinearity. In our recent works [18], we have investigated the dark-in-the-bright (DITB) solitary wave solution of Eq. (1) without fourth-order dispersion. The quintic non-Kerr nonlinear terms are important [19] over the cubic Kerr nonlinearity because non-Kerr nonlinearities are responsible for the stability of localized solutions. In this study, we shall investigate the MI gain of the standard generalized NLS equation, which contains higher-order dispersion, self-steepening, and Raman terms, as well as higher-order non-Kerr nonlinearities. The non-Kerr quintic nonlinear terms in contemporary optics are very important to the upcoming applications in ultrafast signal routing systems, double doped optical fiber, optical switching, etc.

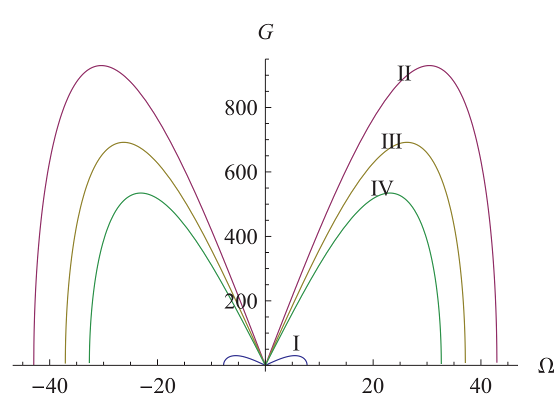

FIG. 2. (Color online) Variation of the MI gain G (km^{-1}) spectrum given by Eq. (10) for the initial power $P_0 = 15$ W. Here the parameter values are the same as in Fig. 1.

II. LINEAR STABILITY ANALYSIS AND MODULATIONAL GAIN

In order to study the modulational instability of Eq. (1) we use the standard linear stability analysis. For this, we consider the propagation of the cw or quasi-cw signal of the initial power P_0 inside a long fiber. The steady-state solution of Eq. (1), corresponding to the cw signal, can be written as

$$E(z,t) = \sqrt{P_0}e^{-i\Phi_{\rm NL}z}, \tag{2}$$

and Eq. (2) shows that the cw signal is amplified exponentially and acquires a nonlinear phase shift, $\phi_{\rm NL} = (\gamma_1 P_0 + \gamma_2 P_0^2)$, induced by self-phase modulation (SPM) and a non-Kerr quintic nonlinear term. To study the MI of the cw wave solution, we introduce the perturbed field of the form

$$E(z,t) = [\sqrt{P_0} + a(z,t)]e^{-i\Phi_{\rm NL}z}, \tag{3}$$

where the complex field $|a(z,t)| \ll \sqrt{P_0}$. If the perturbed field grows exponentially, then the steady state becomes unstable. Substituting Eq. (3) in Eq. (1), we obtain the linearized equation of the perturbed field,

$$\begin{aligned} &a_z + i\beta_2 a_{tt} + \beta_3 a_{ttt} + i\beta_4 a_{tttt} + i\gamma_1 P_0(a + a^*) \\ &\quad + 2i\gamma_2 P_0^2(a + a^*) + \alpha_1 P_0(2a_t + a_t^*) + \alpha_2 P_0(a_t + a_t^*) \\ &\quad + \alpha_3 P_0(3a_t + 2a_t^*) + 2\alpha_4 P_0(a_t + a_t^*). \end{aligned} \tag{4}$$

Here, $a^*(z,t)$ is the complex conjugate of the perturbed field. Now we assume the following ansatz for the perturbed field:

$$a(z,t) = U(z)e^{-i\Omega t} + V(z)e^{i\Omega t}, \tag{5}$$

where $U(z)$ and $V(z)$ are the complex perturbation fields and Ω is the modulation frequency. By substituting Eq. (5) into the linearized Eq. (4), we obtain the equations of the perturbed fields in matrix form

$$i\frac{d}{dz}\begin{pmatrix} U(z) \\ V^*(z) \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix}\begin{pmatrix} U(z) \\ V^*(z) \end{pmatrix}. \tag{6}$$

In Eq. (6), $V^*(z)$ is the complex conjugate of the field $V(z)$, and

$$\begin{aligned} T_{11} = &P_0(\gamma_1 + 2\gamma_2 P_0) - P_0(2\alpha_1 + \alpha_2 + 3\alpha_3 P_0 + 2\alpha_4 P_0)\Omega \\ &- \beta_2\Omega^2 + \beta_3\Omega^3 + \beta_4\Omega^4), \end{aligned} \tag{7a}$$

$$T_{12} = P_0[\gamma_1 + 2\gamma_2 P_0 - (\alpha_1 + \alpha_2 + 2\alpha_3 P_0 + 2\alpha_4 P_0)]\Omega, \tag{7b}$$

$$T_{21} = -P_0[\gamma_1 + 2\gamma_2 P_0 + (\alpha_1 + \alpha_2 + 2\alpha_3 P_0 + 2\alpha_4 P_0)]\Omega, \tag{7c}$$

and

$$\begin{aligned} T_{22} = &-[P_0(\gamma_1 + 2\gamma_2 P_0) + P_0(2\alpha_1 + \alpha_2 \\ &+ 3\alpha_3 P_0 + 2\alpha_4 P_0)\Omega - \beta_2\Omega^2 - \beta_3\Omega^3 + \beta_4\Omega^4]. \end{aligned} \tag{7d}$$

The eigenvalues of the matrix $T = \left(\begin{smallmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{smallmatrix}\right)$ determine the wave number K of the perturbed wave and can be calculated through the characteristics equation $\mathrm{Det}|T - KI| = 0$, where I is a 2×2 unit matrix. The eigenvalues can be evaluated by the dispersion relation determined by the characteristics equation, and is given by

$$K = \tfrac{1}{2}[T_{11} + T_{22} + \sqrt{(T_{11} - T_{22})^2 + 4T_{12}T_{21}}]. \tag{8}$$

MI occurs when the wave number possesses a nonzero imaginary part leading to an exponential growth of the perturbed amplitude. The MI is measured by power gain, and it is defined at any pump frequency Ω as

$$G(\Omega) = 2\mathrm{Im}(K), \tag{9}$$

where $\mathrm{Im}(K)$ represents the imaginary part of K and is given by

$$G(\Omega) = 2\Omega\sqrt{-\left[P_0 M_1 + \left(\beta_2^2 + 2\beta_4 P_0 m_2\right)\Omega^2 - 2\beta_2\beta_4\Omega^4 + \beta_4^2\Omega^6\right]}, \tag{10}$$

where $M_1 = (P_0 m_1^2 - 2\beta_2 m_2)$, $m_1 = [\alpha_1 + \alpha_2 + 2(\alpha_3 + \alpha_4)P_0]$, and $m_2 = (\gamma_1 + 2\gamma_2 P_0)$. The gain attains its peak values when the modulated frequency reaches its optimum value, i.e., its optimum modulation frequency (OMF). The OMF corresponding to the gain spectrum (10) is given by

$$\Omega_{\rm op} = \pm\sqrt{\frac{\beta_2}{2\beta_4} + \frac{2^{\frac{1}{3}}\beta_2^2}{\mathcal{E}^{\frac{1}{3}}} - \frac{4\left(2^{\frac{1}{3}}\right)\beta_4 m_2 P_0}{\mathcal{E}^{\frac{1}{3}}} + \frac{\mathcal{E}^{\frac{1}{3}}}{12\left(2^{\frac{1}{3}}\right)\beta_4^2}}, \tag{11a}$$

where

$$\mathcal{E} = \left(\mathcal{E}_1 + \sqrt{4\mathcal{E}_2^3 + \mathcal{E}_1^2}\right), \tag{11b}$$

with $\mathcal{E}_1 = -432(\beta_4^4 M_1 P_0 + 2\beta_2\beta_4^4 m_2 P_0)$ and $\mathcal{E}_2 = 12(-\beta_2^2\beta_4^2 + 4\beta_4^3 m_2 P_0)$. The MI gain can reach the peak value when $\Omega = \Omega_{\rm op}$.

In Fig. 1, we have shown the variation of optimum modulation frequency (OMF), computed from Eq. (11a) as a function of the GVD (β_2) and/or FOD (β_4) parameters. The parameter values we have used are given in the figure caption.

A. Nonlinear effects on MI gain

Higher-degree non-Kerr nonlinear terms have major influences on the MI gain. We have shown in Fig. 2 four MI gain spectra as a function of the modulation frequency for a particular value of the initial power (P_0) to show the effect of non-Kerr nonlinearity. The gain spectra shown in Fig. 2 denoted by I, II, III, and IV are so divided depending on the terms involved in Eq. (1). Gain spectrum I, in Fig. 2, shows the MI sidebands for the standard NLS equation (for parameter values $\beta_2 = \frac{1}{2}$ ps^2/km, $\gamma_1 = 1$ W^{-1}/km). Gain spectrum II shows the MI spectra for the cubic-quintic nonlinear Schrödinger (NLS) equation with FOD and stimulated Raman scattering

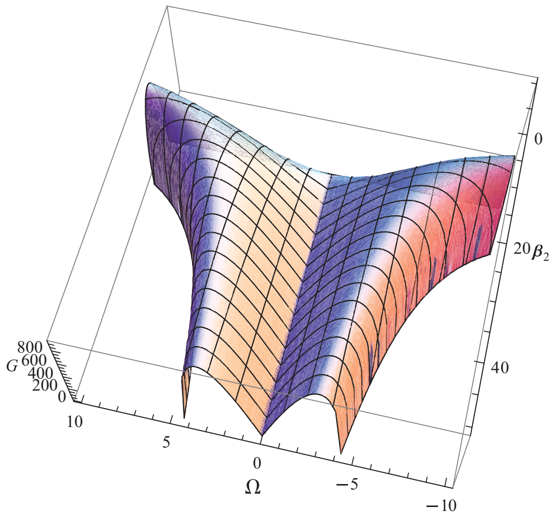

FIG. 3. (Color online) MI gain G (km^{-1}) as a function of β_2 (in ps^2/km) and Ω [in (2π) THz] for fixed power $P_0 = 15$ W and $\beta_4 = -\frac{5}{24} \times 10^{-5}$ ps^4/km < 0. The other parameters are the same as in Fig. 1.

(SRS) terms. Here we have used only the parameters $\beta_2 = \frac{1}{2}$ ps^2/km, $\gamma_1 = 1$ W^{-1}/km, $\gamma_2 = 1$ W^{-2}/km, $\beta_4 = -\frac{5}{24} \times 10^{-5}$ ps^4/km, $\alpha_1 = -0.0247$ W^{-1}/[(2π) km THz], and $\alpha_2 = 0.03705$ W^{-1} (fs/km). For the MI gain (III) in Fig. 1, we have included an extra term related to the quintic nonlinearity, $\alpha_3 = -0.0247$ W^{-2}/[(2π) km THz], and other parameter values are similar to that in Fig. 1 for gain spectrum II. It is clear from Fig. 2 that due to the quintic nonlinear term related to α_3, the peak of the gain spectrum decreases for II to III. In gain spectrum IV, we have shown the effect of the nonlinear term related to α_4 in Eq. (1). Here we have taken the value of $\alpha_3 = 0$ and $\alpha_4 = 0.030875$ W^{-2} (fs/km), and other parameter values are similar to that in Fig. 1 for gain spectrum II. The effect of non-Kerr nonlinearities related to α_3 and α_4 in Eq. (1) on MI gain are very crucial and decrease the MI gain peak value,

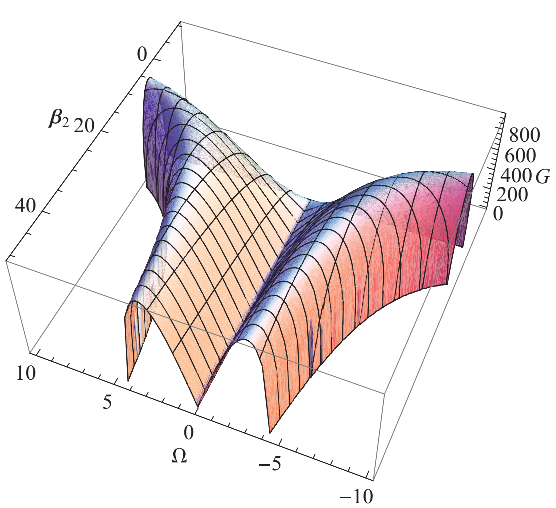

FIG. 4. (Color online) MI gain G (km^{-1}) as a function of β_2 (in ps^2/km) and Ω [in (2π) THz] for fixed power $P_0 = 15$ W and $\beta_4 = \frac{5}{24} \times 10^{-5}$ ps^4/km > 0. The other parameters are the same as in Fig. 1.

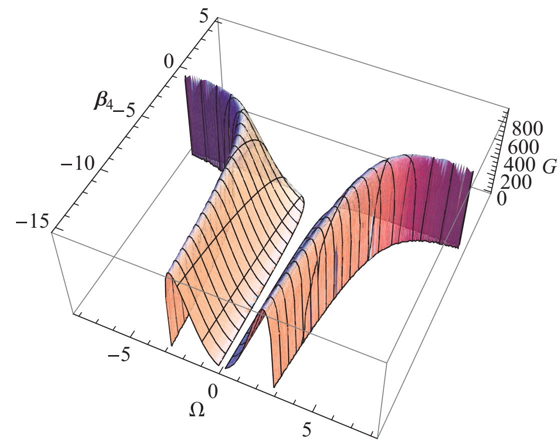

FIG. 5. (Color online) MI gain G (km^{-1}) spectrum as a function of β_4 (in ps^4/km) and Ω [in (2π) THz] for fixed power $P_0 = 15$ and $\beta_2 = -\frac{1}{2}$ ps^2/km The other parameters are the same as in Fig. 1.

indicating the greater stability of the system and stable soliton pulse of Eq. (1). In the above study regarding MI gain, we have taken the initial power $P_0 = 15$ W.

B. Dispersion effects on MI gain

Now we will see the impact of dispersion on the MI gain. It is clear from the expression of the MI gain given in Eq. (10) that the third-order dispersion term has no effect on the MI condition, but GVD and FOD terms have an influence on the MI condition. Figures 3 and 4 show the MI gains as a function of β_2 and Ω for $\beta_4 = -\frac{5}{24} \times 10^{-5}$ ps^4/km and $\beta_4 = \frac{5}{24} \times 10^{-5}$ ps^4/km, respectively, with a fixed initial power $P_0 = 15$ W. In Figs. 3 and 4, we obtain two similar symmetric side lobes due to MI, which appear only on the anomalous GVD regime regardless of the sign of the FOD parameter β_4, and the side lobes vanish in the normal GVD regime $\beta_2 < 0$ in the presence of non-Kerr nonlinearities. On the other hand, Figs. 5 and 6 display the MI gains as a function of β_4 and the modulation frequency Ω for a fixed initial power $P_0 = 15$ W and constant β_2 value. Here MI gains depend

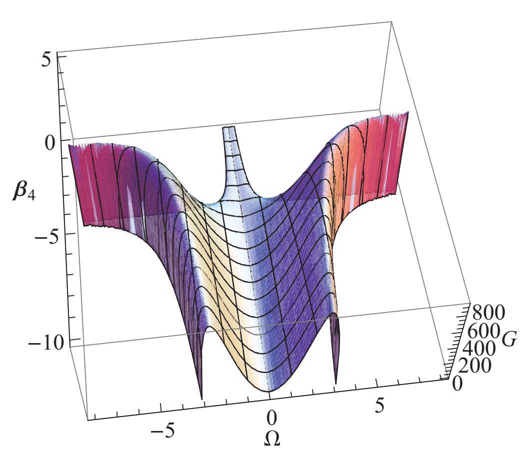

FIG. 6. (Color online) MI gain G (km^{-1}) spectrum as a function of β_4 (in ps^4/km) and Ω [in (2π) THz] for fixed power $P_0 = 15$ W and $\beta_2 = \frac{1}{2}$ ps^2/km. The other parameters are the same as in Fig. 1.

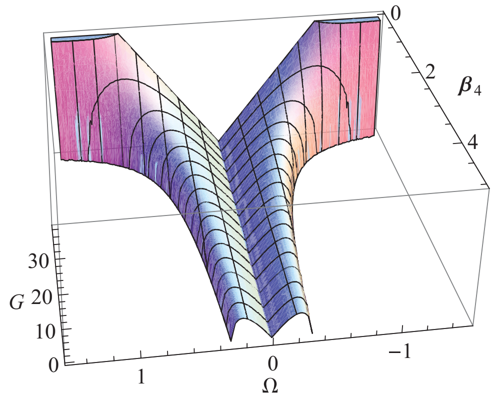

FIG. 7. (Color online) MI gain G (km^{-1}) spectrum as a function of β_4 (in ps^4/km) and Ω [in (2π) THz] for fixed power $P_0 = 15$ W and $\beta_2 = \frac{1}{2}$ ps^2/km. Here $\beta_4 > 0$. The other parameters are the same as in Fig. 1.

crucially on the sign of the GVD parameter (β_2). We observe two symmetrical side lobes in Fig. 5 for $\beta_2 = -\frac{1}{2}$ ps^2/km, but the two lobes vanish here when $\beta_4 > 0$. We also observe that the two side lobes are separate from each other, as opposed to Figs. 3 and 4. Figure 6 displays a more interesting feature. Here we can see that for $\beta_2 = \frac{1}{2}$ ps^2/km, two MI gain sidebands exist regardless of the sign of β_4, and, consequently, the product $\beta_2\beta_4$. In Fig. 7, we show the enlarged picture of MI gain for only the positive values of the FOD parameter, as already shown in Fig. 6. We observe that the maximum gain decreases as β_4 goes to positive values compared to the negative FOD parameter as in Fig. 6. In contrast to Fig. 5, here the two side lobes are attached to each other.

III. CONCLUSION

MI by which a plane wave breaks up into high filament intensity is a ubiquitous phenomenon that occurs in many branches of physics [2–5]. It is a symmetry-breaking instability so that a small perturbation on top of a constant amplitude background experiences exponential growth, and this leads to wave breakup in either space or time. When the cw breakup evolves into a train of subpicosecond, femtosecond, and even autosecond pulses, the higher-order dispersion and the higher nonlinear effects strongly affect the pulse evolution in the optical fibers. We have studied the very interesting MI phenomena of Eq. (1) in the context of optics. We mainly studied the effect of non-Kerr nonlinearities on the MI gain spectra in the presence and absence of FOD. Our study clearly shows that the non-Kerr quintic nonlinear effect reduces the maximum value of the gain and bandwidth. Besides that, in particular, we also have studied the effect of GVD and FOD on MI gain in the presence of non-Kerr nonlinearities. Our investigation shows the existence of sidebands due to MI gain for the negative FOD term in a normal ($\beta_2 < 0$) as well as anomalous dispersion ($\beta_2 > 0$) regime. But, for positive FOD, the sideband gain spectra by MI appears only for the anomalous dispersion ($\beta_2 > 0$) regime. As reported in many other works, in our present study we also find that the third-order dispersion effect does not contribute to MI, and we show that MI can exist not only for an anomalous GVD regime, but also in the normal GVD regime in the HNLS equation in the presence of higher-order dispersion and non-Kerr quintic nonlinearities, as opposed to the standard NLS equation in which MI gain occurs only in the anomalous dispersion regime. We have seen that in both normal and anomalous GVD regimes, strong MI sidebands occur; as a result, when the perturbation becomes large enough, the linear stability of the cw breaks down in terms of a train of short-pulse solitary waves and the evolution of the modulated state is then governed by the HNLS equation given in Eq. (1). We have investigated that the quintic nonlinear effects are more sensitive than the Kerr-nonlinearity on cw, which leads to cw breaking into a stable wave train generation of ultrashort optical pulses. As MI has great application in soliton generation in the fiber [6,14], physically, the equation studied by us is very important and will be useful in modeling ultrashort (beyond femtosecond, and even autosecond) optical pulse propagation through a highly nonlinear medium because the study of MI shows that the quintic non-Kerr nonlinear terms are important over the cubic Kerr nonlinearity for the stability of localized coherent nonlinear structures and the formation of a wave train of ultrashort pulses with a greatly enhanced spectral bandwidth as well as the generation of localized temporal solitons giving rise to stable supercontinuum white-light coherent sources.

ACKNOWLEDGEMENT

K.P. acknowledges DST, DAE-BRNS, UGC, the Government of India, for financial support through major projects.

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