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Coherent control of quantum tunneling in an open double-well system

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We investigate how to apply a high-frequency driving field to the quantum control of a single particle in an open double-well system. The linear stability analysis points out that the stability depends on the external-field parameters and the loss (or gain) coefficients of the system, and the instability leads to a transition of the Floquet quasienergy from real to complex values and results in decaying probabilities for the particle to be in the double well. By combining analytical solutions in the high-frequency approximation with numerical calculations based on an accurate model, we exhibit quantum-dynamical behavior of the particle such as Floquet oscillation, coherent destruction of tunneling, quasi-NOON-state population, partial one-particle tunneling, and the decay of the probabilities of occupation, which are due to the competition and balance between the quantum coherence and the loss (or gain) effect. The results suggest an experimental method for testing quantum motion in an open system by adjusting the driving field.

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I. INTRODUCTION

Periodically driven double-well systems have received much attention over the last few years [1,2]. The main research interest has been motivated by the desire to show coherent control of quantum tunneling through such a system [3–6]. For applications, open quantum systems have recently become the subject of extensive studies [7]. The non-Hermiticity was due to the presence of various gain or loss mechanisms in open systems [8–11], which brought much new content to the study of quantum control [12–15]. For a single particle, the non-Hermitian double-well system (or the mathematically equivalent two-level system) is a basic system that is useful for researching coherent control [16–18]. It is not only a simple extension of the corresponding Hermitian one-body system, but can also be used to simulate non-Hermitian many-body systems without interactions [19], whereas the zero interaction strength can be realized by the Feshbach resonance technique [20]. However, most previous works on non-Hermitian double-well systems took into account the effect of a static field on the stationary states [16–18] and the incoherent control of quantum tunneling [21]; only a few works concerning the coherent control via a periodic external field have been reported [22].

In this paper, we study a different non-Hermitian system with a single particle held in an open and high-frequency-driven double well and seek the analytical solutions and their boundedness conditions. By applying the coherent-control method of the Hermitian system [23,24] to the non-Hermitian system, we explore the competition and balance between the coherent enhancement or suppression of tunneling and the loss (dissipation) or gain from the environment and further apply them to manipulate the stable quantum motions. Under the high-frequency approximation, our analytical results reveal the effects of the external-field parameters and loss or gain coefficient on the system's stability and display that the loss of stability leads to the transition of the Floquet quasienergy spectrum from real (exact phase) to complex (broken phase) values [8] and the corresponding decay of particle's occupied probabilities [21,25]. Due to the competition and balance between quantum coherence and loss or gain, quantum effects are shown such as Floquet oscillations of quantum states with real energies, coherent destruction of tunneling (CDT), quasi-NOON-state population [26], one-particle partial tunneling, Schrödinger-cat-like states, and decaying probabilities. The numerical computations from the accurate model confirm agreement with the analytical results. Based on the capacity of the current setups [5,27,28], we expect that the quantum motions of the open system can be experimentally tested by adjusting the driving parameters.

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II. GENERAL ANALYTICAL SOLUTION IN HIGH-FREQUENCY APPROXIMATION

We consider a single particle held in an open double well, whose quantum dynamics is dominated by the $\mathcal{PT}$-symmetric non-Hermitian Hamiltonian [8]

$$H(t) = \varepsilon_1(t)a_1^\dagger a_1 - \varepsilon_2(t)a_2^\dagger a_2 + \nu(a_1^\dagger a_2 + a_2^\dagger a_1),\\ \varepsilon_j(t) = \alpha\cos(\omega t) - i\beta_j, \quad j = 1,2, \tag{1}$$

where a_j ($a_j^\dagger$) is the annihilation (creation) operator for the atom in the jth well with $j = 1, 2$, ν is the coupling parameter which presents the tunneling rate between the two wells, and $\varepsilon_j(t)$ contains the driving field with amplitude α and frequency ω and the jth well's loss coefficient for $\beta_j > 0$ and/or gain coefficient for $\beta_j < 0$. To simplify, Eq. (1) has been treated as a dimensionless equation in which the reference frequency $\omega_0 \sim 10^2$ Hz and $\hbar = 1$ are set such that the parameters α, β_j, and ν are in units of ω_0 and time is normalized in units of ω_0^{-1}. Obviously, when $\beta_j = 0$ for $j = 1,\ 2$ are taken, system (1) becomes the familiar Hermitian system of a single particle in a double well [23,24]. In particular, when $\omega = 0$, $\alpha = \varepsilon$, $\beta_i = 0$, $\beta_j \neq 0$, $i \neq j$ or $\omega = 0$, $\beta_1 = \beta_2 = \gamma$ are selected, we arrive at the non-Hermitian many-particle Hamiltonian without interaction [19].

Using the localized states $|1\rangle$ and $|2\rangle$ as the basis, the quantum state $|\psi\rangle$ of system (1) can be expanded as [23,24]

$$|\psi\rangle = C_1(t)|1\rangle + C_2(t)|2\rangle, \tag{2}$$

where C_i for $i = 1,2$ denote the time-dependent probability amplitudes in the two wells. Inserting Eqs. (1) and (2) into the Schrödinger equation $i\frac{\partial|\psi\rangle}{\partial t} = H|\psi\rangle$ produces the coupled equations

$$\begin{aligned} i\dot{C}_1(t) &= \varepsilon_1(t)C_1(t) + \nu C_2(t),\\ i\dot{C}_2(t) &= -\varepsilon_2(t)C_2(t) + \nu C_1(t), \end{aligned} \tag{3}$$

of the probability amplitudes. Although Eq. (3) is very simple, no analytical solution in a finite form exists, because of the presence of the periodic functions $\varepsilon_j(t)$. However, for high-frequency driving with $\omega \gg \nu, \beta_j$, we can get the approximate analytical solution. To do this, we introduce the slowly varying functions $d_i(t)$ for $i = 1,2$ and make the function transformations [23,24]

$$\begin{aligned} C_1(t) &= \exp\left[-i\frac{\alpha}{\omega}\sin(\omega t)\right]d_1(t),\\ C_2(t) &= \exp\left[i\frac{\alpha}{\omega}\sin(\omega t)\right]d_2(t), \end{aligned} \tag{4}$$

which transform Eq. (3) into the coupled equations between the slowly varying functions,

$$\begin{aligned} i\dot{d}_1(t) &= -i\beta_1 d_1(t) + \nu\exp\left[i\frac{2\alpha}{\omega}\sin(\omega t)\right]d_2(t),\\ i\dot{d}_2(t) &= i\beta_2 d_2(t) + \nu\exp\left[-i\frac{2\alpha}{\omega}\sin(\omega t)\right]d_1(t). \end{aligned} \tag{5}$$

In the high-frequency limit, the slowly varying functions $d_i(t)$ can be treated approximately as constant during a short period $2\pi/\omega$. Thus, the rapidly varying exponential functions of Eq. (5) can be replaced by their time average [29] so that Eq. (5) simplifies to [23]

$$\begin{aligned} i\dot{d}_1(t) &= -i\beta_1 d_1(t) + Jd_2(t),\\ i\dot{d}_2(t) &= i\beta_2 d_2(t) + Jd_1(t), \end{aligned} \tag{6}$$

where $J = \nu J_0(2\alpha/\omega)$ is the effective or renormalized tunneling rate with $J_0(2\alpha/\omega)$ being the zero-order Bessel function of $2\alpha/\omega$, which depends on the field parameters through the ratio of driving strength and frequency. We here consider the strong-field case [5] in which the ratio of field parameters obeys $0.45 \leqslant \alpha/\omega < 2.6$. This gives the field amplitude α to be on the order of ω. In the high-frequency regime, such a driving strength means that the driving field used is a strong field. From the first of Eqs. (6), we arrive directly at

$$Jd_2 = i[\dot{d}_1(t) + \beta_1 d_1(t)]. \tag{7}$$

Combining Eq. (6) with Eq. (7), the former equation is decoupled to the form

$$\ddot{d}_1(t) + (\beta_1 - \beta_2)\dot{d}_1(t) + (J^2 - \beta_1\beta_2)d_1(t) = 0. \tag{8}$$

Clearly, Eq. (8) is a second-order linear equation with two constant coefficients; the smaller "damping factor" $\Gamma = \beta_1 - \beta_2$ and the lower "quadratic frequency" $\omega_l^2 = J^2 - \beta_1\beta_2$ for the slowly varying function $d_1(t)$. The physical bounded solutions of Eq. (8) exist only under the boundedness conditions $\Gamma \geqslant 0$ and $\omega_l^2 \geqslant 0$, where the equality means some balance between driving and damping. The former condition is fixed by the external environment, and the latter is controlled by the effective tunneling rate. The constant ω_l describes oscillation frequency of the slowly varying functions $d_i(t)$ and satisfies the inequality $\omega_l \ll \omega$.

The general solution of Eq. (8) is mathematically well known:

$$d_1 = F_1\exp(\lambda_1 t) + F_2\exp(\lambda_2 t), \tag{9}$$

where λ_1 and λ_2 are the characteristic values associated with the linear equations (6), F_1 and F_2 are undetermined constants determined by the initial conditions and normalization. The substitution of Eq. (9) into Eq. (7) gives another slowly varying function:

$$d_2 = \frac{i}{J}[F_1(\lambda_1 + \beta_1)\exp(\lambda_1 t) + F_2(\lambda_2 + \beta_1)\exp(\lambda_2 t)]. \tag{10}$$

Inserting Eqs. (9) and (10) into Eq. (6) and setting $F_1 = 0$ or $F_2 = 0$, respectively, we obtain the characteristic values

$$\lambda_{1,2} = \tfrac{1}{2}[(\beta_2 - \beta_1) \pm \sqrt{(\beta_1 - \beta_2)^2 - 4(J^2 - \beta_1\beta_2)}], \tag{11}$$

with λ_1 and λ_2 corresponding to the positive and negative signs, respectively. Given the general solutions (9) and (10), we can easily exhibit the time evolutions of probability $P_i = |d_i|^2$ for the particle localized in the ith well.

The stability analysis [30] on the linear equations (6) and (7) reveals that the real parts of the characteristic values λ_1 and λ_2 are related to stability of the linear system. Writing λ_1 and λ_2 in the form $\lambda_1 = \mathrm{Re}(\lambda_1) + i\mathrm{Im}(\lambda_1)$ and $\lambda_2 = \mathrm{Re}(\lambda_2) + i\mathrm{Im}(\lambda_2)$, we know that two special situations of λ_1 and λ_2 are associated with the particular properties of the solutions. First, if the boundedness conditions $\Gamma \geqslant 0$ and $\omega_l^2 \geqslant 0$ cannot be satisfied, the characteristic values with $\mathrm{Re}(\lambda_n) > 0$ ($n = 1$ and/or 2) appear such that the probability of the particle being in the jth well tends to infinity, $\lim_{t\to\infty}|d_j(t)|^2 \to \infty$. This means that the system loses its stability in the sense of Lyapunov [30]. The instability causes the Floquet quasienergy to transit from real to complex values. The corresponding solutions of Eqs. (9) and (10) do not satisfy the requirement of the probability interpretation in quantum mechanics and should therefore be dropped. Second, when the real parts $\mathrm{Re}(\lambda_1)$ and $\mathrm{Re}(\lambda_2)$ are equal to zero under the boundedness conditions, the imaginary parts correspond to the Floquet quasienergies [31,32] $E_1 = -\mathrm{Im}(\lambda_1)$ and $E_2 = -\mathrm{Im}(\lambda_2)$ for $F_1 = 0$ and $F_2 = 0$, respectively. Inserting d_j with $F_1 = 0$ or $F_2 = 0$ into Eqs. (2) and (4) gives the corresponding Floquet state $|\psi_1\rangle$ or $|\psi_2\rangle$. Therefore, the quantum state in Eq. (2) with the general solutions (9) and (10) may be a coherent superposition of the two Floquet states, $|\psi\rangle = D_1|\psi_1\rangle + D_2|\psi_2\rangle$, with D_1, D_2 being constants adjusted by the initial conditions and normalization. The superposition states imply quantum interferences and *may cause the coherent enhancement or suppression of tunneling*, whose degree is described by the value of the effective tunneling rate J.

Generally, for the appropriate environment with $\Gamma \geqslant 0$, we can obtain the physically meaningful solutions of Eq. (2)

from the general solutions (9) and (10) by adjusting the field parameters to obey the boundedness conditions $\omega_l^2 \geqslant 0$. We divide the physical solutions into three cases as follows:

Case A: CDT and quasi-NOON-state populations under dissipation balance. By the dissipation balance we mean that the loss (gain) coefficients of the two wells take the same values, $\beta_1 = \beta_2$. Such a balance could be established by making the two wells in the same environment. For such a case the adjustment to the field parameters or equivalent J in Eq. (11) could lead to $\mathrm{Re}(\lambda_n) = 0$, $(n = 1$ and $2)$ such that the probability amplitudes $d_j(t)$ $(j = 1$ and $2)$ in Eqs. (9) and (10) are the periodic functions $[\mathrm{Im}(\lambda_n) \neq 0]$ or constants $[\mathrm{Im}(\lambda_n) = 0]$. The corresponding quantum states in Eq. (2) become the Floquet states with a real quasienergy spectrum or zero energy, which describe the particle's quasistationary states whose populations do not change and the same occupied probability in any well is kept constant. Such quasistationary states are called the atomic quasi-NOON states, compared to the NOON state as a stationary state with the same probability of occupation in each well [26]. The invariable populations mean that CDT results in stable populations of particles in the initially occupied states.

Case B: Stable populations without dissipation balance. The unbalanced dissipation infers $\beta_1 \neq \beta_2$ and the partial loss of the stability, which will cause changes in the Floquet quasienergy from real to complex values [8]. By adjusting the effective tunneling rate J, we get $\mathrm{Re}(\lambda_n) < 0$, $\lambda_{n'} = 0$, $(n \neq n')$ which is associated with the probabilities $|d_j(t)|^2$ for $j = 1$ and 2 decreasing and increasing, respectively, from the initial values to the different final values. For any well the probability difference between the initial and final states indicates single-direction tunneling with different degrees. The populations may tend to a stable Schrödinger-cat-like state with a total probability of finding the particle in the double well of less than unity. But a special ratio between the two loss parameters could make the total probability tend to unity.

Case C: Instability and decaying probabilities. For the unbalanced dissipations and under the boundedness conditions, regulations to the field parameters can make $\mathrm{Re}(\lambda_n) < 0$ for $n = 1$ and 2 so that the $|d_j|^2$ for $j = 1$ and 2 decrease exponentially fast. The latter means loss of the specific stability of quantum mechanics [33]. Thus the survival probability [21,25] of the particle in an initial state and the total probability of finding the particle in the double well decay rapidly to zero.

In the next section, we will show that some balance conditions between the coherent enhancement or suppression of tunneling and the loss or gain from the environment can be realized by adjusting the field parameters, and such adjustments can control the particle's instability. Therefore, we can arrive at the physical solutions in the three above-mentioned cases and can manipulate the corresponding quantum motions of the open system via the periodic field.

III. COHERENT CONTROL OF QUANTUM TUNNELING VIA EXTERNAL FIELD

From Eqs. (9), (10), and (11), we know that the populations of the particle in the open double well depend not only on the effective tunneling rate J determined by the amplitude α and frequency ω of the external field, but also on the loss or gain coefficients from the environment, β_1 and β_2, respectively. The "quadratic frequency" $\omega_l^2 = J^2 - \beta_1\beta_2$ reflects the competition between coherent enhancement or suppression of tunneling and the loss or gain from the environment. The condition $\omega_l^2 = 0$ $(J^2 = \beta_1\beta_2)$ means the corresponding balance, which differs from the above-mentioned dissipation balance. For a fixed environment with the given loss or gain coefficient, the characteristic values λ_1 and λ_2 are adjusted only by the effective tunneling rate J. Therefore, in order to produce a required quantum state, we can select a higher driving frequency ω and then regulate the driving strength α to change the value of ω_l^2 and to control the competition. Because the different states are associated with different atomic populations, the quantum state (2) with the analytical solutions (9) and (10) are referable for experimentally researching the quantum tunneling and localization of the particle in the open double well. We will enter into details of the three above-mentioned situations.

A. CDT and quasi-NOON-state populations under dissipation balance

First, we consider the general situation of case A from Sec. II: $\mathrm{Re}(\lambda_n) = 0$ and $\mathrm{Im}(\lambda_n) \neq 0$. Because of dissipation balance, we can set $\beta_1 = \beta_2 = \beta$ and apply this to Eq. (11) to yield

$$\lambda_{1,2} = \pm\sqrt{-(J^2 - \beta^2)}. \tag{12}$$

For a weak loss or gain with small β value, we can adjust the field parameter $2\alpha/\omega$ to satisfy the competition condition $J^2 - \beta^2 > 0$ between J and β, such that Eq. (12) becomes

$$\lambda_{1,2} = \pm i\sqrt{J^2 - \beta^2} = \pm i\omega_l, \tag{13}$$

where ω_l is the lower frequency of slowly varying function $d_j(t)$. The high-frequency condition implies $\omega_l \ll \omega$. Inserting Eq. (13) into Eqs. (10) and (11) gives the periodic solutions

$$d_1 = F_1 e^{i\omega_l t} + F_2 e^{-i\omega_l t},$$
$$d_2 = \frac{i}{J}[F_1(i\omega_l + \beta)e^{i\omega_l t} + F_2(-i\omega_l + \beta)e^{-i\omega_l t}]. \tag{14}$$

We already know that this solution pair is stable according to the linear stability analysis of the previous section. Using the normalization condition

$$\begin{aligned} |d|^2 &= |d_1|^2 + |d_2|^2 \\ &= 2(|F_1|^2 + |F_2|^2) + \frac{4\beta F_1 F_2}{J^2}[\beta\cos(2\omega_l t) - \omega_l \sin(2\omega_l t)] \\ &= 1, \end{aligned} \tag{15}$$

we establish the relationships between constants F_1 and F_2 as

$$2(|F_1|^2 + |F_2|^2) = 1, \quad 4\beta F_1 F_2 = 0. \tag{16}$$

Given Eq. (16), we assert that, for a Hermitian system without dissipation $(\beta = 0)$, constants F_1 and F_2 are constrained only by the first equation of Eqs. (16), and the corresponding solution pair (14) contains more selections for the constants. The solutions without dissipation have been discussed previously and will not be considered here. For our non-Hermitian system, $\beta \neq 0$, the second equation of Eqs. (16) needs $F_i = 0$ for $i = 1$ or 2, and the first equation of Eqs. (16)

gives $F_i = 0$, $F_j = e^{i\phi}/\sqrt{2}$ for $i, j = 1,2$ and $i \neq j$ with ϕ being a constant. Neglecting the immaterial phase factor $e^{i\phi}$, we get two sets of solutions as follows: The first set of solutions from Eq. (14) with $F_1 = 0$ reads

$$d_1 = \frac{1}{\sqrt{2}} e^{-i\omega_l t}, \quad d_2 = \frac{(\omega_l + i\beta)}{\sqrt{2}\, J} e^{-i\omega_l t}, \tag{17}$$

which periodically change in time with frequency ω_l with $\omega_l \ll \omega$. Inserting Eqs. (17) into Eqs. (4) and (2) yields the quantum state

$$|\psi_1\rangle = \frac{e^{-i\omega_l t}}{\sqrt{2}} \left[e^{-i\frac{\alpha}{\omega}\sin(\omega t)}|1\rangle + \frac{(i\beta + \omega_l)}{J} e^{i\frac{\alpha}{\omega}\sin(\omega t)}|2\rangle \right]. \tag{18}$$

Similarly, the second set of solutions from Eq. (14) with $F_2 = 0$ leads to the quantum state

$$|\psi_2\rangle = \frac{e^{i\omega_l t}}{\sqrt{2}} \left[e^{-i\frac{\alpha}{\omega}\sin(\omega t)}|1\rangle + \frac{(i\beta - \omega_l)}{J} e^{i\frac{\alpha}{\omega}\sin(\omega t)}|2\rangle \right]. \tag{19}$$

Equations (18) and (19) denote a pair of the oscillating Floquet states whose phases change periodically and the corresponding Floquet quasienergies read $E_\pm = \pm\omega_l$, respectively. From Eq. (13) we know $J^2 = \beta^2 + \omega_l^2$. Making use of this in Eqs. (17) produces the constant probabilities $|d_j|^2 = 1/2$ for $j = 1$ and 2. They describe the quasi-NOON-state population with the same probability of the particle occupying each well [26]. The initially probabilities of occupation are kept and no quantum tunneling happens; this just is the well-known CDT. It is interesting to compare our CDT condition $J^2 = \beta^2 + \omega_l^2$ with the CDT condition $J = 0$ for a Hermitian double-well system [23,24]. The former condition means quantum coherence and environment damping are balanced.

If the field parameter $2\alpha/\omega$ is regulated to reach the balance condition $J^2 - \beta^2 = \omega_l^2 = 0$ ($J = \beta$), from Eq. (13) we arrive at the special situation of case A, $\text{Re}(\lambda_n) = 0$ and $\text{Im}(\lambda_n) = \pm\omega_l = 0$. Inserting this into Eqs. (18) and (19) yields the quasistationary state with zero Floquet quasienergy and invariable population,

$$\begin{aligned} |\psi\rangle &= |\psi_1\rangle = |\psi_2\rangle \\ &= \frac{1}{\sqrt{2}} e^{-i\frac{\alpha}{\omega}\sin(\omega t)} \left\{ |1\rangle + \exp i\left[\pi/2 + \left(\frac{2\alpha}{\omega}\right)\sin(\omega t) \right] |2\rangle \right\}, \end{aligned} \tag{20}$$

which describes the different quasi-NOON state.

The quasi-NOON states (18)–(20) with periodic phases are the standard single-particle NOON states at any fixed time [26]. They can be regarded as Schrödinger-cat-like states of a single particle in two-mode entanglement, which offers a new approach for investigating many-body entanglement and single-particle cat states [26]. Our results reveal the existence of the quasi-NOON state and provide a theoretical reference for experimentally preparing a quasi-NOON state in an open system.

B. Stable populations without dissipation balance

When the system is in case B of Sec. II, instability causes the transition of the Floquet quasienergy from real to complex values [8]. By setting the field parameters, from Eq. (11) we can get $\text{Re}(\lambda_n) < 0$ and $\lambda_{n'} = 0$ for $n \neq n'$. Applying such characteristic values to the general solutions (9) and (10), it is expected that the particle evolves from a given initial state to a stationary final state with a certain probability. Single-direction tunneling and decaying can occur simultaneously, which leads to the another kind of Schrödinger-cat-like state with total probability of finding the particle in the double well being less than unity. However, the special ratio $\beta_1/\beta_2 = 3$ could make the total probability tend to unity. The tunneling of the particle in the open double well depends on the competition between the coherent enhancement or suppression of tunneling and the loss or gain from the environment, so it is an interesting phenomenon that differs from that of the corresponding isolated system.

According Eq. (11), for the dissipation coefficients obeying $\beta_1 - \beta_2 > 0$ (or $\beta_1/\beta_2 > 1$) we can take $\lambda_1 = 0$ and $\lambda_2 = \beta_2 - \beta_1 < 0$ by adjusting the field parameter $2\alpha/\omega$ to get the new balance between the quantum coherence and loss, $\omega_l^2 = J^2 - \beta_1\beta_2 = 0$. Thus, the general solutions (9) and (10) become

$$\begin{aligned} d_1 &= F_1 + F_2 \exp[(\beta_2 - \beta_1)t], \\ d_2 &= \frac{i}{J}\{F_1\beta_1 + F_2\beta_2 \exp[(\beta_2 - \beta_1)t]\}. \end{aligned} \tag{21}$$

When the particle is initially located in the first well, we have the initial conditions $|d_1(0)| = 1$, $|d_2(0)| = 0$. Inserting them into Eq. (21) yields the undetermined constants F_1 and F_2 in the forms

$$F_1 = \frac{\beta_2}{\beta_2 - \beta_1}, \quad F_2 = \frac{\beta_1}{\beta_2 - \beta_1}. \tag{22}$$

Combining Eqs. (22) with Eqs. (21) results in the total probability finding the particle in the two wells:

$$\begin{aligned} P &= |d|^2 = |d_1|^2 + |d_2|^2 \\ &= \frac{(\beta_1 + \beta_2)[\beta_2 + \beta_1 e^{2(\beta_2 - \beta_1)t}] - 4\beta_1\beta_2 e^{(\beta_2 - \beta_1)t}}{(\beta_2 - \beta_1)^2}. \end{aligned} \tag{23}$$

It is well known that the probability interpretation of quantum mechanics requires the total probability to be less than or equal to one. For an open system, the survival probability of the particle may decay [21,25], and the initial normalized total probability may decrease with time. Therefore, it is necessary to confine the maximal value of P to $P_{\max} \leqslant 1$.

From $dP/dt|_{t=t_j} = 0$ we find that the total probability given in Eq. (23) has the three extrema in time $t_1 = 0$, $t_2 = \frac{1}{\beta_2 - \beta_1}\ln(\frac{2\beta_2}{\beta_1 + \beta_2})$, $t_3 = \infty$. At $t_1 = 0$, Eq. (23) gives $P_{\max}(0) = |d(0)|^2 = 1$, which agrees with the initial condition. When $t = t_2$ is reached, the total probability satisfies $P_{\max}(t_2) = \frac{\beta_2}{\beta_1 + \beta_2} < 1$. As time increasing to $t \to \infty$, Eq. (23) denotes the total probability of the final state, $P_{\max}(t_3) = |d(\infty)|^2 = \frac{\beta_2(\beta_1 + \beta_2)}{(\beta_1 - \beta_2)^2}$. The physical requirement $P_{\max}(t_3) \leqslant 1$ means that the corresponding parameter range, $\beta_1/\beta_2 \geqslant 3$, which agrees with the previous confining condition $\beta_1/\beta_2 > 1$ for Eqs. (21). When $\beta_1/\beta_2 = 3$ is set, the biggest final-state probability reads $|d(\infty)|^2 = 1$. If β_j are limited in the range $\beta_1/\beta_2 > 3$, the total probability of finding the particle is less than unity, but the particle still can be confined in the double well with a certain probability. As the value of β_1/β_2, is increased, Eq. (23) shows that the total probability $|P(\infty)|$ of the final state will decrease.

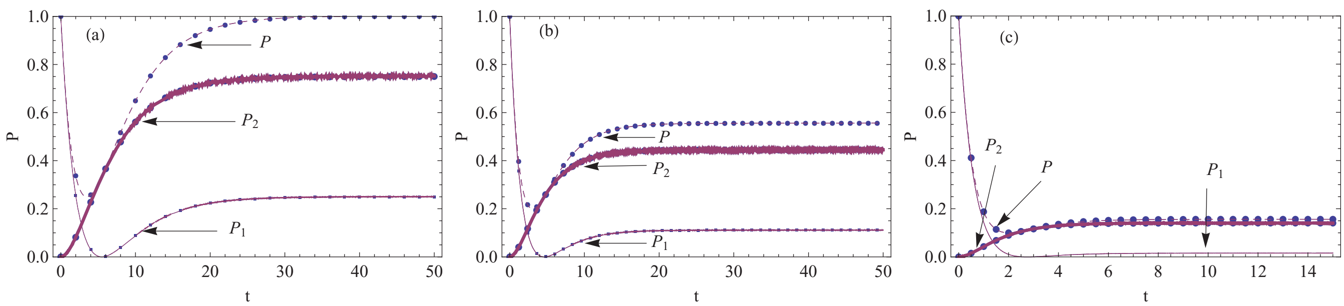

FIG. 1. (Color online) Time-evolution curves of probabilities $P_j = |d_j(t)|^2$ for the particle in the jth well and total probability $P = P_1 + P_2$, under the balance condition $J^2 - \beta_1\beta_2 = 0$. The circular points give the analytical solutions and the curves represent the numerical results for the high frequency $\omega = 80$ and the parameters (a) $\beta_1 = 0.3$, $\beta_2 = 0.1$, $\nu = 1$, $\alpha/\omega = 2.506\,69$, and $\beta_1/\beta_2 = 3$; (b) $\beta_1 = 0.4$, $\beta_2 = 0.1$, $\nu = 1$, $\alpha/\omega = 2.465\,37$, and $\beta_1/\beta_2 = 4$; (c) $\beta_1 = 0.9$, $\beta_2 = 0.1$, $\nu = 1$, $\alpha/\omega = 2.292\,42$, and $\beta_1/\beta_2 = 9$. The probabilities are dimensionless and time is in units of $\omega_0^{-1} = 0.01$ s.

Now let us numerically illustrate the above results. We take three sets of the parameters (ω, $2\alpha/\omega$, ν, β_1, β_2) to satisfy $J^2 - \beta_1\beta_2 = 0$ and $\beta_1/\beta_2 = 3$, 4, 9, respectively, and from Eqs. (21), (22), and (23) we plot the time-evolution figures of the probabilities $P_j = |d_j(t)|^2$ for the particle in the jth well and the total probability $P = |d(t)|^2$, as labeled by the circular points in Figs. 1(a), 1(b), and 1(c). With the same parameter conditions and based on the accurate model (3), we numerically make the time-evolution figures of $P_j = |d_j(t)|^2$ and $P = |d(t)|^2$, as shown by the curves in Figs. 1(a), 1(b), and 1(c). Obviously, in the high-frequency regime, the analytical and numerical solutions are in good agreement. Hereafter, an immaterial difference between both is that for the considered driving frequency $\omega = 80(\omega_0) = 8 \times 10^3$ Hz; the numerical solutions oscillate around the analytical solutions with high-frequency and small amplitude.

In Fig. 1(a) with the special ratio $\beta_1/\beta_2 = 3$, we can see that, after a transient decay, the total probability monotonically tends to the biggest value $|P(\infty)| = 1$, and the particle will be stably confined in the double well. The probability $P_1 = |d_1(t)|^2$ of the initially occupied state decays quickly for a short time, then increases slowly to approach the final value 0.3. The probability $P_2 = |d_2(t)|^2$ of the particle in the second well monotonically increases and will approach 0.7 for a sufficiently large time. This means that the particle will tunnel partly from the first well to the second well with probability 0.7. For the parameter ratio $\beta_1/\beta_2 = 4$, the comparison between Figs. 1(b) and 1(a) displays that the final value of any one of the probabilities P_j and P has decreased a little. The final total probability is less than unity, and the tunneling probability from the first well to the second well is less than 0.5. If $\beta_1/\beta_2 = 9$ is set, in Fig. 1(c) we show that the probabilities for every well and the total probability are further reduced. The probability P_1 in the initial state is finally close to zero, and the probability to tunnel to the second well tends to 0.2, which approximately equates with the total probability.

Similarly, if the particle is initially localized in the second well, $|d_1(0)| = 0$, $|d_2(0)| = 1$, the total probability is

$$P = \frac{(\beta_1 + \beta_2)[\beta_1 + \beta_2 e^{2(\beta_2-\beta_1)t}] - 4\beta_1\beta_2 e^{(\beta_2-\beta_1)t}}{(\beta_2 - \beta_1)^2}. \quad (24)$$

It is easy to prove that, under the match condition $\beta_1/\beta_2 > 1$ of Eqs. (21), the total probability in Eq. (24) may be greater than unity, which is physically unallowable and must be dropped. Therefore, for the field parameters satisfying $J^2 - \beta_1\beta_2 = 0$, the particle initially occupying well 2 cannot be stably trapped in the double well.

It is worth noting that, under the balance condition $J^2 = \beta_1\beta_2$, the change of β_j from $\beta_1 > \beta_2$ to $\beta_1 < \beta_2$ will lead to the nonphysical case in which one of the $\mathrm{Re}(\lambda_n)$ is greater than 0 and the two d_j tend to infinity. The physical requirement $\beta_1 > \beta_2$ implies that the external damping of the left well 2 is weaker, compared to that of the right well 1. Note that the direction of the external field is toward the right.

C. Instability and decaying probabilities

When the system is in case C of Sec. II, $\mathrm{Re}(\lambda_i) < 0$, ($i = 1$ and 2), the total probability of finding the particle in the double well will exponentially decay to zero. Thus the system loses the special stability of quantum mechanics, which is an important case for an open system [33].

The above case corresponds to the loss or gain coefficient obeying $\beta_1 - \beta_2 > 0$ and the field parameter $2\alpha/\omega$ obeying the new competition condition $4(J^2 - \beta_1\beta_2) > (\beta_1 - \beta_2)^2$ such that, from Eq. (11), we have

$$\lambda_{1,2} = \tfrac{1}{2}[(\beta_2 - \beta_1) \pm i\varpi], \quad (25)$$

with $\varpi = \sqrt{4J^2 - (\beta_1 + \beta_2)^2}$. For such a case, Eq. (25) gives the real part of λ_j to be always less than zero. According to the stability analysis, we know that, for any initial conditions, the system will lose its quantum mechanical stability and the total probability of finding the particle in the system will tend to zero. To simplify, we take the initial conditions $|d_1(0)| = 0$, $|d_2(0)| = 1$ as an example. Inserting the initial conditions into Eqs. (9) and (10) and using the normalization conditions produces the undetermined constants

$$F_1 = \pm\frac{J}{\varpi}, \quad F_2 = -F_1. \quad (26)$$

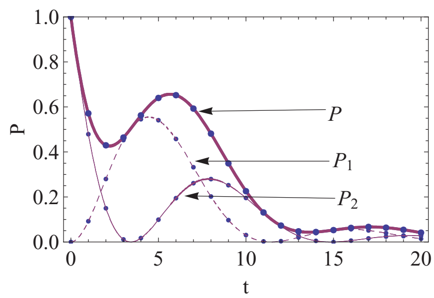

FIG. 2. (Color online) Time-evolution curves showing the decaying probabilities of the particle in the double well. The circular points describe the analytical results from Eq. (27) and the curves indicate the numerical solutions based on Eq. (3). The system parameters and the initial conditions are the same for the analytical and numerical solutions. The probabilities are dimensionless and time is in units of $\omega_0^{-1} = 0.01$ s.

Then, applying the F_1 and F_2 to Eqs. (9) and (10) yields the probabilities of the particle to be in the two wells:

$$|d_1|^2 = e^{-(\beta_1-\beta_2)t}\left\{\frac{2J^2[1-\cos(\varpi t)]}{\varpi^2}\right\},$$
$$|d_2|^2 = e^{-(\beta_1-\beta_2)t}\left[\frac{\varpi\cos\left(\frac{1}{2}\varpi t\right)+(\beta_1+\beta_2)\sin\left(\frac{1}{2}\varpi t\right)}{\varpi}\right]^2. \tag{27}$$

As an example, we take the parameter set (ω, $2\alpha/\omega$, ν, β_1, β_2) to match the condition $4(J^2-\beta_1\beta_2) > (\beta_1-\beta_2)^2$. Adopting such parameters, we illustrate the time evolution of the probabilities $P_j = |d_j(t)|^2$ and $P = P_1 + P_2$ from Eq. (27), as labeled by the circular points in Fig. 2. In the same conditions, we numerically solve the exact model (3), which is shown by the different curves of Fig. 2. Obviously, the analytical and numerical solutions are in good agreement. From Fig. 2 we can see that, after some transitory oscillations, the probability of the particle in any well and the total probability finally tend to zero. So in this case, the particle cannot exist in the double well for a long time.

In order to conveniently describe the above behavior, people employ the conception of survival probability in an initial state [21,25] to investigate how particles populate an open system. The survival probability of a particle in the initial state $|\psi(0)\rangle$ is defined as [21,25]

$$P_{\text{surv}}(t) = |\langle\psi(0)|\psi(t)\rangle|^2. \tag{28}$$

In the initial conditions $|d_1(0)|^2 = 0$, $|d_2(0)|^2 = 1$, combining Eqs. (2) and (4) with Eq. (27), from Eq. (28) we obtain $P_{\text{surv}}(t) = |\langle d_2(0)|d_2(t)\rangle|^2 = P_2$, which is exhibited in Fig. 2. Clearly, after a transitory oscillation the survival probability of initial state $|\psi(0)\rangle = |2\rangle$ decays and tends to zero.

IV. CONCLUSION AND DISCUSSION

We have considered a single particle held in an open and high-frequency-driven double well. The coherent-control method of quantum tunneling for the Hermitian system [23,24] is applied to the non-Hermitian system, which leads to the analytical solutions and their boundedness conditions under the high-frequency limit. By using the analytical results we show the effects of the field parameters and loss or gain coefficient on the system's stability and exhibit how to manipulate the stable quantum motions. We have revealed that the loss of stability leads to the transition of the Floquet quasispectrum from real to complex values [8] and the corresponding decay of particle's probabilities of occupation [21,25]. The competition and balance between the coherent enhancement or suppression of tunneling and the loss (dissipation) or gain from the environment are found, and quantum effects are shown such as the Floquet oscillations of the quantum states with real quasienergies, coherent destruction of tunneling for the new balance conditions, quasi-NOON-state population [26], partial one-particle tunneling, Schrödinger-cat-like states, and the decay of occupation probabilities. By comparing the analytical solutions with the numerical computations from an accurate model, we find good agreement between them, which emphasizes the correctness of the conclusions from the different methods and the suitability of the high-frequency approximation method for the open system. Based on the capacity of the current setups [5,27,28], we expect to experimentally test the quantum motions of the open system via a high-frequency driving field.

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