Reversible absorption of weak fields revealed in coherent transients

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It is shown that the absorbtion of a weak field in a thick resonant absorber is a reversible process even in the case of the homogeneous broadening of the absorption line. As an example, the propagation of a long rectangular pulse with sharp edges in an optically dense resonant medium is studied theoretically in the linear response approximation. Transient nutation (TN), free induction decay (FID), and transients, induced by the phase switch of the incident pulse, are considered. It is shown that in exact resonance the amplitude of FID increases with the length of the medium. FID arises due to the scattered radiation field (dipoles ringing). In a thick medium the scattered field is almost of the same amplitude but opposite in phase with the incident radiation field. Both fields interfere destructively to produce what is seen as radiation damping at the output of the medium. The scattered field needs time to develop. Therefore, the leading edge of the pulse is not absorbed, demonstrating temporal transparency followed by TN. Phase shift (180°) of the pulse brings the incident pulse in phase with the scattered radiation. Constructive interference of the pulse with the scattered radiation field produces a short pulse with an amplitude that is two times larger than the amplitude of the incident pulse. If the input pulse is detuned from resonance, for a particular detuning and optical thickness of the medium the amplitude of the transient pulse, induced by the phase shift, is nearly three times larger than the amplitude of the incident pulse. This is explained by the interference of the scattered field, the phase-shifted input field, and the slowly propagating part of the pulse, developed before the phase shift.

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I. INTRODUCTION

Optical transients such as free induction decay (FID) [1], transient nutation (TN) [2,3], and rotary echo, induced by the phase switch of the exciting pulse, [4] are generally observed in optically thin samples with strong inhomogeneous broadening of the absorption line. The main features of these transients are determined by ringing of individual dipoles with their own frequencies, which are different due to the inhomogeneous broadening. In thin samples these transients provide information about the homogeneous dephasing time T_2 of the dipoles, spectral diffusion, and the lifetime T_1 of the excited state.

In thin samples these transients regenerate only a small fraction of the energy contained in the exciting pulse because of the small number of particles interacting with the field. Therefore, thin samples are inapplicable to produce a noticeable pulse transformation or pulse shaping. Meanwhile, it is known that optically thick samples strongly modify the shape of the pulse at the output. For example, a rectangular pulse is split into the Sommerfeld precursor whose front arrives at time $t_s = L/c$, followed by a rapidly decaying part, and a delayed steady-state response, which is built up at $t_g = L/v_g$, where L is the sample length and v_g is the group velocity at the carrier frequency ω_c of the pulse (see, e.g., Refs. [5,6]). Another example of the pulse compression is a train of pulses of the same amplitude and different durations, applied at the appropriate time sequence. In a thick sample it generates a pulse of amplitude significantly larger than that of the input field [7]. It is important to notice that this kind of pulse transformations is performed in a linear propagation regime, which is even applicable to an extremely weak radiation field containing only one photon (see, e.g., Ref. [8]). Thus, thick samples are capable of modifying strongly the shape of the pulse, its duration, and amplitude, with no need of high input power.

In regard to practical applications, in addition to pulse compression, just the formation of the Sommerfeld precursor itself has been suggested for use for deeper penetration into a material, which may be applied to underwater communication [9] or imaging through biological tissue [10] . The penetration capability of the precursor is explained as a result of a slow reaction of the material to the exciting field, which takes time to build up the refractive index, and so the pulse front escapes out of the material without interaction [6]. Experimentally, precursors were observed in the microwave domain with waveguides whose dispersion is similar to that of the resonantly absorptive medium [11], in sound propagation measurements in superfluid ^{3}He [5], in propagation of optical pulses through semiconductors close to excitonic resonances [12,13], and in deionized water [9]. Recently the optical precursor was also directly observed in a cloud of cold potassium atoms in a region of anomalous dispersion [14].

Fast switch-off of the weak input radiation field, exciting a thick sample, also generates a short pulse of an appreciable intensity at the output of the sample [15]. This radiation field is FID, generated by a linear response polarization, induced in the absorber just before the field switch-off. The radiation field, generated by the medium, is in antiphase with the input wave, and hence both fields interfere destructively to produce what is seen as the field attenuation (absorption) [16]. If the pulse is suddenly switched off, the induced polarization will continue to radiate. This is seen as a sudden rise of intensity of the radiation, generated by the medium, which is no more compensated by the input field. Sudden rise of the radiation amplitude at the output of the absorber can be achieved also by the instantaneous 180° phase shift of the input field [17], which brings it in phase with the field, generated by the medium.

In this paper the formation of the Sommerfeld precursor, FID, and TN, induced by the 180° phase shift of the input field, in a thick absorber are studied for the radiation whose frequency is detuned from resonance. It is shown that depending on the optical thickness of the absorber and detuning from resonance the transients can be appreciably enhanced or quenched. This study opens new perspectives to generate strong pulses in a linear regime. Moreover, one can use weak radiation pulses, which are strongly attenuated in a thick resonant medium. If the attenuation level is such that the output radiation is not observable, then sudden switch-off of the input field, for example, by a shutter or deflector, and/or sudden change of the phase of the input field may produce a radiation field of the detectable intensity at the output of the absorber. Such pulses could be used for the information transmission at the low cost of the radiation power. In this proposal the radiation power is accumulated in the thick absorber. Then, accumulated power is immediately released on demand by the phase shift or shutting of the weak, feeding radiation field. So, the information is transmitted (or coded) by these events (i.e., by the radiation spikes). This idea was inspired by the experiments reported in Refs. [18–20].

II. GENERAL EQUATIONS FOR THE PULSE PROPAGATION IN ABSORPTIVE MEDIUM

In this section general formulas describing the pulse propagation in the slowly varying amplitude (SVA) approximation are presented. We consider only the linear regime, which is well described by the linear response (LR) approximation for the density matrix ρ_{mn} of resonant atoms in the absorber. The population change of the ground (g) and excited (e) states of the atom is neglected in LR approximation and only the equation for the nondiagonal element, $\rho_{eg} = \sigma_{eg} \exp(-i\omega_s t + ik_s z)$, is considered in the form,

$$\dot{\sigma}_{eg} = (i\Delta - \gamma)\sigma_{eg} + i\Omega(z,t), \tag{1}$$

where ω_s and k_s are the frequency and the wave number of the input radiation field, z is the distance, counted from the input face of the absorber inside, γ is the decay rate of the atomic coherence, $\Delta = \omega_s - \omega_0$ is the detuning from the resonant frequency ω_0 of atoms, $\Omega(z,t) = d_{eg}E_s(z,t)/2\hbar$ is the Rabi frequency, d_{eg} is the matrix element of the atomic dipole, induced in the transition $e - g$, and $E_s(z,t)$ is the slowly varying amplitude of the radiation field. The LR approximation is valid if $\Omega^2(z,t) \ll \gamma\gamma_e$, where γ_e is a decay rate of the excited state e. Then saturation of the atomic transition and large excursion of the Bloch vector do not happen.

In SVA approximation the wave equation is reduced to

$$\widehat{L}E_s(z,t) = i\hbar\alpha\sigma_{eg}(z,t)/d_{eg}, \tag{2}$$

where $\widehat{L} = \partial_z + c^{-1}\partial_t$, $\alpha = 4\pi\omega_s N|d_{eg}|^2/\hbar c$ is the coupling constant, and N is the density of resonant atoms in the absorber. The coupling constant α is related to the Beer's law absorption coefficient as $\alpha_B = \alpha/\gamma$. α_B is usually defined for a monochromatic radiation tuned in resonance. Below we use the wave equation in the form $\widehat{L}\Omega(z,t) = i\alpha\sigma_{eg}(z,t)/2$. By means of the Fourier transform,

$$F(\nu) = \int_{-\infty}^{+\infty} f(t)e^{i\nu t}dt, \tag{3}$$

Eqs. (1) and (2) are reduced to

$$\sigma_{eg}(z,\nu) = -\frac{\Omega(z,\nu)}{\nu + \Delta + i\gamma}, \tag{4}$$

$$\left[\frac{\partial}{\partial z} - \frac{i\nu}{c} + A(\nu)\right]\Omega(z,\nu) = 0, \tag{5}$$

where

$$A(\nu) = \frac{i\alpha/2}{\nu + \Delta + i\gamma}. \tag{6}$$

The solution of Eq. (5) is

$$\Omega(z,\nu) = \Omega(0,\nu)\exp[(i\nu z/c) - A(\nu)z], \tag{7}$$

whose inverse Fourier transform gives the familiar expression for the development of the radiation field in the resonant absorber in SVA and LR approximations, that is,

$$\Omega(z,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \Omega(0,\nu)\exp[-i\nu(t - z/c) - A(\nu)z]d\nu. \tag{8}$$

In some references (see, e.g., Refs. [6,14,21,22]), this integral is calculated by the method of contour integration, and the result is expressed in terms of the infinite sum of the Bessel functions of ascending order, multiplied by the complex coefficients, depending on the parameters α, Δ, γ, and t. Actually, there are two such expressions, one is for $\alpha z/2\gamma < \Delta t$ [6] (or $\alpha z/2\gamma < \gamma t$ in exact resonance [14]) and the other is for $\alpha z/2\gamma > \Delta t$ [6] (or $\alpha z/2\gamma > \gamma t$ in exact resonance [14]). Both expressions converge very slowly and, for example, if the resonant detuning is large one has to take many terms (between 50 and 100) of these sums to obtain an accurate approximation of the integral in Eq. (8).

To simplify calculation of the integral in Eq. (8) it is usually reduced with the help of the convolution theorem to (see, e.g., Refs. [5,7,18,19,23])

$$\Omega(z_0,t) = \int_{-\infty}^{+\infty} \Omega(0,t-\tau)R(z_0,\tau)d\tau, \tag{9}$$

where $R(z_0,\tau)$ is the output radiation from the absorber of length z_0, if the input radiation is a very short pulse whose shape is described by the Dirac delta function, $\delta(t)$ [i.e., $R(z_0,\tau)$ is a response function of the absorber of thickness z_0 to a very short pulse]. This function is [5,7,18,19,22–25]

$$R(z_0,t) = \delta(t) - e^{(i\Delta-\gamma)t}\Theta(t)\sqrt{\frac{b_0}{t}}J_1(2\sqrt{b_0 t}), \tag{10}$$

where $\Theta(t)$ is the Heaviside step function, $J_1(x)$ is the first-order Bessel function, and $b_0 = \alpha z_0/2 = \alpha_B z_0\gamma/2$. It should be noted that for $\Delta \neq 0$ the response function of the form, Eq. (10), is found only in Refs. [5,18,25].

The response function $R(z_0,t)$ is a sum of the input field $\delta(t)$ and FID (i.e., dipoles ringing, induced by the short pulse). It is educative to derive the FID part of the response function from qualitative arguments. The short pulse $\delta(t)$ induces single-sided exponential ringing of the dipoles in the absorber. It is described by the solution of Eq. (1) for the nondiagonal element of the density matrix with $\Omega(z,t)=\delta(t-z/c)$, which gives $\sigma_{eg}(z,t)=i\Theta(t-z/c)\exp[(i\Delta-\gamma)(t-z/c)]$, where z is a coordinate of a particle in the absorber. Below for simplicity of notations we disregard a small value z/c in the mathematical expressions.

After short-pulse excitation each dipole in the absorber can be considered as a radiation source. According to the solution of the wave equation [Eq. (2)] the field, emitted by a very thin layer of atoms at the front face of thickness dz, emits a field $\Omega_{\exp}(0,t)=-(\alpha/2)\Theta(t)\exp(i\Delta t-\gamma t)dz$. Here an exponential factor $\exp(i\Delta t)$ appears because dipoles oscillate with their own frequency ω_0 not equal to the carrier frequency ω_s of the exciting pulse. The field, which is emitted by dipoles, located at the front face of the absorber, is resonant for all other atoms in the absorber. Due to multiple scattering on the other atoms this field transforms at the output z_0 of a thick absorber to (see Ref. [21])

$$\Omega_{\exp}(z_0,t)=-\frac{\alpha}{2}\Theta(t)e^{(i\Delta-\gamma)t}J_0(\sqrt{2\alpha z_0 t})dz. \qquad (11)$$

This result is obtained in Ref. [21] by calculating the integral in Eq. (8) for the single-sided exponential input field $\Omega_{\exp}(0,t)$. Dipoles, located at distance $+z$ from the front face of the absorber in a thin layer of thickness dz, produce a field,

$$\Omega_{\exp}(z_0-z,t)=-\frac{\alpha}{2}\Theta(t)e^{(i\Delta-\gamma)t}J_0(\sqrt{2\alpha(z_0-z)t})dz, \quad (12)$$

at the output z_0 of the absorber. In LR approximation the fields emitted by different particles do not interact and they only interfere at the output. The result of the interference of the fields, produced by all particles of the absorber, is $\overline{\Omega}_{\exp}(z_0,t)=\langle\Omega_{\exp}(z_0-z,t)\rangle_z$, where

$$\begin{aligned}&\langle\Omega_{\exp}(z_0-z,t)\rangle_z\\&\quad=-\frac{\alpha}{2}\Theta(t)e^{(i\Delta-\gamma)t}\int_0^{z_0}J_0(\sqrt{2\alpha(z_0-z)t})dz. \quad (13)\end{aligned}$$

The integration (see Ref. [26]) gives

$$\overline{\Omega}_{\exp}(z_0,t)=-\Theta(t)e^{(i\Delta-\gamma)t}\sqrt{\frac{\alpha z_0}{2t}}J_1(\sqrt{2\alpha z_0 t}). \qquad (14)$$

This result is identical to the FID part of Eq. (10). It is important to notice that the phase of FID is 180° shifted with respect to the phase of the incident field.

III. RESONANT EXCITATION

Physical processes, which take place in TN, induced by a step pulse, FID following a rectangular pulse, and transient nutations, induced by a phase switch of the input field, can all be easily understood just from the analysis of the propagation of the step pulse. This is a basic element, whose interaction with a thick absorber contains all the processes actual for the listed phenomena. This is because (i) the rectangular pulse of the duration t_p, that is, $\Omega_R(0,t)=\Omega_0[\Theta(t)-\Theta(t-t_p)]$ can be considered as a sum of two infinite step pulses $\Omega_\Theta(0,t)$ and $-\Omega_\Theta(0,t-t_p)$, which are in antiphase, and they are applied at different moments of time, where $\Omega_\Theta(0,t)=\Omega_0\Theta(t)$, and (ii) the field whose phase suddenly changes by π at time t_d can be considered as a sum of two infinite step pulses $\Omega_\Theta(0,t)$ and $-2\Omega_\Theta(0,t-t_p)$, which are in antiphase and the amplitude of the second pulse is doubled with respect to the first one.

Moreover, since we consider the atom-field interaction in a linear response approximation, knowledge of the amplitude of the step pulse at the output of a thick absorber, $\Omega_\Theta(z_0,t)$, may give us the amplitude of the scattered field at the output in a very simple way.

To estimate the amplitude of the scattered field one can address the argument given in Feynman lectures [16]. According to Feynman, the light, transmitted by any sample, can be considered as a result of the interference of the input wave, as if it would propagate in vacuum, with the secondary wave radiated by the linear polarization induced in the medium. Then, following literally this argument, one can express the output field for the input step pulse as follows $\Omega_\Theta(z_0,t)=\Omega_\Theta(0,t)+\Omega_\Theta^{sc}(z_0,t)$, where $\Omega_\Theta^{sc}(z_0,t)$ is the scattered field, which is

$$\Omega_\Theta^{sc}(z_0,t)=\Omega_\Theta(z_0,t)-\Omega_\Theta(0,t). \qquad (15)$$

This field is just FID, observed at the end of the rectangular pulse. Also knowledge of the amplitude and phase of the scattered field helps to estimate the first maximum of the transients induced by the phase switch of the input field.

A. Rectangular pulse

In this subsection we consider transients, induced by a weak rectangular pulse with sharp edges if the pulse is in exact resonance, $\Delta=0$. According to Eq. (9) the front of the pulse, which is the step pulse, $\Omega_\Theta(0,t)$, is transformed at the output to

$$\Omega_\Theta(z_0,t)=\Omega_0\Theta(t)\left[1-\int_0^t e^{-\gamma\tau}\sqrt{\frac{b_0}{\tau}}J_1(2\sqrt{b_0\tau})d\tau\right]. \qquad (16)$$

Integration by parts, the integral in Eq. (16) reduces this expression to

$$\begin{aligned}\Omega_\Theta(z_0,t)=\Omega_0\Theta(t)\Big[&e^{-\gamma t}J_0(2\sqrt{b_0 t})\\&+\gamma\int_0^t e^{-\gamma\tau}J_0(2\sqrt{b_0\tau})d\tau\Big]. \qquad (17)\end{aligned}$$

As it is shown in Ref. [5] this representation of the output amplitude simplifies the analysis. The amplitude consists of two components, the first is a function decaying to zero and the second has the asymptote (see Ref. [26]),

$$\lim_{t\to+\infty}\left[\gamma\int_0^t e^{-\gamma\tau}J_0(2\sqrt{b_0\tau})d\tau\right]=e^{-b_0/\gamma}. \qquad (18)$$

The power of exponent in the right-hand side of Eq. (18) is $b_0/\gamma=\alpha_B z_0/2$. This exponent describes the Beer's low attenuation of the amplitude of the step pulse to the value $\Omega_0\exp(-\alpha_B z_0/2)$ at $t\to+\infty$.

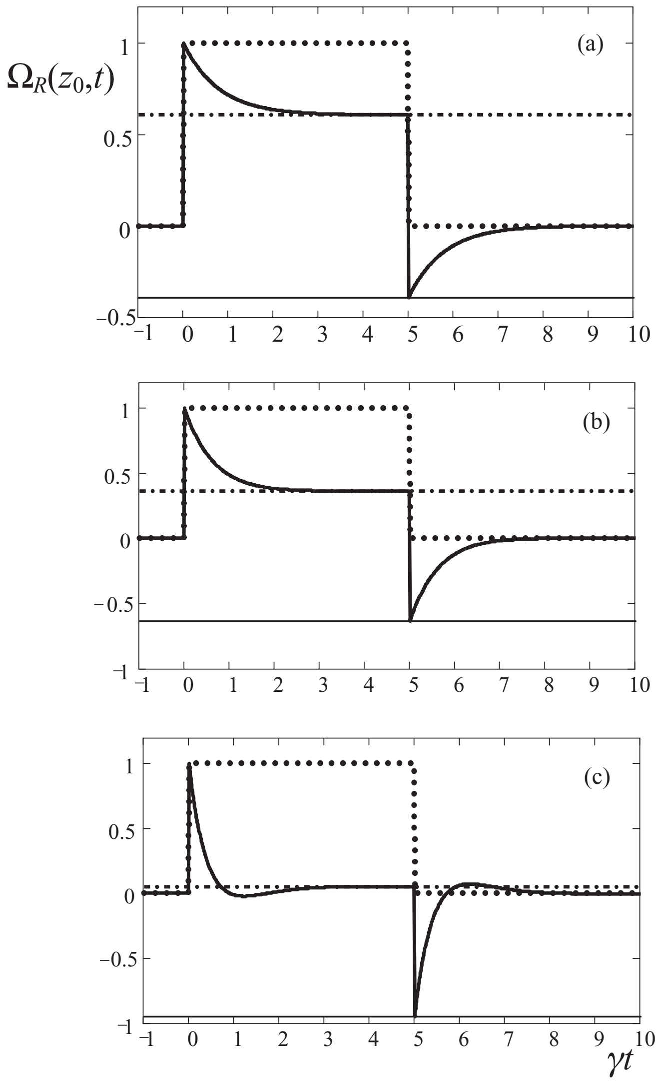

FIG. 1. Time evolution of the output pulse (thick solid line) for different values of the absorber thickness, $\alpha_B z_0$, which is 1 for (a), 2 for (b), and 6 for (c). Input pulse is shown by dots. Dash-dotted line shows the Beer's law attenuation level. Thin solid line shows the asymptote of the scattered field amplitude, Eq. (20). The amplitude is normalized to Ω_0.

Knowledge of the output for the step pulse from a thick resonant absorber allows one to find the output for the rectangular pulse, which is

$$\Omega_R(z_0,t) = \Omega_\Theta(z_0,t) - \Omega_\Theta(z_0,t-t_p). \tag{19}$$

Figure 1 shows time evolution of the output pulse (thick solid line) for different values of optical thickness of the absorber, $\alpha_B z_0$, which is equal to 1 for (a), 2 for (b), and 6 for (c). It is worth the reader's attention that the leading edges of the pulses, shown in Figs. 1(a) and 1(b), look very similar to the experimentally observed optical precursors in Ref. [14] (see Fig. 1 in this reference).

The plots clearly show that the sharp front of the rectangular pulse (whose shape is shown by dots in Fig. 1) escapes out of the absorber without interaction. This is because the polarization of the absorber, which is $P(z,t) = N d_{ge}\rho_{eg}(z,t)$, needs time to build up. Then, with time inversely proportional to b_0 this polarization rises producing a scattered field (or dipoles ringing), which is in antiphase with the input field [16] [see Eq. (15)]. Destructive interference of the input and scattered fields reduces the amplitude of the output field. Development time and the amplitude of the scattered field depend on the optical thickness of the absorber. The thicker the sample, the shorter the time and the larger the amplitude of the scattered field are.

The development rate of the scattered field is defined by the parameter b_0. The amplitude of the scattered field is zero for $t = 0$. With time this amplitude tends to the value,

$$\Omega_\Theta^{sc}(z_0,+\infty) = \Omega_0[\exp(-\alpha_B z_0/2) - 1], \tag{20}$$

and the amplitude of the output field tends to asymptote $\Omega_0 \exp(-\alpha_B z_0/2)$, shown by the dash-dotted line in Fig. 1. At $t = t_p$, when the field is switched off only the scattered field, $\Omega_\Theta^{sc}(z_0,t)$, is seen at the output. The asymptote of the amplitude of the scattered field, Eq. (20), is shown in Fig. 1 by the thin solid line.

As seen from Fig. 1(c), for a thick absorber the output field just before $t = t_p$ is almost zero. This observation tells us that the field is completely absorbed by the sample and we usually assume that this process is irreversible if the dephasing of the dipoles ($d_{ge}\rho_{eg}$), induced in the absorber, is irreversible. However, fast switch-off of the field is followed by the FID pulse whose amplitude is nearly the same as the amplitude of the input pulse if the absorber is very thick. Thus, the absorption is not an irreversible process and FID regenerates the field energy accumulated in the absorber in a form of the scattered field or dipoles ringing. In the next section another transient is considered whose amplitude is even greater.

Approximate expressions for $\Omega_\Theta(z_0,t)$ simplifying the analysis of the output field features and conventional derivations of the expression for $\Omega_R(z_0,t)$, which are different from the response function technique, Eq. (9), are given in the appendix, Secs. A and B. These derivations clarify the physics of the scattered field development, give the distribution of the polarization along the absorber, and help to propose new experiments in thick samples.

It should be noted that the approximation of $\Omega_\Theta(z_0,t)$, given in the appendix, Eq. (A6), is expressed as a sum of three terms proportional to the Bessel functions of the zero, first, and second orders. In many other papers on this topic the solution for the step pulse, $\Omega_\Theta(z_0,t)$, is given in a form of an infinite sum of the Bessel functions of ascending integer order. Even in the case when this solution is expressed via two Lommel's functions (see Ref. [5]), finally they are again expressed as the infinite sums of the Bessel functions with the coefficients $s_0^n(t/b_0)^{n/2}\exp(-s_0 t)$ or $(-s_0)^{-n}(t/b_0)^{-n/2}\exp(-s_0 t)$, where $s_0 = \gamma - i\Delta$ and n is the order of the Bessel function. This is because the Lommel's functions have the only representation via the expansion in terms of the Bessel functions.

B. Instantaneous phase shift of the incident field

We consider the propagation of the step pulse and assume that its phase φ ($\varphi = 0$) suddenly changes to π at time $t_d > 0$.

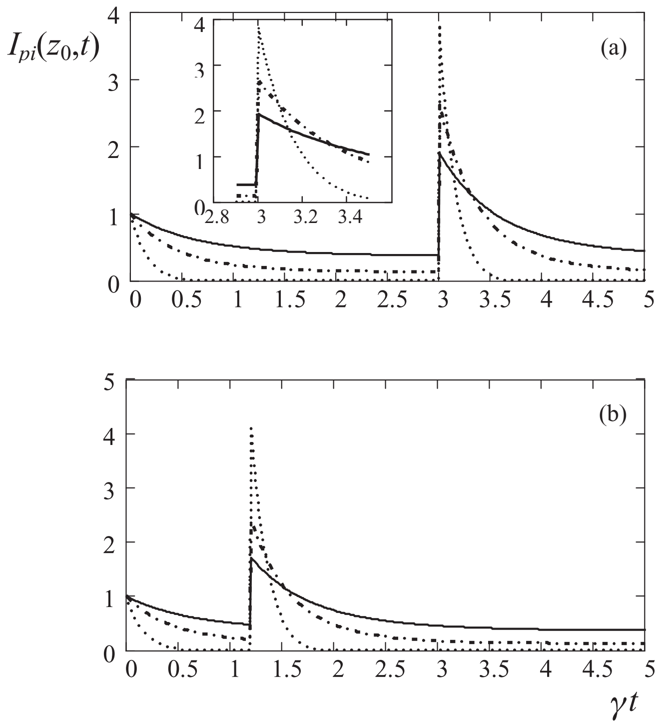

FIG. 2. Time evolution of the output step field, whose phase is shifted to π at $t_d = 3T_2$ (a), and $t_d = 1.2T_2$ (b). Solid line is for the absorber with $\alpha_B z_0 = 1$, dash-dotted line is for $\alpha_B z_0 = 2$, and dotted line is for $\alpha_B z_0 = 6$. The insert in (a) is zoom-in of the domain of the burst. The intensity is normalized to I_0.

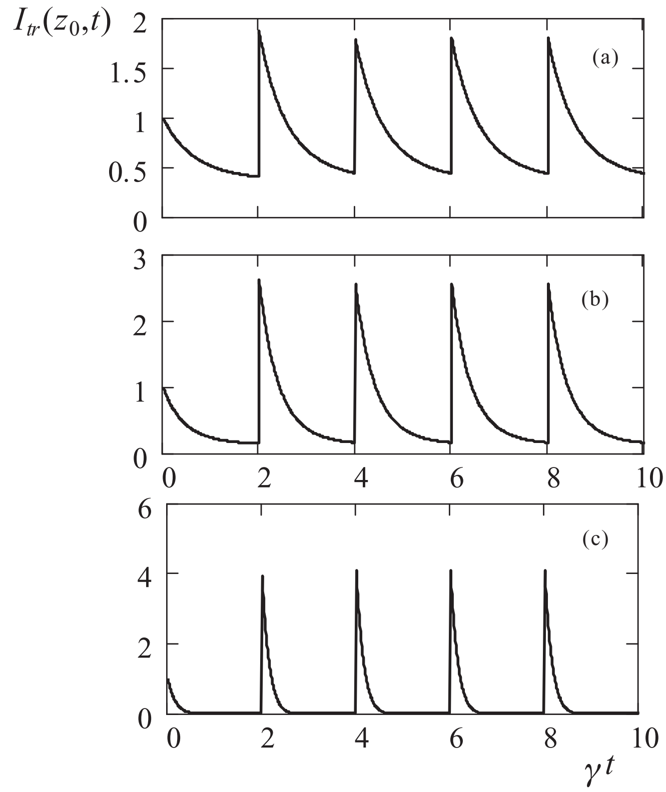

FIG. 3. Pulse train of the output field, induced by the sudden change of the phase of the input field at time intervals multiple to $t_d = 2T_2$. The optical thickness is 1 for (a), 2 for (b), and 6 for (c). The intensity is normalized to I_0.

The amplitude of such a field at the input of the resonant absorber can be described by the expression,

$$\Omega_{pi}(0,t) = \Omega_\Theta(0,t) - 2\Omega_\Theta(0,t - t_d). \qquad (21)$$

Then, with the help of the response function technique, one can easily derive the expression for the output field,

$$\Omega_{pi}(z_0,t) = \Omega_\Theta(z_0,t) - 2\Omega_\Theta(z_0,t - t_d), \qquad (22)$$

where $\Omega_\Theta(z_0,t)$ is defined in Eq. (17).

Figure 2 shows the time evolution of the output intensity $I_{pi}(z_0,t) = |\Omega_{pi}(z_0,t)|^2$ of the step pulse experiencing instantaneous π shift of its phase at $t = t_d$. This phase shift induces a radiation burst whose maximum intensity is even larger than the intensity of the input field, $I_0 = |\Omega_0|^2$. If t_d is relatively long with respect to the dephasing time of the atomic coherence $T_2 = 1/\gamma$, for example, it is equal to $3T_2$ as in Fig. 2(a), then the maximum amplitude of the transient nutation, induced by the phase shift, is estimated as follows. A shift of the field phase by π can be considered as a sudden switch-off of the field $\Omega_\Theta(0,t)$ and simultaneous switch-on of the field with the opposite phase and the same amplitude [i.e., $-\Omega_\Theta(0,t - t_d)$]. For large t_d the first process produces FID with maximum amplitude $\Omega_0[\exp(-\alpha_B z_0/2) - 1]$ at $t = t_d$. The field, which is switched on at t_d, is not absorbed in the very beginning and hence the amplitude of the radiation burst, $\Omega_{\max}$, is

$$\Omega_{\max} = \Omega_0[\exp(-\alpha_B z_0/2) - 2], \qquad (23)$$

and its intensity is $I_{\max} = I_0[\exp(-\alpha_B z_0/2) - 2]^2$. For a thick absorber ($\alpha_B z_0 \gg 1$) the intensity of the radiation burst exceeds nearly 4 times the intensity of the input field. The radiation enhancement arises due to the constructive interference of the phase-shifted field with the scattered field, produced by the field just before its phase shift. Both fields have phase π.

For shorter time t_d [see Fig. 2(b)] and for relatively thick samples (e.g., for $\alpha_B z_0 = 6$) the maximum intensity of the radiation burst is slightly more than 4 times larger with respect to I_0. This is because the amplitude of the scattered field $\Omega_\Theta^{sc}(z_0,t)$ for $\alpha_B z_0 = 6$ and at $t_d = 1.2T_2$ slightly exceeds the amplitude of the input field, Ω_0. Since they have opposite phases, their interference is seen as a small negative dip in Fig. 1(c).

It is interesting to notice that a train of successive phase shifts of the input field is capable of producing a train of spikes. If, for example, at the end of each time interval t_d the phase of the input field suddenly increases by π, then the output field amplitude changes as follows,

$$\Omega_{tr}(z_0,t) = \Omega_\Theta(z_0,t) + 2\sum_{k=1}^{n}(-1)^k\Omega_\Theta(z_0,t - kt_d), \qquad (24)$$

where n is the number of phase shifts. The plots for the intensity of the output field, $I_{tr}(z_0,t) = |\Omega_{tr}(z_0,t)|^2$, if $t_d = 2T_2$ are shown in Fig. 3 for the absorbers with optical thickness 1 (a), 2 (b), and 6 (c). For thick absorbers it is possible to use even a shorter time interval t_d (see Fig. 4), where t_d is T_2 for (a) and $T_2/2$ for (b).

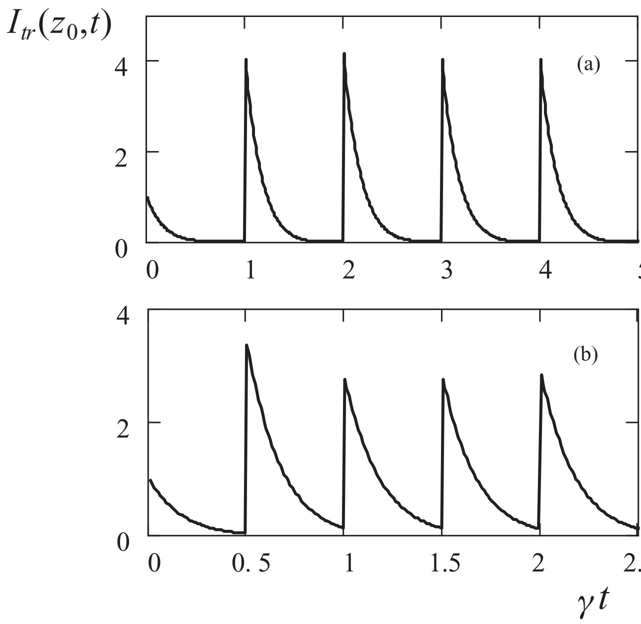

FIG. 4. The same pulse train as in Fig. 3 for $t_d = T_2$ (a) and $t_1 = T_2/2$ (b). Optical thickness is 6. The intensity is normalized to I_0.

Such a train of pulses can be used for a new type of communication. If we take, for example, an optical fiber, doped by resonant impurities, then a train of the phase-shift events produced with the continuous wave (cw) input field transforms into the train of the radiation bursts at the output with the same dwell time t_d. The information in this transmission line is coded in the presence or absence of the radiation burst at each particular moment of time $t_k = kt_d$, where k is a natural number. This way the information transmission looks robust against noise since no field is present between successive moments of time, t_k and t_{k+1}. Another advantage is low intensity of the input cw field and high contrast of the signal. This advantage comes from the fact that the energy of the signal pulse at the output is taken from the transmitting medium accumulating the energy of the cw field within the time interval t_d between signal pulses (i.e., within the dwell time of the phase shifts).

C. Slow change of the field phase

In practice the phase shift of the input field is not instantaneous and takes a finite time. To estimate the influence of the time scale of the phase change on the amplitude and duration of the radiation burst we model the phase evolution by the function,

$$\varphi(t) = \tan^{-1}[\delta\omega(t - t_d)] + \pi/2, \tag{25}$$

where $\delta\omega$ quantifies the rate of the phase change. According to Eq. (25) the phase rises from 0 to π, and if $\delta\omega$ is much larger than γ and b_0, one can consider $\varphi(t)$ as a step function. A time derivative of $\varphi(t)$, which is an instantaneous frequency of the field, is also a simple function,

$$\dot{\varphi}(t) = \frac{\delta\omega}{1 + \delta\omega^2(t - t_d)^2}. \tag{26}$$

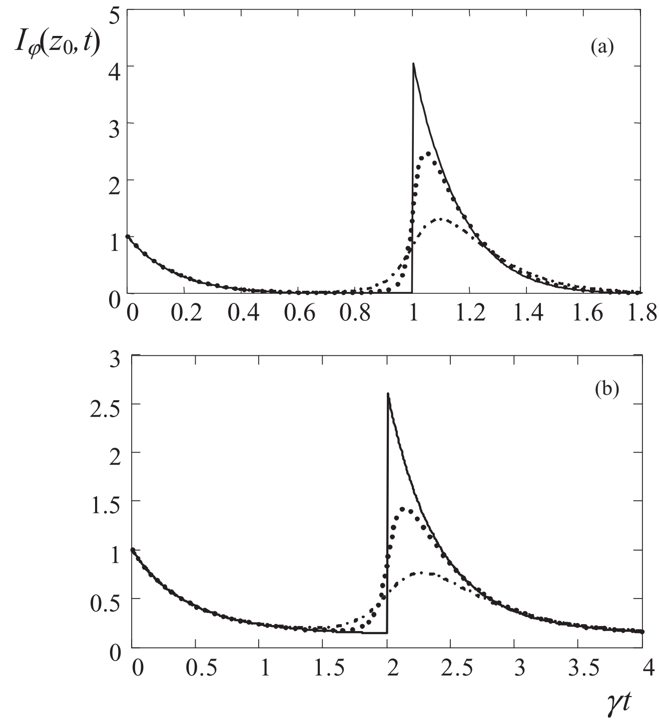

FIG. 5. The shape of the radiation burst, $I_\varphi(z_0,t) = |\Omega_\varphi(z_0,t)|^2$, for different rates of the phase shift δ, and different thickness of the absorber, specified by the parameter b_0. They are $b_0 = 3\gamma$, $\delta = 30\gamma$ (dotted line), and $\delta = 9\gamma$ (dash-dotted line) in (a), and $b_0 = \gamma$, $\delta = 9\gamma$ (dotted line), and $\delta = 3\gamma$ (dash-dotted line) in (b). The radiation intensity is normalized to $I_0 = \Omega_0^2$. Solid line shows the intensity for the instantaneous phase shift. Time t_d is $1/\gamma$ in (a) and $2/\gamma$ in (b).

We express the amplitude of the input step pulse, whose phase changes according to Eq. (25) in a time domain around $t_d > 0$, as follows $\Omega_\varphi(0,t) = \Omega_\Theta(0,t)\exp[i\varphi(t)]$. With the help of Eq. (9) one can derive for the resonant absorber that the output field is

$$\Omega_\varphi(z_0,t) = \Omega_0\Theta(t)\left[e^{i\varphi(t)} - b_0\int_0^t e^{i\varphi(t-\tau)-\gamma\tau}\frac{J_1(2\sqrt{b_0\tau})}{\sqrt{b_0\tau}}d\tau\right]. \tag{27}$$

If $\delta\omega \gg b_0$, the function $\varphi(t-\tau)$ can be approximated by the step function $\pi\Theta(t-\tau-t_d)$. Then, the integration domain of the integral in Eq. (27) is divided into two subdomains $(0,t_d)$ and (t_d,t). Integrating by parts these two integrals we find that Eq. (27) is reduced to Eq. (22) valid for the instantaneous phase shift.

For arbitrary relation between $\delta\omega$ and b_0 Eq. (27) is reduced to

$$\Omega_\varphi(z_0,t) = \Omega_0\Theta(t)[e^{-\gamma t+i\varphi(0)}J_0(2\sqrt{b_0 t}) + f_\varphi(t)], \tag{28}$$

where

$$f_\varphi(t) = \int_0^t [\gamma + i\dot{\varphi}(t-\tau)]e^{i\varphi(t-\tau)-\gamma\tau}J_0(2\sqrt{b_0\tau})d\tau. \tag{29}$$

Numerical calculation of the integral shows that, for example, for $\delta\omega = 100b_0$ the shape of the radiation burst is almost the same as for the instantaneous phase shift, except small smoothening of the initial peak. Figure 5 demonstrates two examples of the radiation burst, induced in the samples with different optical thickness [i.e., $b_0 = 3\gamma$ (a) and $b_0 = \gamma$ (b)]. We found that when δ is roughly 10 times larger than b_0, the amplitude of the radiation burst is large. For smaller δ, for example, for $\delta = 3b_0$, the amplitude of the radiation burst drops and its shape broadens.

Thus, to induce the radiation burst with large intensity the rate of the phase change is to be larger than the rate of the formation of the scattered field, which is defined by the parameter b_0.

To induce the radiation burst with intensity comparable to the intensity of the input field, the rate δ of the phase change should be at least three times larger than b_0. Therefore, to reduce the absolute value of this necessary rate it is preferable to use samples with moderate thickness, for example, $b_0 = \gamma$ as in Fig. 5(b). However, for samples with moderate thickness the absolute value of the amplitude of the scattered field decreases according to Eq. (20). For example, for $b_0 = \gamma$ the asymptote of this amplitude is $\Omega^{sc}_{\Theta}(z_0, +\infty) = -0.63\Omega_0$. Therefore, the maximum intensity of the radiation burst after the instantaneous phase switch at $t_d \gg 1/b_0$ is $I_{pi}(z_0,t_d) \approx [\Omega^{sc}_{\Theta}(z_0, +\infty) - \Omega_0]^2$. If, for example, $b_0 = \gamma$, the maximum intensity of the spike is $2.7I_0$ [see Fig. 5(b), solid line], which is appreciably smaller than the maximum intensity of the spike at the output of a thick absorber [see Fig. 5(a), solid line].

D. Rectangular pulse with smooth edges

It is interesting to notice that the expression for the amplitude of the output radiation field for the step pulse, $\Omega_\Theta(z_0,t)$, Eq. (17), is a sum of two terms. The first term, $\Omega_0\Theta(t)\exp(-\gamma t)J_0(2\sqrt{b_0 t})$, corresponds to the output of the radiation field whose input envelope is a single-sided exponent [i.e., $\Omega_1(t) = \Omega_0\Theta(t)\exp(-\gamma t)$]. The counterpart of the second term is $\Omega_2(t) = \Omega_0\Theta(t)[1 - \exp(-\gamma t)]$. Thus, the step pulse can be considered as a sum of the pulse $\Omega_1(t)$ with a sharp front and the smoothly rising pulse $\Omega_2(t)$. Their time dependencies at the output of a thick absorber are qualitatively different. The pulse $\Omega_1(t)$ demonstrates a precursor at the output of a thick absorber since the scattered field does not develop instantly. Then this pulse $\Omega_1(t)$ decays to zero with time. The pulse $\Omega_2(t)$ rises smoothly from 0 to Ω_0. Therefore the scattered field has time to develop in a thick absorber and it extinguishes the pulse $\Omega_2(t)$ due to their destructive interference. Just the pulse $\Omega_2(t)$ defines the amplitude of the step pulse at the output when $t \gg 1/b_0$ and obviously $\Omega_2(t)$ does not show a precursor at a time interval $0 < t < 1/b_0$ if $b_0 \geqslant \gamma$.

Actually the step pulse, as well as the rectangular pulse, are idealizations. Their edges are not infinitely sharp. We model the step pulse with finite rise time of its front as

$$\Omega_{\Theta f}(t) = \Omega_0\Theta(t)[1 - \exp(-rt)], \tag{30}$$

where r quantifies the rate of the pulse rise. This pulse is analogous to the pulse $\Omega_2(t)$, whose front rises with the rate γ. When the rate r of the pulse rise is small ($r \leqslant \gamma$), one can expect that the precursor would not be observed since the atomic dipoles have enough time to develop the response field.

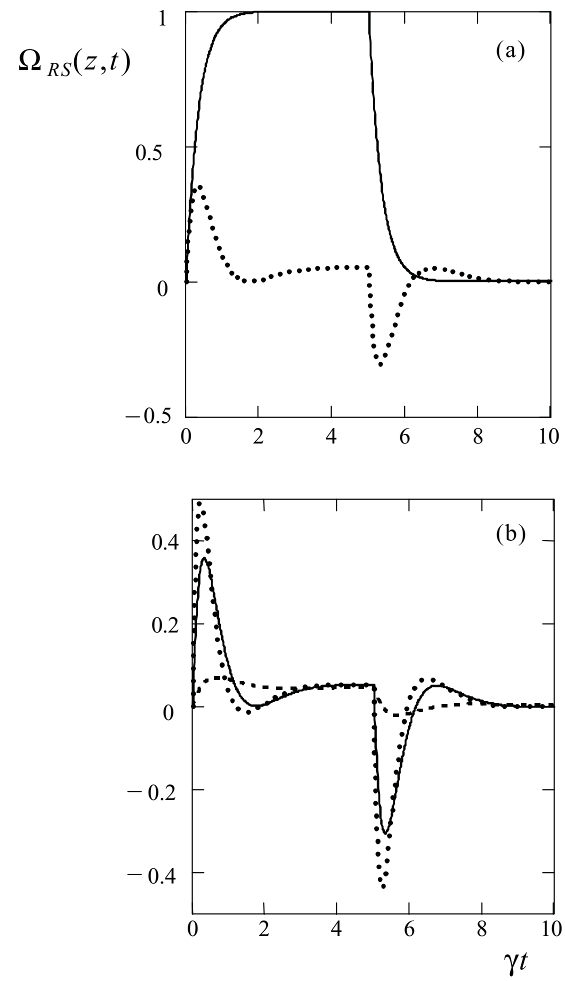

FIG. 6. (a) Time evolution of the rectangular pulse with smooth edges at the input (solid line) and output (dotted line) of the thick absorber with $b_0 = 3\gamma$. Pulse duration is $t_p = 5/\gamma$. The rate of the rise and fall of the pulse edges is $r = 3\gamma$. (b) Comparison of the time evolution of the pulse front at the output for $r = 0.3\gamma$ (dashed line), $r = 3\gamma$ (solid line), and $r = 6\gamma$ (dotted line). The field amplitude is normalized to Ω_0.

According to Eq. (9) the pulse amplitude at the output of a thick absorber is

$$\Omega_{\Theta f}(z_0,t) = \Theta(t)\Omega_0 \int_0^t [\gamma + (r - \gamma)e^{-r(t-\tau)}] \times e^{-\gamma\tau} J_0(2\sqrt{b_0\tau})d\tau. \tag{31}$$

The rectangular pulse with smooth edges we model by the function,

$$\Omega_{\mathrm{RS}}(t) = \Omega_{\Theta f}(t) - \Omega_{\Theta f}(t - t_p), \tag{32}$$

where t_p is a pulse duration. Its amplitude at the output of a thick absorber is

$$\Omega_{\mathrm{RS}}(z_0,t) = \Omega_{\Theta f}(z_0,t) - \Omega_{\Theta f}(z_0,t - t_p). \tag{33}$$

Examples of the amplitude evolution of the pulse are shown in Fig. 6. If $r \geqslant b_0 > \gamma$ the amplitudes of the precursor and FID are appreciably larger than the attenuated amplitude of the output radiation typical for the steady-state absorption [i.e., than $\Omega_0 \exp(-b_0/\gamma)$]. If $r \gg b_0$, the output field $\Omega_{\mathrm{RS}}(z_0,t)$

almost coincides with that [Eq. (19)] for the rectangular pulse with sharp edges. On the contrary, if $r \ll b_0$ the amplitude of the precursor and FID are negligibly small [see Fig. 6(b), dashed line]. In this case the scattered field has time to develop and it always compensates the incident field down to the level, defined by Beer's law, even when the pulse is switched on and off.

IV. NONRESONANT EXCITATION

It was shown in Ref. [20] that a nonresonant single-sided exponential pulse demonstrates qualitatively different transients in thick absorbers. In this section we show that the step pulse, rectangular pulse, and step pulse, whose phase instantly changes to π at $t = t_d$, also reveal unusual transients out of resonance.

A. Transient nutations induced by the step pulse

According to Eqs. (9) and (10) the nonresonant step pulse is transformed at the output of a thick absorber to

$$\Omega_\Theta(z_0,t) = \Omega_0\Theta(t)[e^{(i\Delta-\gamma)t}J_0(2\sqrt{b_0 t}) + f_\Theta(t)], \quad (34)$$

where

$$f_\Theta(t) = (\gamma - i\Delta)\int_0^t e^{(i\Delta-\gamma)\tau}J_0(2\sqrt{b_0\tau})d\tau. \quad (35)$$

The field $\Omega_\Theta(z_0,t)$ consists of two components, that is,

$$\Omega_r(z_0,t) = \Omega_0\Theta(t)e^{(i\Delta-\gamma)t}J_0(2\sqrt{b_0 t}), \quad (36)$$

and

$$\Omega_{nr}(z_0,t) = \Omega_0\Theta(t)f_\Theta(t). \quad (37)$$

The first component, $\Omega_r(z_0,t)$, decays to zero with the rate b_0. This component coincides with the output radiation field whose input envelope is a single-sided exponent $\Omega_r(0,t) = \Omega_0\Theta(t)\exp(i\Delta t - \gamma t)$. This field is in resonance with the absorber since its frequency coincides with the atomic frequency ω_0 [i.e., $\Omega_r(0,t)\exp(-i\omega_s t) = \Omega_0\Theta(t)\exp(-i\omega_0 t - \gamma t)$].

The second component coincides with the output field whose input envelope is $\Omega_{nr}(0,t) = \Omega_0\Theta(t)[1 - \exp(i\Delta t - \gamma t)]$. It has no sharp front. If $|\Delta| \gg \gamma$ the amplitude of this field rises with the rate Δ. This component is out of resonance. Since the function $f_\Theta(t)$ in Eq. (37) has the asymptote (see Ref. [26]),

$$\lim_{t\to+\infty} f_\Theta(t) = \exp\left(-\frac{b_0}{\gamma - i\Delta}\right), \quad (38)$$

the amplitude of the output field $\Omega_{nr}(z_0,t)$ tends to nonzero, constant value. In resonance, $\Delta = 0$, the amplitude of the second component tends to a very small value, defined by Beer's law, if $b_0 \gg \gamma$. If $|\Delta| \gg \gamma$, the second component experiences small absorption and acquires a phase, which depends on the parameters b_0 and Δ.

The nonresonant component of the field, $\Omega_{nr}(0,t)$, propagates with reduced group velocity (see Refs. [27,28]). Therefore, the front of $\Omega_{nr}(0,t)$ experiences appreciable delay seen at the output as a slow development rate of this component [i.e., $\Omega_{nr}(z_0,t)$]. The resonant component $\Omega_r(0,t)$ escapes from interaction for very short t close to zero, demonstrating precursor at the output of a thick absorber. Then, for longer times $\Omega_r(z_0,t)$ experiences fast decay. Superposition of these components is seen as the precursor, decaying fast, and then a slow rise of the field amplitude to almost the same value as at the input. Examples of such an evolution are shown in Fig. 7. For larger ratio $|\Delta|/b_0$ the slow component develops faster. A rough estimate of the development rate of the slow field is given in the appendix, Sec. C.

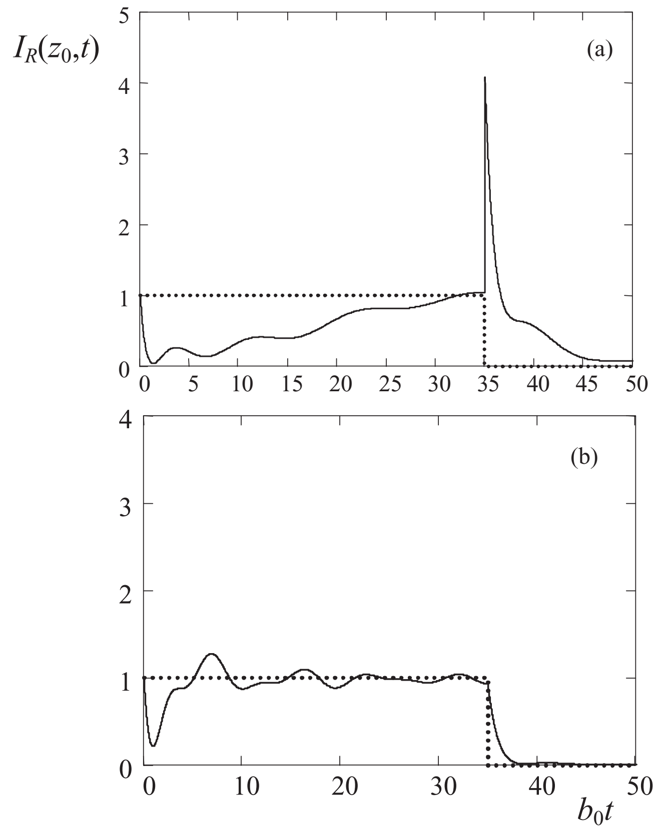

FIG. 7. Time evolution of the rectangular pulse at the output of absorber with thickness parameter $b_0 = 100\gamma$ (solid line). The resonant detuning is $\Delta = 31.8\gamma$ in (a) and $\Delta = 100\gamma$ in (b). The input pulse is shown by dots. The intensity is normalized to I_0.

B. FID at the end of the rectangular pulse

At the end of the rectangular pulse the output field is described by Eq. (19). Therefore, FID, following switch-off of the nonresonant field, has two main contributions. The first is mostly defined by the term $-\Omega_r(z_0,t - t_p)$ and the second comes from $\Omega_{nr}(z_0,t)$. If $b_0 t_p \gg 1$ the amplitude of the first component at $t = t_p$ is $\overline{\Omega}_1(t_p) = -\Omega_0$, and the amplitude of the second component is $\overline{\Omega}_2(t_p) \approx \Omega_0\exp[-b_0/(\gamma - i\Delta)]$. Thus, maximum amplitude of FID at $t = t_p$ is

$$\Omega_{\rm FID}(z_0,t_p,t_p) \approx -\Omega_0 + \overline{\Omega}_2(t_p). \quad (39)$$

In resonance the second component is almost zero if $b/\gamma \gg 1$. Out of resonance $\overline{\Omega}_2(t_p)$ is large and it is defined by the field propagating with reduced group velocity. The phase of this field,

$$\varphi_{sl} = -\frac{\Delta b_0}{\Delta^2 + \gamma^2}, \tag{40}$$

equals $\pm\pi$ if $\Delta = \mp\Delta_\pi$ and $b_0 > 2\pi\gamma$, where

$$\Delta_\pi = \frac{b_0}{2\pi} + \sqrt{\left(\frac{b_0}{2\pi}\right)^2 - \gamma^2}. \tag{41}$$

The amplitude of this field, $\Omega_0 \exp(-\pi\gamma/\Delta_\pi)$, is close to Ω_0 if $\Delta_\pi \gg \pi\gamma$. Thus, at $t = t_p$ and for $b_0 t_p \gg 1$, $\Delta = \mp\Delta_\pi$ FID has the amplitude $-2\Omega_0$ and its intensity is $4I_0$. This is a result of constructive interference of the fast and slow components of the field. Such a constructive interference of two fields is shown in Fig. 7(a) where $\Delta = \Delta_\pi$. An opposite example, when constructive interference of the two fields does not happen, is shown in Fig. 7(b). For the chosen parameters ($b_0 = \Delta = 100\gamma$) the intensity of FID at $t = t_p$ is $I_{\text{FID}}(t_p) = |-\Omega_0 + \overline{\Omega}_2(t_p)|^2 = 0.91 I_0$ since $\overline{\Omega}_2(t_p) = \Omega_0(0.535 - i0.833)$.

C. Transients induced by the instantaneous phase shift

If the phase of the step pulse $\varphi = 0$ instantly changes to π at time t_d, the field at the output of a thick absorber acquires fast transients, which are described by Eq. (22). Two examples of these transients are shown in Fig. 8. The amplitude of the transients at $t = t_d$ is defined by two contributions. They are FID, induced by the pulse switch-off, $\Omega_{\text{FID}}(z_0,t,t_d) = \Omega_\Theta(z_0,t) - \Omega_\Theta(z_0,t - t_d)$, and transient nutations, induced by the step-pulse switch-on with opposite phase, $\Omega_{TN}(z_0,t,t_d) = -\Omega_\Theta(z_0,t - t_d)$. As shown in the previous subsection, FID also has two contributions (i.e., fast and slow). Transient nutations, induced by the phase-shifted field, have large amplitude at $t = t_d$, $\Omega_{\text{TN}}(z_0,t_d,t_d) = -\Omega_0$, because the front of the phase-shifted field escapes from interaction with the absorber. Thus, if $b_0 t_d \gg 1$, the total amplitude of the output field at $t = t_d$ is

$$\Omega_{pi}(z_0,t_d) \approx -2\Omega_0 + \Omega_{sl}(t_d), \tag{42}$$

where $\Omega_{sl}(t_d) \approx \Omega_0 \exp[-b_0/(\gamma - i\Delta)]$ is a slow component of the field developed before t_d. If $\Delta = \Delta_\pi$, the phase of the slow field $\Omega_{sl}(t_d)$ is π and all three fields interfere constructively. They are the fast and slow components of FID, and the fast component produced by the phase-shifted field. If $\Delta_\pi \gg \pi\gamma$, the amplitude of the radiation burst at $t = t_d$ is three times larger than the amplitude of the input radiation field, and its intensity is nine times larger than the intensity of the input field [see Fig. 8(a)]. If the absolute value of the phase of the slow field, φ_{sl}, is much smaller than π, the slow field interferes destructively with two other components of the output field and the intensity of the radiation burst decreases. An example of reduced transients at $t = t_d$ is shown in Fig. 8(b) where $\varphi_{sl} = -0.5$ radians.

It is interesting to compare the distribution of the atomic coherence $\sigma_{eg}(z,t)$ along the absorber just before the phase shift of the field for resonant and nonresonant excitations. For the resonant excitation spatial and temporal dependence of the atomic coherence is derived in the appendix, Sec. B [see Eq. (A14)]. It is easy to show that for the nonresonant excitation one can obtain the expression for $\sigma_{eg}(z,t)$ simply

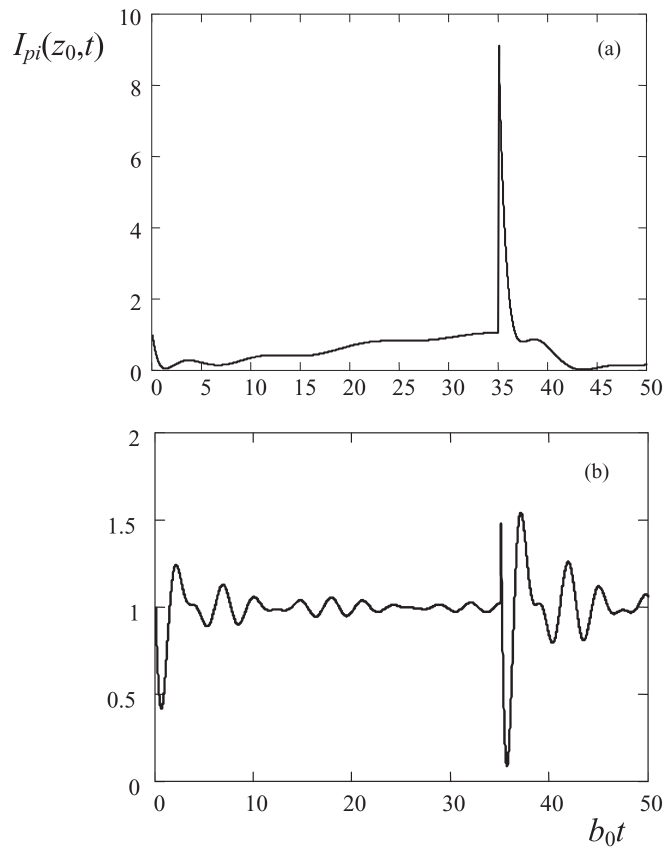

FIG. 8. Transients induced by the π shift of the field phase at $t_d = 35/b_0$. The parameter of the absorber thickness is $b_0 = 100\gamma$. The resonant detuning is $\Delta = 0.318 b_0$ in (a) and $\Delta = 2b_0$ in (b). The field intensity is normalized to I_0.

replacing the parameter γ in Eq. (A14) with $\gamma - i\Delta$. The spatial dependence of the absolute value of the coherence, $|\sigma_{eg}(z,t)|$ at $t = 0.35/\gamma$ is shown in Fig. 9(a) for the resonant (solid line) and nonresonant (dashed line) excitations. The nonresonant detuning is $\Delta = 31.8\gamma$, which corresponds to Δ_π for $b = 100\gamma$. For the nonresonant excitation the atomic coherence is relatively homogeneously distributed along the absorber, while in resonance it is mostly concentrated in the front domain of the absorber. In resonance the atomic coherence has pure imaginary value whose sign changes with distance. As a result the absorber is divided into domains such that in neighboring domains the atomic coherence has opposite phases [see Fig. 9(b)]. Out of resonance the imaginary part of the atomic coherence is almost homogeneous along the absorber. The real part of the atomic coherence changes almost linearly with distance crossing zero at a particular coordinate.

D. Dephasing of the atomic coherence far from resonance

In gases as well as in solids the radiative broadening of the absorption line is not the only mechanism of the dephasing of the atomic coherence. In many cases the nonradiative contribution to the dephasing rate γ is dominant, i.e., $\gamma = \gamma_r + \gamma_{nr}$ and $\gamma_{nr} \gg \gamma_r$, where γ_r is the contribution of the spontaneous decay of the excited state e and γ_{nr} is the contribution of the nonradiative decay of the atomic coherence $\sigma_{eg}(t)$. Usually the absorbtion coefficient in Beer's law, $\alpha_B = \alpha/\gamma$, is defined for

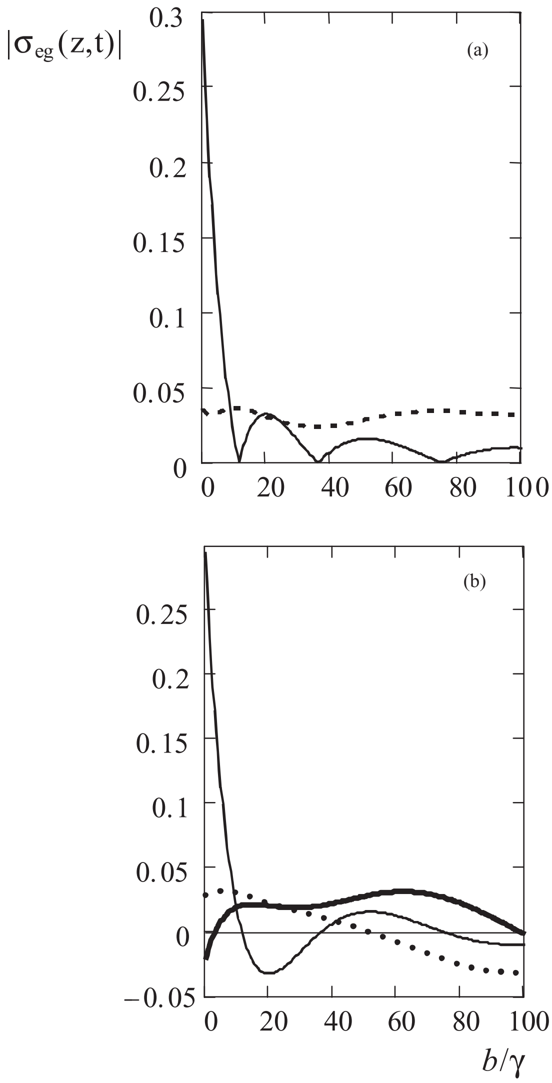

FIG. 9. (a) The distribution of the absolute value of the atomic coherence along the absorber for resonant (solid line) and nonresonant (dashed line) excitations. (b) The distribution of the atomic coherence for resonant excitation (thin solid line). The distribution of the real (dotted line) and imaginary (thick solid line) components of the atomic coherence along the absorber for nonresonant excitation. Resonant detuning is $\Delta = 31.8\gamma$. The duration of the excitation is $t_p = 0.35/\gamma$. The value of the atomic coherence is normalized to Ω_0/γ.

the monochromatic radiation tuned in resonance and the decay rate in this formula is the total decay rate γ. In Ref. [28] it is shown that far from resonance, $|\Delta\tau_c| \gg 1$, where τ_c is the correlation time of the stochastic processes, responsible for the nonradiative broadening, the contribution of γ_{nr} is canceled, and the dephasing rate is determined only by the radiative decay rate γ_r. Since the Beer's law coefficient is defined in exact resonance, far from resonance the effective thickness of the absorber increases substantially due to the suppression of the nonradiative decay, i.e., $\alpha_{\mathrm{Bout}} = \alpha_{\mathrm{Bin}}\gamma_{nr}/\gamma_r \gg \alpha_{\mathrm{Bin}}$ if $\gamma_{nr} \gg \gamma_r$, where $\alpha_{\mathrm{Bin}} = \alpha/\gamma$ is the absorption coefficient in resonance and α_{Bout} is the absorption coefficient [without the spectral factor, see Eq. (6)] far from resonance. The actual thickness parameter b is not modified out of resonance since $b = \alpha_{\mathrm{Bout}}\gamma_r/2 = \alpha_{\mathrm{Bin}}\gamma_{nr}/2$. However the condition $b \gg \gamma_r$, considered in this section, is easily satisfied for moderately thick samples if $|\Delta\tau_c| \gg 1$.

Inhomogeneous broadening is also not effective if $|\Delta| > 3\Delta_{inh}$, where Δ_{inh} is a half width of the inhomogeneously broadened absorption line (see Refs. [28–30]). This is because far wings of the inhomogeneously broadened absorption line with Gaussian distribution of the resonant frequencies are actually Lorentzian wings caused by radiative broadening.

For the resonant excitation of the inhomogeneously broadened absorption line with a Gaussian shape the response function, given in Eq. (10), is not applicable. Its analytical derivation is not trivial and up to date only numerical calculation of the integral in Eq. (8) is possible to perform analysis of optical transients. This analysis is quite lengthy and time consuming, and it is planned for future.

Meanwhile, as an example, inhomogeneous broadening with Lorentzian distribution of resonant frequencies was considered in Ref. [22]. Since the convolution of two Lorentzians is Lorentzian, the response function of such a medium coincides with that, given in Eq. (10), where γ is substituted by $\gamma + \Delta_{inh}$.

V. CONCLUSION

Interaction of a weak radiation field with thick resonant absorbers is studied theoretically. Coherent excitation of the atomic dipoles in the absorber induces their coherent ringing seen as a coherently scattered radiation field. Since the phase of the coherently scattered radiation is opposite the phase of the incident radiation, these fields interfere destructively. As a result the radiation field decreases substantially at the output of a thick absorber.

If the input pulse has a sharp leading edge its front escapes the attenuation because the scattered field needs time to develop. Therefore the decrease of the intensity of the output radiation is not instantaneous. It develops later with the rate defined by the absorber thickness. Thus, the optical precursor appears at the output, followed by optical transients decaying to a steady-state intensity, which is consistent with Beer's law attenuation. Fast switch-off of the incident radiation field produces large intensity FID. It is just coherent ringing of the induced dipoles continuing to produce a coherently scattered field. This field is not compensated by the input field after its switch-off. Therefore a thick absorber produces FID whose amplitude is almost the same as the amplitude of the input field and whose phase is opposite.

Another transient is produced by a fast phase switch of the field from zero to π. Then FID and the phase-shifted input field interfere constructively producing a radiation burst whose intensity is four times larger than the intensity of the input field. Such a phase shift is proposed for use in communication. The information can be coded in a train of pulses (radiation bursts), induced by the phase switch. Between the signals the energy is stored in a transmitting line in a form of the destructively interfering input field and dipoles ringing. An optical fiber, doped with resonant impurities, is proposed as a transmitting line.

If the step-pulse input field is far from resonance its front also escapes from interaction forming the optical precursor.

Then, the start of the dipoles ringing with the frequency of the input field form a pulse propagating with a reduced group velocity and a phase different from the phase of the input pulse. As a result of the delayed atomic response the front of the step pulse is transformed into the smoothly rising pulse, whose front is delayed and spread in time. Thus, optical transients, induced by the step pulse, are split into the optical precursor, followed by a dip, and a delayed part whose intensity is close to the intensity of the input pulse.

Fast switch-off of the input field leaves the atomic dipoles free and then they start to oscillate with their own frequency, which is different from the frequency of the input field. In this case two coherent fields are developed, i.e., FID oscillating with the resonant frequency of atoms and the slowly propagating field, which is still present in the absorber and whose frequency coincides with the frequency of the input field. The phase of FID is π. The phase of the slow field depends on the optical thickness of the absorber and resonant detuning. If they are chosen such that the phase of the slow field is also π, FID and the slow field interfere constructively after switch-off of the input field. Then, the radiation burst is observed whose intensity is four times larger than the intensity of the input field.

A fast phase switch of the field to π brings the input field, FID, and the slow field in phase. Their constructive interference produces a radiation burst whose intensity is nine times larger than the intensity of the input field.

The effect of the phase switch was experimentally studied for the resonant γ quanta in Refs. [18–20]. Anomalous radiation burst after the phase switch was observed in Ref. [20] at the output of the thick absorber for the nonresonant excitation by γ quanta.

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APPENDIX

A. Step-pulse analysis

To facilitate calculation of the integral in Eq. (8) for the output of the step pulse from a thick sample one can apply the Laplace transform,

$$F(s) = \int_0^{+\infty} e^{-sb} f(b) db, \tag{A1}$$

to the function $\Omega_\Theta(z,t)$, which is assumed to be a function of the single real variable $b = \alpha z/2$. Here s is a complex Laplace variable. For the step pulse the Fourier transform $\Omega(0,\nu)$ in Eq. (8) is $\Omega_\Theta(0,\nu) = i\Omega_0/\nu$. The Laplace transformation of the transmission function,

$$\exp\left(-i\nu - \frac{ib}{\nu + i\gamma}\right), \tag{A2}$$

reduces the Laplace transform of Eq. (8) to

$$\Omega_\Theta(s,\nu) = \frac{i}{2\pi}\int_{-\infty}^{+\infty} \frac{e^{-i\nu t}}{\nu\left(s + \frac{i}{\nu+i\gamma}\right)} d\nu. \tag{A3}$$

This integral is easily calculated by the method of residues. The result is

$$\Omega_\Theta(s,\nu) = \frac{\gamma}{\gamma s + 1} + \frac{e^{-\gamma t - t/s}}{s(\gamma s + 1)}. \tag{A4}$$

The inverse Laplace transform of Eq. (A4) is found with the help of the convolution theorem and tables of Laplace transforms (see, e.g., Ref. [31]),

$$\Omega_\Theta(z,t) = e^{-b/\gamma} + \frac{e^{-\gamma t}}{\gamma}\int_0^b e^{-\beta/\gamma} J_0(2\sqrt{(b-\beta)t}) d\beta. \tag{A5}$$

The main contribution to the integral in Eq. (A5) is given by the domain where β is small. Therefore, if one takes only the first three terms of the Taylor expansion of the function $J_0(2\sqrt{(b-\beta)t})$ in the vicinity of $\beta = 0$, then the integral is easily calculated. The approximated result is

$$\Omega_\Theta(z,t) = e^{-b/\gamma} + e^{-\gamma t}\left[f_0(b) J_0(2\sqrt{bt}) + f_1(b,t)\frac{J_1(2\sqrt{bt})}{\sqrt{bt}} + f_2(b,t)\frac{J_2(2\sqrt{bt})}{bt}\right], \tag{A6}$$

where

$$f_0(b) = 1 - e^{-b/\gamma}, \tag{A7}$$

$$f_1(b,t) = \gamma t\left[1 - \left(1 + \frac{b}{\gamma}\right) e^{-b/\gamma}\right], \tag{A8}$$

$$f_2(b,t) = (\gamma t)^2\left[1 - \left(1 + \frac{b}{\gamma} + \frac{b^2}{2\gamma^2}\right) e^{-b/\gamma}\right]. \tag{A9}$$

Comparison of the approximated time evolution of $\Omega_\Theta(z_0,t)$, Eq. (A6), with exact function, Eq. (17), for the output of the step pulse from absorbers with different optical thickness Th $= \alpha_B z_0$ is shown in Fig. 10. Over a wide range of values of the parameter Th these functions are quite close to each other.

B. Conventional derivation of the FID signal

In this subsection FID signal, induced by the resonant rectangular pulse ($\Delta = 0$), is derived if the distribution of the polarization along the sample, when the pulse is switched off, is known. This derivation is useful for the qualitative understanding of the formation of FID in a thick sample.

According to Eq. (4) the atomic coherence $\sigma_{eg}(z,t)$, induced by a pulse $\Omega(z,t)$, is

$$\sigma_{eg}(z,t) = i\int_{-\infty}^{+\infty} e^{-\gamma(t-\tau)}\Theta(t-\tau)\Omega(z,\tau) d\tau. \tag{A10}$$

For the step pulse the distribution of the field amplitude, $\Omega_\Theta(z,\tau)$, along the sample is derived in Sec. III [see Eq. (16)]. Substituting $\Omega_\Theta(z,\tau)$ into Eq. (A10) one obtains

$$\sigma_{eg}(z,t) = i\Omega_0\Theta(t)e^{-\gamma t}\left[F_1(z,t) + F_2(z,t)\right], \tag{A11}$$

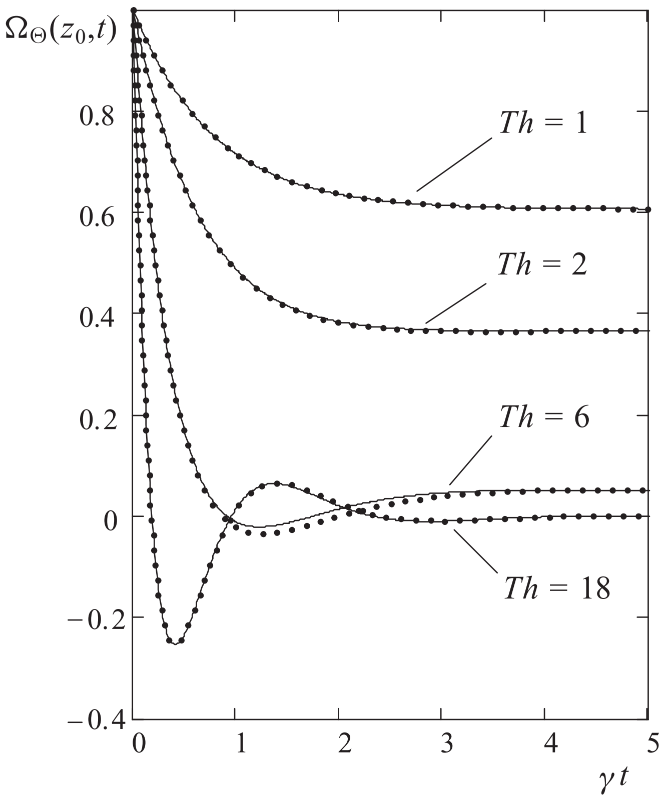

FIG. 10. Time evolution of the front of the step-pulse output for different values of the absorber thickness $\mathrm{Th} = \alpha_B z_0$. Thin solid line represents the function, which is given by Eq. (17). The approximation, Eq. (A6), is shown by dots. The amplitude of the output pulse is given in units of the amplitude of the input pulse Ω_0.

where

$$F_1(z,t) = \int_0^t J_0(2\sqrt{b\tau})d\tau. \tag{A12}$$

$$F_2(z,t) = \gamma \int_0^t d\tau_1 \int_0^{\tau_1} d\tau_2 e^{\gamma(\tau_1-\tau_2)} J_0(2\sqrt{b\tau_2}). \tag{A13}$$

Change of the order of the integrals in $F_2(z,t)$ and integration of the integral with a variable τ_2 gives

$$\sigma_{eg}(z,t) = i\Omega_0\Theta(t)\int_0^t e^{-\gamma\tau} J_0(2\sqrt{b\tau})d\tau. \tag{A14}$$

Figure 11 shows a distribution of the coherence $\sigma_{eg}(z,t)$ along the sample at different moments of time t elapsed from the pulse switch on, i.e., $T_2/2$, T_2, and $3T_2$, where $T_2 = 1/\gamma$ is a homogeneous dephasing time of the atomic coherence. It is obvious that with time the atomic polarization of a noticeable amplitude is mostly concentrated in the front domain of the absorber, i.e., for $0 < D < 5$, where $D = \alpha_B z$ is the optical depth measured from the input. For $D > 5$ the value of this coherence is almost negligible.

After switch-off of the pulse at $t = t_p$ the atomic coherence decays as

$$\sigma_{eg}(z,t)|_{t\geqslant t_p} = e^{-\gamma(t-t_p)}\sigma_{eg}(z,t_p), \tag{A15}$$

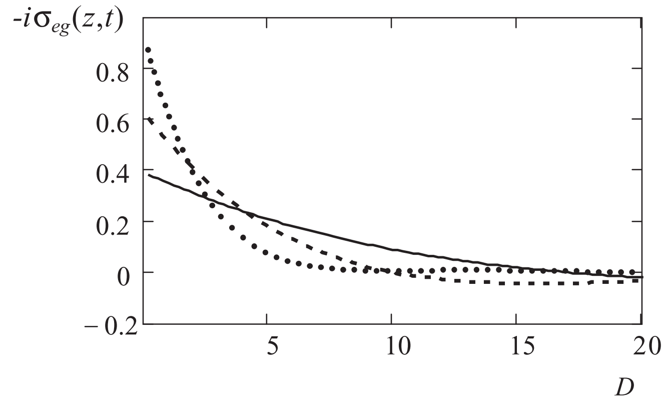

FIG. 11. Distribution of the coherence $\sigma_{eg}(z,t)$ along z in units of nondimensional optical depth $D = \alpha_B z$ at different moments of time t, which are $T_2/2$ (solid line), T_2 (dashed line), and $3T_2$ (dotted line). The value of the coherence is normalized to Ω_0/γ.

where $\sigma_{eg}(z,t_p)$ is defined in Eq. (A14). Each thin slice of the sample of thickness dz emits a radiation field,

$$\Omega_r(z,t,t_p) = i\frac{\alpha}{2}e^{-\gamma(t-t_p)}\sigma_{eg}(z,t_p)dz, \tag{A16}$$

which is a result of the dipoles ringing in a thin slice. The field, which is produced by dipoles in the slice, located at distance $+z$ from the front face of the absorber, propagates a distance $z_0 - z$ through the absorber with resonant particles. According to the arguments given in Sec. II [see Eqs. (11) and (12)] this field transforms to

$$\Omega_{\mathrm{out}}(z_0 - z,t,t_p) = \Omega_r(z,t,t_p)J_0\left(\sqrt{2\alpha(z_0 - z)(t - t_p)}\right), \tag{A17}$$

at the output of the absorber with coordinate z_0. The sum of all these fields gives the FID signal, which is

$$\Omega_{\mathrm{FID}}(z_0,t,t_p) = \int_0^{z_0} \Omega_{\mathrm{out}}(z_0 - z,t,t_p)dz. \tag{A18}$$

The explicit form of this expression is

$$\Omega_{\mathrm{FID}}(z_0,t,t_p) = -\Omega_0\int_0^{b_0} db \int_0^{t_p} d\tau e^{-\gamma(t-t_p+\tau)} F_3(t-t_p,\tau,b), \tag{A19}$$

where

$$F_3(t-t_p,\tau,b) = J_0(2\sqrt{b\tau})J_0(2\sqrt{(b_0-b)(t-t_p)}). \tag{A20}$$

With the help of the Laplace transform, Eq. (A1), and the convolution theorem one finds that

$$\int_0^{b_0} J_0(2\sqrt{b\tau})J_0(2\sqrt{(b_0-b)(t-t_p)})db \risingdotseq \frac{1}{s^2}e^{(t-t_p+\tau)/s}, \tag{A21}$$

where the right-hand side of the equation is the Laplace transform of the left-hand side integral. The inverse Laplace

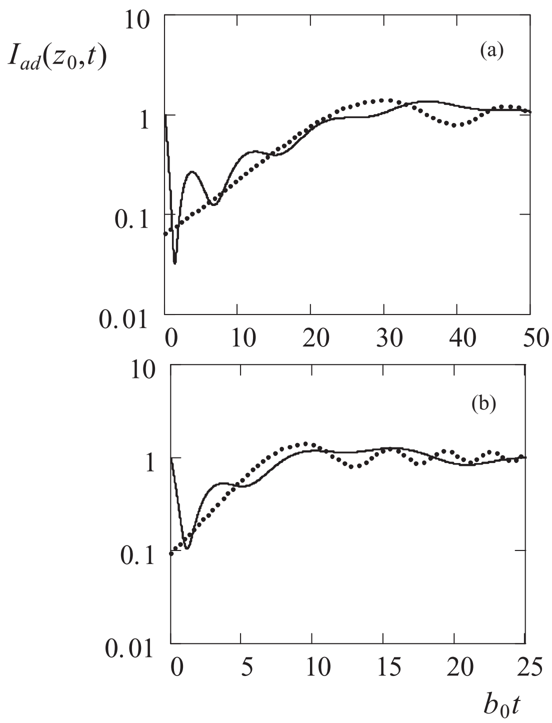

FIG. 12. Intensity of the slow adiabatic component of the step pulse, $I_{ad}(z_0,t) = |\Omega_{ad}(z_0,t)|^2$, is shown by dots. Output intensity of the step pulse is shown by the solid line. Both are normalized to I_0. The ratio of Δ/b_0 is 0.3 in (a) and 0.6 in (b).

transformation of the obtained result allows one to reduce Eq. (A19) to

$$\Omega_{\mathrm{FID}}(z_0,t,t_p) = -\Omega_0 b \int_0^{t_p} d\tau e^{-\gamma(t-t_p+\tau)} \frac{J_1(2\sqrt{b(t-t_p+\tau)})}{\sqrt{b(t-t_p+\tau)}}. \quad \text{(A22)}$$

Integration by parts of the integral in Eq. (A22) gives the expression, which is identical to Eq. (19) obtained in Sec. II for FID signal with the help of the response function technique.

C. Slow part of the step pulse

With the help of the adiabatic following approximation, Refs. [27,28], we derive the expression describing time evolution of the slow component of the step pulse, $\Omega_{\Theta}(z_0,t)$, whose front experiences time delay and spreading. For simplicity we consider the case when the parameters b_0 and $\Delta$are much larger than γ. Then Eq. (8) can be approximated as

$$\Omega_{\Theta}(z_0,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{i}{\nu} \exp\left(-i\nu t - \frac{ib_0}{\nu+\Delta}\right) d\nu. \quad \text{(A23)}$$

In Ref. [27] it is shown that, for example, the nonresonant rectangular pulse is split at the output of a thick absorber into two components (i.e., the fast nonadiabatic and slow adiabatic components). The propagation of the slow adiabatic component is well described by Eq. (8) where the transmission function $A(\nu)$ is approximated by its expansion in power series near $\nu = 0$. In Ref. [28] it is found that it is enough to take only three terms of this expansion, that is,

$$\frac{ib_0}{\nu+\Delta} \approx \frac{ib_0}{\Delta}\left(1 - \frac{\nu}{\Delta} + \frac{\nu^2}{\Delta^2}\right). \quad \text{(A24)}$$

The first term, ib_0/Δ, describes the phase shift of the field. The second term, $-ib_0\nu/\Delta^2$, gives a time delay of the pulse front due to the reduced group velocity. The third term describes the group velocity dispersion.

The integral in Eq. (A23), where the adiabatic approximation (A24) is taken into account, is calculated in Ref. [20]. The result for the adiabatic part, $\Omega_{ad}(z_0,t)$, is

$$\Omega_{ad}(z_0,t) = \frac{\Omega_0 e^{-ib_0/\Delta}}{2}\left\{1 + (1-i)\left[C\left(\frac{t-t_d}{t_{br}}\right) + iS\left(\frac{t-t_d}{t_{br}}\right)\right]\right\}, \quad \text{(A25)}$$

where $C(x)$ and $S(x)$ are Fresnel integrals [31], $t_d = b_0/\Delta^2$ is a delay time of the pulse, and $t_{sp} = \sqrt{2\pi b_0/\Delta^3}$ quantifies a time spreading of the pulse front due to the group velocity dispersion.

A time when the intensity of the step pulse at the output of the nonresonant absorber reaches its input value (i.e., when the slow component is formed) can be estimated as $t_d + t_{sp}$. For the numerical examples, given in Fig. 12, this estimate coincides quite well with time when intensity of the step pulse at the output, $I_{\Theta}(z_0,t)$, reaches its value at the input I_0. This time, $t_d + t_{sp}$, is $26.4/b_0$ in (a) and $8.2/b_0$ in (b). In Fig. 12 the function $I_{\Theta}(z_0,t)$ (solid line) is plotted according to Eq. (34) where $\gamma = 0$.

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