PHYSICAL REVIEW A **85**, 031801(R) (2012)

Anderson localization of light at the interface between linear and nonlinear dielectric media with an optically induced photonic lattice

Dragana M. Jović,[1,2] Milivoj R. Belić,[3] and Cornelia Denz[2]
[1]*Institute of Physics, University of Belgrade, P.O. Box 68, 11001 Belgrade, Serbia*
[2]*Institut für Angewandte Physik and Center for Nonlinear Science (CeNoS), Westfälische Wilhelms-Universität Münster, 48149 Münster, Germany*
[3]*Texas A&M University at Qatar, P.O. Box 23874, Doha, Qatar*
(Received 15 November 2011; published 12 March 2012)

In a numerical study, we observe Anderson localization of surface modes at the interface between a linear and a nonlinear dielectric medium, containing an optically induced disordered photonic lattice. We discover the threshold for the existence of such localized modes. The influence of Kerr nonlinearity and disorder levels on the transverse localization of light at such an interface is discussed. We demonstrate the suppression of localization for lower disorder levels, as compared to both completely linear and completely nonlinear medium. We also reveal Anderson localization at the linear-nonlinear interface in the presence of a phase-slip defect, and demonstrate the suppression of localization in that case as well.

DOI: 10.1103/PhysRevA.85.031801   PACS number(s): 42.25.Dd, 72.15.Rn

Introduction. Surface modes—special types of electromagnetic waves localized near an interface separating two different media—have recently attracted great attention, owing to their novel physics and possible interesting applications in all-optical switching and sensing [1,2]. In optics, surface waves are known to exist at an interface between continuous and periodic media [3], between different waveguide arrays [4], and also at the interface between a homogeneous lattice and a superlattice [5]. The insertion of defects may provide an additional physical mechanism for light confinement and a possibility to control light propagation in photonic lattices. There exist different types of lattice defects, but recently the existence of self-localized structures was demonstrated at the interface between two arrays connected by a defect channel, the so-called phase-slip defect [6,7].

Disordered systems have played a central role in condensed-matter physics and Anderson localization is among the most fascinating and universal phenomena in the physics of such systems [8–10]. Despite its 50-year history [11], Anderson localization still excites much interest in a variety of systems, including light waves in different materials. Competition between nonlinearity and disorder was investigated experimentally in fiber arrays [12] and in disordered two-dimensional photonic lattices [13]. Lahini *et al*. [14] experimentally demonstrated the effect of the medium's nonlinearity on light localization and concluded that the nonlinearity in a disordered medium favors the occurrence of localization on a shorter distance scale.

In this Rapid Communication, we extend these concepts to the transverse light localization at an interface between a linear and a nonlinear dielectric medium, with an optically induced disordered photonic lattice. The properties of light propagating along the interface between linear and nonlinear media have been explored in a few earlier studies [15–17]. Here, we extend the investigation of light localization at similar interfaces, but with the inclusion of an optically induced photonic lattice and a slip-phase defect. The system of interest is depicted in Fig. 1. For investigation, we pick the square lattice.

First, we introduce the model and study numerically the effect of the interface on Anderson localization of light. Second, we address one of the most important issues, namely, the interplay of medium's nonlinearity and the degree of disorder in such a system. We reveal that the localization at the linear-nonlinear interface occurs when the strength of disorder in the lattice and the strength of nonlinearity exceed critical values. The crossover from nonlocalized to localized modes is observed in the plane of two parameters: the lattice disorder level and the nonlinearity strength in the nonlinear part of the medium. Finally, we discuss briefly the effect of phase-slip defects on the disorder-induced localization in the system with the linear-nonlinear interface [Fig. 1(b)]. The phase-slip defect represents an interruption in the lattice periodicity, in which the lattices in the linear and the nonlinear part of the medium are separated by some distance larger than the lattice constant. We assume that the distance between lattice sites across the phase slip is a noninteger number in units of the lattice constant.

This Rapid Communication is organized as follows. We introduce the model which describes the propagation of light at the interface between linear and nonlinear dielectric media with an optically induced disordered photonic lattice. We summarize our numerical results at such an interface and present a comparison with the localization in both completely linear and completely nonlinear media. Then we study localization at the same interface, but with a phase-slip defect. Finally, we conclude the Rapid Communication.

Theoretical model and system geometry. We consider the propagation of light at an interface between linear and nonlinear dielectric media with an optically induced disordered photonic lattice. The propagation of a light beam along the z axis is described using the paraxial wave equation for the slowly varying electric field amplitude F:

$$i\frac{\partial F}{\partial z} = -\Delta F - \gamma |F|^2 F - VF, \quad -\infty \leqslant x \leqslant 0, \tag{1}$$

$$i\frac{\partial F}{\partial z} = -\Delta F - VF, \quad 0 \leqslant x \leqslant \infty, \tag{2}$$

where $\Delta = \partial_x^2 + \partial_y^2$ is the *transverse* Laplacian, γ is the dimensionless strength of the nonlinearity (in the nonlinear part of the medium), and $V = V(x,y)$ is the induced optical

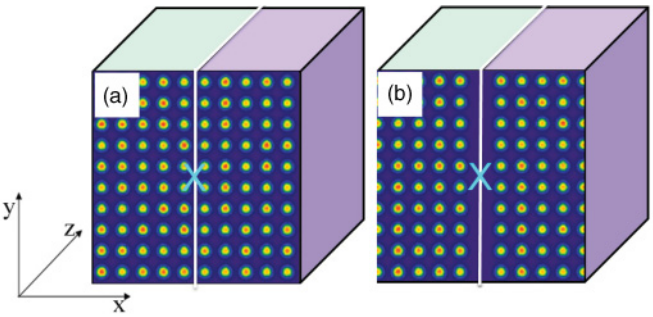

FIG. 1. (Color online) (a) Sketch of the system, with an interface between the linear and nonlinear parts of a dielectric medium and with a photonic lattice induced in the medium. (b) The same geometry, but with the phase-slip defect introduced at the interface. The crosses mark the locations of input Gaussian beams.

lattice potential, which only depends on the transverse coordinates. We choose this potential in the form of a sum of Gaussian beams, uniform along the z direction, with the peak intensity V_0. A scaling of coordinates is introduced, $x/x_0 \to x$, $y/x_0 \to y$, $z/L_D \to z$, to obtain a dimensionless equation, where x_0 is the typical full width at half maximum (FWHM) beam waist and L_D is the corresponding diffraction length. For the nonlinearity, we adopt the simple Kerr nonlinearity, although other models may be utilized as well [18]. Thus, the propagation equation in the nonlinear part is of the nonlinear Schrödinger equation type. The propagation equation is solved numerically by employing a beam propagation method developed earlier [18].

We launch a narrow Gaussian beam along the propagation direction (z), positioned at the interface (marked by crosses in Fig. 1). The width of the input beam is comparable to the lattice constant. While propagating in the medium, the beam is subject to different competing influences: It broadens in the linear and defocusing nonlinear media and it focuses in the nonlinear focusing medium. It also filaments in the optically induced lattice and displays a tendency to stick to the surface, as a surface wave. We want to know what happens to the beam when, additionally, it is subject to Anderson localization.

For Anderson localization to occur, disorder must be introduced in the lattice. We realize disorder using a randomized lattice intensity $V_r(x,y)$, imposed on the periodic potential V. Since the lattice is uniform in the z direction, randomness is confined to the transverse plane. This places a stringent requirement on the transverse localization: The potential must *not* vary along the propagation direction, otherwise Anderson localization would not occur. Each Gaussian beam in the lattice is multiplied by a random lattice peak intensity V_{0r}. This random peak intensity takes values from the interval $(1 - NR)V_0 < V_{0r} < (1 + NR)V_0$, where R is the random number generator, and N determines the degree of disorder. They both may take values from the interval $[0,1]$. We quantify the disorder level by the ratio between the intensity of the random lattice and the intensity of the periodic lattice.

The study of Anderson localization requires many realizations of a randomized system. In our system they are realized by starting each simulation with a different seed of the random number generator. All quantities of interest characterizing Anderson localization [19,20], such as the inverse participation ratio and the effective beam width, are evaluated as ensemble averages over 100 disorder realizations.

Transverse Anderson localization at the linear-nonlinear interface. To start with, we vary the medium's nonlinearity in the nonlinear part of the medium and increase the level of disorder, keeping all other parameters fixed. With an increasing level of disorder, the output intensity beam profile narrows down and the exponentially decaying tails become an indication of localization. The output intensity profiles are averaged over multiple different realizations for each disorder level, and are fitted with linear functions on the logarithmic scale. The crossover to localization becomes evident by the transition from a broad Gaussian quadratically shaped profile to a linearly decaying exponential intensity profile. This transition is most easily discerned in the direction perpendicular to the interface, which is x in our geometry. In that case the profile is approximated by a function of the form $I \sim \exp(-2|x|/\xi)$, where ξ is then the localization length in the x direction. There naturally exists the localization length in the y direction, along the interface, but for a uniform transverse lattice (no phase-slip defect) the two values are not much different. Introducing the phase-slip defect makes the difference between the two larger. In our numerics, when fitting to the exponential profile, we choose the error in the estimate to be up to 10%. This leads to a transition region, rather than a sharp threshold.

We consider both the focusing and defocusing medium's nonlinearity (γ positive and γ negative). The corresponding localization regions are summarized in Fig. 2, which are based

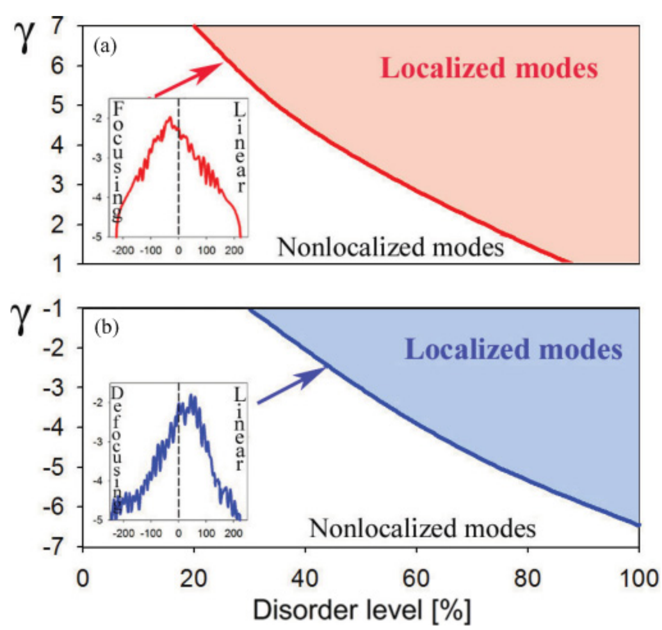

FIG. 2. (Color online) Crossover from the nonlocalized to the localized surface modes. The strength of the nonlinearity in the nonlinear part of medium vs the disorder level is shown for focusing (a) and defocusing (b) nonlinearity. Examples of averaged intensity profiles in the x direction, on the logarithmic scale, are presented as insets. Note the displacement of the peaks from the interface. Physical parameters are as follows: the crystal length $L = 20$ mm; input lattice intensity $V_0 = 1$; lattice period $d = 15$ μm; input beam intensity $|F_0|^2 = 0.5$; input beam FWHM $= 13$ μm.

RAPID COMMUNICATIONS

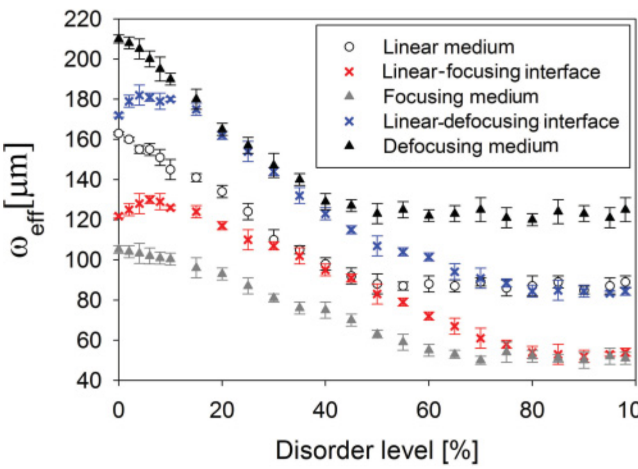

FIG. 3. (Color online) Comparison between localizations in the presence of an interface and with no interface. Effective beam widths at the lattice output vs the disorder level are presented for two different interfaces and for the completely linear and nonlinear cases. The points are ensemble averages and error bars depict the spread in the values coming from the statistics. Parameters are as in Fig. 2.

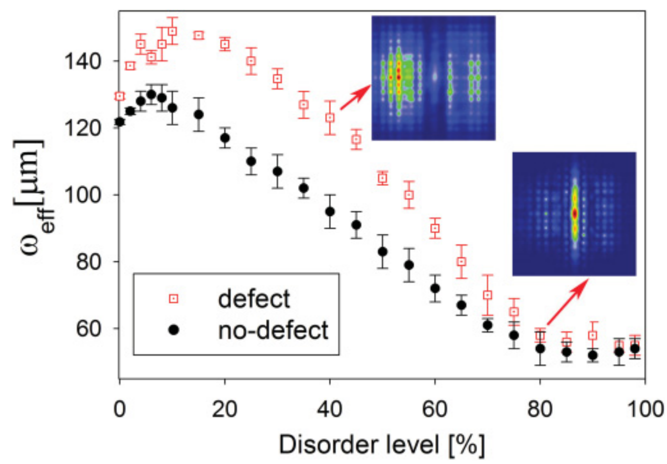

FIG. 4. (Color online) Influence of a phase-slip defect on the localization. The effective beam width at the lattice output is shown as a function of the disorder level. The red squares represent localization with the phase-slip defect and the black circles represent the case at the interface with no defect. The focusing nonlinearity is chosen, with $\gamma = 5$. The insets depict the surface modes; the nonlinear medium is to the left-hand side. Here, the thickness of the defect equals 2.2 lattice constants. Other parameters are as in Fig. 2.

on the results of many numerical simulations in the plane of two parameters: the medium's nonlinearity and the disorder level. We observe the threshold curves for the existence of surface localized modes at the linear-nonlinear interface in this parameter space. For low values of the nonlinearity strength γ and lower disorder levels, only nonlocalized modes exist. When either medium's nonlinearity or the disorder level are increased, one observes the transition to localized modes. As mentioned, the transition is not sharp. This is most evident in the region of low nonlinearities ($|\gamma| < 1$), where it is difficult to determine the existence of surface states, because of the closeness to the linear regime. Therefore, the region of $-1 < \gamma < 1$ is not presented in the figure.

For both the linear-focusing and linear-defocusing interfaces, typical examples of localized modes are presented as averaged intensity profiles on the logarithmic scale (insets in Fig. 2). We find that different strengths of the medium nonlinearity can lead to different relative positions of the surface modes. It is seen that the localized modes at the linear-focusing interface are pushed into the focusing nonlinear part, whereas in the case of the linear-defocusing interface, they are pulled into the linear part of the medium.

Comparison with the completely linear or completely nonlinear medium. To estimate how the linear-nonlinear interface affects the localization process, we compare the corresponding surface modes with the modes in the completely linear or the completely nonlinear medium. We consider concurrently linear-focusing and linear-defocusing interfaces, and also pure linear and nonlinear (either focusing or defocusing) bulk media. Naturally, pure media display bulk transverse localized modes, whereas interfaces display surface localized modes. Our results are summarized in Fig. 3.

A relevant quantity for the characterization of the localization level is the effective beam width $\omega_{\rm eff} = P^{-1/2}$, where $P = \int I^2(x,y,L)dxdy\{\int I(x,y,L)dxdy\}^{-2}$ is the inverse participation ratio [19]. We measure the effective beam width at the lattice output. In the completely linear or completely nonlinear media, the effective beam width gets smaller as the level of disorder increases. But for both interfaces, increasing the level of disorder initially leads to the enhanced expansion of the beam, and then to the localization at higher levels of disorder. For high disorder, it is seen that the effective beam width for the linear-defocusing interface becomes equal to the width of the completely linear case, whereas for the linear-focusing interface it becomes equal to the width of the completely focusing case (cf. Fig. 3).

We use the strength of the nonlinearity $\gamma = 5$ as representative for the focusing, and $\gamma = -5$ as representative for the defocusing case. To compare the localization in different regimes, averaged effective widths are normalized to the corresponding values without disorder. We use such a normalized quantity to indicate the strength of localization. It is a geometric criterion, in that when the effective beam width is more reduced, the localization is more pronounced. At lower disorder levels, the localization is more pronounced in both completely linear and completely nonlinear cases than at the interfaces. As the strength of disorder is further increased, the localization effects become relatively less pronounced in the completely linear and completely nonlinear cases.

Localization at the interface with a phase-slip defect. Finally, we analyze in some detail the localization effects in the system with a phase-slip defect (Fig. 4). We demonstrate the suppression of Anderson localization at the linear-nonlinear interface with the phase-slip defect, so that a stronger disorder is needed to obtain the same localization as in the case with no defect. This surprising result is nonetheless consistent with the observations reported earlier for Anderson localization of light near lattice boundaries [21]. As the width of the defect gets larger, our results go over to the results obtained in Ref. [21] for a straight lattice edge with Kerr nonlinearity.

We find that the level of suppression depends on the strength of disorder; the suppression is more pronounced at weak disorder levels. For strong disorder levels, the localization is slightly enhanced and becomes almost as effective as the localization with no defect. Typical localized modes are presented as insets in Fig. 4. There are two different localization lengths along the two transverse directions for the localized mode in the presence of such a defect [20]. It is also observed that the localization is less pronounced for larger defect widths (defined as the distance between the two lattice sites across the defect).

Conclusions. In conclusion, we have observed numerically the surface localized modes at the linear-nonlinear interface with an optically induced square photonic lattice. The crossover from the nonlocalized to the localized modes at such an interface is presented in the plane of two parameters: the disorder level and the nonlinearity strength in the nonlinear part of the medium. We have demonstrated the suppression of localization at the interface for lower disorder levels, as compared to both the completely linear and the completely nonlinear medium. We have analyzed numerically how the presence of the phase-slip defect in the two-dimensional photonic lattice modifies the phenomenon of Anderson localization of light.

This work is supported by the Ministry of Education and Science of the Republic of Serbia (Project No. OI 171036) and the Qatar National Research Foundation (Project No. NPRP 09-462-1-074). D.M.J. expresses gratitude to the Alexander von Humboldt Foundation for the Fellowship for Postdoctoral Researchers.

[1] F. Lederer, G. Stegeman, D. N. Christodoulides, G. Assanto, M. Segev, and Y. Silberberg, Phys. Rep. **463**, 1 (2008).
[2] Yu. S. Kivshar, Laser Phys. Lett. **5**, 703 (2008).
[3] S. Suntsov, K. G. Makris, G. A. Siviloglou, R. Iwanow, R. Schiek, D. N. Christodoulides, G. I. Stegeman, R. Morandotti, H. Yang, G. Salamo, M. Volatier, V. Aimez, R. Ars, M. Sorel, Y. Min, W. Sohler, X. Wang, A. Bezryadina, and Z. Chen, J. Nonlinear Opt. Phys. Mater. **16**, 401 (2007).
[4] A. Szameit, Y. V. Kartashov, F. Dreisow, M. Heinrich, V. A. Vysloukh, T. Pertsch, S. Nolte, A. Tünnermann, F. Lederer, and L. Torner, Opt. Lett. **33**, 663 (2008).
[5] M. Heinrich, Y. V. Kartashov, L. P. R. Ramirez, A. Szameit, F. Dreisow, R. Keil, S. Nolte, A. Tünnermann, V. A. Vysloukh, and L. Torner, Phys. Rev. A **80**, 063832 (2009).
[6] A. Szameit, M. I. Molina, M. Heinrich, F. Dreisow, R. Keil, S. Nolte, and Yu. S. Kivshar, Opt. Lett. **35**, 2738 (2010).
[7] A. Kanshu, C. E. Rüter, D. Kip, P. P. Beličev, I. Ilić, M. Stepić, and V. M. Shandarov, Opt. Express **19**, 1158 (2011).
[8] P. W. Anderson, Phys. Rev. **109**, 1492 (1958).
[9] S. John, Phys. Rev. Lett. **53**, 2169 (1984).
[10] H. De Raedt, Ad Lagendijk, and P. de Vries, Phys. Rev. Lett. **62**, 47 (1989).
[11] A. Lagendijk, B. Tiggelen, and D. S. Wiersma, Phys. Today **62**, 24 (2009).
[12] T. Pertsch, U. Peschel, J. Kobelke, K. Schuster, H. Bartelt, S. Nolte, A. Tünnermann, and F. Lederer, Phys. Rev. Lett. **93**, 053901 (2004).
[13] T. Schwartz, G. Bartal, S. Fishman, and M. Segev, Nature (London) **446**, 52 (2007).
[14] Y. Lahini, A. Avidan, F. Pozzi, M. Sorel, R. Morandotti, D. N. Christodoulides, and Y. Silberberg, Phys. Rev. Lett. **100**, 013906 (2008).
[15] W. J. Tomlinson, Opt. Lett. **5**, 323 (1980).
[16] D. Mihalache, G. I. Stegeman, C. T. Seaton, R. Zanoni, A. D. Boardman, and T. Twardowski, Opt. Lett. **12**, 187 (1987).
[17] Z. Shi, H. Li, and Qi Guo, Phys. Rev. A **83**, 023817 (2011).
[18] M. Belić, M. Petrović, D. Jović, A. Strinić, D. Arsenović, K. Motzek, F. Kaiser, Ph. Jander, C. Denz, M. Tlidi, and P. Mandel, Opt. Express **12**, 708 (2004).
[19] D. M. Jović and M. R. Belić, Phys. Rev. A **81**, 023813 (2010).
[20] D. M. Jović, M. R. Belić, and C. Denz, Phys. Rev. A **84**, 043811 (2011).
[21] D. M. Jović, Yu. S. Kivshar, C. Denz, and M. R. Belić, Phys. Rev. A **83**, 033813 (2011).