Efficient single-photon-assisted entanglement concentration for partially entangled photon pairs

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We present two realistic entanglement concentration protocols (ECPs) for pure partially entangled photons. A partially entangled photon pair can be concentrated to a maximally entangled pair with only an ancillary single photon with a certain probability, while the conventional ECPs require two copies of partially entangled pairs at least. Our first protocol is implemented with linear optics and the second protocol is implemented with cross-Kerr-nonlinearities. Compared with other ECPs, they do not need to know the accurate coefficients of the initial state. With linear optics, it is feasible with current experiments. With cross-Kerr-nonlinearities, it does not require sophisticated single-photon detectors and can be repeated to get a higher success probability. Moreover, the second protocol can get the higher entanglement transformation efficiency and it may be the most economical protocol by far. Meanwhile, both protocols are more suitable for multiphoton system concentration because they need less operations and classical communications. All these advantages make the two protocols useful in current long-distance quantum communications.

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I. INTRODUCTION

Entanglement plays an important role in current quantum information processing, such as quantum computation [1], quantum key distribution [2,3], quantum teleportation [4], controlled teleportation [5], dense coding [6], and quantum-state sharing [7]. In the past ten years, a large number of experiments have reported that quantum computation and quantum communication are more powerful in many aspects than their classical counterparts. In order to complete such quantum-information-processing protocols, maximally entangled states are usually required. However, in practical transmission and storage, the entanglement inevitably will contact the environment, and the noise will degrade the entanglement. Generally speaking, the maximally entangled state such as the Bell state $|\phi^+\rangle = \frac{1}{\sqrt{2}}(|H\rangle|H\rangle + |V\rangle|V\rangle)$ may become a mixed state. That is,

$$\rho = F|\phi^+\rangle\langle\phi^+| + (1-F)|\psi^+\rangle\langle\psi^+|. \quad (1)$$

Here, $|\psi^+\rangle = \frac{1}{\sqrt{2}}(|H\rangle|V\rangle + |V\rangle|H\rangle)$. It also may become a partially entangled state $|\varphi\rangle = \alpha|H\rangle|H\rangle + \beta|V\rangle|V\rangle$, with $|\alpha|^2 + |\beta|^2 = 1$. Here $|H\rangle$ ($|V\rangle$) represents the horizontal (vertical) photon polarization.

Entanglement purification can distill a set of mixed entangled states into a subset of highly entangled states with local operation and classical communication [8–15]. However, it can only improve the quality of the mixed state and cannot get the maximally entangled state. On the other hand, entanglement concentration, which will be detailed later, can be used to convert the partially entangled pairs to the maximally entangled pairs [16–27]. In 1996, Bennett *et al.* proposed the entanglement concentration protocol (ECP), which is called the Schmidt decomposition protocol. In their protocol, they use collective measurements which are difficult to manipulate experimentally [16]. Bose *et al.* also proposed an ECP based on entanglement swapping [17], but their protocol requires the collective Bell-state measurement. Moreover, they need to know the coefficients in order to reconstruct the same entangled states. In 2001, Zhao *et al.* and Yamamoto *et al.* proposed two similar ECPs independently with polarization beam splitters (PBSs) [19–22]. They simplified the Schmidt projection method and adopted the parity check to substitute the original collective measurement. Here we call it PBS1 protocol. This idea was developed to reconstruct the ECP with the cross-Kerr-nonlinearity, which can be used to construct quantum nondemolition detectors (QND) [25]. Here, we call it the QND1 protocol.

The current ECPs have in common that, for instance, in Refs. [19,21,23–27] they need initially at least two copies of less-entangled (or, say, partially entangled) pairs. But, after performing the protocols, at most one pair of maximally entangled states can be obtained, or both of each should be discarded, according to the measurement results by classical communication. Local operation and classical communication cannot be used to increase entanglement. Therefore, these ECPs are essentially the transformation of entanglement and the previous works of entanglement concentration are not optimal. Actually, two copies of less-entangled pairs are not necessary. Using one copy of a less-entangled pair to distillate high-quality entanglement has been proposed in continuous-variable system. In Ref. [28], Opatrný *et al.* showed the improvement on teleportation of continuous variables by photon subtraction via a conditional measurement. In 2008, He and Bergou proposed a general probabilistic approach for transforming a single copy of a discrete entanglement state without classical communication [29].

In this paper, we describe two single-photon-assisted ECPs in which only one pair of less-entangled states and one single photon are required. The two ECPs are focused on the practical

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discrete less-entangled photon pairs and are implemented with linear optics and the cross-Kerr-nonlinearity, respectively. Comparing with current ECPs, the single-photon-assisted ECPs are more economical. The first ECP, with linear optics, can reach the same success probability as the protocol of Ref. [21] but requires only one pair of less-entangled states. In the second ECP, with the help of the cross-Kerr-nonlinearity, it can be repeated to get a higher success probability. These advantages make the two protocols more feasible in practical applications.

This paper is organized as follows: In Sec. II, we first explain the basic principle of single-photon-assisted ECP with linear optics. We call it the PBS2 protocol. In Sec. III, we extend this protocol to the system of cross-Kerr-nonlinearity. We call it the QND2 protocol. We show that sophisticated single-photon detectors are not required and the discarded items in the PBS2 protocol can also be reused to perform concentration. A success probability higher than other protocols can be obtained. In Sec. IV, we first calculate the entanglement transformation efficiency and then present a discussion and summary.

II. SINGLE-PHOTON-ASSISTED ENTANGLEMENT CONCENTRATION WITH LINEAR OPTICS

The basic principle of our PBS2 protocol is shown in Fig. 1. The less-entangled pair of photons emitted from S_1 are sent to Alice and Bob. Photon a belongs to Alice and photon b belongs to Bob. The initial photon pair is in the following unknown state:

$$|\Phi\rangle_{a1b1} = \alpha|H\rangle_{a1}|H\rangle_{b1} + \beta|V\rangle_{a1}|V\rangle_{b1}. \tag{2}$$

The other source S_2 emits a single photon of the form

$$|\Phi\rangle_{a2} = \alpha|H\rangle_{a2} + \beta|V\rangle_{a2}. \tag{3}$$

Here, $|\alpha|^2 + |\beta|^2 = 1$ and $a1$, $b1$, and $a2$ are different spatial modes.

The initial state of the three photons can be written as

$$\begin{aligned}|\Psi\rangle = |\Phi\rangle_{a1b1} \otimes |\Phi\rangle_{a2} = \alpha^2|H\rangle_{a1}|H\rangle_{b1}|H\rangle_{a2} \\ +\alpha\beta|H\rangle_{a1}|H\rangle_{b1}|V\rangle_{a2} + \alpha\beta|V\rangle_{a1}|V\rangle_{b1}|H\rangle_{a2} \\ +\beta^2|V\rangle_{a1}|V\rangle_{b1}|V\rangle_{a2}.\end{aligned} \tag{4}$$

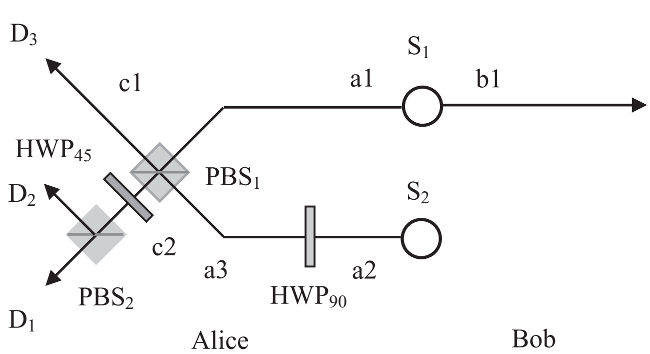

FIG. 1. Schematic of the single-photon-assisted ECPs with linear optics. S_1 is the partial entanglement source and S_2 is the single-photon source. PBSs transmit the horizontal polarization component and reflect the vertical component. HWP_{90} and HWP_{45} rotate the polarization of the state by 90° and 45°, respectively.

Alice first rotates the polarization state of the single photon $|\Phi\rangle_{a2}$ by 90° by half-wave plate (HWP_{90} in Fig. 1). Then the state can be rewritten as

$$\begin{aligned}|\Psi\rangle' &= |\Phi\rangle_{a1b1} \otimes |\Phi'\rangle_{a3} = \alpha^2|H\rangle_{a1}|H\rangle_{b1}|V\rangle_{a3} \\ &\quad +\alpha\beta|H\rangle_{a1}|H\rangle_{b1}|H\rangle_{a3} \\ &\quad +\alpha\beta|V\rangle_{a1}|V\rangle_{b1}|V\rangle_{a3} + \beta^2|V\rangle_{a1}|V\rangle_{b1}|H\rangle_{a3} \\ &= \alpha^2|H\rangle_{a1}|V\rangle_{a3}|H\rangle_{b1} + \beta^2|V\rangle_{a1}|H\rangle_{a3}|V\rangle_{b1} \\ &\quad +\alpha\beta(|H\rangle_{a1}|H\rangle_{a3}|H\rangle_{b1} + |V\rangle_{a1}|V\rangle_{a3}|V\rangle_{b1}).\end{aligned} \tag{5}$$

From above equation, it is evident that the items $|H\rangle_{a1}|H\rangle_{a3}|H\rangle_{b1}$ and $|V\rangle_{a1}|V\rangle_{a3}|V\rangle_{b1}$ will lead to two output modes $c1$ and $c2$, both containing exactly one photon. However, item $|H\rangle_{a1}|V\rangle_{a3}|H\rangle_{b1}$ will lead to two photons both in mode $c2$, and item $|V\rangle_{a1}|H\rangle_{a3}|V\rangle_{b1}$ will lead to both photons in mode $c1$. Therefore, by choosing the three-mode cases (i.e., each of modes $c1$, $c2$, and $b1$ contain exactly one photon), the initial state is projected into a maximally three-photon-entangled state:

$$|\Psi\rangle'' = \frac{1}{\sqrt{2}}(|H\rangle_{c1}|H\rangle_{c2}|H\rangle_{b1} + |V\rangle_{c1}|V\rangle_{c2}|V\rangle_{b1}), \tag{6}$$

with a probability of $2|\alpha\beta|^2$.

In order to generate a maximally entangled Bell state between Alice and Bob, they could perform a 45° polarization measurement on the $c2$ photon. In Fig. 1, with the quarter-wave plate (HWP_{45} in Fig. 1), it can make

$$\begin{aligned}|H\rangle_{c2} &\to \frac{1}{\sqrt{2}}(|H\rangle_{c2} + |V\rangle_{c2}), \\ |V\rangle_{c2} &\to \frac{1}{\sqrt{2}}(|H\rangle_{c2} - |V\rangle_{c2}).\end{aligned} \tag{7}$$

After the rotation, Eq. (6) evolves to

$$\begin{aligned}|\Psi\rangle''' = \tfrac{1}{2}(|H\rangle_{c1}|H\rangle_{b1} + |V\rangle_{c1}|V\rangle_{b1})|H\rangle_{c2} \\ +(|H\rangle_{c1}|H\rangle_{b1} - |V\rangle_{c1}|V\rangle_{b1})|V\rangle_{c2}.\end{aligned} \tag{8}$$

Now Alice lets photon $c2$ pass through PBS_2. Clearly, if detector D_1 fires, the photon pair is left in the state

$$|\phi^+\rangle_{a1b1} = \frac{1}{\sqrt{2}}(|H\rangle_{c1}|H\rangle_{b1} + |V\rangle_{c1}|V\rangle_{b1}). \tag{9}$$

If detector D_2 fires, the photon pair is left in the state

$$|\phi^-\rangle_{a1b1} = \frac{1}{\sqrt{2}}(|H\rangle_{c1}|H\rangle_{b1} - |V\rangle_{c1}|V\rangle_{b1}). \tag{10}$$

Both Eqs. (9) and (10) are maximally entangled states. One of them says Alice or Bob only needs to perform a phase flip to convert Eq. (10) to (9), and the whole concentration process is finished. It has the success probability of $2|\alpha\beta|^2$, which is the same as Ref. [21]. During the whole process, they do not require two copies of the less-entangled pairs, and only one pair and a single photon is required. Meanwhile, they do not need to know the exact coefficients of the initial states $|\Phi\rangle_{a1b1}$ and $|\Phi\rangle_{a2}$, but only require them to equal.

From Fig. 1, it is straightforward to extend this protocol to reconstruct maximally entangled multipartite Greenberg-Horne-Zeilinger (GHZ) states from partially entangled GHZ states.

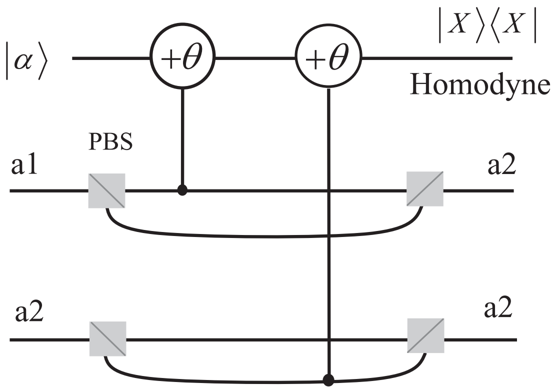

FIG. 2. Schematic of the principle of the quantum nondemolition detector(QND) constructed by the cross-Kerr-nonlinearity. It can also be shown in Ref. [25]. One can distinguish the state $|HH\rangle$ and $|VV\rangle$ from $|HV\rangle$ and $|VH\rangle$ by the different phase shift of the coherent state.

The partially multipartite entangled GHZ state with N photons can be written as

$$|\Phi\rangle_N = \alpha|H\rangle_A H\rangle_B \cdots H\rangle + \beta|V\rangle_A V\rangle_B \cdots V\rangle. \quad (11)$$

Here the subscriptions A, B, ... denote the parties Alice, Bob, Charlie, etc. Each one owns one photon. With the same principle described above, Alice needs another single photon of the same form as Eq. (3). After passing through the PBS, if the output modes of Alice's PBS both contain exactly one photon, then the whole system collapses into an $N+1$ maximally entangled state. Then, with the same principle, they can obtain the N-photon maximally entangled state by measuring one photon after rotating 45°.

Interestingly, this protocol seems more feasible for the concentration of a multiphoton GHZ system. First, in this protocol, only Alice needs to perform the protocol, while in the conventional protocols [19,21,25], all parties should perform the same operations as Alice. Second, in this protocol, only Alice asks other parities to retain or discard their photons, and they do not need to check their measurement results. It greatly simplifies the complication of classical communication. Third, multiphoton GHZ states are more difficult to generate in the current condition. We can have the same success probability but need only one pair of a less-entangled GHZ state.

III. SINGLE-PHOTON-ASSISTED ENTANGLEMENT CONCENTRATION WITH CROSS-KERR-NONLINEARITY

So far, we have briefly described the single-photon-assisted ECP based on linear optics. In the above description, Alice exploits the PBS and sophisticated single-photon detectors to distinguish $|HH\rangle$ and $|VV\rangle$ from $|HV\rangle$ and $|VH\rangle$. It is essentially the parity-check measurement of polarized photons. Unfortunately, with current technology, sophisticated single-photon detectors are not likely to be available, which makes this protocol unachievable with only linear optics. In this section, we introduce the cross-Kerr-nonlinearity to construct a QND, which can also be used to implement this protocol. The cross-kerr nonlinearity has been widely studied in the construction of CNOT gates [30], performance of

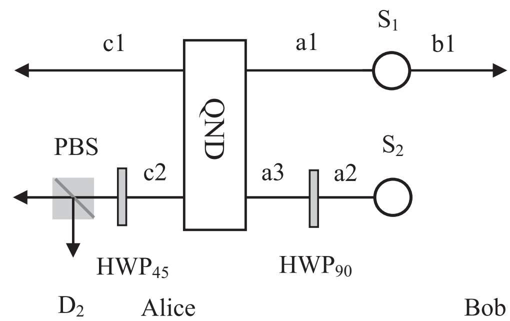

FIG. 3. Schematic of the ECP with QND. Here we substitute the PBS shown in Fig. 1 with the QND. This ECP can obtain a higher success probability.

complete Bell-state analysis [31], entanglement purification [13], and so on [32–40]. The Hamiltonian of a cross-Kerr-nonlinear medium can be written as $H = \hbar\chi \hat{n_a}\hat{n_b}$. Here, the $\hbar\chi$ is the coupling strength of the nonlinearity, which is decided by the cross-Kerr-material, and $\hat{n_a}$ ($\hat{n_b}$) is the number operator for mode a (b).

If we consider a coherent state $|\alpha\rangle$ and a single photon $|\psi\rangle = \gamma|0\rangle + \delta|1\rangle$ that interacts via the cross-Kerr-nonlinearity, the whole system can be described as [30,31]

$$\begin{aligned} U_{ck}|\psi|\alpha\rangle &= (\gamma|0\rangle + \delta|1\rangle)|\alpha\rangle \\ &= \gamma|0\rangle|\alpha\rangle + \delta|1\rangle|\alpha e^{i\theta}\rangle. \end{aligned} \quad (12)$$

Here, $|0\rangle$ and $|1\rangle$ are Fock states, which means no photon and one photon, respectively, and $\theta = \chi t$, where t is the interaction time. It is obvious that the phase shift of the coherent state is directly proportional to the number of photons.

In Fig. 3, we adopt the QND shown in Fig. 2 to substitute the PBS. Then the whole system $|\Phi\rangle_{a1b1} \otimes |\Phi'\rangle_{a3}$ with the coherent state $|\alpha\rangle$ can be written as

$$\begin{aligned} &|\Psi\rangle'|\alpha\rangle \\ &= |\Phi\rangle_{a1b1} \otimes |\Phi'\rangle_{a3}|\alpha\rangle = (\alpha^2|H\rangle_{a1}|H\rangle_{b1}|V\rangle_{a3} \\ &\quad + \alpha\beta|H\rangle_{a1}|H\rangle_{b1}|H\rangle_{a3} + \alpha\beta|V\rangle_{a1}|V\rangle_{b1}|V\rangle_{a3} \\ &\quad + \beta^2|V\rangle_{a1}|V\rangle_{b1}|H\rangle_{a3})|\alpha\rangle \\ &\to \alpha^2|H\rangle_{a1}|V\rangle_{a3}|H\rangle_{b1}|\alpha e^{i2\theta}\rangle + \beta^2|V\rangle_{a1}|H\rangle_{a3}|V\rangle_{b1}|\alpha\rangle \\ &\quad + \alpha\beta(|H\rangle_{a1}|H\rangle_{a3}|H\rangle_{b1} + |V\rangle_{a1}|V\rangle_{a3}|V\rangle_{b1})|\alpha e^{i\theta}\rangle. \end{aligned} \quad (13)$$

In above evolution, the items $|H\rangle_{a1}|V\rangle_{a3}$ and $|V\rangle_{a1}|H\rangle_{a3}$ pick up a 2θ phase shift and no phase shift, respectively. But the items $|H\rangle_{a1}|H\rangle_{a3}$ and $|V\rangle_{a1}|V\rangle_{a3}$ both pick up a θ phase shift. Therefore, if the phase shift of homodyne measurement is θ, Alice asks Bob to keep the whole state. Otherwise they discard the state. The remaining state is essentially the state described in Eq. (6). Therefore, following the same step described above, one can ultimately obtain the maximally entangled state $|\phi^+\rangle_{c1b1}$ if D_1 fires and gets $|\phi^-\rangle_{c1b1}$ if D_2 fires.

In above description, Alice only picks up the instance with phase shift θ on her coherent state and discards the other instances. If a suitable cross-Kerr-medium is available and Alice can control the interaction time t exactly, which makes the phase shift $\theta = \pi$, one cannot distinguish in this way the phase shift 0 from 2π. The discarded items in the above equation are

$$|\Phi'\rangle = \alpha^2|H\rangle_{c1}|V\rangle_{a3}|H\rangle_{b1} + \beta^2|V\rangle_{c1}|H\rangle_{a3}|V\rangle_{b1}, \quad (14)$$

with probability $|\alpha|^4 + |\beta|^4$. Alice uses HWP_{45} to rotate the photon in $c2$ and finally detects it by D_1 or D_2. Equation (14) becomes

$$|\Phi''\rangle = \alpha^2|H\rangle_{c1}|H\rangle_{b1} + \beta^2|V\rangle_{c1}|V\rangle_{b1} \quad (15)$$

if D_1 fires and becomes

$$|\Phi'''\rangle = \alpha^2|H\rangle_{c1}|H\rangle_{b1} - \beta^2|V\rangle_{c1}|V\rangle_{b1} \quad (16)$$

if D_2 fires.

$|\Phi''\rangle$ and $|\Phi'''\rangle$ are both the partially entangled states which can also be used to reconcentrate to the maximally entangled states. For instance, if they get $|\Phi''\rangle$, Alice only needs to choose another single photon of form $\alpha^2|H\rangle_{a2} + \beta^2|V\rangle_{a2}$ and follows the same method as described above. That is,

$$\begin{aligned}
&(\alpha^2|H\rangle_{c1}|H\rangle_{b1} + \beta^2|V\rangle_{c1}|V\rangle_{b1})\\
&\quad\otimes(\alpha^2|H\rangle_{a2} + \beta^2|V\rangle_{a2}) \otimes |\alpha\rangle\\
&\to (\alpha^2|H\rangle_{c1}|H\rangle_{b1} + \beta^2|V\rangle_{c1}|V\rangle_{b1})\\
&\quad\otimes(\alpha^2|V\rangle_{a2} + \beta^2|H\rangle_{a2}) \otimes |\alpha\rangle\\
&\to \alpha^4|H\rangle_{a1}|V\rangle_{a3}|H\rangle_{b1}|\alpha e^{i2\theta}\rangle + \beta^4|V\rangle_{a1}|H\rangle_{a3}|V\rangle_{b1}|\alpha\rangle\\
&\quad + (\alpha\beta)^2(|H\rangle_{a1}|H\rangle_{a3}|H\rangle_{b1} + |V\rangle_{a1}|V\rangle_{a3}|V\rangle_{b1})|\alpha e^{i\theta}\rangle.
\end{aligned} \quad (17)$$

Alice can also pick up the phase shift θ and get the same state as Eq. (6). The probability of success for obtaining Eq. (6) is

$$P_2 = \frac{2|\alpha\beta|^4}{|\alpha|^4 + |\beta|^4}. \quad (18)$$

We can also get

$$\begin{aligned}
P_3 &= \frac{2|\alpha\beta|^8}{|\alpha|^8 + |\beta|^8},\\
&\cdots,\\
P_N &= \frac{2|\alpha\beta|^{2^N}}{|\alpha|^{2^N} + |\beta|^{2^N}},
\end{aligned} \quad (19)$$

where N is the iteration number of our concentration processes.

The total success probability to get a maximally entangled state from the initial partially entangled state is

$$P = P_1 + P_2 + \cdots + P_N = \sum_{N=1}^{\infty} P_N. \quad (20)$$

Interestingly, if $\alpha = \beta = \frac{1}{\sqrt{2}}$, $P = \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^N} = 1$. But if $\alpha \neq \beta$, $P < 1$. Figure 4 shows that the relationship between the coefficient of the initial partially entangled state α and the total success probability P. From Fig. 4, it is shown that the success probability is not a fixed value. It is related with the entanglement of the initial state, and it increases with the entanglement of the initial partially entangled state.

We should point out that, during a practical operation, the longer interaction time will induce decoherence from losses. It will make the output state become a mixed state. Therefore, controlling the longer interaction time to make the phase shift

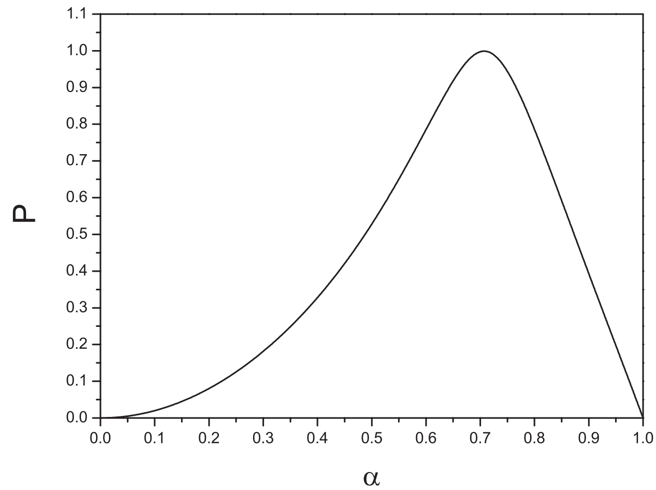

FIG. 4. Success probability P of getting a maximally entangled state after performing the QND2 protocol $N (N \to \infty)$ is altered with the entanglement of the initial partially entangled state (i.e., α). For the numerical simulation, we chose $N = 10$ as a good approximation.

$\theta = \pi$ may not seem like an efficient way. Fortunately, a better alternative is to rotate the coherent state in Eq. (13) by θ. After rotation, Eq. (13) becomes

$$\begin{aligned}
&\to \alpha^2|H\rangle_{a1}|V\rangle_{a3}|H\rangle_{b1}|\alpha e^{i\theta}\rangle\\
&\quad + \beta^2|V\rangle_{a1}|H\rangle_{a3}|V\rangle_{b1}|\alpha e^{-i\theta}\rangle\\
&\quad + \alpha\beta(|H\rangle_{a1}|H\rangle_{a3}|H\rangle_{b1} + |V\rangle_{a1}|V\rangle_{a3}|V\rangle_{b1})|\alpha\rangle.
\end{aligned} \quad (21)$$

From the above description, if the coherent state picks up no phase shift, the remaining state is also the same as Eq. (6). Otherwise, one can use $|X\rangle\langle X|$ homodyne detection [30], which makes $|\alpha e^{\pm i\theta}\rangle$ so that it cannot be distinguished. In this way, the discarded state is also the same as described in Eq. (14). Moreover, with the help of the QND, this protocol can also be extended to multiphoton systems and can be used to reconstruct maximally entangled multiphoton GHZ states. It has the same success probability as shown in Fig. 4.

IV. DISCUSSION AND SUMMARY

Thus far, we have fully described our protocols with both PBS and QND. In each protocol, we only require one pair of less-entangled photons and a single photon. It is known that local operation and classical communication cannot increase the entanglement. Therefore, entanglement concentration is essentially the transformation of entanglement. We define the entanglement transformation efficiency η as

$$\eta = \frac{E_c}{E_0}. \quad (22)$$

Here, E_0 is the entanglement of the initial partially entangled state and E_c is the entanglement of the state after concentrating one time. E_c can be described as

$$E_c = P_s \times 1 + (1 - P_s)E'. \quad (23)$$

The first term of Eq. (23) means that, after concentration, we get the maximally entangled state with success probability P_s. The second term means that the concentration is a failure, and we get a lesser-entangled pair. Obviously, if we use the PBS to perform the concentration, the second term is 0 because it collapses to a separated state $|HV\rangle$ or $|VH\rangle$ in different spatial modes [19,21]. For a two-body pure entangled state, von Neumann entropy is suitable to describe the entanglement. Therefore, the entanglement of the initial state in Eq. (2) can be described as

$$E = -|\alpha|^2 \log_2 |\alpha|^2 - |\beta|^2 \log_2 |\beta|^2. \tag{24}$$

We calculate the η of the PBS1 protocol as [21]

$$\eta_{\rm PBS1} = \frac{2|\alpha\beta|^2 \times 1}{2E} = \frac{|\alpha\beta|}{E}. \tag{25}$$

The “2” in the denominator means that initially we need two copies of less-entangled states with entanglement E.

For the QND1 protocol [25],

$$\eta_{\rm QND1} = \frac{E'_{\rm QND1}}{2E}, \tag{26}$$

with

$$E'_{\rm QND1} = 2|\alpha\beta|^2 + (|\alpha|^4 + |\beta|^4) \times \left(-\frac{|\alpha|^4}{|\alpha|^4+|\beta|^4}\log_2\frac{|\alpha|^4}{|\alpha|^4+|\beta|^4} - \frac{|\beta|^4}{|\alpha|^4+|\beta|^4}\log_2\frac{|\beta|^4}{|\alpha|^4+|\beta|^4}\right). \tag{27}$$

In our protocol, we only need one pair of less-entangled states to perform the protocol. Therefore, in the PBS2 protocol,

$$\eta_{\rm PBS2} = \frac{2|\alpha\beta|^2}{E} = 2\eta_{\rm PBS1} \tag{28}$$

and, in the QND2 protocol,

$$\eta_{\rm QND2} = \frac{E'_{\rm QND1}}{E} = 2\eta_{\rm QND1}. \tag{29}$$

The relationship between the coefficient α and entanglement transformation efficiency is shown in Fig. 5. It is shown that η is also not a fixed value but increases with the initial entanglement. In the QND2 protocol, η can reach the maximum value 1 with $\alpha = \frac{1}{\sqrt{2}}$, which means that the initial state is the maximally entangled state. But in traditional protocols [21,25], $\eta \leqslant 0.5$.

We also calculate the limit of the entanglement transformation efficiency of the QND2 protocol by iterating the protocol $N(N \to \infty)$ times:

$$\eta^{N\to\infty}_{\rm QND2} = \frac{\sum_{N=1}^{\infty} E_N P_N}{E_0} = \frac{P}{E_0}, \tag{30}$$

where E_N gives the entanglement of the remaining states after performing a successful concentration in the Nth iteration. It is a maximally entangled state with $E_N = 1$. Figure 6 shows the relationship between α and the transformation probability η. Obviously, η monotonically increases with the entanglement of the initial state and can reach the maximum value 1 when the initial state is maximally entangled, because $\alpha = \frac{1}{\sqrt{2}}$.

In this paper, the basic elements for us to complete the task are the PBS and QND. In fact, both of them play the same role (i.e., a parity check). In Refs. [21] and [25], they also resort to PBS and QND to perform the concentration. But in each step, they require two pairs of less-entangled states. Our protocol shows that, with only one pair of less-entangled states and a single photon, we can also perform this task. This good feature gives these protocols a higher entanglement transformation efficiency than others. In the process of describing our concentration protocol, we exploit the entanglement source and the single-photon source. An ideal single-photon source should emit exactly one and only one photon when the device is triggered. However, no single-photon source or entanglement source will be ideal. With current technology, a practical pulse generated by a source may contain no photons or multiple photons, with different probabilities. We denote as P_m the probability of emitting m photons. Interestingly, P_0 means no

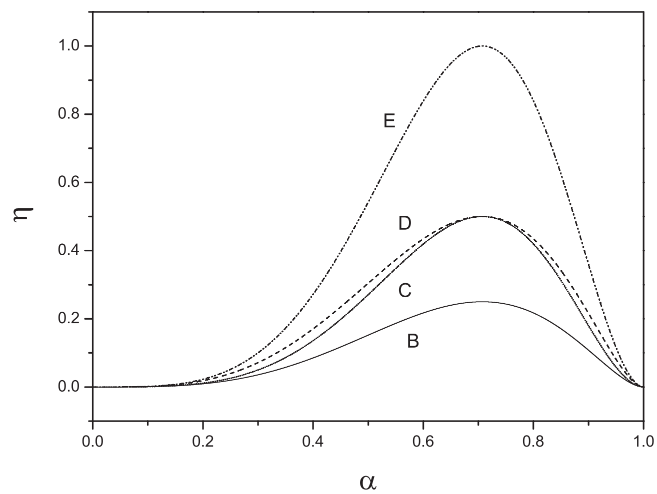

FIG. 5. Entanglement transformation efficiency η is altered by the coefficient α after performing each protocol one time. Curves B, C, D, and E correspond to the protocols PBS1 [21], QND1 [25], PBS2, and QND2, respectively. All the curves show that η increases with entanglement of the initial entangled state. The QND2 protocol has the highest transformation efficiency. It can reach the maximum value 1 when $\alpha = \frac{1}{\sqrt{2}}$.

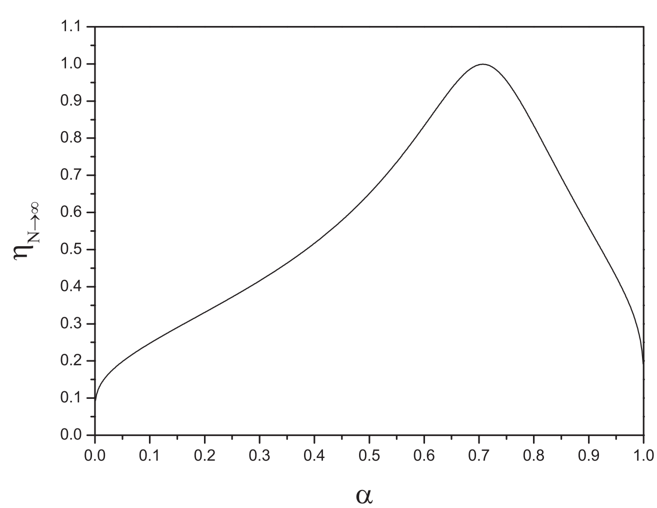

FIG. 6. Entanglement transformation efficiency η plotted against α after performing each protocol $N(N \to \infty)$ in the QND2 protocol. For the numerical simulation, we chose $N = 10$ as a good approximation.

photon and will not give rise to errors. It only decreases the success probability of the protocol because only two photons cannot satisfy the three-mode cases. However, $m \geqslant 2$ will give rise to errors. For example, $m = 2$ will lead both modes $c1$ and $c2$ to contain one photon in Fig. 1, which is a success event in our protocol. Fortunately, it is possible to make the probability of such events rather small. In Ref. [41], a single-photon source was reported whose P_0 and P_2 were 14% and 0.08% respectively. Current spontaneous parametric-down-conversion entanglement sources are analogous to the single-photon source. It generates an entangled pair of the form [19]

$$|\Upsilon\rangle = \sqrt{g}(|\text{vac}\rangle + \gamma|\phi^+\rangle + \gamma^2|\phi^+\rangle^2 + \cdots). \qquad (31)$$

The multiphoton items $|\phi^+\rangle^2$ can also cause errors. In practical teleportation experiments, $\gamma^2 \sim 10^{-4}$ [42,43], and the errors of multiphoton items are negligible.

In Sec. III, we exploit the cross-Kerr-nonlinearity to implement our protocol. Although a lot of works have studied the area of cross-Kerr-nonlinearities [25,26,30–40], we should acknowledge that it is still a quite-controversial assumption to have a clean cross-Kerr-nonlinearity in the optical single-photon regime. In 2002, Kok *et al.* pointed out that the Kerr phase shift is only $\tau \approx 10^{-18}$ in the optical single-photon regime [44,45]. In 2003, Hofmann showed that, with a single two-level atom in a one-sided cavity, a large phase shift of π can be achieved [46]. Gea-Banacloche showed that large shifts via the giant Kerr effect with a single-photon wave packet are impossible with current technology [47]. The results of previous work of Shapiro and Razavi are consistent with Gea-Banacloche [48,49]. Recently, He *et al.* discussed how the feasibility of QNDs relies on the compatibility of small phase shifts with large coherent-state amplitude. They developed a general theory of iteration between continuous-mode photonic pulses and applied it to the case of single photons interacting with a coherent state. They showed that, if the pulses can fully pass through each other and the unwanted transverse-mode effects can be suppressed, the high fidelities, nonzero conditional phases, and high photon numbers are compatible [37]. Recent research also shows that, by using weak measurements, it is possible to amplify a cross-Kerr-phase-shift to an observable value, which is much larger than the intrinsic magnitude of the single-photon-level nonlinearity [50].

In summary, we have presented two different protocols for nonlocal entanglement concentration of partially entangled states. We exploit both the PBS and the cross-Kerr-nonlinearity to achieve the task. Our protocols have several advantages: First, they do not need to know the exact efficiency α and β of the less-entangled pairs. Second, they also do not resort to collective measurements. Third, with QNDs, the parties are not required to adopt sophisticated single-photon detectors, and it can be iterated to get a higher success probability. Fourth, compared with previous works, the most significant advantage is that, in each step, we only need one pair of the less-entangled state. It allows our protocols to obtain higher entanglement transformation efficiencies than others. Fifth, these protocols are more feasible for multiphoton GHZ state concentration because they greatly reduce the practical operations and simplify the complication of classical communication for each parties. All these advantages may make our protocols more useful in practical applications.

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