Fidelity of fermionic-atom number states subjected to tunneling decay

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Atom number states are a valuable resource for ultracold chemistry, atom interferometry, and quantum information processing. Recent experiments have achieved their deterministic preparation in trapped few-fermion systems. We analyze the tunneling decay of these states, in terms of both the survival probability and the nonescape probability, which can be extracted from measurements of the full counting statistics. At short times, both probabilities exhibit deviations from the exponential law. The decay is governed by the multiparticle Zeno time, which exhibits a signature of quantum statistics and contact interactions. The subsequent exponential regime governs most of the dynamics, and we provide accurate analytical expressions for the associated decay rates. Both dynamical regimes are illustrated in a realistic model. Finally, a global picture of multiparticle quantum decay is presented.

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I. INTRODUCTION

Successful production and preservation of atomic states containing an exactly known number of particles (so-called atomic Fock or atom number states) are of importance for ultracold chemistry, atom interferometry, quantum information processing, and investigating the foundations of quantum physics. Among the methods proposed for creating atom number states are *atom culling* in time-dependent traps [1–6] and the use of similar techniques in optical lattices [7,8]. Spectacular experimental progress in the deterministic preparation of few-atom states has been reported in [9,10]. But when the confining potential has finite potential barriers, which allow tunneling leakage, an initial trapped state with a well-defined atom number eventually evolves into a mixture of several atom number states, as shown in Fig. 1. This process can be quantitatively characterized by the fidelity decay. The fidelity is often defined as the probability to persist in the initial atom number state or, somewhat less stringently, by the nonescape probability, i.e., the probability that the total number of atoms within the trap remains the same. Unlike the integrated density profile of an atomic cloud [11,12], both probabilities refer to *all* particles occupying a certain subspace of a Hilbert space simultaneously and are, in this sense, multiparticle observables.

The purpose of this paper is to investigate the fidelity decay of an atomic Fock state due to tunneling leakage. Hence, it is both of fundamental interest to understand the quantum decay of multiparticle systems and of practical relevance to determine the decay rates of trapped Fock states [1–6,9,10]. We focus on ultracold atomic vapors confined in tight, effectively one-dimensional, waveguides and, more specifically, on polarized fermions and related fermionized systems, such as a bosonic cloud in the Tonks-Girardeau (TG) regime [13]. Since the rate of atom losses due to three-body collisions is suppressed due to spatial antibunching [14,15], these systems are well suited for investigating the quantum dynamics of multiparticle tunneling decay. Although the tunneling decay is expected to roughly follow an exponential law, deviations are expected both at short and long times on theoretical grounds. Long-time deviations from exponential decay in the tunneling dynamics of multiparticle systems have been studied in [16], and we shall focus on the short-time and exponential regime. Additional relevant works include studies of the fidelity decay in a multiparticle Loschmidt echo [17,18] and different scenarios of two-particle quantum decay [19–23]. The rest of the paper is organized as follows: in Sec. II we define the nonescape and survival probabilities for a general quantum system. In Sec. III we analyze their short-time evolution and the Zeno effect [24–26]. In Sec. IV we discuss the decay of one-dimensional trapped fermions and related systems and investigate the effect of quantum statistics in terms of the multiparticle Zeno time. Section V is devoted to the regime governed by an exponential decay law, an accurate semiclassical approximation for the corresponding decay rates is provided. Section VI illustrates the different dynamical regimes in a trap model of relevance to recent experiments [9,10]. A general picture of multiparticle quantum decay is outlined in Sec. VII, and the paper ends with a set of conclusions in Sec. VIII.

II. NONESCAPE AND SURVIVAL PROBABILITIES

We start by considering the time evolution of the probability that a multiparticle quantum system, initially prepared in a (possibly mixed) state ϱ_0, will be found inside a certain subspace $\mathcal{D} \subset \mathcal{H}$ of the Hilbert space $\mathcal{H}$. The ability to experimentally measure the full counting statistics (FCS) [9] motivates the introduction of the multiparticle nonescape probability [16]. Consider the full atom number distribution $p(n,t) = \langle \delta(\hat{n}_{\mathcal{D}} - n)\rangle_t$, where $\hat{n}_{\mathcal{D}}$ is the atom number operator in the subspace of interest $\mathcal{D}$, $n = 1, \ldots, N$ is an integer, and the expectation value is taken with respect to a time-evolving state. $p(n,t)$ represents the probability of finding exactly n particles at time t in the subspace $\mathcal{D}$. Its typical behavior is exhibited in Fig. 1 for an N-particle metastable Fock state. The probability to preserve the N-particle Fock state at time t is given by $p(n = N,t)$, and we shall denote it by $P_{\mathcal{D}}(t)$ in the following.

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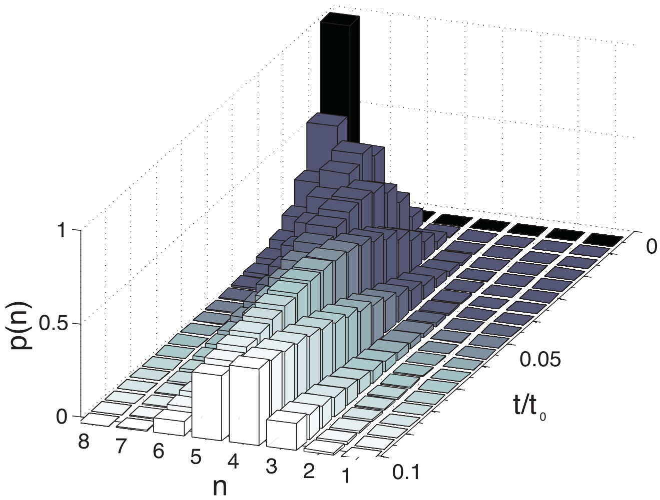

FIG. 1. (Color online) Fidelity decay of an atom number state of polarized fermions induced by tunneling losses. The probability to find n atoms inside the trap $p(n,t)$ is initially peaked at $n=8$. As the time of evolution goes by, $p(n,t)$ broadens and shifts to lower values of the mean atom number. The trapping potential model is further discussed in Sec. VI.

Letting $\hat{\Pi}$ be the projector on $\mathcal{D}$, we write this probability as

$$\mathcal{P}_{\mathcal{D}}(t)=\mathrm{tr}[\varrho_0\hat{\Pi}(t)],\quad \hat{\Pi}(t)=\exp(i\hat{H}t)\hat{\Pi}\exp(-i\hat{H}t). \quad (1)$$

Let $|n\rangle$ be a basis on $\mathcal{H}$, so that an initial state $\varrho_0=\sum_n p_n|n\rangle\langle n|$. Let $\hat{\Lambda}=\sum_n|n\rangle\langle n|$ be the projector associated with the space spanned by the initial state ϱ_0, such that $\hat{\Lambda}\varrho_0\hat{\Lambda}=\varrho_0$. The behavior of $P_{\mathcal{D}}(t)$ largely depends on whether ϱ_0 is contained within $\mathcal{D}$. If it is, $\hat{\Pi}$ can be decomposed as the sum of projectors

$$\hat{\Pi}=\hat{\Lambda}+\hat{Q},\quad \hat{Q}\varrho_0\hat{Q}=0. \quad (2)$$

In this case we will refer to $\mathcal{P}_{\mathcal{D}}(t)$, satisfying $\mathcal{P}_{\mathcal{D}}(0)=1$, as the *nonescape probability* $P(t)$, which must be a nonincreasing function, at least for short times. In the special case of $\hat{Q}=0$, $P_{\mathcal{D}}(t)$ becomes the *survival probability*, i.e., that for the system to remain in its initial state. This we will denote by $S(t)$, bearing in mind that $S(0)=1$. Note that $P(t)\geqslant S(t)$ since the condition for the system to stay within $\mathcal{D}$ is less strict than that for staying in one specific state ϱ_0. An obvious example is a wave packet contained within a spatial region Δ, for which the nonescape condition means not leaving Δ, as opposed to remaining in the initial state (survival). Finally, if ϱ_0 is not contained in $\mathcal{D}$, we have

$$\varrho_0=\hat{\Pi}\varrho_0\hat{\Pi}+\delta\varrho_0, \quad (3)$$

where $\delta\varrho_0$ is the component of the initial state orthogonal to $\mathcal{D}$. In this case $P_{\mathcal{D}}(t)$ may increase even at short times, a simple example being a wave packet arriving in an initially empty region of space. Next we consider the short-time evolution of the two probabilities.

III. THE SHORT-TIME LIMIT

The short-time expansion of the probability to remain in the subspace $\mathcal{D}$ takes the form

$$\mathcal{P}_{\mathcal{D}}(t)=\mathrm{tr}(\varrho_0\hat{\Pi})-i\,\mathrm{tr}(\varrho_0[\hat{\Pi},\hat{H}])t - \mathrm{tr}\left[\varrho_0\left(\frac{1}{2}\{\hat{\Pi},\hat{H}^2\}-\hat{H}\hat{\Pi}\hat{H}\right)\right]t^2+\mathcal{O}(t^3), \quad (4)$$

where $\{\hat{A},\hat{B}\}$ denotes the anticommutator of $\hat{A}$ and $\hat{B}$. As is well known (see, e.g., [24,25]), vanishing of the term linear in t leads to the Zeno effect: frequent checks of whether the system is still contained in $\mathcal{D}$ (e.g., projection measurements of $\hat{\Pi}$) would prevent it from leaving the subspace. In particular, with the interval between checks being τ, $\mathcal{P}_{\mathcal{D}}(t)$ would typically exhibit an effective exponential decay,

$$\mathcal{P}_{\mathcal{D}}(t)\sim\exp(-\gamma t),\quad \gamma\equiv\tau/\tau_Z^2, \quad (5)$$

where the Zeno time τ_Z is determined by the coefficient multiplying t^2 in Eq. (4),

$$\tau_Z=\{\mathrm{tr}[\varrho_0(\{\hat{\Pi},\hat{H}^2\}/2-\hat{H}\hat{\Pi}\hat{H})]\}^{-1/2}. \quad (6)$$

Nonetheless, we shall be concerned with quantum decay in the absence of measurements. It is readily seen that the linear-in-time term in Eq. (4) vanishes provided the initial state is contained in $\mathcal{D}$, i.e., the orthogonal component $\delta\varrho_0=0$, or can be neglected, $||\delta\varrho_0||\ll 1$. The decay of $\mathcal{P}_{\mathcal{D}}(t)$ becomes then quadratic in time and governed by τ_Z according to

$$\mathcal{P}_{\mathcal{D}}(t)=1-(t/\tau_Z)^2+\mathcal{O}(t^3). \quad (7)$$

Under this condition, from Eq. (6) for the Zeno time, we have

$$\tau_Z=[\Delta\hat{H}_{\varrho_0}-\mathrm{tr}(\varrho_0\hat{H}\hat{Q}\hat{H})]^{-1/2}, \quad (8)$$

and not merely given by the inverse of the energy variance of the initial state, $\Delta\hat{H}_{\varrho_0}=\mathrm{tr}[\varrho_0\hat{H}^2]-\mathrm{tr}[\varrho_0\hat{H}]^2$. The last term in the square brackets vanishes in the case of the survival probability, $\hat{Q}=0$. Thus, a survival Zeno time never exceeds that in the nonescape case, reflecting the fact that it is easier to maintain a system within a larger subset of its Hilbert space. [Note that τ_Z in Eq. (8) becomes infinite, as it should, if $\mathcal{D}$ is chosen to coincide with $\mathcal{H}$]. Next we proceed to the case of several particles confined in a potential trap with tunneling leakage.

IV. MULTIPARTICLE ZENO TIMES OF FERMIONIZED SYSTEMS

Consider N noninteracting particles initially trapped in a potential well $V_I(x)$, which, at $t=0$, is instantly converted into a trapping potential with a finite barrier $V(x)$, as shown in Fig. 2, thus allowing the particles to escape into the continuum. The Hamiltonian of the system reads

$$\hat{H}(x_1,\dots,x_N)=\sum_{i=1}^{N}\hat{h}(x_i), \qquad \hat{h}(x_i)=-\partial_{x_i}^2/2+V(x_i),\quad i=1,2,\dots,N. \quad (9)$$

Spin-polarized fermions fall within this description, and certain strongly interacting systems can be described in a similar way. This is the case of bosonic atoms in the

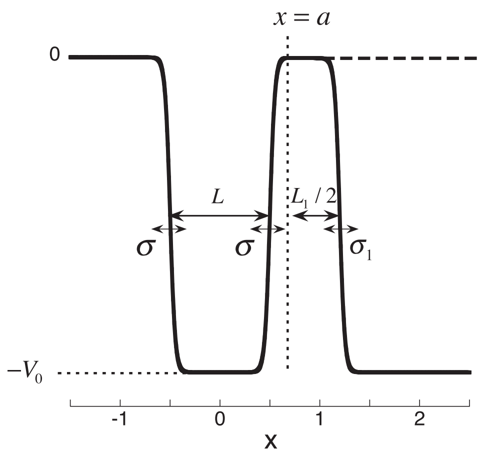

FIG. 2. The “bathtub” trapping potential $V(x)$ (solid line). Also shown is the initial well $V_I(x)$ (dashed line).

Tonks-Girardeau regime, where strong zero-range hard-core interaction leads to fermionization [13]. In this case there is a one-to-one correspondence (the Bose-Fermi mapping [13]) between the symmetric state of strongly interacting bosons $\Psi^{TG}(x_1,\ldots,x_N)$ and the antisymmetric state of the dual system of noninteracting fermions $\Psi^F(x_1,\ldots,x_N)$,

$$\Psi^{TG}(x_1,\ldots,x_N) = \mathcal{A}\Psi^F(x_1,\ldots,x_N), \tag{10}$$

where the antisymmetric unit function $\mathcal{A} = \prod_{1\leqslant j<k\leqslant N} \mathrm{sgn}(x_k - x_j)$. It follows that for the Tonks-Girardeau gas both the nonescape and the survival probabilities coincide with those calculated for the corresponding system of noninteracting fermions [16],

$$P^{TG}(t) = P^F(t), \quad S^{TG}(t) = S^F(t). \tag{11}$$

Both of these systems were found to be optimal for the preparation of atomic Fock states using atom-culling techniques [1–6,9,10]. In the following discussion we will simply refer to a “fermionic” system, bearing in mind that the results apply both to the Tonks-Girardeau gas with infinitely strong contact interactions and to a system of noninteracting polarized fermions.

The ground state of a fermionic system is given by the Slater determinant, and we choose

$$\Psi_0^F(x_1,\ldots,x_N) = \frac{1}{\sqrt{N!}}\mathrm{det}_{n,k=1}^N[\phi_n(x_k)], \tag{12}$$

where $|\phi_n\rangle$, $n = 1,\ldots,N$, are the N lowest eigenstates of the one-particle Hamiltonian, $\hat{h}|\phi_n\rangle = \epsilon_n|\phi_n\rangle$. In order to study the effect of quantum statistics on the Zeno time it is instructive to consider also the ground state for distinguishable particles,

$$\Psi_0^{\mathrm{dist}}(x_1,\ldots,x_N) = \prod_{n=1}^N \phi_n(x_n), \tag{13}$$

and the somewhat artificial case of a bosonic excited state where the same first N levels are occupied by noninteracting particles obeying Bose-Einstein statistics,

$$\Psi_0^B(x_1,\ldots,x_N) = \frac{1}{\sqrt{N!}}\mathrm{per}_{n,k=1}^N[\phi_n(x_k)], \tag{14}$$

where per stands for the permanent, i.e., the sum on the right-hand side of all permutations of indices k for a fixed order of indices n. Note that these three states, Eqs. (12), (13), and (14), all have the same energy.

The survival Zeno times [Eq. (8) with $\hat{Q} = 0$] for the three cases are calculated by inserting in Eq. (8) the appropriate initial state, (12), (13), or (14). For distinguishable particles the calculation is straightforward, for polarized fermions the matrix elements in Eq. (8) can be evaluated using the Slater-Condon rules [27], and for bosons an extension of the latter is required to symmetric states. The result can be written in a compact form,

$$\tau_Z = \left\{\sum_n \Delta\hat{h}_n + 2\alpha\sum_{n<k}|\langle\phi_n|\hat{h}|\phi_k\rangle|^2\right\}^{-\frac{1}{2}}, \tag{15}$$

where $\Delta\hat{h}_n = \langle\phi_n|\hat{h}^2|\phi_n\rangle - \langle\phi_n|\hat{h}|\phi_n\rangle^2$ and $\alpha = 0$ for distinguishable particles, $\alpha = -1$ for fermions, and $\alpha = 1$ for excited bosons.

In Eq. (15), the first sum, common to all statistics, depends on the spread of the energy in the initial one-particle states induced by the tunneling decay. This contribution dominates the Zeno time. The second sum arises from the indistinguishability of the particles and the associated symmetrization of the initial state and, more precisely, from its immanant (either determinant or permanent) structure. While it reduces the survival Zeno time for noninteracting excited bosons, it increases τ_Z in polarized fermions and bosons in the TG regime, leading to a slowing down of their decay. The symmetrization imposed by exchange quantum statistics plays no role, as it follows from the fact that the multiparticle Zeno time is shared by dual systems related by the Bose-Fermi mapping. The different corrections to τ_Z arise only for indistinguishable particles and are manifest thanks to the effect that contact interactions (including as such the Pauli exclusion principle) have on the energy dispersion of the initial state. We recall that short-time decay $\mathcal{P}_\mathcal{D}(t) = 1 - (t/\tau_Z)^2 = \mathcal{O}(t^3)$ is governed by the Zeno time. Also, we recall that when the system is frequently observed, its lifetime γ^{-1} becomes proportional to τ_Z^2; see Eq. (5). It follows that a fermionic state (or bosons in the TG regime) decays over short times more slowly than noninteracting excited bosons. This result is somewhat counterintuitive in nature, given that as a result of the Pauli exclusion principle (or hard-core contact interactions in the TG gas) fermionized systems exhibit spatial antibunching, while noninteracting bosons prefer to group together (bunching). However, this intuition applies only in subsequent stages of evolution, while the short-time dynamics is exclusively governed by the energy dispersion of the initial state.

In the following section we shall describe the rates associated with the subsequent exponential decay, where the indistinguishability of the particles and density-density correlations play a subdominant role with respect to the energy distribution. The intuition based on spatial bunching applies to the long-time asymptotics of multiparticle quantum decay. At long times, deviations from the exponential decay occur due to the possibility that the decay products recombine to reconstruct the initial state. As a result, spatial bunching and antibunching effects play the dominant role, and fermionized systems decay according to a power law $\mathcal{P}_\mathcal{D}(t) \propto 1/t^\alpha$ with an exponent $\alpha > 0$ about N times larger than in the case of noninteracting bosons [16].

V. EXPONENTIAL REGIME AND DECAY RATES OF FERMIONIZED SYSTEMS

We shall now turn our attention to the regime characterized by exponential decay, most easily observed in experiments. While manipulating cold atoms, one may be interested in maintaining exactly N of them in a specified region of space Δ, usually large enough to enclose the trap subspace. The corresponding nonescape probability is obtained from Eq. (1) by choosing the projector onto the corresponding part of the multiconfigurational space,

$$\hat{\Pi} = \prod_{n=1}^{N} \int_{\Delta} |x_n\rangle\langle x_n| dx_n. \tag{16}$$

For a fermionic system starting in its ground state (12) the nonescape probability reads [16] (we drop the superscript F in the following and include a subscript for the particle number N)

$$P_N(t) = \det_{n,k=1}^{N}[\langle\phi_n(t)|\chi_\Delta|\phi_k(t)\rangle], \tag{17}$$

where $\langle\phi_n(t)|\chi_\Delta|\phi_k(t)\rangle = \int_\Delta dx \phi_n^*(x,t)\phi_k(x,t)$, while for the probability to survive in state (12), $S_N(t)$, one has [16–18]

$$S_N(t) = \left|\det_{n,k=1}^{N}[\langle\phi_n(0)|\phi_k(t)\rangle]\right|^2. \tag{18}$$

Once the initial state is prepared, after the Zeno time scale, each of the single-particle states $\{|\phi_k\rangle\}$ involved in the Slater determinant (12) experiences a nearly exponential decay, i.e., $|\phi_k(t)\rangle \approx \exp(-\gamma_k t)||\phi_k(0)\rangle$, $k = 1,\dots,N$. Then, the N-particle survival and nonescape probabilities decay exponentially,

$$P_N(t) \approx \exp(-\Gamma t)\mathcal{P}_N(0), \quad S_N(t) \approx \exp(-\Gamma t)\mathcal{S}_N(0), \tag{19}$$

with a decay rate

$$\Gamma = 2\sum_{k=1}^{N} \gamma_k. \tag{20}$$

Individual decay constants γ_k are given by the imaginary parts of the complex energies E_k corresponding to the first N leading poles q_k of the reflection amplitude for the potential trap in the complex momentum q plane, $\gamma_k = \mathrm{Im}E(q_k) = \mathrm{Re}q_k\mathrm{Im}q_k$.

One can avoid precise determination of the momentum pole positions by making instead a simple semiclassical estimate [28]. Let E_k be the energy of the kth bound state in the initial trap and $x_0^k < x_1^k < x_2^k$ be the three turning points in the quenched potential satisfying $V(x_i^k) = E_k$ $(i = 0,1,2)$, e.g., for a potential where the left barrier is arbitrarily wide so that tunneling losses occur only through the right barrier, as in the case depicted in Fig. 2.

The semiclassical probability to tunnel in one attempt across the potential barrier is given by (see, e.g, [29])

$$T(E_k) \approx \frac{\exp[-2S(E_k)]}{\{1 + \exp[-2S(E_k)]/4\}^2}, \tag{21}$$

where $S(E_k) \equiv \int_{x_1^k}^{x_2^k} dx \{2[V(x) - E_k]\}^{1/2}$ is the complex action corresponding to the classically forbidden region $x_1 < x < x_2$. Since the period of the bound motion is given by $\tau_k = 2\int_{x_0^k}^{x_1^k} dx\{2[V(x) - E_k]\}^{-1/2}$, the particle impacts on the barrier with an approximate frequency $n_k = 1/\tau_k$. Multiplying T by the number of impacts per unit time gives individual decay rates

$$2\gamma_k \approx \tau_k^{-1} T(E_k), \quad k = 1,\dots,N, \tag{22}$$

which together with Eq. (20) yield the desired rate of the exponential decay in Eqs. (19).

VI. THE MODEL AND RESULTS

As a realistic model of a one-dimensional trap we consider a smooth bathtub potential (Fig. 2, dashed line)

$$V_I(x) = -\frac{1}{2}V_0\left[1 - \tanh\left(\frac{|x| - L/2}{\sigma_1}\right)\right], \tag{23}$$

whose right wall is instantly turned into a barrier of finite width at $t = 0$ (Fig. 2, solid line),

$$V(x,t) = -\frac{1}{2}V_0\left[1 - \tanh\left(\frac{|x| - L/2}{\sigma}\right)\right]\Theta(a - x) - \frac{1}{2}V_0\left[1 + \tanh\left(\frac{x - a - L_1/2}{\sigma_1}\right)\right]\Theta(x - a). \tag{24}$$

Here V_0 is the depth of the initial well (and also the barrier height of the metastable potential), L and L_1 are the widths of the well and barrier, respectively, and σ (σ_1) determine the smoothness of the inner (outer) potential walls. It is convenient to introduce the dimensionless variables (we reintroduce Planck's constant $\hbar$)

$$x \to x/L, \quad t \to t/t_0, \quad V_0 \to V_0 t_0/\hbar, \tag{25}$$

with $t_0 \equiv mL^2/\hbar$. The absorbing potential introduced by Manolopoulos [30,31] is employed to avoid unphysical reflections at the boundaries of the numerical grid $L_{1,2}^{\mathrm{box}}$. Bound one-particle eigenstates of the initial well $\phi_n(x,0)$ are obtained by a standard finite-difference technique and then evolved in time using the Crank-Nicolson scheme to yield $\phi_n(x,t)$ required in Eqs. (17) and (18). We use $L_1/L = 0.08$, $\sigma/L = \sigma_1/L = 0.01$, $a/L = 0.55$, and $V_0 t_0/\hbar = C^2\pi^2$, where C is the capacity of the well, i.e., the maximum number of bound states it supports. We also note that, for ^{23}Na atoms in a potential well with $L = 80\ \mu$m, t_0 is about 2.39 s. Finally, we chose $L_1^{\mathrm{box}}/L = -20$, $L_2^{\mathrm{box}}/L = 30$, and the absorbing potential identical to that used in [6]. For the calculation of the nonescape probability (1) for a fermionized system in its ground state (12) we chose the spatial region Δ in Eq. (16) to include most of the trap,

$$\Delta = (-\infty,a], \tag{26}$$

so that $\delta\varrho_0$ in Eq. (3), now associated with exponential tails of the $\phi_n(x)$ extending into the right classically forbidden region of $V_I(x)$, is small. Figure 3 shows the decay dynamics of P_N and S_N for different fermionic-atom number states when the initial well supports a maximum of $C = 8$ bound states. The two curves are remarkably similar, given that S_N is sensitive to the population of individual one-particle states, while P_N only

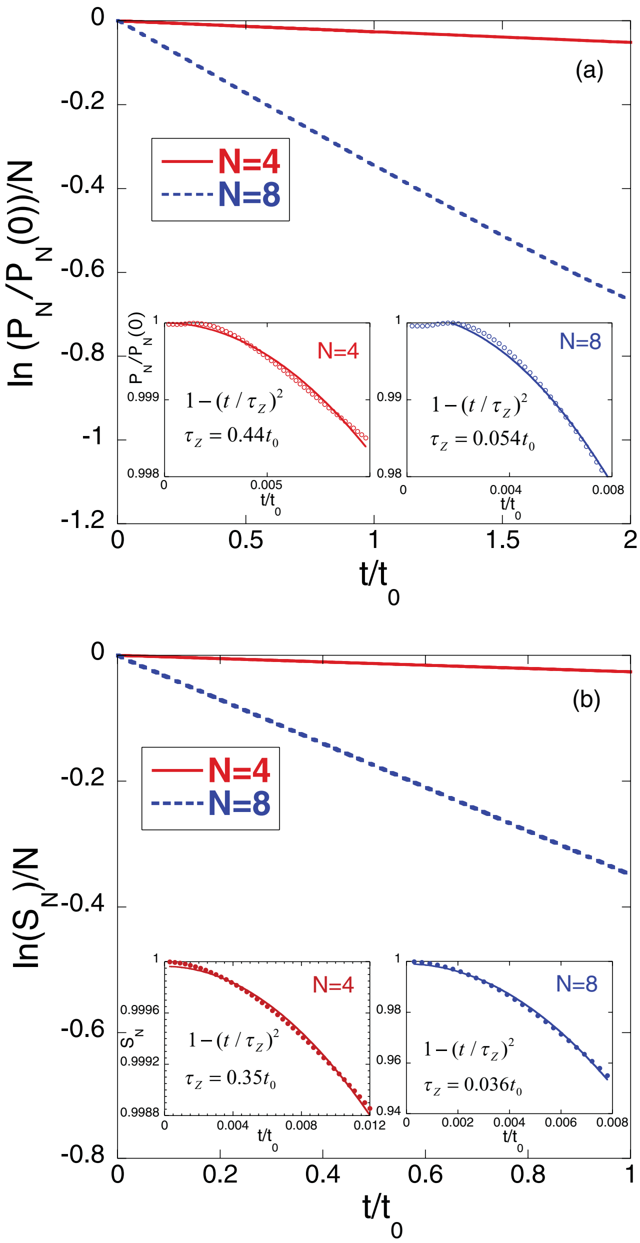

FIG. 3. (Color online) Nonescape and survival probabilities for fermionic-atom number states with $N = 4$ and $N = 8$ leaking out of potential (24) with $C = 8$. The insets show the quadratic behavior of $P_{\mathrm{N}}(t)$ and $S_{\mathrm{N}}(t)$ in the short-time regime (solid line) and the fitted quadratic parabola (dots).

depends on whether atoms have left the region Δ. In agreement with the discussion in the previous sections, two regimes can be identified. The short-time behavior is characterized by a decay quadratic in time, as shown in the inset of Fig. 3. Subsequently, as dictated by Eq. (19), the exponential law holds. The associated transition is illustrated in Fig. 4 for an $N = C = 12$ fermionic Fock state and occurs approximately on the time scale

$$\tau_q \approx \tau_Z^2 \Gamma, \tag{27}$$

where the Zeno time τ_Z and decay rate Γ are given by Eqs. (15) and (20), respectively.

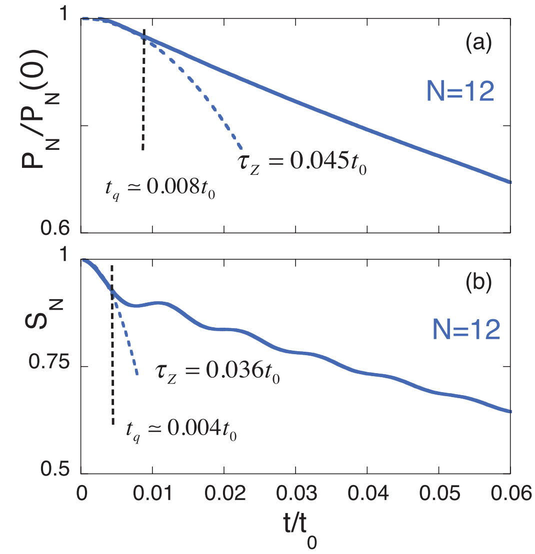

FIG. 4. (Color online) (a) Nonescape and (b) survival probabilities for a fermionic 12-atom state leaking out of potential trap in Eq. (24) with capacity $C = 12$ (solid line) (all times are in units of t_0). Also shown are the short-time quadratic fits (dashed line) and the times t_q beyond which the quadratic approximation fails (vertical dashed line).

We first focus on the short-time dynamics, where the characteristic scale of the decay is given by the Zeno time τ_Z. As the energy dispersion of the initial state with respect to the Hamiltonian for $t > 0$ increases, the corresponding τ_Z is reduced. In particular, this is expected from Eq. (15) for increasing particle number N. Figure 5 shows the survival Zeno time τ_Z as a function of N. Its dependence on the particle number is enhanced as N approaches C, and the initial Fock state involves energy components closer to the brim of the trap, which result in the spatial extension of the atomic cloud

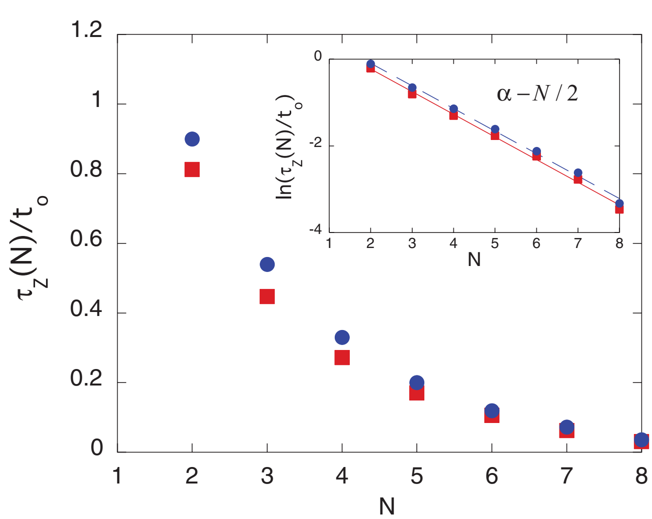

FIG. 5. (Color online) Dependence of the survival Zeno time τ_Z on the number of trapped fermions N for a potential (24) with $C = 8$. The analytical Zeno times given by Eq. (15) (squares) are shown together with the numerical values extracted from a parabolic fit to the exact short-time decay dynamics (dots). The inset shows the corresponding exponential fits to the dependence of τ_Z on N.

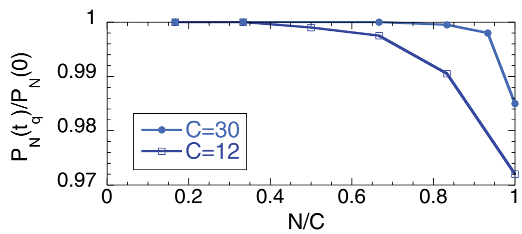

FIG. 6. (Color online) Loss of fidelity for fermionic-atom number states leaking out of potential trap described by Eq. (24) for two different capacities $C = 12{,}30$ ($N \leqslant C$). The fidelity is evaluated at the transition time t_q beyond which the quadratic approximation fails (cf. Fig. 4), that is, right before the exponential regime sets in.

beyond the trap region Δ due to the existence of tunneling tails for $x > a$. Indeed, it is found numerically that the suppression of the Zeno-time is approximately exponential with respect to N, $\tau_Z(N) \approx \tau_Z(1)\exp(-N/2)$; see the inset in Fig. 5.

This rapid decline of $\tau_Z(N)$ might suggest that the short-time deviations from the exponential law should be irrelevant for Fock states with already moderate numbers of atoms. However, this is not completely the case, as is illustrated in Fig. 4, which demonstrates that the fidelity decay of a Fock state also accelerates with the number of atoms and becomes significant already at the end of the quadratic evolution, provided N is sufficiently close to C. To quantify this effect, we evaluated the transition time $t_q(N)$ (Fig. 4, vertical dashed line) such that for $t \gtrsim t_q$ the quadratic approximation to $P_N(t)$ or $S_N(t)$ starts to fail. (Note that t_q is not the same as τ_Z, which determines the curvature of the short-time parabolic decay.) The plot of the fidelity factor $P_N(t_q)/P_N(0)$ vs N shown in Fig. 6 confirms that as N/C approaches unity a significant fidelity decay of an atom number state occurs before the exponential regime sets in. The effect is more pronounced for small traps, the goal of current experimental efforts.

We next focus on the characterization of the exponential regime. The dependence of the decay rate (20) on the number of particles N is in good agreement with the semiclassical estimate derived from Eq. (22), as shown in Fig. 7. This is to be contrasted with the case of a trap with thin potential barriers [11], where the decay rate of any single-particle excited state is soon governed by the longest-lived resonance, favoring the observation of multiple exponential regimes

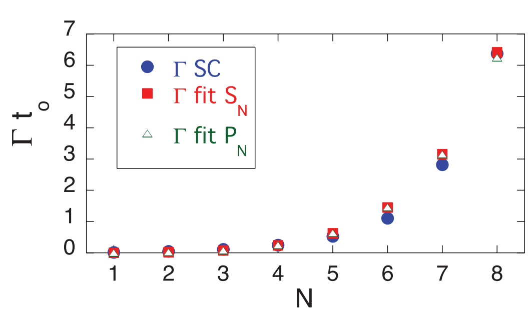

FIG. 7. (Color online) Dependence of the exponential decay rates for $S_N(t)$ (squares) and $P_N(t)$ (triangles) on the number of trapped atoms N for a potential trap in Eq. (24) of capacity $C = 8$. The observed survival decay rates are in excellent agreement with the analytical prediction based on Eq. (22).

(for $N = 2$ with a δ barrier; see [21]). In such a case the multiparticle decay rate becomes $\Gamma = 2\gamma_1 N$, linear in N. Note that expression (20) for the decay rates equally applies to distinguishable particles or excited states of noninteracting bosons. Its success in reproducing the decay rates extracted from exponential fits to the exact decay dynamics points out that the indistinguishability of the particles and the many-body correlations among them plays a subdominant role in the exponential regime. Nonetheless, as the initial state includes states closer to the brim of the trap, Eq. (21) loses accuracy.

VII. DISCUSSION

Given the recent flurry of works dealing with few-body quantum decay [11,12,16–23], it is interesting to establish the current understanding and point out some aspects deserving further studies. In particular, combining the results obtained in this manuscript with those in [16], we identified the following stages in multiparticle quantum decay of systems with contact interactions. As in the single-particle decay, three regimes are found.

Short-time asymptotics: Zeno regime. A quadratic-in-time quantum decay occurs that is characterized by the multiparticle Zeno time, i.e., the inverse of the energy variance of the quenched Hamiltonian (that is, for $t > 0$) evaluated in the initial state, Eq. (15). The main contribution to this time scale arises from the energy spread of the initial state. Nonetheless, in the presence of interactions there is a counterintuitive correction reflecting the indistinguishability of the particles. This correction slows down the decay of polarized fermions with respect to degenerated states of noninteracting bosons. It would be clarifying to consider systems with generalized exclusion statistics [32] and compare the multiparticle quantum decay between systems with finite-interactions related by Bose-Fermi duality [33].

Exponential regime. Following the Zeno regime, exponential decay sets in, characterized by multiparticle decay rates well described in a semiclassical approximation, i.e., Eq. (22). The dependence of the decay rates is dominated by the energy spread of the initial state. This is expected to be the general behavior for smooth potentials, but for finite interactions the question remains as to the validity of the semiclassical approximation. For example, considering a one-dimensional Bose gas [22], does Eq. (22) evaluated at the quasimomenta of the initial state (Bethe roots) describe accurately the decay rate, or are there important corrections arising from correlations among the particles? Interestingly enough, the exact decay dynamics of two fermions for arbitrarily thin potential barriers exhibits a transition among different decay rates [21]. The observability of multiple exponential regimes for arbitrary particle number N and as a function of the potential features well deserves further studies.

Long-time asymptotics: postexponential power-law decay. The subsequent regime is characterized by a power-law decay and arises due to the possibility that the decay products recombine to form the initial state in a classical sense. This regime is governed by the short-range density-density correlations, which lead to a dramatic dependence of the power-law exponents on the particle number N [16]. The current understanding would benefit from studies for systems with finite and long-range interparticle interactions as well as two- and three-dimensional systems with degeneracy arising from angular momentum. The power-law exponents in these systems are expected to exhibit a rich dependence on N.

We note that a deep understanding of multiparticle quantum decay is relevant across very different fields of physics, beyond quantum foundations. For instance, a good control of the particle number might be key to advancing the field of quantum simulation with ultracold gases, and this goal is being currently pursued in different laboratories [9,10]. Similarly, the short-time quantum decay is at the heart of certain applications, such as dynamical decoupling schemes for high-fidelity quantum memories, quantum gates, quantum computing, and generally extending the coherence times in quantum systems [34]. At the same time, the existence of different stages of quantum decay may have important cosmological implications [35].

VIII. CONCLUSIONS

In summary, we have analyzed the many-body tunneling decay of trapped fermionic-atom number (Fock) states in a realistic model of relevance to recent experiments [9,10]. We focused our attention on the fidelity decay resulting from tunneling leakage, quantified as a decrease in the nonescape or survival probability, and identified the signatures of contact interactions and quantum statistics in the short-time multiparticle decay. Even though the survival and nonescape Zeno times rapidly decrease with the number of atoms N, as N approaches the maximum capacity of the trap a substantial loss of fidelity is found already at the end of the quadratic (Zeno) evolution, before the exponential regime sets in. Moreover, explicit expressions for the decay rates in the exponential regime have been provided. Our results are amenable to experimental verification by standard techniques, e.g., by registering the time evolution of the full counting statistics in a trap which allows leaking by quantum tunneling (see, for example, Ref. [9]). Finally, we note that strong s-wave scattering in ultracold bosons or spin polarization in fermions suppress three-body losses, facilitating the study of genuinely quantum losses in these systems.

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