

Using quantum state protection via dissipation in a quantum-dot molecule to solve the Deutsch problem

M. M. Santos, F. O. Prado, H. S. Borges, A. M. Alcalde, J. M. Villas-Bôas, and E. I. Duzzioni
Universidade Federal de Uberlândia, Caixa Postal 593, 38400-902 Uberlândia, MG, Brazil

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The wide set of control parameters and reduced size scale make semiconductor quantum dots attractive candidates to implement solid-state quantum computation. Considering an asymmetric double quantum dot coupled by tunneling, we combine the action of a laser field and the spontaneous emission of the excitonic state to protect an arbitrary superposition state of the indirect exciton and ground state. As a by-product we show how to use the protected state to solve the Deutsch problem.

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I. INTRODUCTION

One important feature of quantum mechanics is the superposition principle. This fundamental characteristic for pure states can be summarized by the quantum entanglement phenomenon [1,2]. It is believed that such a property can give to the quantum world the advantage of processing information in a more efficient way than its classical counterpart [3]. However, in a realistic scenario, decoherence destroys the superposition of states leading the system state to a statistical mixture [4]. A number of methods has been proposed to circumvent this difficulty; among them we highlight the quantum error-correcting codes [5], decoherence-free subspaces in collective systems [6], and dynamical decoupling methods [7]. Differently from the methods cited above, the engineering reservoir technique [8] makes use of incoherent control of Markovian reservoirs to drive the system state to a desired superposition of pure states in the asymptotic limit [9]. Therefore, the effective interaction between the system and reservoir must be carefully engineered to drive the system to equilibrium with the reservoir. Recently this theme received a great deal of attention in order to obtain pure entangled protected states [9]. Besides the recent generalization of the engineering reservoir technique to protect nonstationary superposition states [10–12], two important applications of this technique have emerged: (i) the construction of robust quantum memories [13] and (ii) the implementation of quantum computation via dissipation (QCD) [14]. In the former case the quantum information can be stored for long times at the nodes of a quantum network, while in the later case the quantum processor is insensitive to external perturbations or decoherence. Particularly, in Ref. [14] it was shown that QCD is universal and can efficiently simulate a given quantum circuit.

One of the clearest demonstration of the power of quantum processing is the quantum solution for the Deutsch problem [15]. Later improved by Deutsch and Jozsa [16] and by Collins, Kim, and Holton [17], the optimized version of the Deutsch algorithm is able to decide whether a binary function is constant or balanced with a single measurement, while in the classical case two measurements are required. The Deutsch problem was proved using different experimental setups, such as nuclear magnetic resonance [18,19], optical systems [20], circuit quantum electrodynamics architectures [21], trapped ions [22], the nitrogen-vacancy defect center [23], and quantum dots [24].

In this work we investigate the protection of superposition states in a physical system composed by two asymmetric quantum dots coupled by tunneling under the application of laser fields, where the interplay of the optical field and the tunneling creates a structure of two excitonic levels: direct and indirect, as described in Ref. [25]. The results obtained here generalize the result obtained in Ref. [26], where a robust state of the indirect exciton was found. Through the protection of an arbitrary coherent superposition of the ground and excited (indirect exciton) states, we propose the implementation of the optimized version of the Deutsch algorithm [17] using QCD. In general, the proposals for implementing quantum computing protocols make use of coherent control of quantum systems. In contrast, our proposal to solve the Deutsch problem has the advantage of being controlled incoherently by the reservoir, becoming immune to decoherence processes.

The manuscript is divided as follows. In Sec. II we obtain analytically the protected asymptotic state of the system which is parametrized on the Bloch sphere. Such a state is used to solve the Deutsch problem in Sec. III. Conclusions and perspectives for this work are presented in Sec. IV. In the Appendix we deduce the effective master equation leading to the protected state.

II. PROTECTED PURE STATE

We consider two asymmetric quantum dots coupled by tunneling, which under the application of a static electric field along the growth direction can be simplified to a three-level system [25]: the ground state $|0\rangle$ (no excitation on the system), the exciton state $|1\rangle$ (one electron-hole pair in the same dot), and the indirect exciton state $|2\rangle$ (the electron in one dot and the hole in another). The exciton state can be created from the ground state by the application of a laser field with the right frequency and the indirect exciton from the exciton state by tunneling of one electron, as experimentally demonstrated in Ref. [27]. In this way, the Hamiltonian in the rotating wave approximation for this system is ($\hbar = 1$)

$$H(t) = \sum_{j=0}^2 \omega_j |j\rangle\langle j| + T_e(|1\rangle\langle 2| + |2\rangle\langle 1|) + \Omega[e^{i(\varphi+\omega_L t)}|0\rangle\langle 1| + e^{-i(\varphi+\omega_L t)}|1\rangle\langle 0|], \quad (1)$$

where ω_j is the energy of the j th level and T_e is the tunneling coupling between the levels $|1\rangle$ and $|2\rangle$. We tune the gate voltage to have $\omega_1 = \omega_2 = \omega_L$ and $\omega_0 = 0$. ω_L and φ are the frequency and phase of the laser field, respectively. $\Omega = \langle 0|\vec{\mu} \cdot \vec{E}|1\rangle$ is the dipole coupling between the excitonic transition, with $\vec{\mu}$ being the electric dipole moment and \vec{E} the incident electric field of the laser [28]. According to Ref. [26], the dominant decoherence processes can effectively be described by spontaneous emission from states $|1\rangle$ to $|0\rangle$ and $|2\rangle$ to $|0\rangle$ with decay rates Γ_1 and Γ_2 , respectively. This approximation remains valid if we consider that (i) the system is at very low temperatures so that the pure dephasing induced by acoustic phonons does not modify significantly the system dynamics [29]; (ii) the creation of the lower energy exciton state is made by the application of a resonant laser field, which inhibits transitions mediated by phonons [30,31]; (iii) the optical phonon effects are negligible because they have quite different frequencies (~ 30 meV) compared with that of our system (see the main text) [29]. Then, the dynamics of the system can be described by the master equation

$$\frac{\partial \rho(t)}{\partial t} = -i[H(t), \rho] + \mathcal{L}(\rho), \quad (2)$$

with $H(t)$ being the Hamiltonian defined in Eq. (1) and $\mathcal{L}(\rho)$ the dissipative Liouvillian, given by

$$\mathcal{L}(\rho) = \mathcal{L}_1(\rho) + \mathcal{L}_2(\rho) = \sum_{i=1}^2 \frac{\Gamma_i}{2} (2|0\rangle\langle i|\rho|i\rangle\langle 0| - \rho|i\rangle\langle i| - |i\rangle\langle i|\rho). \quad (3)$$

The steady states of the system can be found through the condition

$$\lim_{t \rightarrow \infty} \frac{\partial \rho(t)}{\partial t} = 0, \quad (4)$$

where $t \rightarrow \infty$ means $t \gg 1/\min \Gamma_i$. The right-hand side of Eq. (2) fulfills the condition (4) for a pure state $|\Psi\rangle$ (also called dark state) if $\mathcal{L}(|\Psi\rangle\langle\Psi|) = 0$ and $H|\Psi\rangle = E|\Psi\rangle$ [9]. For a master equation in the Lindblad form [32],

$$\mathcal{L}(\rho) = \frac{\gamma}{2} (2O\rho O^\dagger - O^\dagger O\rho - \rho O^\dagger O),$$

where γ is the decay rate and O the jump operator, the state $|\Psi\rangle$ is the only protected state if $O|\Psi\rangle = 0$ [$\mathcal{L}(|\Psi\rangle\langle\Psi|) = 0$] and there is no further eigenstate $|\phi\rangle$ of O such that $[O, O^\dagger]|\phi\rangle = 0$ [33]. The method used here to obtain the dark state of the system is not unique. For instance, in Refs. [34,35] the dark state conditions are met when the absorption spectrum is null. Already in Ref. [36] a signature of the dark state is found in the second-order correlation function. In order to remove the time dependence of the right-hand side of Eq. (2), we move to a rotating frame defined by the unitary transformation

$$U(t) = \exp\left[\frac{i\omega_L t}{2}(|1\rangle\langle 1| + |2\rangle\langle 2| - |0\rangle\langle 0|)\right]. \quad (5)$$

In this frame the Hamiltonian becomes

$$H_{int} = \Omega(e^{i\varphi}|0\rangle\langle 1| + e^{-i\varphi}|1\rangle\langle 0|) + T_e(|1\rangle\langle 2| + |2\rangle\langle 1|), \quad (6)$$

the form of $\mathcal{L}(\rho)$ remains unchanged, and ρ is replaced by $\rho_{int} = U^\dagger \rho U$. Considering the requirements to obtain a protected pure state, we initially find the eigenvectors of H_{int}

$$|E_+(\theta, \varphi)\rangle = \frac{\sin(\theta/2)|0\rangle + e^{-i\varphi}|1\rangle + e^{-i\varphi} \cos(\theta/2)|2\rangle}{\sqrt{2}}, \quad (7a)$$

$$|E_0(\theta, \varphi)\rangle = \cos(\theta/2)|0\rangle - e^{-i\varphi} \sin(\theta/2)|2\rangle, \quad (7b)$$

$$|E_-(\theta, \varphi)\rangle = \frac{\sin(\theta/2)|0\rangle - e^{-i\varphi}|1\rangle + e^{-i\varphi} \cos(\theta/2)|2\rangle}{\sqrt{2}}, \quad (7c)$$

with eigenvalues $E_\pm = \pm\sqrt{\Omega^2 + T_e^2}$ and $E_0 = 0$. φ is the laser phase defined above and $\cos(\theta/2) = T_e/\sqrt{\Omega^2 + T_e^2}$. Through the condition $\mathcal{L}(|\Psi\rangle\langle\Psi|) = 0$ we observe that the dissipative Liouvillians \mathcal{L}_1 and \mathcal{L}_2 have $\{|0\rangle, |2\rangle\}$ and $\{|0\rangle, |1\rangle\}$ as their dark states, respectively. As the relation between the decay rates is about $\Gamma_2 = 10^{-4}\Gamma_1$ with Γ_1 of the order of 0.33–6.6 μeV [37–39], we conclude that the dissipative dynamics is basically governed by $\mathcal{L}_1(\rho)$. Note that the eigenvector $|E_0(\theta, \varphi)\rangle$ is composed only by $\{|0\rangle, |2\rangle\}$ states. Therefore, $|E_0(\theta, \varphi)\rangle$ is a dark state of the system, which is in good agreement with our numerical calculations performed in the regime of parameters defined above. In the Appendix we analytically show how to obtain the effective master equation whose protected state is $|E_0(\theta, \varphi)\rangle$. Naturally the protected state is not pure; the deviation from $|E_0(\theta, \varphi)\rangle$ introduced by the indirect exciton decay channel (Γ_2) can be obtained through the fidelity $\mathcal{F}(\infty)$ in the stationary regime, defined by

$$\mathcal{F}(\infty) \equiv \lim_{t \rightarrow \infty} [\langle E_0(\theta, \varphi)|\rho(t)|E_0(\theta, \varphi)\rangle] = \frac{1 + \frac{\Gamma_2}{\Gamma_1} \frac{T_e^2}{T_e^2 + \Omega^2}}{1 + \frac{\Gamma_2}{\Gamma_1} \frac{T_e^4 + 2\Omega^4}{T_e^2(T_e^2 + \Omega^2)}}. \quad (8)$$

A simple analysis of the particular case $T_e \sim \Omega$ shows that the fidelity $\mathcal{F}(\infty) \simeq 1 - \Gamma_2/\Gamma_1$ attains values next to 1, even for Γ_2 one order of magnitude lower than Γ_1 .

The state $|E_0(\theta, \varphi)\rangle$ can be represented on the Bloch sphere, where θ and φ are polar and azimuthal angles. The experimentally accessible values of $\Omega \simeq 0.05$ –1.0 meV [38,40] and $T_e \simeq 0.01$ –10 meV [41,42] enable θ to vary approximately from 0.5° to 179° , while the laser phase φ is easily controlled in the range $[0, 2\pi)$. The dependence of $|E_0(\theta, \varphi)\rangle$ with respect to Ω and T_e is analyzed considering three particular cases:

(i) $\Omega \gg T_e$: In this case the protected state becomes $|E_0(\pi, \varphi)\rangle = |2\rangle$ provided that $\theta \rightarrow \pi$. This is achieved increasing the laser amplitude, since for $\Gamma_1 \gg \Gamma_2$ the lifetime of the indirect exciton state goes to infinity. This result is in accordance with Ref. [26].

(ii) $\Omega \ll T_e$: In the opposite scenario where $\theta \rightarrow 0$ the protected asymptotic state is the ground state of the system $|E_0(0, \varphi)\rangle = |0\rangle$. Since the laser amplitude is weak, the direct and indirect exciton states are not populated.

(iii) $\Omega = T_e$: For $\theta = \pi/2$ the protected state $|E_0(\pi/2, \varphi)\rangle = (|0\rangle - e^{-i\varphi}|2\rangle)/\sqrt{2}$ is a coherent superposition of states $|0\rangle$ and $|2\rangle$. In this last case, the

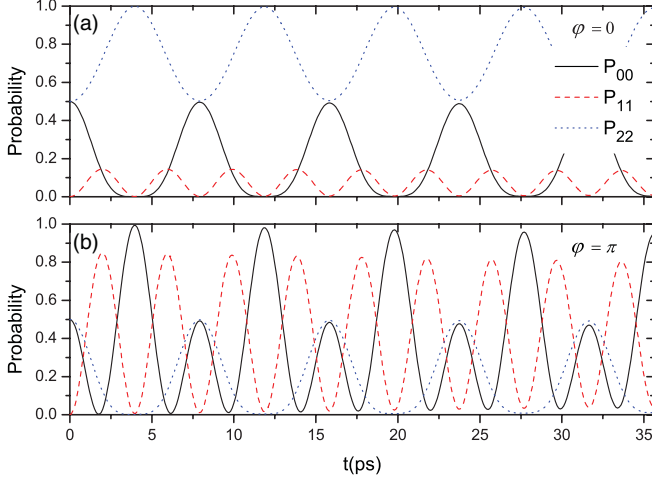


FIG. 1. (Color online) Time evolution of the population of the states $|0\rangle$ (black solid line), $|1\rangle$ (red dashed line), and $|2\rangle$ (blue dotted line) considering the application of a laser field with amplitude $\bar{\Omega} = \Omega(\sqrt{2} + 1)$, relative phase $\bar{\varphi} = 0$, and frequency $\bar{\omega} = \omega_L$. The initial state of the system is $|E_0(\pi/2, \varphi)\rangle$ for (a) $\varphi = 0$ and (b) π . The physical parameters are $\Omega = T_e = 200 \mu\text{eV}$, $\Gamma_1 = 3 \mu\text{eV}$, and $\Gamma_2 = 10^{-4}\Gamma_1$.

control of the laser relative phase φ will enable us to implement the Deutsch algorithm, as shown below.

III. DEUTSCH ALGORITHM VIA DISSIPATIVE QUANTUM COMPUTATION

The Deutsch algorithm was one of the first quantum algorithms to make explicit use of the quantum parallelism [15,17]. Such an algorithm was built to decide whether a given binary function $f : \{0,1\} \rightarrow \{0,1\}$ is constant [$f(0) = f(1)$] or balanced [$f(0) \neq f(1)$]. Differently from the original solution to the Deutsch problem [15], which is probabilistic, we present here a deterministic algorithm due to Collins, Kim, and Holton [17]. Besides being deterministic, the approach to the Deutsch problem used in Ref. [17] is interesting because only one query to the oracle is made and auxiliary qubits are unnecessary.

To implement the Deutsch algorithm we use the protected state $|E_0(\pi/2, \varphi)\rangle$ with the laser phase φ being 0 or π . In order to clarify the execution of the algorithm, we make the correspondence between the function domain $\{0,1\}$ and the states of the system $\{|0\rangle, |2\rangle\}$ so that $0 \rightarrow |0\rangle$ and $1 \rightarrow |2\rangle$. We define next the parameter $\varepsilon \equiv f(|2\rangle) - f(|0\rangle)$, which can take the values $\{-1, 0, 1\}$. Therefore, the state $|E_0(\pi/2, \varphi)\rangle$ can be

TABLE I. The dependence of $\Delta P_{00}(t_n)$ on the ratio $\epsilon = \Gamma_2/\Gamma_1$ for $\varphi = 0$.

	$\epsilon = 0.0001$	$\epsilon = 0.001$	$\epsilon = 0.01$
$\Delta P_{00}(t_1)$	-4.9913×10^{-5}	-4.9847×10^{-4}	-4.9178×10^{-3}
$\Delta P_{00}(t_2)$	-4.9757×10^{-5}	-4.9689×10^{-4}	-4.9010×10^{-3}
$\Delta P_{00}(t_3)$	-4.9597×10^{-5}	-4.9527×10^{-4}	-4.8838×10^{-3}
$\Delta P_{00}(t_4)$	-4.9434×10^{-5}	-4.9363×10^{-4}	-4.8665×10^{-3}

TABLE II. The dependence of $\Delta P_{00}(t_n)$ on the ratio $\epsilon = \Gamma_2/\Gamma_1$ for $\varphi = \pi$.

	$\epsilon = 0.0001$	$\epsilon = 0.001$	$\epsilon = 0.01$
$\Delta P_{00}(t_1)$	9.9108×10^{-5}	9.8974×10^{-4}	9.7653×10^{-3}
$\Delta P_{00}(t_2)$	9.7382×10^{-5}	9.7249×10^{-4}	9.5942×10^{-3}
$\Delta P_{00}(t_3)$	9.5694×10^{-5}	9.5563×10^{-4}	9.4268×10^{-3}
$\Delta P_{00}(t_4)$	9.4043×10^{-5}	9.3913×10^{-4}	9.2361×10^{-3}

rewritten as

$$\begin{aligned}
 |E_0(\pi/2, \varphi)\rangle &= \frac{|0\rangle - e^{-i\pi\varepsilon}|2\rangle}{\sqrt{2}} = \frac{|0\rangle - (-1)^\varepsilon|2\rangle}{\sqrt{2}} \\
 &= \frac{|0\rangle - (-1)^{f(|2\rangle) - f(|0\rangle)}|2\rangle}{\sqrt{2}}. \quad (9)
 \end{aligned}$$

Therefore, if the function is constant [$f(|2\rangle) = f(|0\rangle)$] then $\varphi = 0$. Otherwise, if the function is balanced [$f(|2\rangle) \neq f(|0\rangle)$] then $\varphi = \pi$. It is assumed that only the oracle has information about the function $f(i)$. In this case the oracle is the laser phase φ programmer. The last part of an algorithm is the readout of the solution. This is made here, first, by replacing the current laser field by another one with amplitude $\bar{\Omega} = \Omega(\sqrt{2} + 1)$, relative phase $\bar{\varphi} = 0$, and frequency $\bar{\omega} = \omega_L$ resonant to the transition $|0\rangle \rightarrow |1\rangle$. In this new configuration, $|E_0(\pi/2, \varphi)\rangle$ becomes the initial state of system and the evolved state now is $\bar{\rho}(t)$. In Fig. 1 we observe the time evolution of the populations $P_{ii}(t) = \langle i|\bar{\rho}(t)|i\rangle$ of the states $|0\rangle$ (black solid line), $|1\rangle$ (red dashed line), and $|2\rangle$ (blue dotted line) considering the phases $\varphi = 0$ and $\varphi = \pi$ of the first laser. At specific times $t_n = n\pi/(\bar{\Omega}^2 + T_e^2)^{1/2}$ with $n = 1, 3, 5, \dots$, the state of the system will be $\bar{\rho}(t) \simeq |0\rangle\langle 0|$ if the phase of the state $|E_0(\pi/2, \varphi)\rangle$ is $\varphi = \pi$ and $\bar{\rho}(t) \simeq |2\rangle\langle 2|$ if the phase is $\varphi = 0$. Therefore, applying another laser pulse resonant with the transition $|0\rangle \leftrightarrow |1\rangle$ and observing the time-resolved absorption spectrum [24] it is possible to distinguish between the states $|0\rangle$ and $|2\rangle$, provided that if the electron is in state $|2\rangle$ there will be no absorption, while if the electron is in state $|0\rangle$, the light will be absorbed. In summary, discovering whether the phase φ is 0 or π is equivalent to solving the Deutsch problem.

To finish our analysis of the Deutsch algorithm, we will show that the error introduced by the fact that $|E_0(\pi/2, \varphi)\rangle$ is not the perfect steady state of the system is negligible. We define the quantity $\Delta P_{00}(t_n) \equiv |\langle 0|\bar{\rho}(t_n, \rho(t \rightarrow \infty)) - \bar{\rho}(t_n, |E_0(\pi/2, \varphi)\rangle)\langle E_0(\pi/2, \varphi)||0\rangle|$ as the difference of population in the state $|0\rangle$ for the density operator $\bar{\rho}(t_n)$ considering two different initial conditions, the approximate state $|E_0(\pi/2, \varphi)\rangle$ and the exact state $\rho(t \rightarrow \infty)$, where the later is obtained by numerical calculation. Tables I and II show the dependence of $\Delta P_{00}(t_n)$ on the ratio $\epsilon \equiv \Gamma_2/\Gamma_1$ for $\varphi = 0$ and π , respectively.

IV. CONCLUSIONS AND PERSPECTIVES

In summary, we showed the existence of a pure protected state in a system composed by two asymmetric quantum

dots coupled by tunneling and driven by a laser field. The direct exciton state is under strong spontaneous decay. The asymptotic protected state is a superposition of the ground and the indirect exciton state. By controlling the ratio between the laser amplitude and tunneling rate it is possible to control the polar angle, while controlling the laser phase enables the control of the azimuthal angle of the Bloch sphere. The scheme for state protection remains true at low temperatures ($T \sim 0$ K). For high temperatures the phonon contribution becomes important so that the phonon-induced dephasing will destroy the superposition of the protected state. As an application of dissipative quantum computation we proposed the implementation of the Deutsch algorithm in this system, which basically consists of distinguishing the relative phase 0 or π between the states $|0\rangle$ and $|2\rangle$. Differently from the usual proposals for implementing quantum algorithms, which are based on unitary evolutions, here we make use of incoherent evolution of the Markovian reservoir. This approach is interesting because it is naturally immune to external perturbations and decoherence processes. A similar scheme might protect the state of two or more qubits which can be used to implement more sophisticated algorithms, such as Deutsch-Jozsa and Grover algorithms.

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APPENDIX: EFFECTIVE MASTER EQUATION

For the regime of parameters considered here, the master equation (2) has an analytical solution. However, the method developed in this appendix can be useful to obtain the effective master equation for a more elaborate problem. First we rewrite Eq. (2) in the interaction picture according to the unitary transformation (5):

$$\frac{\partial \rho_{int}(t)}{\partial t} = -i[H_{int}, \rho_{int}] + \mathcal{L}_{int}(\rho_{int}), \quad (\text{A1})$$

where H_{int} is given by Eq. (6) and the dissipative Liouvillian by Eq. (3) with ρ replaced by ρ_{int} . Performing a change of basis to the Hamiltonian eigenstates, H_{int} in Eq. (A1) can be expressed as

$$H_{int} = E_+ (|E_+\rangle\langle E_+| - |E_-\rangle\langle E_-|), \quad (\text{A2})$$

and the dissipative Liouvillians $\mathcal{L}_{int}(\rho_{int}) = \mathcal{L}_1(\rho_{int}) + \mathcal{L}_2(\rho_{int})$ as

$$\begin{aligned} \mathcal{L}_1(\rho_{int}) = & \frac{\Gamma_1}{4} [\sin^2(\theta/2) [\sqrt{2} \cot(\theta/2) (|E_0\rangle\langle E_+| - |E_0\rangle\langle E_-|) + |E_+\rangle\langle E_+| - |E_-\rangle\langle E_-| + |E_-\rangle\langle E_+| - |E_+\rangle\langle E_-|] \rho_{int} \\ & \times [\sqrt{2} \cot(\theta/2) (|E_+\rangle\langle E_0| - |E_-\rangle\langle E_0|) + |E_+\rangle\langle E_+| - |E_-\rangle\langle E_-| - |E_-\rangle\langle E_+| + |E_+\rangle\langle E_-|] \\ & - \{\rho_{int}, |E_+\rangle\langle E_+| + |E_-\rangle\langle E_-| - |E_-\rangle\langle E_+| - |E_+\rangle\langle E_-|\}] \end{aligned} \quad (\text{A3})$$

and

$$\begin{aligned} \mathcal{L}_2(\rho_{int}) = & \frac{\Gamma_2}{4} [\sin^2(\theta/2) \cos^2(\theta/2) [\sqrt{2} \cot(\theta/2) (|E_0\rangle\langle E_-| - |E_-\rangle\langle E_0| - |E_+\rangle\langle E_0| + |E_0\rangle\langle E_+|) \\ & + |E_+\rangle\langle E_+| + |E_-\rangle\langle E_-| - 2|E_0\rangle\langle E_0| + |E_-\rangle\langle E_+| + |E_+\rangle\langle E_-|] \rho_{int} \\ & \times [-\sqrt{2} \cot(\theta/2) (|E_0\rangle\langle E_-| - |E_-\rangle\langle E_0| - |E_+\rangle\langle E_0| + |E_0\rangle\langle E_+|) \\ & + |E_+\rangle\langle E_+| + |E_-\rangle\langle E_-| - 2|E_0\rangle\langle E_0| + |E_-\rangle\langle E_+| + |E_+\rangle\langle E_-|] \\ & - \cos^2(\theta/2) \{\rho_{int}, |E_+\rangle\langle E_+| + |E_-\rangle\langle E_-| + |E_-\rangle\langle E_+| + |E_+\rangle\langle E_-| \\ & - \sqrt{2} \tan(\theta/2) (|E_0\rangle\langle E_-| + |E_-\rangle\langle E_0| + |E_+\rangle\langle E_0| + |E_0\rangle\langle E_+|) + 2 \tan^2(\theta/2) |E_0\rangle\langle E_0|\}], \end{aligned} \quad (\text{A4})$$

where $\{a, b\} = ab + ba$ states for the anticommutator. The action of a unitary transformation $\tilde{U}(t) = \exp(-iH_{int}t)$ on Eq. (A1) is able to remove the unitary part of its dynamics. Such procedure is interesting because the operators of the form $|E_i\rangle\langle E_j|$ with $i, j = \{+, -, 0\}$ for $i \neq j$ in Eq. (A1) will oscillate quickly as shown here:

$$\tilde{U}^\dagger(t) |E_+\rangle\langle E_0| \tilde{U}(t) = |E_+\rangle\langle E_0| e^{iE_+t}, \quad (\text{A5a})$$

$$\tilde{U}^\dagger(t) |E_-\rangle\langle E_0| \tilde{U}(t) = |E_-\rangle\langle E_0| e^{-iE_+t}, \quad (\text{A5b})$$

$$\tilde{U}^\dagger(t) |E_+\rangle\langle E_-| \tilde{U}(t) = |E_+\rangle\langle E_-| e^{2iE_+t}. \quad (\text{A5c})$$

Since $\Gamma_2 \ll \Gamma_1 \ll \Omega, T_e$ it is possible to perform the rotating wave approximation, leading to an effective master

equation

$$\frac{\partial \rho_{\text{eff}}(t)}{\partial t} = \mathcal{L}_{\text{eff}}(\rho_{\text{eff}}). \quad (\text{A6})$$

The dissipative Liouvillian $\mathcal{L}_{\text{eff}}(\rho_{\text{eff}})$ can be written in the Lindblad form as

$$\begin{aligned} \mathcal{L}_{\text{eff}}(\rho_{\text{eff}}) = & \sum_{\alpha=1}^2 \sum_{i=1}^5 \frac{\Gamma_{\alpha,i}}{2} (2O_{\alpha,i} \rho_{\text{eff}} O_{\alpha,i}^\dagger - O_{\alpha,i}^\dagger O_{\alpha,i} \rho_{\text{eff}} \\ & - \rho_{\text{eff}} O_{\alpha,i}^\dagger O_{\alpha,i}), \end{aligned} \quad (\text{A7})$$

where

$$\begin{aligned}
 \Gamma_{1,1} &= \frac{\Gamma_1}{8} \sin^2(\theta/2) & O_{1,1} &= |E_+\rangle\langle E_+| - |E_-\rangle\langle E_-| \\
 \Gamma_{1,2} &= \Gamma_{1,1} & O_{1,2} &= |E_-\rangle\langle E_+| \\
 \Gamma_{1,3} &= \Gamma_{1,1} & O_{1,3} &= |E_+\rangle\langle E_-| \\
 \Gamma_{1,4} &= \frac{\Gamma_1}{4} \cos^2(\theta/2) & O_{1,4} &= |E_0\rangle\langle E_-| \\
 \Gamma_{1,5} &= \Gamma_{1,4} & O_{1,5} &= |E_0\rangle\langle E_+| \\
 \Gamma_{2,1} &= \frac{\Gamma_2}{8} \sin^2(\theta/2) \cos^2(\theta/2) & O_{2,1} &= |E_+\rangle\langle E_+| - 2|E_0\rangle\langle E_0| + |E_-\rangle\langle E_-| \\
 \Gamma_{2,2} &= \Gamma_{2,1} & O_{2,2} &= |E_-\rangle\langle E_+| \\
 \Gamma_{2,3} &= \Gamma_{2,1} & O_{2,3} &= |E_+\rangle\langle E_-| \\
 \Gamma_{2,4} &= 2\Gamma_{2,1} & O_{2,4} &= \cot(\theta/2)|E_0\rangle\langle E_+| - \tan(\theta/2)|E_-\rangle\langle E_0| \\
 \Gamma_{2,5} &= 2\Gamma_{2,1} & O_{2,5} &= \cot(\theta/2)|E_0\rangle\langle E_-| - \tan(\theta/2)|E_+\rangle\langle E_0|.
 \end{aligned}$$

In Eq. (A7), the index α refers to the direct (Γ_1) and indirect (Γ_2) exciton decay rates and index i enumerates the operators. From this effective master equation it is easy to see that $|E_0\rangle$ is the only eigenstate with null eigenvalue of operators $O_{1,i}$. Although the operators $O_{2,i}$ presented in $\mathcal{L}_{\text{eff}}(\rho_{\text{eff}})$ rotate the state $|E_0\rangle$, the decay rates $\Gamma_{2,i}$ are much lower than $\Gamma_{1,i}$. In summary, the dissipative dynamics is dictated by the direct

exciton decay, ensuring that the only protected pure state is $|E_0\rangle$. To prove the uniqueness of the protected state, we solve Eq. (A1) using the software Wolfram MATHEMATICA7 for an arbitrary initial state with density matrix elements $\rho_{ij}(0)$ with $i, j = 0, 1, 2$ so that $\sum_{i=0}^2 \rho_{ii}(0) = 1$. We get the expression for the density matrix elements in the asymptotic time ($t \rightarrow \infty$):

$$\begin{aligned}
 \rho_{00}(\infty) &= \frac{T_e^4(T_e^2 + \Omega^2)\Gamma_1 + (T_e^6 + \Omega^6)\Gamma_2}{(T_e^2 + \Omega^2)[T_e^2(T_e^2 + \Omega^2)\Gamma_1 + (T_e^4 + 2\Omega^4)\Gamma_2]}, \\
 \rho_{11}(\infty) &= \frac{\Omega^4\Gamma_2}{T_e^2(T_e^2 + \Omega^2)\Gamma_1 + (T_e^4 + 2\Omega^4)\Gamma_2}, \\
 \rho_{22}(\infty) &= \frac{T_e^2\Omega^2(T_e^2 + \Omega^2)(\Gamma_1 + \Gamma_2)}{(T_e^2 + \Omega^2)[T_e^2(T_e^2 + \Omega^2)\Gamma_1 + (T_e^4 + 2\Omega^4)\Gamma_2]}, \\
 \rho_{02}(\infty) &= -e^{i\varphi} T_e \Omega \frac{T_e^2(T_e^2 + \Omega^2)\Gamma_1 + (T_e^4 - \Omega^4)\Gamma_2}{(T_e^2 + \Omega^2)[T_e^2(T_e^2 + \Omega^2)\Gamma_1 + (T_e^4 + 2\Omega^4)\Gamma_2]}, \\
 \rho_{01}(\infty) &= \rho_{12}(\infty) = 0.
 \end{aligned}$$

Note that the elements $\rho_{ij}(\infty)$ are independent of $\rho_{ij}(0)$. Considering the range of values for the parameters Γ_1 , Γ_2 , T_e , and Ω used in the paper, $\Gamma_1 \gg \Gamma_2$ and $T_e \geq \Omega$, the matrix element $\rho_{11}(\infty) \ll \rho_{00}(\infty), \rho_{02}(\infty), \rho_{22}(\infty)$. Then, the protected state is approximately $|E_0(\theta, \varphi)\rangle$ independently of the initial state of the system. Another way to prove the uniqueness of the protected state $|E_0(\theta, \varphi)\rangle$ is using theorem 2 of Ref. [9]. Defining the jump operator $c \equiv |0\rangle\langle 1|$ and the ‘‘subspace’’ $S \equiv \{\gamma|E_-(\theta, \varphi)\rangle + \delta|E_+(\theta, \varphi)\rangle$ with

$\gamma, \delta \in \mathbb{C}$ and $|\gamma|^2 + |\delta|^2 = 1$, we observe that $S \neq cS$ for every γ, δ, θ , and φ .

The time scale necessary for the system state to be stationary is $t_{ss} = 1/\min \Gamma_{1,i}$ ($i = 1, \dots, 5$), which depends on Γ_1 , T_e , and Ω . Returning to the Schrödinger picture, we observe that the unitary transformation $\tilde{U}(t)$ does not affect the protected state, while $U(t)$ introduces a relative phase between the states $|0\rangle$ and $|2\rangle$.

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