Steady-state entanglement and normal-mode splitting in an atom-assisted optomechanical system with intensity-dependent coupling

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In this paper, we study theoretically bipartite and tripartite continuous variable entanglement as well as normal-mode splitting in a single-atom cavity optomechanical system with intensity-dependent coupling. The system under consideration is formed by a Fabry-Pérot cavity with a thin vibrating end mirror and a two-level atom in the Gaussian standing wave of the cavity mode. We first derive the general form of the Hamiltonian describing the tripartite intensity-dependent atom-field-mirror coupling due to the presence of the cavity mode structure. We then restrict our treatment to the first vibrational sideband of the mechanical resonator and derive a tripartite atom-field-mirror Hamiltonian. We show that when the optical cavity is intensely driven, one can generate bipartite entanglement between any pair in the tripartite system and that, due to entanglement sharing, atom-mirror entanglement is efficiently generated at the expense of optical-mechanical and optical-atom entanglement. We also find that in such a system, when the Lamb-Dicke parameter is large enough, one can simultaneously observe the normal mode splitting into three modes.

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I. INTRODUCTION

Cavity optomechanics is a rapidly growing field of research that is concerned with the interaction between a mechanical resonator (MR) and the radiation pressure of an optical cavity field [1–6]. Optomechanical coupling is widely employed for a large variety of applications [7], more commonly as a sensor for the detection of weak forces [8] and small displacements [9] or as an actuator in integrated electrical, optical, and optoelectronic systems [10,11]. However, the most experimental and theoretical efforts are devoted to cooling and trapping such MRs in their quantum ground state, which recently has been done successfully [12]. Furthermore, in Ref. [13] the authors proposed a different scheme to enhance the cooling process by using photothermal (bolometric) force [14]. They took into account noise effects due to the granular nature of photon absorption and, finally, showed that the MR can achieve the lowest phonon occupation number by means of this procedure. Moreover, it seems promising for the realization of long-range interaction between qubits in future quantum information hardware [15], and for probing of quantum mechanics at increasingly large mass and length scales [16]. The coupling of an MR via radiation pressure to a cavity field shows interesting similarities to an intracavity nonlinear Kerr-like interaction [6] or even a more complicated form of nonlinearity [17].

To observe and control quantum behavior in an optomechanical system, it is essential to increase the strength of the coupling between the mechanical and the optical degrees of freedom. However, the form of this coupling (e.g., linear or nonlinear) is crucial in determining which phenomena can be observed in such a system. Thanks to the rapid progress of nanotechnology, it has been possible to manipulate the optomechanical coupling in quantum optomechanical hybrid systems. In this direction, most experimental and theoretical efforts are devoted to entangling an MR either with a single atom [18–22] or with atomic ensembles [23–28], entangling a nanomechanical oscillator with a Cooper-pair box [29], and entangling two charge qubits [30] or two Josephson junctions [31] via nanomechanical resonators. Alternatively, schemes for entangling a superconducting coplanar waveguide field with a nanomechanical resonator, either via a Cooper-pair box within the waveguide [32] or via direct capacitive coupling [33], have been proposed.

In Ref. [24] the authors proposed a scheme for the realization of a hybrid, strongly quantum-correlated system consisting of an atomic ensemble surrounded by a high-finesse optical cavity with a vibrating mirror. They have shown that, in an experimentally accessible parameter regime, the steady state of the system shows both tripartite and bipartite continuous variable (CV) entanglement. More recently, the dynamics of a movable mirror of a cavity coupled through radiation pressure to the light scattered from ultracold atoms in an optical lattice was investigated [34]. The author showed that in the presence of atom-atom interaction as a source of nonlinearity [35], the coupling of the mechanical oscillator, the cavity field fluctuations, and the condensate fluctuations (Bogoliubov mode) leads to the splitting of the normal mode into three modes (normal-mode splitting (NMS) [36–40]). The system described there shows a complex interplay among three distinct systems, namely, the nanomechanical cantilever, the optical microcavity, and the gas of ultracold atoms.

Optomechanical NMS is one of the fascinating phenomena arising from the strong coupling between a cavity and a mechanical mirror [41–43]. In Ref. [42] it was shown that the cooling of mechanical oscillators in the resolved sideband regime with a high-driving-power laser can entail the appearance of NMS. Moreover, the dynamics of a movable

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mirror of a nonlinear optical cavity is considered in Ref. [43]. It has been shown that a $\chi^{(3)}$ medium with a strong Kerr nonlinearity placed inside the cavity inhibits NMS due to the photon blockade mechanism (this happens only if the Kerr nonlinearity is much greater than the cavity decay rate). As the authors showed in Refs. [34,43], the nonlinearity plays a crucial role in the appearance of NMS in optomechnical systems.

The main purpose of the present paper is to study the quantum behavior of an atom-assisted cavity optomechanical system in which a single two-level atom is trapped in the standing-wave light field of a single-port Fabry-Perot cavity. The infinite set of optical modes of the cavity can be described by Hermite-Gauss modes. As we will see, the intracavity mode structure can be employed to realize a type of intensity-dependent coupling of the single atom to the vibrational mode of MR. The presence of such intensity-dependent interaction modifies the dynamics of the system, the entanglement properties, and the displacement spectrum of MR. We show that in the first vibrational sideband of MR, a stationary, i.e., long-lived, atom-mirror entanglement can be generated by properly matching the Lamb-Dicke parameter (LDP). This parameter plays an important role in our investigation in the sense that it determines the strength of the nonlinearity in the system. We show that bipartite entanglement between the subsystems depends greatly on the LDP. It is also remarkable that, in the steady-state condition, a high resolution of NMS in the form of three-mode splitting is approached. In particular, the appearance of intensity-dependent coupling leads to a progressive increase in NMS due to the strong nonlinear atom-field-mirror interaction.

The paper is organized as follows. In Sec. II we derive an intensity-dependent Hamiltonian describing the triple coupling of atom-field-mirror through j-phonon excitations of the vibrational sideband. In Sec. III, we derive the quantum Langevin equations (QLEs) and linearize them around the semiclassical steady state. In Sec. IV we study the steady state of the system and quantify the entanglement properties of the system by using the logarithmic negativity. In Sec. V we investigate the appearance of NMS in the displacement spectrum of the mirror. Our conclusions are summarized in Sec. VI.

II. MODEL

The system studied in this paper is sketched in Fig. 1. It consists of a hybrid system formed by a single two-level atom with transition frequency $\omega_{\rm e}$ which is trapped in the standing-wave light field of a single-port Fabry-Perot cavity with a movable mirror coated on the plane side of an MR. The geometry of the resonator determines the spatial structure of the acoustic modes. The movable mirror is treated as a quantum mechanical harmonic oscillator with effective mass m, frequency $\omega_{\rm m}$, and energy decay rate $\gamma_{\rm m}$. The system is also coherently driven by a laser field with frequency ω_l through the cavity mirror with amplitude $\mathcal{E}$. We assume that the single atom is indirectly coupled to the mechanical oscillator via the common interaction with the intracavity field at frequency $\omega_{\rm c}$.

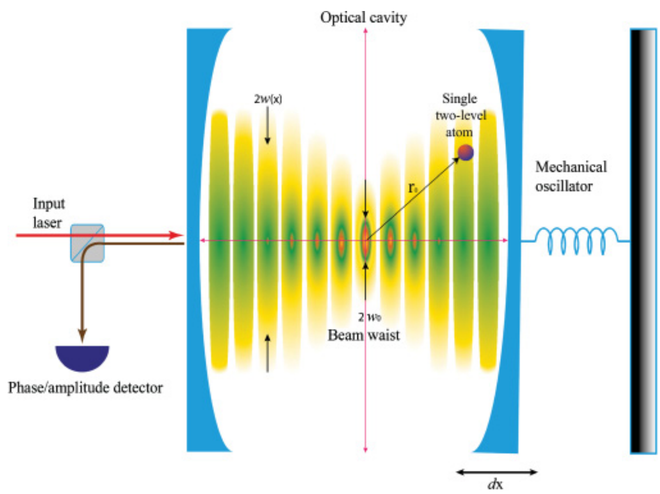

FIG. 1. (Color online) Schematic of the atom-assisted optomechanical system. It contains an optical cavity ending in a fixed mirror and a slightly moving mirror which is attached to a spring. Inside the cavity there is a two-level atom. The system is coherently driven by a laser field

In our investigation we can restrict the model to the case of single-cavity and mechanical modes. This is justified when the cavity free spectral range is much larger than the mechanical frequency $\omega_{\rm m}$ (i.e., cavities are not too large). In this case, scattering of photons from the driven mode into other cavity modes is negligible [44] and the input laser successfully drives only one cavity mode. This guarantees the fact that only one cavity mode participates in the optomechanical interaction and the neighboring modes are not excited by a single central frequency input laser. In addition, one can restrict to a single mechanical mode when the detection bandwidth is chosen such that it includes only a single, isolated, mechanical resonance and mode-mode coupling is negligible [45].

A. Hamiltonian of the system

In the absence of dissipation and fluctuations, the total Hamiltonian of the system is given by the sum of three terms: the free evolution term [19,24]

$$H_0 = \hbar\omega_{\rm c} a^\dagger a + \hbar\omega_{\rm m} b^\dagger b + \frac{\hbar\omega_{\rm e}}{2}\sigma^z, \tag{1}$$

the interaction term

$$H_{\rm int} = -\hbar\xi_0(b+b^\dagger)a^\dagger a + \hbar\chi_{mnl}(\vec{r}_0,x)[a\sigma^+ + {\rm h.c.}], \tag{2}$$

and the laser driven term

$$H_{\rm dri} = i\hbar\mathcal{E}(a^\dagger {\rm e}^{-{\rm i}\omega_l t} - a{\rm e}^{{\rm i}\omega_l t}), \tag{3}$$

where a ($[a,a^\dagger]=1$) is the annihilation operator of the cavity field with decay rate κ, $b([b,b^\dagger]=1)$ is the motional annihilation operator of the MR, and the single two-level atom is described by the spin-1/2 algebra of the Pauli matrices σ^-, σ^+, and σ^z which satisfy the commutation relations $[\sigma^+,\sigma^-]=\sigma^z$ and $[\sigma^z,\sigma^\pm]=\pm 2\sigma^\pm$. It should be noted that the free Hamiltonian, (1), has been written within the Raman-Nath approximation [51], i.e., in the limit when the atom is allowed only to move over a distance which is much less than the wavelength of the light. Therefore, in this approximation, one can neglect the kinetic energy of the atom. The first term of $H_{\rm int}$ is the optomechanical coupling with the radiation-pressure coupling constant $\xi_0 = (\omega_c/L)x_{\rm ZPF}$, in which $x_{\rm ZPF} = \sqrt{\hbar/m\omega_{\rm m}}$ is the zero-point fluctuation of the mechanical oscillator. The second term of $H_{\rm int}$ denotes "three-body" interactions among the atom, the cavity field, and the vibration of the mirror. The field-atom coupling rate in terms of an infinite set of optical modes is well described by the Hermite-Gauss modes [46,47]

$$\chi_{mnl}(\vec{r}) = g_0 K_{mnl}(x,y,z)\sin\left[\psi_{mnl}(x,y,z) - \frac{l\pi}{2}\right], \quad (4)$$

where, for $m,n = 0,1,\ldots, l = 1,2,\ldots,$

$$K_{mnl}(x,y,z) = \frac{H_n\left[\frac{\sqrt{2}y}{w(x)}\right]H_m\left[\frac{\sqrt{2}z}{w(x)}\right]\exp\left[-\frac{z^2+y^2}{w^2(x)}\right]}{w(x)\sqrt{\pi 2^{n+m-2}m!n!L}}, \quad (5)$$

$$\psi_{mnl}(x,y,z) = kx - \phi(x)(m+n+1) + k\frac{z^2+y^2}{2R(x)}. \quad (6)$$

Here $g_0 = \mu\sqrt{\omega_c/\epsilon_0 V}$, ϵ_0 is the vacuum permittivity, V shows the volume of the cavity, and μ is the electric-dipole transition matrix element. $H_n(y)$ is the nth Hermite polynomial; $w(x) = w_0[1+(\frac{x}{x_R})^2]^{\frac{1}{2}}$ is the beam waist at x, which is defined as the distance out from the axis center of the beam where the irradiance drops to $1/e^2$ of its values on the axis; $R(x) = x + x_R^2/x$ is the radius of curvature of the wavefront at x; $\phi(x) = \arctan(x/x_R)$ is the Gouy phase shift [46]; w_0 is the cavity waist radius, which depends on the geometry of the Fabry-Perot cavity; and $x_R = w_0^2 k/2$ is the Rayleigh range, which combines the wavelength and waist radius into a single parameter and completely describes the divergence of the Gaussian beam. Note that the Rayleigh range is the distance from the beam waist to the point at which the beam radius has increased to $\sqrt{2}w_0$. The coupling rate $\chi_{mnl}(\vec{r}_0,x)$ depends on the initial atomic position $\vec{r}_0$ (measured from the cavity waist) as well as the displacement $x = x_{\rm ZPF}(b+b^\dagger)$ of the mirror due to $k = \omega_{\rm eff}(x)/c$, where $\omega_{\rm eff}(x) = \omega_c(1-\frac{x}{L})$. As we will see in the next section, this dependence on the position of MR is responsible for the appearance of a new type of optomechanical nonlinearity. Finally, the Hamiltonian $H_{\rm dri}$ describes the input driving by a laser with frequency ω_l and amplitude $|\mathcal{E}| = \sqrt{2\kappa P/\hbar\omega_l}$, where P is the input laser power and κ is the cavity loss rate through its input port.

B. Nonlinear atom-field-mirror coupling

As we have seen, the Gaussian standing-wave structure of the cavity mode leads to the field-atom coupling rate $\chi_{mnl}(r_0,x)$. Such field-atom coupling in the presence of the mode structure of the field has been studied extensively in the literature and it has been shown that a certain type of nonlinearity is prepared in the field-atom system. For instance, in Refs. [48,49] the influences of the atomic motion and field-mode structure on atomic dynamics were investigated. It was shown that the atomic motion and the field-mode structure give rise to nonlinear transient effects in the atomic population which are similar to self-induced transparency and adiabatic effects. In our treatment, the spatial field-mode structure leads to the appearance of an intensity-dependent interaction among the intracavity optical mode, the MR, and the single atom. To show this, we assume that the atom is well located at the transverse (polar) coordinate (measured from the cylindrically symmetric cavity axis along the x direction) $\rho_0 = \sqrt{x_0^2+y_0^2} = \mu\, w(x_0)$, where $0 \leqslant \mu \leqslant 1$. In the x direction the localization of the atom can be expressed as $k_0 x_0 = \epsilon\pi$ for $\epsilon > 0$, where $k_0 = \omega_c/c$.

At the lowest order of the optical modes, i.e., $m = n = 0, l = 1$, the tripartite coupling rate reduces to

$$\chi_{001}(\vec{r}_0,x) \equiv \chi(x) = \frac{2g_0}{e^{\mu}w(x_0)\sqrt{\pi L}}\sin \times \left[kx_0 - \phi(x_0) - \frac{\pi}{2} + \frac{2\mu x_0}{kw_0^2}\right], \quad (7)$$

which can be rewritten in terms of the mirror position by using the position dependence of the wavelength $k = k_0(1 - x/L)$ as

$$\chi(x) = \frac{2g_0}{e^{\mu}w(x_0)\sqrt{\pi L}}\sin[\theta + \eta_0 x], \quad (8)$$

where $\eta_0 = \frac{2\mu x_0}{w_0^2 k_0 L}$ and

$$\theta = \left(1 + \frac{2\mu}{w_0^2 k_0^2}\right)k_0 x_0 - \phi(x_0) - \frac{\pi}{2}. \quad (9)$$

By substituting $x = x_{\rm ZPF}(b+b^\dagger)$ in Eq. (8), we obtain

$$\chi(b,b^\dagger) = \frac{g_0}{ie^{\mu}w(x_0)\sqrt{\pi L}}\{e^{i\theta}\exp[i\eta(b+b^\dagger)] - \text{h.c.}\}, \quad (10)$$

where the parameter

$$\eta = \eta_0 x_{\rm ZPF} = \frac{2\pi\mu\epsilon}{w_0^2 k_0^2 L}\sqrt{\frac{\hbar}{m\omega_m}} \quad (11)$$

is the so-called LDP. By using the Baker-Campbell-Hausdorff theorem in Eq. (10) and expanding the exponential terms in terms of b and $b^\dagger$, the coupling rate can be written as

$$\chi(b,b^\dagger) = \frac{g_0 e^{-\eta^2/2}}{ie^{\mu}w(x_0)\sqrt{\pi L}}\left\{e^{i\theta}\sum_{m,m'}\frac{(i\eta b^\dagger)^{m'}(i\eta b)^m}{m!m'!} - \text{h.c.}\right\}. \quad (12)$$

By using the bosonic commutation relation of the operators b and $b^\dagger$, the jth term of the field-atom coupling rate is obtained as

$$\chi_j(b,b^\dagger) = \frac{g_0 e^{-\eta^2/2}}{ie^{\mu}w(x_0)\sqrt{\pi L}} \times \left[e^{i\theta}\sum_m \frac{(i\eta)^{2m+j}(b^\dagger)^{m+j}b^m}{m!(m+j)!} - \text{h.c.}\right] = g_{j,\mu}(b^\dagger)^j f_j(n_b) + \text{h.c.}, \quad (13)$$

where $g_{j,\mu} = \frac{g_0 e^{-\eta^2/2}(i\eta)^j}{ie^{\mu}w(x_0)\sqrt{\pi L}}e^{i\theta}$ describes the effective atom-field-mirror coupling rate, and the Hermitian nonlinearity function

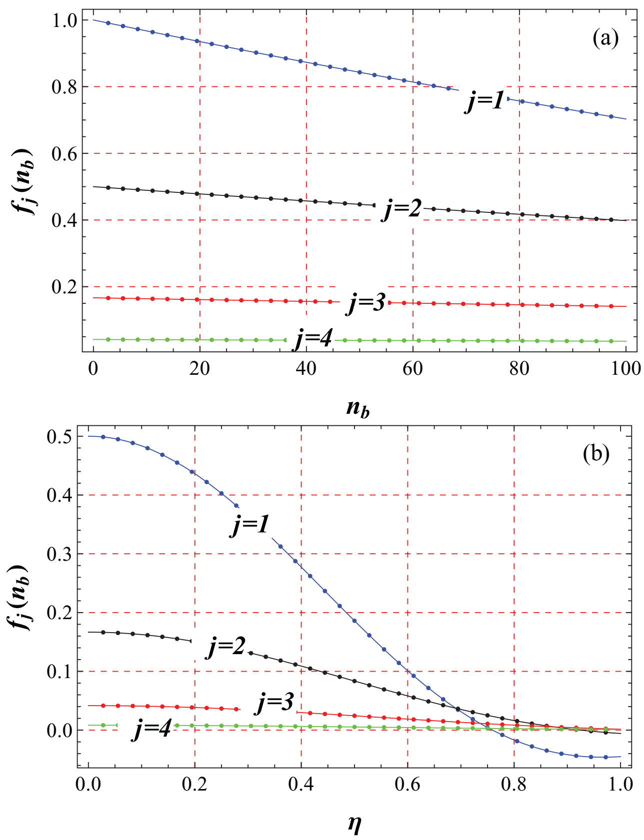

FIG. 2. (Color online) The nonlinearity function $f_j(n_b)$ as a function of (a) phonon number n_b for $\eta = 0.08$ and for different values of vibrational sideband j and (b) the Lamb-Dicke parameter, η, for $n_b = 10$ and for different values of vibrational sideband j.

$f_j(n_b)$ defined by

$$f_j(n_b) = \sum_m \frac{(i\eta)^{2m} n_b!}{m!(m+j)!(n_b-m)!} = \frac{n_b!}{(n_b+j)!} L_{n_b}^{j}(-\eta^2), \tag{14}$$

with $n_b = b^\dagger b$ and $L_{n_b}^{j}(-\eta^2)$ as the associated Laguerre polynomial, describes a nonlinear atom-field-mirror coupling through j-phonon excitations of the vibrational sideband. The nonlinearity function $f_j(n_b)$ has a central role in our treatment. It determines the form of nonlinearity of the intensity dependence of the coupling among the cavity field, the MR, and the single atom. As we will see, this function drastically influences the dynamics of the system, its entanglement properties, and its responsible for the appearance of NMS of a high visibility in the displacement spectrum of the MR. Figure 2(a) shows the nonlinearity function $f_j(n_b)$ as a function of n_b and for different values of phonon excitation number j. As shown, this function makes the maximum contribution around small values of vibrational acoustic excitation n_b. Furthermore, with an increase in the number j, the strength of the nonlinearity function $f_j(n_b)$ decreases considerably. On the other hand, Fig. 2(b) shows that the nonlinearity decreases with increasing LDP. It is remarkable that, for the higher orders of the vibrational sideband, $j \geqslant 3$, this function is no longer sensitive to the LDP.

Now, by substituting Eq. (13) into the interaction Hamiltonian of Eq. (2), we obtain the nonlinear form of the Hamiltonian as

$$H_{\text{int}}^{(j)} = -\hbar\xi_0(b+b^\dagger)a^\dagger a + \hbar[g_{j,\mu}(b^\dagger)^j f_j(n_b) + \text{h.c.}] \times [a\sigma^+ + \text{h.c.}]. \tag{15}$$

Near the *photon-phonon* resonance [19] where the frequencies satisfy $\omega_{\text{m}} + \omega_{\text{e}} - \omega_{\text{c}} \simeq 0$, the rotating-wave approximation reduces the above Hamiltonian to

$$H_{\text{int}}^{(j)} \simeq -\hbar\xi_0(b+b^\dagger)a^\dagger a + \hbar[g_{j,\mu}(b^\dagger)^j f_j(n_b) a\sigma^+ + \text{h.c.}]. \tag{16}$$

This Hamiltonian describes a nonlinear tripartite atom-field-mirror coupling and represents a novel type of optomechanical intensity-dependent interaction. Hamiltonian (16) is general and one can recover the results of Ref. [24] by taking $j = 0$, $f_j(n_b) \to 1$, or the results of Refs. [19,21] by setting $j = 1$, $f_j(n_b) \to 1$. To study the system dynamics we restrict our investigation by considering the first excitation of the vibrational sideband i.e., $j = 1$. In this limit one may use the simple form of the Hamiltonian

$$H_{\text{imt}}^{(j=1)} \simeq -\hbar\xi_0(b+b^\dagger)a^\dagger a + \hbar g_\mu \times [b^\dagger f(n_b) a\sigma^+ + \sigma^- a^\dagger f(n_b) b], \tag{17}$$

where $f(n_b) \equiv f_1(n_b)$ and $g_\mu = \frac{g_0 e^{-\eta^2/2}\eta}{e^\mu w(x_0)\sqrt{\pi L}}$. Since we deal with a well-localized atom we can assume $\theta = \pi$ in Eq. (9), which is realized by choosing a proper value of ϵ corresponding to the position of the atom in the x direction. We pointed out that for the experimentally feasible parameters of the system under consideration [50], i.e., $k_0 \simeq 10^6$ m^{-1}, $m =$ 10 pg, $\omega_m/2\pi = 10$ MHz, and $L = 1\mu$m, and for the smallest achievable value of the cavity beam waist, $w_0 \geqslant 10^{-9}$ m, the LDP is always less than one, i.e., $\eta < 1$. In this limit we can keep terms up to first order in the phonon number n_b and safely truncate the summation of Eq. (14):

$$f_{j=1}(n_b) \simeq 1 - \frac{\eta^2}{2} n_b. \tag{18}$$

By substituting Eq. (18) into the Hamiltonian, (17), one can write the interaction Hamiltonian as

$$H_{\text{int}} = -\hbar\xi_0(b+b^\dagger)a^\dagger a + \hbar g_\mu[b^\dagger a\sigma^+ + \sigma^- a^\dagger b] - \frac{\hbar\eta^2 g_\mu}{2}[b^\dagger n_b a\sigma^+ + \sigma^- a^\dagger n_b b]. \tag{19}$$

Note that the first and second terms in the above Hamiltonian denote the standard tripartite atom-field-mirror coupling which was recently studied in Refs. [19,21]. The third term denotes an intensity-dependent coupling among the three subsystems of atom-field-mirror. This type of nonlinear coupling is attributed to the spatial field-mode structure at the position of the atom.

III. DYNAMICS OF THE SYSTEM

To describe the dynamical behavior of the system under consideration it is necessary to consider the fluctuation-dissipation processes affecting the three subsystems. For this purpose, we first assume the excitation probability of the single atom to be low. In this limit, the dynamics of the atomic polarization can be described in terms of the bosonic operators c and $c^\dagger([c,c^\dagger]=1)$ [24,52], where the atomic annihilation operator is defined as $c=\sigma^-/\sqrt{|\langle\sigma^z\rangle|}$. This is valid in the low atomic excitation limit, i.e., when the atom is initially prepared in its ground state [24]. This means that the single-atom excitation probability should be much less than 1, i.e., $\frac{g_0|\alpha_s|^2}{\Delta_a^2+\gamma_a^2}\ll 1$, where $\Delta_a=\omega_a-\omega_l$ is the atomic detuning with respect to the laser and γ_a is the decay rate of the excited atomic level. Therefore, the bosonization of the atomic operators is valid only if $g_0\ll\Delta_a^2+\gamma_a^2$, that is, the atom is weakly coupled to the cavity.

The dynamics of the system is fully characterized by the following set of nonlinear QLEs, written in a frame rotating at the input laser frequency,

$$\dot{c}=-[\gamma_a+i\Delta_a]c-iG\left(1-\frac{\eta^2}{2}n_b\right)ab^\dagger+\sqrt{2\gamma_a}F_{\rm a},\tag{20a}$$

$$\dot{a}=-[\kappa+i\Delta_{0f}]a+i\xi_0a(b+b^\dagger)-iG\left(1-\frac{\eta^2}{2}n_b\right)bc+\mathcal{E}+\sqrt{2\kappa}a_{\rm in},\tag{20b}$$

$$\dot{b}=-[\gamma_m+i\omega_m]b+i\xi_0a^\dagger a-iG\left[(1-\eta^2n_b)ac^\dagger-\frac{\eta^2}{2}a^\dagger cb^2\right]+\sqrt{2\gamma_m}b_{\rm in},\tag{20c}$$

where $\Delta_{0f}=\omega_c-\omega_l$ is the cavity detuning with respect to the laser, $G=g_\mu\sqrt{|\langle\sigma^z\rangle|}$, and γ_m is the decay rate of the vibrational mode of the MR. The motional quantum fluctuation $b_{\rm in}(t)$ satisfies the relations [53]

$$\begin{aligned}\langle b_{\rm in}(t)b_{\rm in}^\dagger(t')\rangle&=[\langle n_{b,{\rm th}}\rangle+1]\delta(t-t'),\\ \langle b_{\rm in}^\dagger(t)b_{\rm in}(t')\rangle&=\langle n_{b,{\rm th}}\rangle\delta(t-t'),\\ \langle b_{\rm in}(t)b_{\rm in}(t')\rangle&=\langle b_{\rm in}^\dagger(t)b_{\rm in}^\dagger(t')\rangle=0,\end{aligned}\tag{21}$$

where $\langle n_{b,{\rm th}}\rangle$ is the mean number of phonons in the absence of optomechanical coupling, determined by the temperature of the mechanical bath T,

$$\langle n_{b,{\rm th}}\rangle=\frac{1}{e^{\frac{\hbar\omega_m}{k_BT}}-1}.\tag{22}$$

The only nonvanishing correlation function of the noises affecting the atom and the cavity field is $\langle a_{\rm in}(t)a_{\rm in}^\dagger(t')\rangle=\langle F_a(t)F_a^\dagger(t')\rangle=\delta(t-t')$ [53].

A. Linearization of QLEs

Our aim is to study the conditions under which one can efficiently correlate and entangle the atom and the MR by means of the common interaction with the intracavity optical mode. As shown in Refs. [45,54], a straightforward way for achieving stationary and robust entanglement in CV optomechanical systems is to choose an operating point where the cavity is intensely driven so that the intracavity field is strong, which is realized for high-finesse cavities and enough driving power. Therefore, we focus onto the dynamics of the fluctuations around the classical steady state by decomposing each operator in Eqs. (20) as the sum of its steady-state value and a small fluctuation, e.g., $a=\alpha_s+\delta a$, $b=\beta_s+\delta b$, and $c=c_s+\delta c$. The steady-state terms of these operators are given by

$$b_s=\frac{\alpha_s(\xi/2-G_3)}{(\omega_m-i\gamma_m)},\tag{23a}$$

$$c_s=\frac{G_2\alpha_s}{(i\gamma_a-\Delta_a)},\tag{23b}$$

$$\mathcal{E}=\alpha_s\left[i\Delta_f+\kappa-\frac{|G_2|^2}{(\gamma_a+i\Delta_a)}\right],\tag{23c}$$

where $\Delta_f=\Delta_{0f}-2\xi_0{\rm Re}(b_s)$ denotes the effective optomechanical detuning and $\xi=2\xi_0a_s$. The other parameters are defined in the Appendix. In the linearization manner, we also obtain the following linear QLEs for the quantum fluctuations of the triple system

$$\delta\dot{c}=-[\gamma_a+i\Delta_a]\delta c-iG\left[\left(1-\frac{\eta^2}{2}|b_s^2|\right)(a_s\delta b^\dagger+b_s^*\delta a)-\frac{\eta^2}{2}a_sb_s^*(b_s\delta b^\dagger+b_s^*\delta b)\right]+\sqrt{2\gamma_a}F_{\rm a},\tag{24a}$$

$$\begin{aligned}\delta\dot{a}=&-[\kappa+i\Delta_{0f}]\delta a+i\xi_0[\delta a(b_s+b_s^*)+a_s(\delta b+\delta b^\dagger)]\\&-iG\left[\left(1-\frac{\eta^2}{2}|b|^2\right)(c_s\delta b+b_s\delta c)-\frac{\eta^2}{2}b_sc_s(b_s\delta b^\dagger+b_s^*\delta b)\right]+\sqrt{2\kappa}a_{\rm in},\end{aligned}\tag{24b}$$

$$\begin{aligned}\delta\dot{b}=&-[\gamma_m+i\omega_m]\delta b+i\xi_0a_s(\delta a^\dagger+\delta a)-iG\Big[(1-\eta^2|b_s|^2)(a_s\delta c^\dagger+c_s^*\delta a)-\eta^2a_sc_s^*(b_s\delta b^\dagger+b_s^*\delta b)\\&-\frac{\eta^2}{2}\{b_s^2(c_s\delta a^\dagger+a_s\delta c)+2a_sb_sc_s\delta b\}\Big]+\sqrt{2\gamma_m}b_{\rm in},\end{aligned}\tag{24c}$$

in terms of the fluctuations of the quadrature operators,

$$\delta X_a=\frac{1}{\sqrt{2}}(\delta a+\delta a^\dagger),\quad \delta Y_a=\frac{1}{\sqrt{2}i}(\delta a-\delta a^\dagger),\tag{25}$$

$$\delta X_c=\frac{1}{\sqrt{2}}(\delta c+\delta c^\dagger),\quad \delta Y_c=\frac{1}{\sqrt{2}i}(\delta c-\delta c^\dagger),\tag{26}$$

$$\delta q=\frac{1}{\sqrt{2}}(\delta b+\delta b^\dagger),\quad \delta p=\frac{1}{\sqrt{2}i}(\delta b-\delta b^\dagger).\tag{27}$$

The resulting linearized QLEs can be written in the compact matrix form

$$\dot{u}(t) = Au(t) + n(t), \tag{28}$$

where $u(t) = [\delta q(t), \delta p(t), \delta X_a(t), \delta Y_a(t), \delta X_c(t), \delta Y_c(t)]^{\mathsf{T}}$ is the vector of CV fluctuation operators and $n(t) = [\sqrt{2\gamma_m} q^{\text{in}}(t), \sqrt{2\gamma_m} p^{\text{in}}(t), \sqrt{2\kappa} X_a^{\text{in}}(t), \sqrt{2\kappa} Y_a^{\text{in}}(t), \sqrt{2\gamma_a} X_c^{\text{in}}(t), \sqrt{2\gamma_a} Y_c^{\text{in}}(t)]^{\mathsf{T}}$ is the corresponding vector of noises. Moreover, the drift matrix A is a 6×6 matrix

$$A = \begin{pmatrix} -\Gamma_{1m} & \Omega_{1m} & -M_2^I & M_2^R & -M_1^I & M_1^R \\ -\Omega_{2m} & -\Gamma_{2m} & -M_2^R & -M_2^I & -M_3^R & -M_3^I \\ -G_1^I & G_1^R & -\kappa & \Delta_f & -G_2^I & G_2^R \\ \xi - G_3^R & -G_3^I & -\Delta_f & -\kappa & -G_2^R & -G_2^I \\ -N_2^I & N_2^R & -N_1^I & N_1^R & -\gamma_a & \Delta_a \\ -N_3^R & -N_3^I & -N_1^R & -N_1^I & -\Delta_a & -\gamma_a \end{pmatrix}, \tag{29}$$

where O_i^R and O_i^I denote the real and imaginary parts of parameter O_i, respectively. The other matrix elements are defined in the Appendix.

B. Stationary quantum fluctuations

Here we focus our attention on the stationary properties of the system. For this purpose, we should consider the steady-state condition governed by Eq. (28). The steady state is reached when the system is stable, which occurs if and only if all the eigenvalues of the matrix A have a negative real part. These stability conditions can be obtained, for example, by using the Routh-Hurwitz criterion [55].

The steady state is a zero-mean Gaussian state due to the fact that the dynamics of the fluctuations is linearized and all noises are Gaussian. As a consequence, it is fully characterized by the 6×6 stationary correlation matrix (CM) V, with matrix elements

$$V_{ij} = \frac{\langle u_i(\infty) u_j(\infty) + u_j(\infty) u_i(\infty) \rangle}{2}. \tag{30}$$

The formal solution of Eq. (28) yields [45]

$$V_{ij} = \int_0^\infty ds \int_0^\infty ds' M_{ik}(s) M_{jl}(s') D_{kl}(s - s'), \tag{31}$$

where $M(t) = \exp(At)$ and $D(s - s')$ is the diffusion matrix, the matrix of noise correlations, defined as $D_{kl}(s - s') = \langle n_k(s) n_l(s') + n_l(s') n_k(s) \rangle / 2$. For the noise diffusion matrix we have $D(s - s') = D\delta(s - s')$, where $D = \text{diag}[\gamma_m(2\bar{n}_b + 1), \gamma_m(2\bar{n}_b + 1), \kappa, \kappa, \gamma_a, \gamma_a]$. Therefore, Eq. (31) is simplified to

$$V = \int_0^\infty d\omega V(\omega), \tag{32}$$

where

$$V(\omega) = M(\omega) D M(\omega)^{\mathsf{T}}. \tag{33}$$

When the stability conditions are satisfied ($M(\infty) = 0$), one can obtain the following Lyapunov equation:

$$AV + VA^{\mathsf{T}} = -D. \tag{34}$$

Equation (34) is a linear equation for V and can be straightforwardly solved. However, the explicit form of V is complicate and is not reported here.

IV. ENTANGLEMENT PROPERTIES OF THE STEADY-STATE OF THE TRIPARTITE SYSTEM

In this section we examine the entanglement properties of the steady state of the tripartite system under consideration. For this purpose, we consider the entanglement of the three possible bipartite subsystems that can be obtained by tracing over the remaining degrees of freedom. This bipartite entanglement will be quantified by using the logarithmic negativity [56],

$$E_N = \max[0, -\ln 2\eta^-], \tag{35}$$

where $\eta^- \equiv 2^{-1/2}[\Sigma(V_{\text{bp}}) - \sqrt{\Sigma(V_{\text{bp}})^2 - 4\det V_{\text{bp}}}]^{1/2}$ is the lowest symplectic eigenvalue of the partial transpose of the 4×4 CM, V_{bp}, associated with the selected bipartition, obtained by neglecting the rows and columns of the uninteresting mode,

$$V_{\text{bp}} = \begin{pmatrix} B & C \\ C^T & B' \end{pmatrix}, \tag{36}$$

and $\Sigma(V_{\text{bp}}) \equiv \det B + \det B' - 2\det C$.

In Fig. 3 we have plotted the three bipartite logarithmic negativities, E_N^{am} (atom-mirror), E_N^{fa} (field-atom), and E_N^{mf} (mirror-field) versus the normalized atomic detuning Δ_a/ω_m at a fixed temperature of $T = 0.4$ K for two values of the LDP—$\eta = 0.04$ [Fig. 3(a)] and $\eta = 0.08$ [Fig. 3(b)]—and for the experimentally feasible parameters [50], i.e., an MR with oscillation frequency $\omega_m/2\pi = 10$ MHz, quality factor $Q = 11 \times 10^5$, $m = 10$ pg, and an optical cavity with length $L = 1\ \mu$m and damping rate $\kappa = 0.07\omega_m$ driven by a laser with $k_0 \simeq 10^6\ \text{m}^{-1}$ and power $P_c = 800\ \mu$W. The atom damping constant has been taken as $\gamma_a/2\pi = 0.04\omega_m$ with coupling constant $g_0/2\pi = 10^3$ Hz.

The optical cavity detuning has been fixed at $\Delta_f = -\omega_m$, which turns out be the most convenient choice. As seen, by increasing LDP, the two bipartite entanglements of E_N^{am} and E_N^{fa} increase and the bipartite entanglement of E_N^{mf} decreases overall. The reason is that, with an increase in the LPD, the tripartite atom-field-mirror coupling rate increases compared to the coupling rate of the bipartite field-mirror subsystem, or equivalently, the parameter G/ξ is increased. This result reveals that by changing the LDP one can control the tripartite coupling amplitude or even go through a regime in which the tripartite system reduces to an effective bipartite subsystem. However, the three logarithmic negativities do not behave in the same way and the entanglement sharing is evident. In particular, the entanglement of interest, i.e., E_N^{am}, increases at the expense of the mirror-field entanglement, while E_N^{fa} always remains non-negligible.

Figure 4 shows the logarithmic negativity of the three bipartite subsystems versus the normalized atomic detuning Δ_a/ω_m for a fixed value of the LDP, $\eta = 0.04$, and for two temperatures: $T = 1.2$ K [Fig. 4(b)] and $T = 3$ K [Fig. 4(b)]. The optical cavity detuning was again fixed at $\Delta_f = -\omega_m$. As expected, the three kinds of bipartite entanglement decrease with increasing temperature, but the atom-mirror and field-atom entanglements show high temperature robustness.

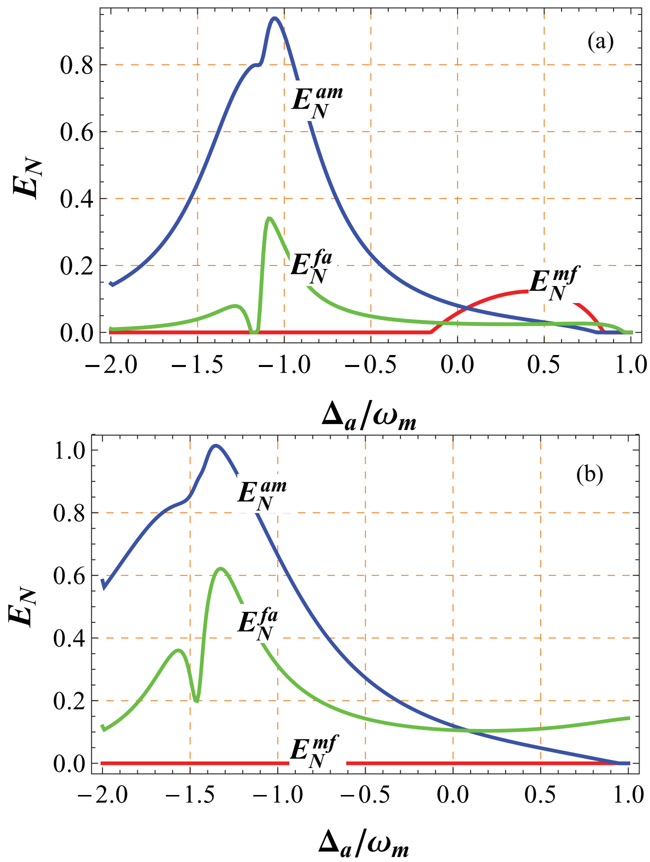

FIG. 3. (Color online) Plot of E_N of the three bipartite subsystems [E_N^{am} (atom-mirror), E_N^{fa} (field-atom), E_N^{mf} (mirror-field)] versus the normalized atomic detuning Δ_a/ω_m at a fixed temperature $T = 0.4$ K and for two values of the LDP: (a) $\eta = 0.04$ and (b)$\eta = 0.08$. The optical cavity detuning was fixed at $\Delta_f = -\omega_m$, while the other parameters were $\omega_m/2\pi = 10$ MHz, $Q = 11 \times 10^5$, $m = 10$ pg, $\kappa = 0.07\omega_m$, $k_0 = 10^6$ m^{-1}, $P_c = 800$ μW, $\gamma_a/2\pi = 0.04\omega_m$, and $g_0/2\pi = 10^3$ Hz.

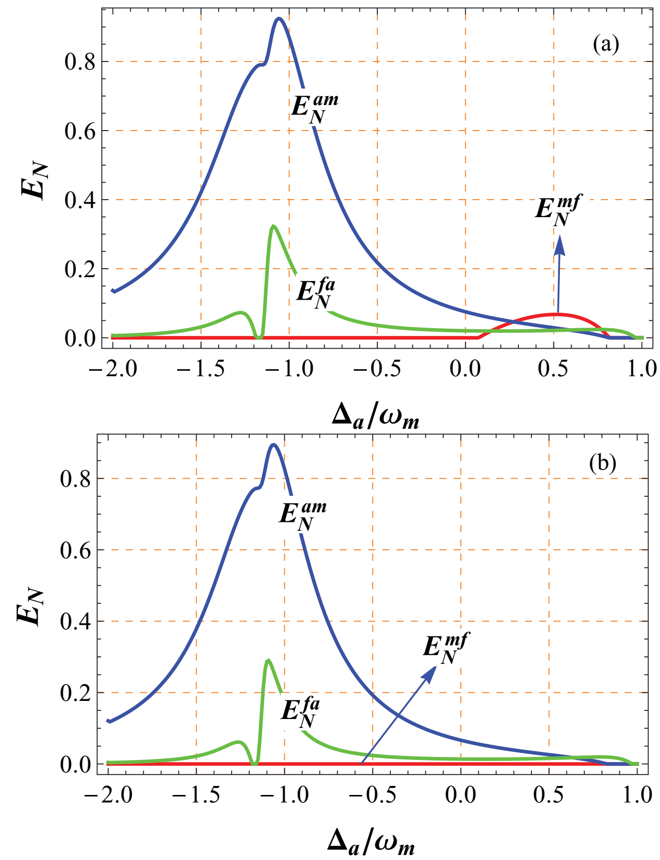

FIG. 4. (Color online) Plot of E_N of the three bipartite subsystems [E_N^{am} (atom-mirror), E_N^{fa} (field-atom), E_N^{mf} (mirror-field)] versus the normalized atomic detuning Δ_a/ω_m for a fixed value of the LDP, $\eta = 0.04$, and for two temperatures: (a) $T = 1.2$ K and (b) $T = 3$ K. The optical cavity detuning was fixed at $\Delta_f = -\omega_m$ and the other parameters are as in Fig. 3.

However, the field-mirror entanglement shows extremely fragile entanglement robustness versus temperature and its logarithmic negativity falls down to 0 at $T = 3$ K.

Generally, the scheme is able to generate appreciable entanglement between the atom and the MR, especially with sharing from the mirror-field entanglement. Similar bipartite entanglement behavior can be observed in other similar tripartite systems, such as the atom-field-mirror scheme proposed in Ref. [24], the microwave-optical-mirror system in Ref. [57], and the two-cavity optomechanical setup in Ref. [58].

The effect of the LDP is also illustrated in Fig. 5, where we have plotted the logarithmic negativity as a function of η at fixed optical cavity detuning $\Delta_f = -\omega_m$ and at atomic detuning $\Delta_a = \omega_m$. It is clear that by increasing the LDP, the field-atom entanglement increases when the entanglement between the intracavity mode and the MR is drastically suppressed. We also observe that the atom-mirror logarithmic negativity E_N^{am} plateaus in the case where η increases.

Figure 6 shows E_N^{fa} and E_N^{am} versus Δ_a/ω_m and γ_a/ω_m for $\eta = 0.04$. As is clear, the E_N^{fa} and E_N^{am} are maximized around sideband $\Delta_a \simeq \omega_m$. By increasing the atomic spontaneous emission rate γ_a as we expected, both logarithmic negativities decrease drastically.

The entanglement features of the tripartite system at the steady state can be observed by experimentally measuring the corresponding CM. This can be done by combining the existing experimental techniques. By homodyning the cavity output one can measure the cavity field quadratures. Reference [3] proposed a scheme to measure the mechanical position and momentum of the MR, in which, by adjusting the detuning and bandwidth of an additional adjacent cavity, both the position and the momentum of the mirror can be measured by homodyning the output of the second cavity. Moreover, by adopting the same scheme as in Ref. [59], the atomic polarization quadratures X_a and Y_a can be measured, i.e., by making a Stokes parameter measurement of a laser beam, shined transversal to the cavity and to the cell and off-resonantly tuned to another atomic transition. Very recently, Ref. [60] demonstrated the proof of principle of the use of a Bose-Einstein condensate (BEC) as a diagnostic tool to determine the elusive mirror-light entanglement in a hybrid optomechanical device. In this case, one does find a working point such that the mirror-light entanglement is reproduced by the BEC-light quantum correlations.

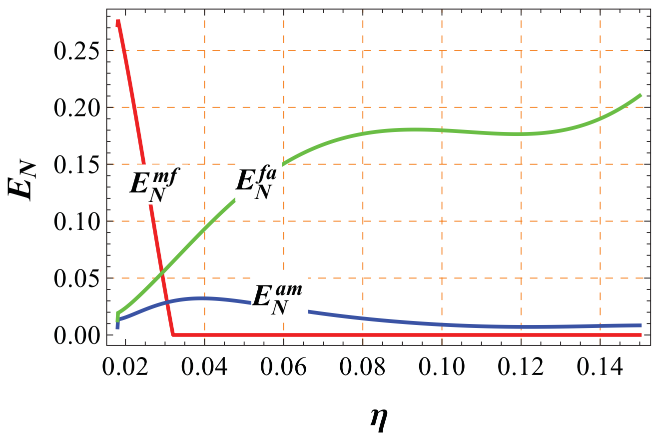

FIG. 5. (Color online) Plot of E_N of the three bipartite subsystems [E_N^{am} (atom-mirror), E_N^{fa} (field-atom), E_N^{mf} (mirror-field)] versus the LDP, η, for a fixed temperature of $T = 0.4$ K. The optical cavity and atomic detuning are $\Delta_f = -\omega_m$ and $\Delta_a = \omega_m$, respectively. The other parameters are as in Fig. 3.

V. NORMAL-MODE SPLITTING IN THE DISPLACEMENT SPECTRUM OF THE MR

In this section, we show that the atom-field-mirror coupling leads to the splitting of the normal mode into three modes (NMS). Optomechanical NMS, however, involves driving four parametrically coupled nondegenerate modes out of equilibrium. The NMS does not appear in the steady-state spectra but, rather, manifests itself in the fluctuation spectra of the mirror displacement. To study the NMS in our system we need to determine the displacement spectrum of the mirror as

$$S_q(\omega) = \frac{1}{2\pi}\int d\Omega e^{-i(\omega+\Omega)t}\langle\delta q(\omega)\delta q(\Omega)+\delta q(\Omega)\delta q(\omega)\rangle = V_{11}(\omega), \quad (37)$$

where $V_{11}(\omega,\Omega) = 1/2\langle\delta q(\omega)\delta q(\Omega)+\delta q(\Omega)\delta q(\omega)\rangle$ is an element of CM which is given by Eq. (33). Unfortunately, the analytical form of the displacement spectrum of the mirror is too complicated to put a clear physical interpretation on it. Thus, in the following, we give and analyze the results obtained by numerical calculations.

Figure 7 shows the displacement spectrum of the MR as a function of the normalized frequency ω/ω_m at $\Delta_f = \omega_m$, $\Delta_a = \omega_m$ and for two values of the LDP: $\eta = 0.016$ and $\eta = 0.04$. For small values of the LDP, we observe the usual NMS into two modes with central peaks at the sidebands $\omega = \pm\omega_m$. Figure 7 shows a highly symmetric structure with respect to $\omega = 0$. As shown, with increasing LDP, the normal mode splits up into three modes.

A more clear illustration of the three-mode splitting is shown in Fig. 8. This figure shows the displacement spectrum of the MR versus the normalized frequency ω/ω_m and atomic detuning Δ_a/ω_m at $\Delta_f = \omega_m$. The three-mode splitting manifests itself mainly at $\Delta_a \simeq \omega$. Upon going through the region far from $\Delta_a \simeq \omega$, three-mode splitting merges into two-mode splitting around $\Delta_a \simeq 0.75\omega_m$ and $\Delta_a \simeq 1.35\omega_m$. The NMS is associated with the mixing among the vibrational mode of the MR, fluctuations of the cavity field around the steady state, and fluctuations of the atomic mode. The origin of the fluctuations of the cavity field is the beat of the pump photons with the photons scattered from the atom. For not so large values of the LDP (small nonlinearity), field-atom coupling is much smaller than field-mirror coupling. Therefore, the system simply reduces to the case of two-mode coupling, i.e., coupling between the mechanical mode and the photon fluctuations [34]. When the LDP is large enough, the mechanical mode, the photon mode, and the atomic mode form a system of three coupled oscillators. The occurrence of splitting of the normal mode into three modes has been analyzed recently in another tripartite system, i.e., a cavity quantum optomechanical system of ultracold atoms in an optical lattice [34]. Furthermore, similar three-coupled-oscillator experimental results, with two coupled cavities, each containing three identical quantum

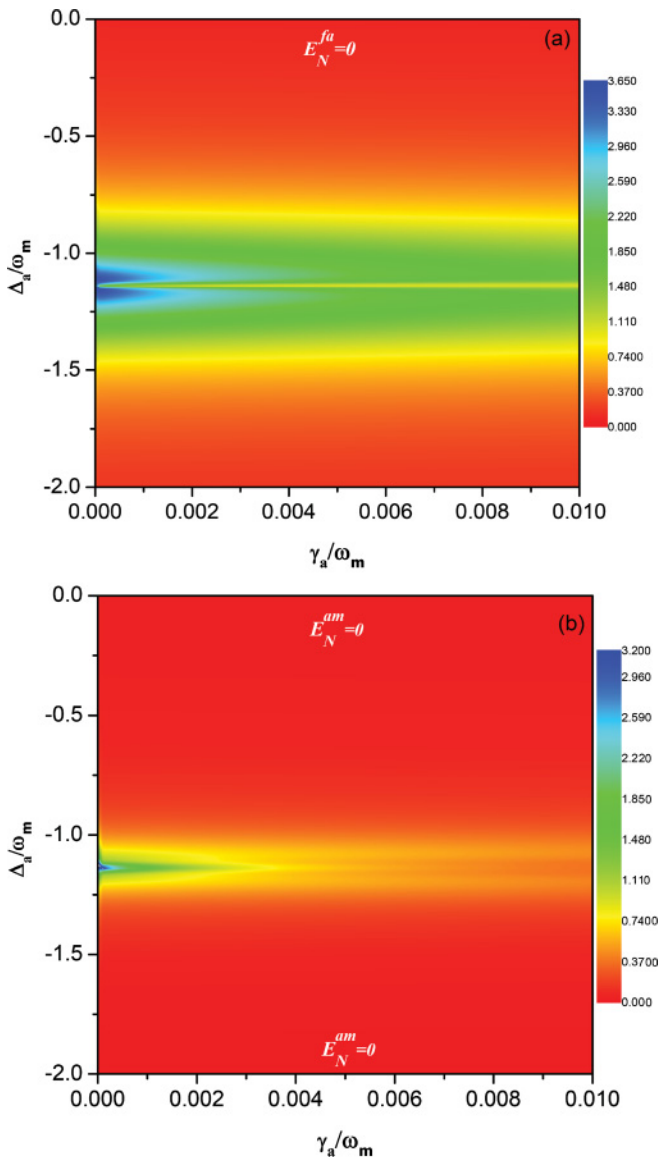

FIG. 6. (Color online) Density plot of E_N of the bipartite subsystems: (a)E_N^{fa} and (b) E_N^{am} versus Δ_a/ω_m and γ_a/ω_m for $\eta = 0.04$ and for $T = 0.4$ K. The optical cavity detuning was fixed at $\Delta_f = -\omega_m$. The other parameters are as in Fig. 3.

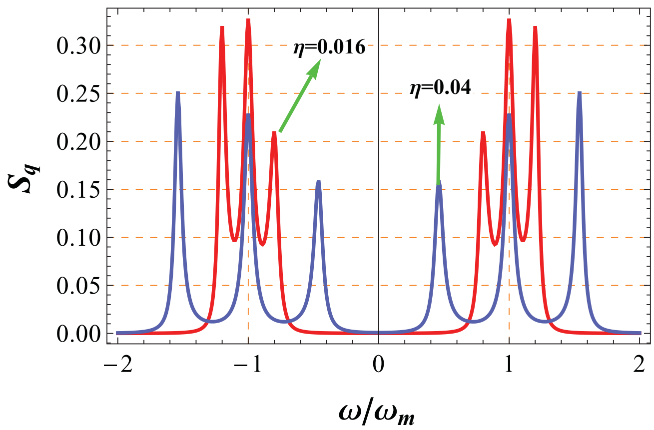

FIG. 7. (Color online) Normalized plot of the displacement spectrum $S_q(\omega)$ versus ω/ω_m at fixed temperatures $T = 0.4$ K and for two values of the LDP: $\eta = 0.016$ (red line) and $\eta = 0.04$ (blue line). The optical cavity and atomic detuning have been fixed at $\Delta_f = \omega_m$ and $\Delta_a = \omega_m$, respectively. The other parameters are as in Fig. 3.

wells [61] and one microcavity containing two quantum wells [62], have been reported.

It should be pointed out that to observe NMS, the energy exchange among the three modes should take place on a time scale faster than the decoherence of each mode. NMS into three modes due to a local increase in the LDP has also been reported in Ref. [63], where the authors have shown that NMS can be observed only if the coupling between the atoms and the cavity is strong enough. This strong coupling can be achieved by increasing the atom numbers. One experimental limitation could be spontaneous emission, which leads to momentum diffusion and hence heating of the atomic sample [64]. In our model, we do not encounter such a limitation

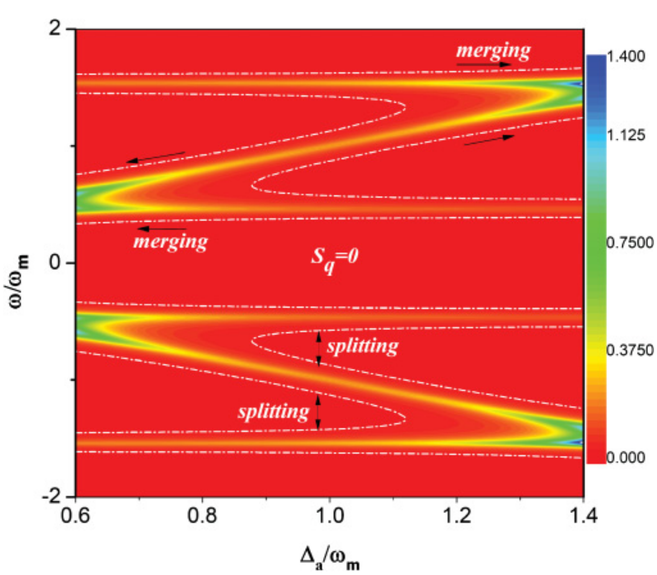

FIG. 8. (Color online) Density plot of the displacement spectrum $S_q(\omega)$ versus ω/ω_m and Δ_a/ω_m for $T = 0.4$ K and $\eta = 0.04$. The optical cavity detuning was fixed at $\Delta_f = \omega_m$. Other parameters are as in Fig. 3.

and three-mode splitting is approached by proper choice of the LDP.

VI. CONCLUSION

In this work, we have proposed a theoretical scheme for the realization of tripartite intensity-dependent coupling among a single mode of a Fabry-Perot cavity with an oscillating mirror, a single two-level atom inside it, and a vibrational mode of the oscillating mirror. We have shown that in the presence of a Gaussian standing wave in the optical cavity mode, a type of tripartite atom-mirror-field coupling can be manifested. To describe this interaction, we then found the general form of the corresponding nonlinear Hamiltonian. We have restricted our investigation to the first vibrational sideband $j = 1$ and have studied its dynamics by adopting a QLE treatment. We have focused our attention on the steady state of the system, and, in particular, on the stationary quantum fluctuations of the system, by solving the linearized dynamics around the classical steady state. We have seen that, in an experimentally accessible parameter regime, the steady state of the system shows both tripartite and bipartite CV entanglement. We have shown that the LDP (as a measure of the strength of nonlinearity) can extremely modify both tripartite and bipartite CV entanglement in the system. In particular, with an increase in the LDP, one can see that field-atom and atom-mirror entanglement increase, at the expense of optical-mechanical entanglement. The intracavity mode is able to mediate for the realization of a robust stationary (i.e., persistent) entanglement between the MR mode and the single two-level atom, which could be extremely useful in quantum information and quantum computer networks in which the MR modes are used for quantum communications [65,66], and the atom is used as a qubit (e.g., solid-state qubits). Furthermore, we have analyzed the occurrence of NMS in the displacement spectrum of the oscillating mirror. As we have shown, for a small value of LDP, the usual NMS into two modes with central peaks at the sidebands $\omega = \pm\omega_m$ is observed, and with increasing LDP, the normal mode splits up into three modes. We have shown that, when the LDP is large enough, the mechanical mode, the photon mode, and the atomic mode form a system of three coupled oscillators. The realization of such a scheme will also open new opportunities for the implementation of quantum teleportation and/or the photon blockade process to prevent two or more photons from entering the cavity at the same time.

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APPENDIX: DEFINITION OF THE ELEMENTS OF THE DRIFT MATRIX OF EQ. (29)

In the drift matrix of Eq. (29) we have defined

$$\Gamma_{1m} = \gamma_m + M_3^I, \quad \Gamma_{2m} = \gamma_m + M_4^I,$$
$$\Omega_{1m} = \omega_m + M_3^R, \quad \Omega_{2m} = \omega_m + M_4^R,$$

$$G_1 = G\left[c_s^*(1-\eta^2|b_s|^2) + \frac{\eta^2}{2}c_s b_s^2\right],$$
$$G_2 = G b_s^*\left(1-\frac{\eta^2}{2}|b_s|^2\right),$$
$$G_3 = G\left[c_s^*(1-\eta^2|b_s|^2) - \frac{\eta^2}{2}c_s b_s^2\right],$$
$$M_1 = -G a_s\left[(1-\eta^2|b_s|^2) + \frac{\eta^2}{2}b_s^{2*}\right],$$
$$M_2 = G a_s\left[(1-\eta^2|b_s|^2) - \frac{\eta^2}{2}b_s^{2*}\right],$$
$$M_3 = G c_s\left[(1-\eta^2|b_s|^2) + \frac{\eta^2}{2}b_s^2\right],$$
$$M_3 = G c_s\left[(1-\eta^2|b_s|^2) - \frac{\eta^2}{2}b_s^2\right],$$
$$M_4 = -G\eta^2 a_s[b_s^* c_s^* + c_s b_s - c_s^* b_s],$$
$$M_5 = -G\eta^2 a_s[b_s^* c_s^* + c_s b_s + c_s^* b_s],$$
$$N_1 = b_s\left(1-\frac{\eta^2}{2}|b_s|^2\right),$$
$$N_2 = -a_s\left[(1-\eta^2|b_s|^2) + \frac{\eta^2}{2}b_s^2\right],$$
$$N_3 = a_s\left[(1-\eta^2|b_s|^2) - \frac{\eta^2}{2}b_s^2\right].$$

[1] C. M. Caves, Phys. Rev. Lett. **45**, 75 (1980).
[2] J. Hwang, M. M. Fejer, and W. E. Moerner, Phys. Rev. A **73**, 021802 (2006).
[3] D. Vitali, S. Gigan, A. Ferreira, H. R. Bohm, P. Tombesi, A. Guerreiro, V. Vedral, A. Zeilinger, and M. Aspelmeyer, Phys. Rev. Lett. **98**, 030405 (2007).
[4] X. Zou and W. Mathis, Phys. Lett. A **324**, 484 (2004); A. N. Cleland and M. R. Geller, Phys. Rev. Lett. **93**, 070501 (2004).
[5] T. J. Kippenberg and K. J. Vahala, Opt. Express **15**, 17172 (2007).
[6] C. Fabre, M. Pinard, S. Bourzeix, A. Heidmann, E. Giacobino, and S. Reynaud, Phys. Rev. A **49**, 1337 (1994); K. Jacobs, P. Tombesi, M. J. Collett, and D. F. Walls, *ibid.* **49**, 1961 (1994).
[7] P. F. Cohadon, A. Heidmann, and M. Pinard, Phys. Rev. Lett. **83**, 3174 (1999); Y. Hadjar, P. F. Cohadon, C. G. Aminoff, M. Pinard, and A. Heidmann, Europhys. Lett. **47**, 545 (1999); S. Mancini, V. Giovannetti, D. Vitali, and P. Tombesi, Phys. Rev. Lett. **88**, 120401 (2002); C. H. Metzger and K. Karrai, Nature (London) **432**, 1002 (2004); B. S. Sheard, M. B. Gray, C. M. Mow-Lowry, D. E. McClelland, and S. E. Whitcomb, Phys. Rev. A **69**, 051801(R) (2004); H. Rokhsari, T. J. Kippenberg, T. Carmon, and K. J. Vahala, IEEE J. Quantum Electron. **12**, 96 (2006); O. Arcizet *et al.*, Nature (London) **444**, 71 (2006); M. Ludwig, B. Kubala, and F. Marquardt, New. J. Phys. **10**, 095013 (2008); A. Xuereb, P. Domokos, J. Asbóth, P. Horak, and T. Freegarde, Phys. Rev. A **79**, 053810 (2009).
[8] C. Bradaschia *et al.*, Nucl. Instrum. Methods Phys. Res. Sec. A **289**, 518 (1990).
[9] M. D. LaHaye, O. Buu, B. Camarota, and K. C. Schwab, Science **304**, 74 (2004).
[10] M. Tsang, Phys. Rev. A **81**, 063837 (2010).
[11] K. R. Brown, J. Britton, R. J. Epstein, J. Chiaverini, D. Leibfried, and D. J. Wineland, Phys. Rev. Lett. **99**, 137205 (2007).
[12] J. D. Teufel, J. W. Harlow, C. A. Regal, and K. W. Lehnert, Phys. Rev. Lett. **101**, 197203 (2008); J. D. Teufel *et al.*, Nature **475**, 359 (2011); **471**, 204 (2011).
[13] S. De Liberato, N. Lambert, and F. Nori, Phys. Rev. A **83**, 033809 (2011).
[14] M. Ludwig, C. Neuenhahn, C. Metzger, A. Ortlieb, I. Favero, K. Karrai, and F. Marquardt, Phys. Rev. Lett. **101**, 133903 (2008).
[15] P. Rabl *et al.*, Nature Phys. **6**, 602 (2010).
[16] W. Marshall, C. Simon, R. Penrose, and D. Bouwmeester, Phys. Rev. Lett. **91**, 130401 (2003).
[17] Sh. Barzanjeh, M. H. Naderi, and M. Soltanolkotabi, J. Phys. B **44**, 105504 (2011); Phys. Rev. A **84**, 023803 (2011).
[18] K. Hammerer, M. Wallquist, C. Genes, M. Ludwig, F. Marquardt, P. Treutlein, P. Zoller, J. Ye, and H. J. Kimble, Phys. Rev. Lett. **103**, 063005 (2009); M. Wallquist, K. Hammerer, P. Zoller, C. Genes, M. Ludwig, F. Marquardt, P. Treutlein, J. Ye, and H. J. Kimble, Phys. Rev. A **81**, 023816 (2010).
[19] Y. Chang, H. Ian, and C. P. Sun, J. Phys. B **42**, 215502 (2009).
[20] A. Glaetzle, K. Hammerer, A. Daley, R. Blatt, and P. Zoller, Opt. Commun. **283**, 758 (2010).
[21] W. Wang, L. C. Wang, and C. P. Sun, J. Kor. Phys. Soc. **57**, 704 (2010).
[22] S. Camerer, M. Korppi, A. Jockel, D. Hunger, T. W. Hansch, and P. Treutlein, Phys. Rev. Lett. **107**, 223001 (2011).
[23] P. Treutlein, D. Hunger, S. Camerer, T. W. Hänsch, and J. Reichel, Phys. Rev. Lett. **99**, 140403 (2007).
[24] C. Genes, D. Vitali, and P. Tombesi, Phys. Rev. A **77**, 050307(R) (2008).
[25] H. Ian, Z. R. Gong, Y.-x. Liu, C. P. Sun, and F. Nori, Phys. Rev. A **78**, 013824 (2008).
[26] K. Hammerer, M. Aspelmeyer, E. S. Polzik, and P. Zoller, Phys. Rev. Lett. **102**, 020501 (2009).
[27] D. Hunger, S. Camerer, T. W. Hänsch, D. Koenig, J. P. Kotthaus, J. Reichel, and P. Treutlein, Phys. Rev. Lett. **104**, 143002 (2010).
[28] R. Kanamoto and P. Meystre, Phys. Scripta **82**, 038111 (2010).
[29] A. D. Armour, M. P. Blencowe, and K. C. Schwab, Phys. Rev. Lett. **88**, 148301 (2002).
[30] X. Zou and W. Mathis, Phys. Lett. A **324**, 484 (2004).
[31] A. N. Cleland and M. R. Geller, Phys. Rev. Lett. **93**, 070501 (2004).
[32] A. K. Ringsmuth and G. J. Milburn, J. Mod. Opt. **54**, 2223 (2007).
[33] D. Vitali, P. Tombesi, M. J. Woolley, A. C. Doherty, and G. J. Milburn, Phys. Rev. A **76**, 042336 (2007).
[34] A. B. Bhattacherjee, Phys. Rev. A **80**, 043607 (2009).
[35] Z. Haghshenasfard, M. H. Naderi, and M. Soltanolkotabi, J. Phys. B: At. Mol. Opt. Phys. **42**, 195501 (2009).

[36] J. J. Sanchez-Mondragon, N. B. Narozhny, and J. H. Eberly, Phys. Rev. Lett. **51**, 550 (1983).
[37] G. S. Agarwal, Phys. Rev. Lett. **53**, 1732 (1984).
[38] M. G. Raizen, R. J. Thompson, R. J. Brecha, H. J. Kimble, and H. J. Carmichael, Phys. Rev. Lett. **63**, 240 (1989).
[39] A. Boca, R. Miller, K. M. Birnbaum, A. D. Boozer, J. McKeever, and H. J. Kimble, Phys. Rev. Lett. **93**, 233603 (2004).
[40] P. Maunz, T. Puppe, I. Schuster, N. Syassen, P. W. H. Pinkse, and G. Rempe, Phys. Rev. Lett. **94**, 033002 (2005).
[41] S. Huang and G. S. Agarwal, Phys. Rev. A **80**, 033807 (2009).
[42] J. M. Dobrindt, I. Wilson-Rae, and T. J. Kippenberg, Phy. Rev. Lett. **101**, 263602 (2008).
[43] T. Kumar, A. B. Bhattacherjee, and ManMohan, Phys. Rev. A **81**, 013835 (2010).
[44] C. K. Law, Phys. Rev. A **51**, 2537 (1995).
[45] C. Genes, D. Vitali, and P. Tombesi, New J. Phys. **10**, 095009 (2008).
[46] A. E. Siegman, *Lasers* (University Science Books, Sausalito, 1986).
[47] C. Biancofiore, M. Karuza, M. Galassi, R. Natali, P. Tombesi, G. DiGiuseppe, and D. Vitali, Phys. Rev. A **84**, 033814 (2011).
[48] A. Joshi and S. V. Lawande, Phys. Rev. A **42**, 1752 (1990).
[49] A. Joshi, Phys. Rev. A **58**, 4662 (1998).
[50] G. Anetsberger, O. Arcizet, Q. P. Unterreithmeier, R. Rivire, A. Schliesser, E. M. Weig, J. P. Kotthaus, and T. J. Kippenberg, Nature Physics **5**, 909 (2009).
[51] W. P. Schleich, *Quantum Optics in Phase Space* (WILEY-VCH 2001).
[52] T. Holstein and H. Primakoff, Phys. Rev. **58**, 1098 (1940).
[53] C. W. Gardiner and P. Zoller, *Quantum Noise* (Springer, Berlin, 2000).
[54] F. Galve, L. A. Pachón, and D. Zueco, Phys. Rev. Lett. **105**, 180501 (2010).
[55] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products* (Academic Press, Orlando, 1980); A. Hurwitz, *Selected Papers on Mathematical Trends in Control Theory*, edited by R. Bellman and R. Kabala (Dover, New York, 1964).
[56] J. Eisert, Ph.D. thesis, University of Potsdam, 2001; G. Vidal and R. F. Werner, Phys. Rev. A **65**, 032314 (2002); G. Adesso, A. Serafini, and F. Illuminati, *ibid.* **70**, 022318 (2004).
[57] Sh. Barzanjeh, D. Vitali, P. Tombesi, and G. J. Milburn, Phys. Rev. A **84**, 042342 (2011).
[58] M. Paternostro, D. Vitali, S. Gigan, M. S. Kim, C. Brukner, J. Eisert, and M. Aspelmeyer, Phys. Rev. Lett. **99**, 250401 (2007).
[59] J. Sherson, H. Krauter, R. K. Olsson, B. Julsgaard, K. Hammerer, J. I. Cirac, and E. S. Polzik, Nature **443**, 557 (2006).
[60] G. De Chiara, M. Paternostro, and G. Massimo Palma, Phys. Rev. A **83**, 052324 (2011).
[61] R. P. Stanley *et al.*, Appl. Phys. Lett. **65**, 2093 (1994).
[62] E. K. Lindmark *et al.*, Opt. Lett. **21**, 994 (1996).
[63] J. Klinner, M. Lindholdt, B. Nagorny, and A. Hemmerich, Phys. Rev. Lett. **96**, 023002 (2006).
[64] K. W. Murch *et al.*, Nat. Phys. **4**, 561 (2008).
[65] S. Pirandola, S. Mancini, D. Vitali, and P. Tombesi, Phys. Rev. A **68**, 062317 (2003).
[66] S. Mancini, D. Vitali, and P. Tombesi, Phys. Rev. Lett. **90**, 137901 (2003).