Analytical study of the self-healing property of Airy beams

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An analytical expression for the optical field of an Airy beam partially blocked by an opaque obstacle is derived. The self-healing properties of the Airy beam are studied and discussed in detail. The study shows that the self-healing process of the Airy beam is affected by many factors such as the opaque obstacle size, propagation distance, wavelength, and parameters of the Airy beam. The self-healing process is caused by the convergence of energy from the side to the position of the opaque obstacle and is finished when the convergence of energy flow disappears. When the propagation distance is short, the main lobe of the Airy beam is affected by the obstruction located near the main lobe. When the propagation distance is long, the main lobe of the Airy beam can be affected by the obstruction located far away from the main lobe. The result agrees with the existing results and can be explained by the caustic of the Airy beam.

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I. INTRODUCTION

In recent years, nondiffracting waves have attracted much attention due to their many interesting properties and potential applications [1–6]. The Airy beam, which was predicted by Berry and Balaze within the context of quantum mechanics, is one type of nondiffracting wave [7]. One of the most interesting properties is the self-healing ability. However, since the Airy beam has infinite energy, a long tail, and decays very slowly, it is very difficult to generate such beams. In practice, truncated Airy beams have been proposed and can be demonstrated experimentally [8,9]. Both theory and experiment indicate that the truncated Airy beams have almost the same properties as the Airy beam, such as the self-healing ability and acceleration [10–12], and exhibit more resilience against perturbations of media [13,14].

The self-healing ability of Airy beams makes them very useful in optical tweezing and many other areas. Both theoretical and experimental works concerning the self-healing properties have been carried out [10]. However, these theoretical studies are restricted in their numerical calculations. To see the self-healing process clearly, an analytical expression for the self-healing of Airy beams has been derived in present paper. With the help of the analytical expression, the self-healing process and the effects of many factors, such as the size and position of an opaque obstacle and propagation distance, on the self-healing can be investigated simply; some interesting results are obtained.

II. ANALYTICAL EXPRESSION FOR AN AIRY BEAM PARTIALLY BLOCKED BY AN OPAQUE OBSTACLE

The optical field of the two-dimensional finite-energy Airy beam at the origin is given by [10–15]

$$U_1(x_1,y_1) = \prod_{\chi=x,y} \mathrm{Ai}\left(\frac{\chi_1}{\omega_0}\right) \exp\left(a\frac{\chi_1}{\omega_0}\right), \tag{1}$$

where $(x_1,\ y_1)$ represents the transverse coordinates of initial plane, Ai( ) denotes the Airy function, ω_0 is an arbitrary transverse scale, and a in the exponential function is a parameter associated with the truncation of the Airy beam. To analytically study the self-healing properties of the Airy beam, we assume that a finite opaque obstacle that partially blocks the initial beam has Gaussian absorption efficiency. Therefore, the transmittance function of the Airy beam has the form

$$T(x_1,y_1) = 1 - \exp\left[-\frac{(x_1-b_x)^2+(y_1-b_y)^2}{w_0^2}\right], \tag{2}$$

where w_0 is the size and $(b_x,\ b_y)$ is the central coordinates of the opaque obstacle. With the help of the Fresnel diffraction integral, the optical field at the receiver plane can be expressed by

$$U_2(x_2,y_2) = \frac{k}{2\pi i z}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} U_1(x_1,y_1)T(x_1,y_1) \times \exp\left\{\frac{ik}{2z}[(x_2-x_1)^2+(y_2-y_1)^2]\right\} dx_1dy_1, \tag{3}$$

where $(x_2,\ y_2)$ is a pair of coordinates on the z plane, $k = 2\pi/\lambda$, and λ is the wavelength. From Eqs. (1)–(3) and using the Babinet principle, the optical field at the z plane can be expressed as

$$U_2(x_2,y_2) = u_2(x_2,y_2) - u_2'(x_2,y_2), \tag{4}$$

where $u_2(x_2,y_2)$ is the optical field of the Airy beam without an obstacle and $u\prime_2(x_2,y_2)$ is the field of the Airy beam passing through an off-axis Gaussian aperture. Performing the

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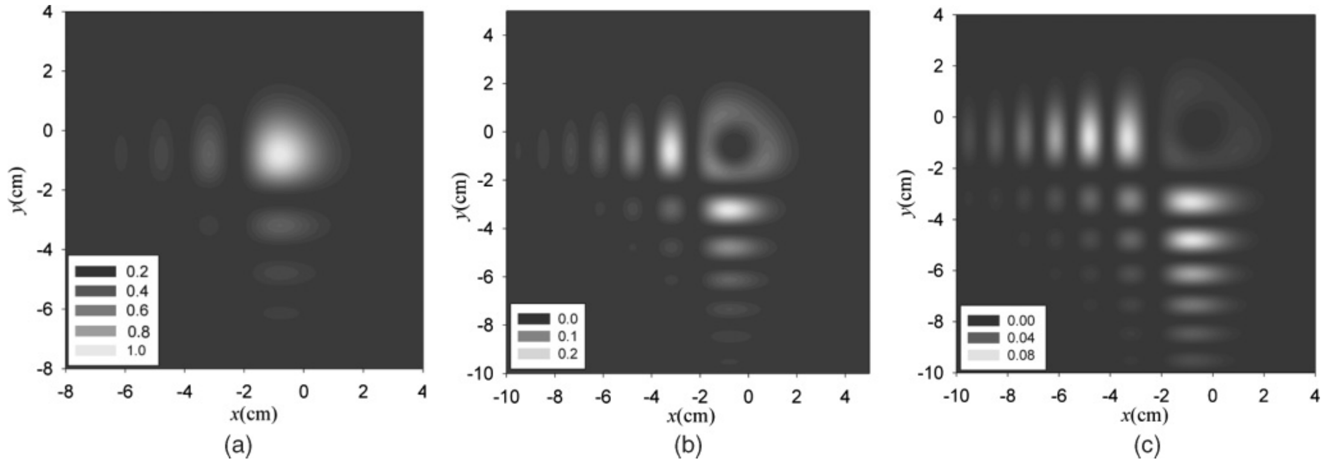

FIG. 1. Intensity distribution of the Airy beam at the initial plane (a) without an obstacle, (b) for $b_x = b_y = -0.6$ cm and $w_0 = 1.5$ cm, and (c) for $b_x = b_y = -0.5$ cm and $w_0 = 3$ cm.

integration in Eq. (3) we obtain

$$u_2'(x_2,y_2) = \frac{1}{1+2i\eta}\prod_{\chi=x,y}\mathrm{Ai}\left[\frac{1}{1+2i\eta}\left(\frac{\chi_2}{\omega_0}+\delta_\chi\right)\right]\exp\left\{\frac{iS_\chi}{(1+2i\eta)}\left(\frac{\chi_2}{\omega_0}+\delta_\chi\right)-\frac{\eta}{\xi(1+2i\eta)}\frac{\chi_2^2}{\omega_0^2}-\frac{b_\chi^2}{w_0^2}\right.$$
$$\left.+\frac{i\xi}{2(1+2i\eta)}\left[\frac{1}{48}\left(\frac{\xi}{1+2i\eta}\right)^2+\left(S_\chi-\frac{\xi}{4+8i\eta}\right)^2\right]\right\}, \tag{5}$$

where $\eta = z/kw_0^2$ and $\xi = z/k\omega_0^2$ are the normalized propagation distance associated with the Gaussian beam and the Airy beam, respectively, $S_\chi = \xi/(2+4i\eta) - i(a + 2b_\chi\omega_0/w_0^2)$, and $\delta_\chi = \xi(ia - S_\chi)/2 + ib_\chi\eta/\omega_0$. From Eq. (5) we can see that $\lim_{\eta\to\infty} u_2'(x_2,y_2) = 0$ and

$$u_2(x_2,y_2) = \lim_{\eta\to 0} u_2'(x_2,y_2) = \prod_{\chi=x,y}\mathrm{Ai}\left[\frac{\chi_2}{\omega_0}-\left(\frac{\xi}{2}\right)^2+ia\xi\right]$$
$$\times\exp\left[\left(a+\frac{i}{2}\xi\right)\frac{\chi_2}{\omega_0}+\frac{ia^2}{2}\xi-\frac{a\xi^2}{2}-\frac{i\xi^3}{12}\right], \tag{6}$$

which agrees with the existing results [10–12]. From Eqs. (4)–(6), the self-healing of the Airy beam blocked by an opaque obstacle can be investigated analytically.

III. ANALYSIS AND DISCUSSION

To see the self-healing process of the Airy beam we set $a = 0.2$ and $\omega_0 = 1$ cm in the following calculation. Contour plots of the intensity distribution for the Airy beam at the initial plane with an opaque obstacle located at a different position are plotted in Fig. 1. As a comparison, the intensity distribution for the Airy beam without an obstacle is plotted in Fig. 1(a).

It should be pointed out that the normalized intensity in the present paper is defined as the intensity divided by its maximum intensity for the Airy beam without an opaque obstacle under the same conditions. Figures 1(c) and 1(b) show the intensity distribution of an Airy beam whose main lobe is partially blocked by an opaque obstacle of a different size. It can be seen that the beam blocked by an opaque obstacle can be analytically expressed by Eqs. (1) and (2).

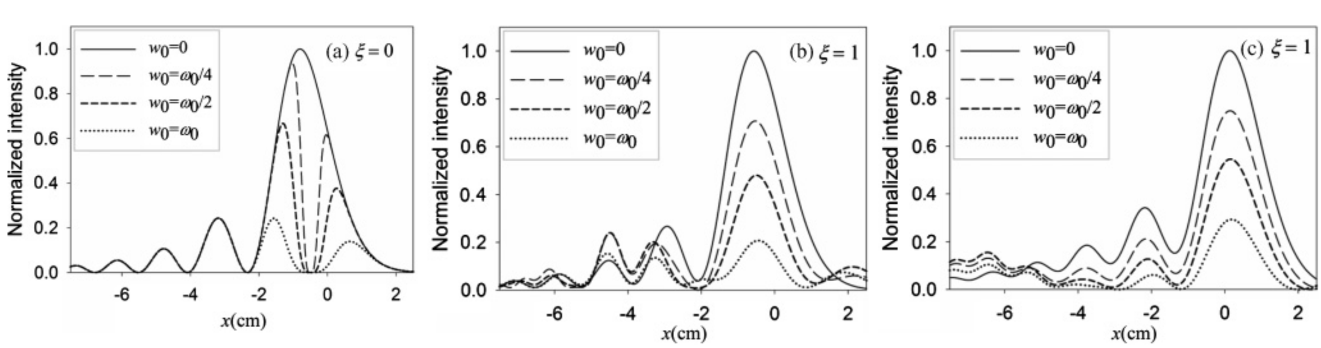

FIG. 2. Self-healing process of the Airy beam during propagation for $a = 0.2$, $\omega_0 = 1$ cm, and $b_x = -0.005$ m at (a) the initial plane, (b) $\xi = 1$, and (c) $\xi = 2$.

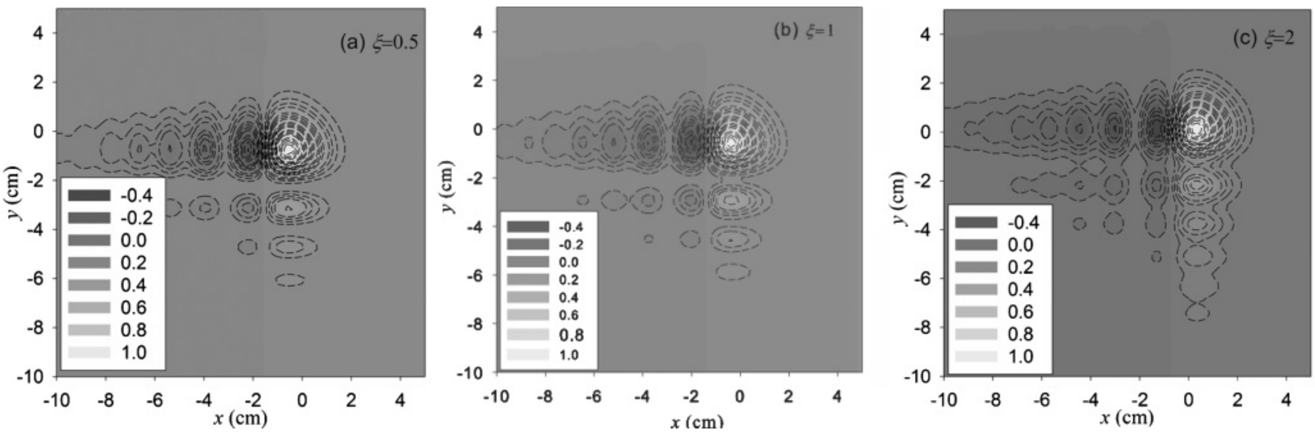

FIG. 3. Distribution of the Poynting vector along the x axis for the Airy beam without an opaque obstacle for $a = 0.2$ and $\omega_0 = 1$ cm at (a) $\xi = 0.5$, (b) $\xi = 1$, and (c) $\xi = 2$.

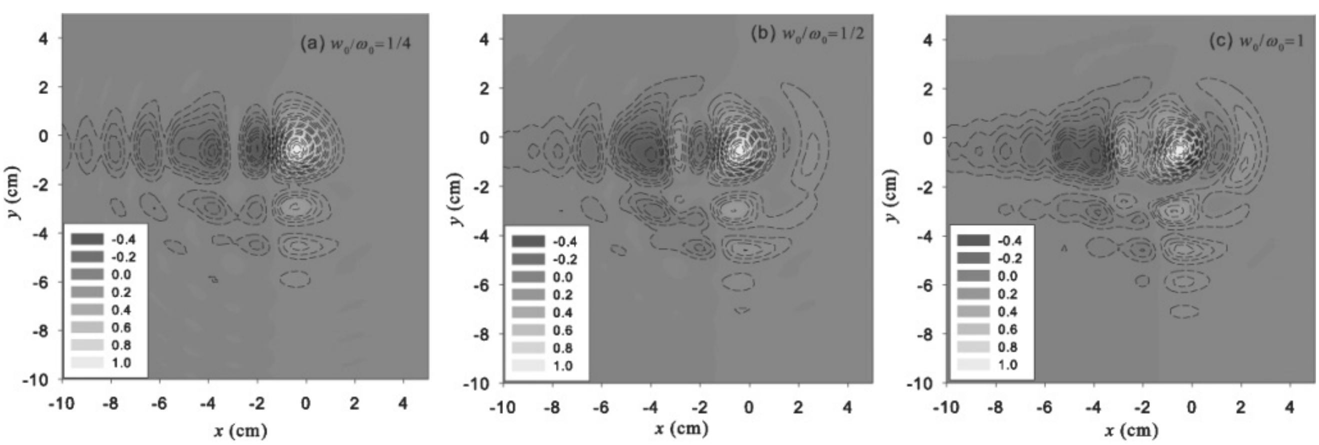

FIG. 4. Distribution of the Poynting vector along the x axis for the Airy beam with a different opaque obstacle for $a = 0.2$, $\omega_0 = 1$ cm, $b_x = -0.5$ cm, and $\xi = 0.5$ at (a) $w_0/\omega_0 = 1/4$, (b) $w_0/\omega_0 = 1/2$, and (c) $w_0/\omega_0 = 1$.

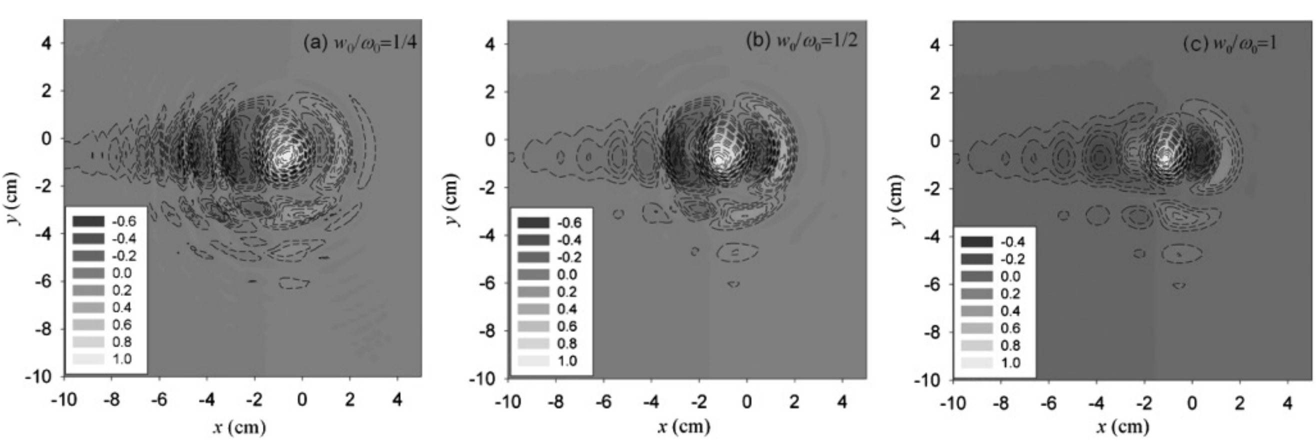

FIG. 5. Distribution of the Poynting vector along the x axis for the Airy beam with a different opaque obstacle for $a = 0.2$, $\omega_0 = 1$ cm, $b_x = -0.5$ cm, and $\xi = 1$ at (a) $w_0/\omega_0 = 1/4$, (b) $w_0/\omega_0 = 1/2$, and (c) $w_0/\omega_0 = 1$.

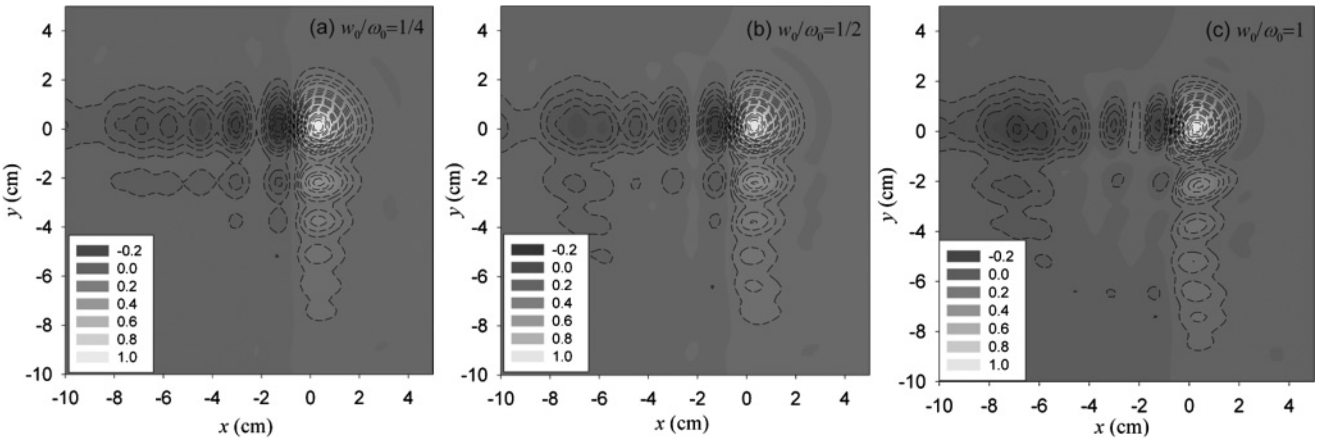

FIG. 6. Distribution of the Poynting vector along the x axis for the Airy beam with a different opaque obstacle for $a = 0.2$, $\omega_0 = 1$ cm, $b_x = -0.5$ cm, and $\xi = 2$ at (a) $w_0/\omega_0 = 1/4$, (b) $w_0/\omega_0 = 1/2$, and (c) $w_0/\omega_0 = 1$.

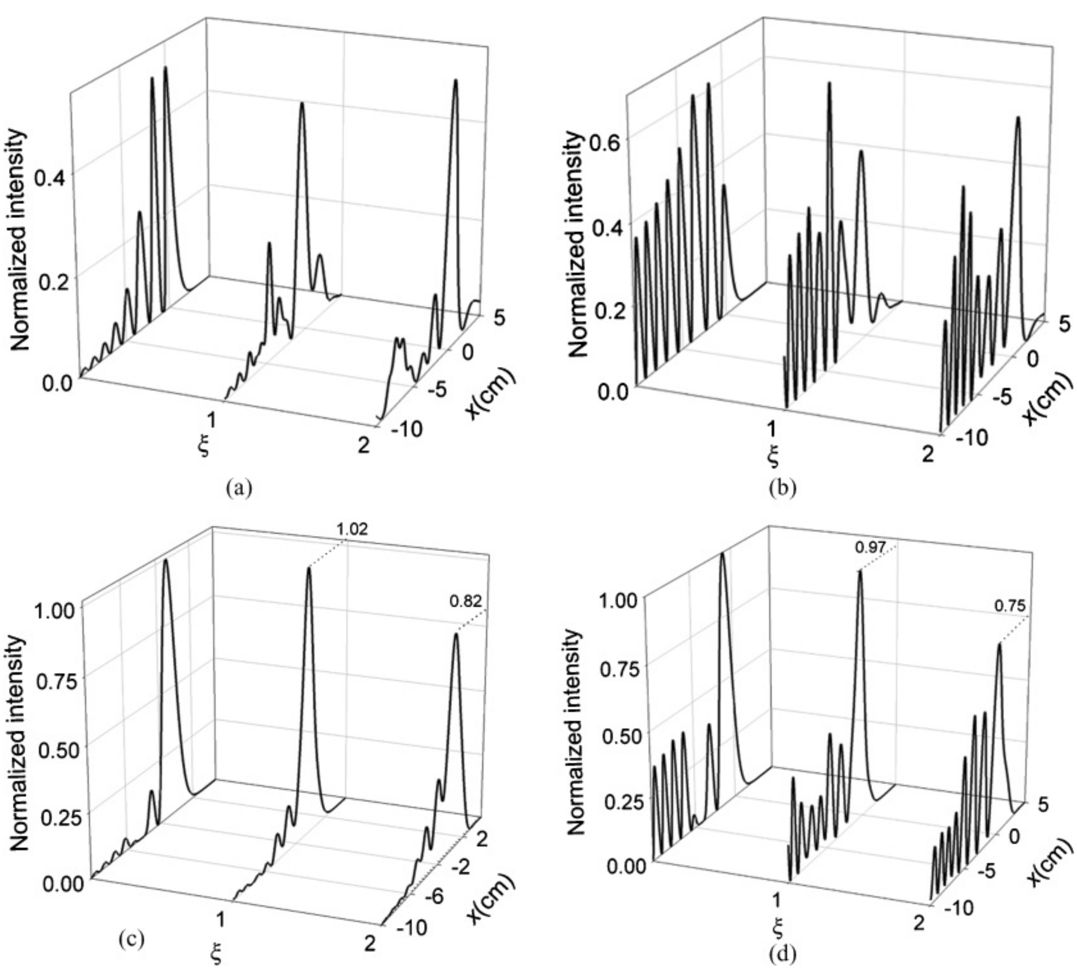

FIG. 7. Variation of the normalized intensity of the Airy beam with a different propagation distance for (a) $a = 0.2$, $\omega_0 = 0.5$ m, and $b_x = 0.73$ cm; (b) $a = 0$ (the exact Airy beam), $\omega_0 = 0.5$ cm, and $b_x = 1$ cm; (c) $a = 0.2$, $\omega_0 = 1$ cm, and $b_x = 4.4$ cm; and (d) $a = 0$, $\omega_0 = 1$ cm, and $b_x = 6.1$ cm.

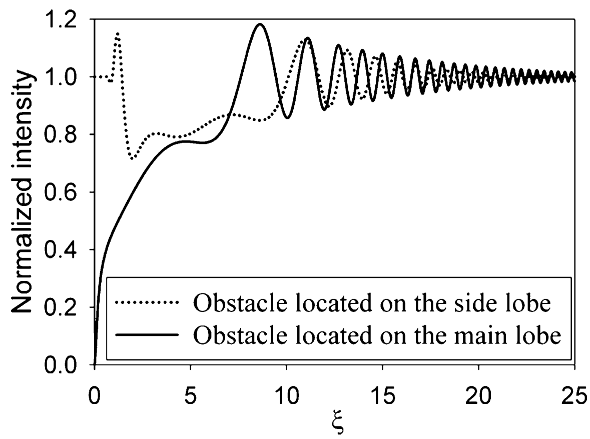

FIG. 8. Variation of the normalized intensity of the exact Airy beam ($a = 0$) along the trajectory $x = (1.02 + \xi^2/4)\omega_0$ with the parameters denoted by the solid and dotted lines the same as in Figs. 7(c) and 7(d), respectively.

To see the self-healing process of the Airy beam during propagating, the evolution of the one-dimensional intensity distribution is shown in Fig. 2. We see that when w_0 is small, the small part of the main lobe is blocked [see Fig. 2(a)] and then the self-healing process is fast. In contrast, a large obstacle causes slow reconstruction. From Fig. 2 we also see that the position of the peak value intensity for the Airy beam with a different obstacle moves isochronously from left to right during propagation.

Because the Airy beam cannot keep its shape unchanged during propagation due to its finite energy, its side lobes gradually disappear and the main lobe evolves into a Gaussian shape with the increase of propagation distance [14]. The self-healing process can also be seen from the Poynting vector, which is defined in the paraxial regime as

$$\vec{S} = \vec{S}_z + \vec{S}_\perp = \frac{1}{2\eta_0}|U_2|^2\hat{z} + \frac{i}{4\eta_0 k}[U_2\vec{\nabla}_\perp U_2^* - U_2^*\vec{\nabla}_\perp U_2], \tag{7}$$

where η_0 is the impedance of free space, $\vec{S}_z$ denotes the component along the z axis of the Poynting vector, and $\vec{S}_\perp = \vec{S}_x + \vec{S}_y$ is the transverse component. Because of the symmetry of the Airy beam, only the Poynting vector along the x axis is investigated in the following. The positive value denotes the energy flow in the direction of the positive x axis and the negative value denotes the opposite direction. The normalized Poynting vector is defined as its value divided by its maximum. As a comparison, the contour plot of $\vec{S}_x$ for the Airy beam without an opaque obstacle is plotted in Fig. 3.

Figure 3 shows that the flow of energy for the Airy beam trends to spread from the center to both sides. From a comparison with Fig. 2 we can see that the boundary for the three cases is a straight line located to the left of the peak value of the main lobe. The position of the boundary of the Poynting vector also moves from left to right like the peak intensity of the Airy beam during propagation.

The contour plots of the Poynting vector of the Airy beam with an opaque obstacle are shown in Figs. 4–6, where one could see that the distribution of the Poynting vector becomes more complex due to disturbances of the obstacle. For example, when the propagation distance is small ($\xi = 0.5$), there is energy flow in the direction of the negative x axis to the right of the boundary line, namely, some energy around the center of the opaque obstacle converges into the center to reform the beam. With the increase of propagation distance, the convergence becomes slow (see Fig. 5). With a further increase of the propagation distance, the convergence gradually disappears, namely, the self-healing process has finished (see Fig. 6). When the opaque obstacle is small, a shorter propagation distance is needed to reform the beam shape.

Besides the flow of energy, a physical explanation of the self-healing process of the Airy beam can also be given by catastrophe optics [16] and geometrical optics. Existing results show that the self-healing property is attributed to the caustic of the wave packet and generated by a continuum of sideways contributions to the field [1,16–18]. These results can also be obtained by using the analytical expression in the present paper. The evolution of the intensity distribution of the Airy beam with a different obstacle is shown in Fig. 7.

We can see that if the main lobe of the Airy beam is blocked partially by an obstacle [see Figs. 7(a) and 7(c)], the distorted main lobe will be reconstructed gradually during propagation. However, if the side lobes are blocked partially [see Figs. 7(b) and 7(d)], the main lobe remains unchanged when the propagation distance is short; however, with an increase of the propagation distance, the normalized intensity for the main lobe decreases. When we further increase the propagation distance, the effects of the obstacle on the main lobe disappear. To see the variation of the main lobe during propagation, the normalized intensity of the exact Airy beam ($a = 0$) along the trajectory $x = (1.02 + \xi^2/4)\omega_0$ is plotted in Fig. 8, where the parameters are the same as in Figs. 7(c) and 7(d), respectively. From Figs. 7 and 8 we can see that the obstruction of the main lobe will affect the intensity of the main lobe of the Airy beam with a short propagation distance and the obstruction far away from the main lobe will affect the intensity of the main lobe for the Airy beam with a longer propagation distance. The result agrees with the existing results and can be explained by the caustic of the Airy beam [1,18].

IV. CONCLUSION

We derive an analytical expression for the optical field of the Airy beam partially blocked by an opaque obstacle. Based on the formula, the self-healing process of the Airy beam in free space has been studied and discussed in detail. The results show that self-healing process is affected by the size of the opaque obstacle. A large opaque obstacle causes slow reform of the Airy beam. Meanwhile, the position of the peak value intensity for the Airy beam with a different obstacle moves isochronously from left to right during propagation. From the variation of the Poynting vector we can see that an opaque obstacle will cause the energy flow and its direction to change. Some energy around the center of the opaque obstacle converges into the center to reform the beam. When the convergence of the energy flow towards a center disappears we can consider the self-healing process to be finished. Although the obstruction of the side lobe far away from the main lobe does not affect the intensity of the main lobe in the initial plane, it will affect the main lobe when the propagation distance is longer. This agrees with the existing results and can be explained by the caustic of the Airy beam.

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