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Entanglement of a two-mode field in a collective three-level atomic system

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The entanglement properties of two-mode field from a laser-driven collective three-level atomic system are investigated by taking into account the spontaneously generated coherence. Under some conditions, it is found that the entanglement between the two cavity modes can be significantly enhanced by the collectivity of the atoms compared to the case of independent atoms when the relative phase $\Delta\phi = \pi$. Moreover, the spontaneously generated coherence can also greatly enhance the entanglement in comparison to the case without this coherence.

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I. INTRODUCTION

Many theoretical and experimental studies of collective effects in the interaction of atoms with a laser field have been carried out since the early work by Dicke [1]. Collective atomic systems have been proved to have various advantages over single (or independent) atomic systems. For example, due to the collectivity, the intensity of superradiant fluorescence in a sample of N atoms is proportional to N^2 and superfluorescence can be enhanced via decay interference [2]. The complete dressed-state population inversion in the strongly driven two-level atomic system can be achieved [3]. The index of refraction and the group velocity may be modified strongly and rapidly [4]. Studies also showed that compared to the case of the independent atoms, the quantum squeezing and nonclassical correlations of the radiation field of the collective systems can be enhanced [5]. Very recently, such a system has been exploited to cool a three-level atomic ensemble [6] and collective spontaneous decay and superradiance can be inhibited via the ensemble's Stark interaction with a vacuum electromagnetic field [7].

On the other hand, recently, the spontaneously generated coherence (SGC) has attracted considerable interest. The effects of SGC on electromagnetically induced transparency, dark states, lasing without inversion, coherent population trapping, resonance fluorescence, transient processes, squeezing spectra, etc., have been extensively studied [8]. For instance, it has been shown that this kind of coherence can change the steady-state response of the medium, modify significantly the absorption or spontaneous emission spectra of a near-degenerate system [9], and enhance Kerr nonlinearity [10]. The existence of such coherence effects depends on the nonorthogonality of the two dipole matrix elements. One of the possibilities is to use sodium dimers, which can be modeled as a five-level molecule in which transitions with parallel and antiparallel dipole moments can be selected [11,12]. An alternative solution is to engineer atomic systems with parallel dipole moments in the bad cavity limit [13]. Experimentally, SGC arising from radiative decay of the trion into the spin states has been confirmed in charged GaAs quantum dots [14].

In recent years, continuous variable (CV) entangled light has attracted much interest because of potential applications in quantum information science, such as CV quantum teleportation [15,16], long-distance quantum communications [17], quantum dense coding [18], and quantum computation [19]. Due to the relative simplicity and high efficiency in the generation, manipulation, and detection, a variety of physical systems presenting CV entanglement have been investigated both theoretically and experimentally [20–30]. Nondegenerate parametric oscillator [21–23] and nondegenerate four-wave mixing [24–28] are some conventional sources of the two-mode entangled light. However, most of the previous works to enhance the entanglement of two cavity modes were carried out in the independent atomic systems without SGC. It is natural to ask whether we can take advantage of the effects of collectivity and SGC on the generation of entanglement. In the recent past, with the collective interactions of two-level atomic ensembles, the generation of atomic entanglement can be achieved and the generation of the robust two-mode entanglement has also been proposed [31]. Therefore, taking the SGC into account, it is thus of interest to discuss the entanglement of the radiation field with the atomic collectivity.

In this paper, we investigate the entanglement of a cavity field generated from a laser-driven V-type collective atomic ensemble inside a two-mode cavity with SGC. After deriving the master equation of the cavity field in the dressed-state picture of the driven atoms, the influences of the atomic collectivity and SGC on the field entanglement are discussed in detail. We show that when SGC is present the dressed-state populations in a collection of atoms are different from those of independent atomic systems. Moreover, under some conditions the entanglement of the cavity field can be significantly enhanced compared to the case without SGC or the atomic collectivity and can vary rapidly with the relative phase between two driven fields in the presence of SGC when the number of involved atoms is large.

II. MODEL AND MASTER EQUATION

We consider a collection of V-type three-level atoms inside a two-mode cavity with one ground state $|3\rangle$ and two excited states $|1\rangle$ and $|2\rangle$ (see Fig. 1). The two dipole-allowed transitions $|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |3\rangle$ are driven independently by two strong laser fields having the same frequency ω_L with Rabi frequencies Ω_1 and Ω_2 and phases ϕ_1 and ϕ_2, respectively. At the same time, the transitions $|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |3\rangle$ are

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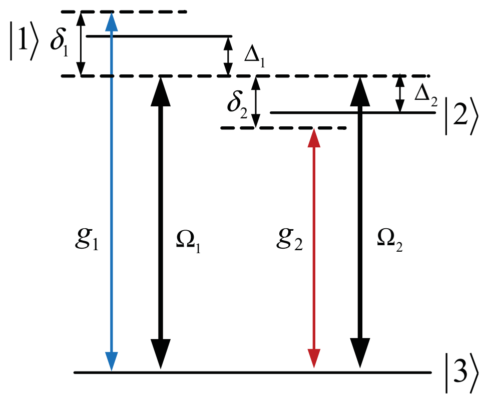

FIG. 1. (Color online) Schematic diagram of the atomic levels. Two laser fields having the same frequency ω_L drive the $|1\rangle \to |3\rangle$ transition with detuning Δ_1 and the $|2\rangle \to |3\rangle$ transition with detuning Δ_2, respectively. Two nondegenerate cavity modes of frequencies ω_1 and ω_2 couple to the driven transition with detunings δ_1 and δ_2 from the laser frequency.

coupled to two cavity modes denoted by annihilation operators a_1 and a_2 with frequencies ω_1 and ω_2, respectively.

In the rotating frame with respect to the laser frequency ω_L, the Hamiltonian of the cavity-atom system is given by

$$H = H_0 + V, \tag{1}$$

where

$$H_0 = \Delta_1 S_{11} + \Delta_2 S_{22} + (\Omega_1 e^{i\phi_1} S_{13} + \Omega_2 e^{i\phi_2} S_{23} + \text{H.c.}), \tag{2}$$

$$V = g_1 a_1 S_{13} e^{-i\delta_1 t} + g_2 a_2 S_{23} e^{i\delta_2 t} + \text{H.c.} \tag{3}$$

Here $\Delta_j = \omega_{j3} - \omega_L$, $\delta_1 = \omega_1 - \omega_L$, and $\delta_2 = \omega_L - \omega_2$, where ω_{j3} being the atomic transition frequencies from the excited states $|j\rangle$ to ground state $|3\rangle$ and g_l are coupling constants between the cavity fields and the atoms. The collective atomic operators S_{ij} are defined as $S_{ij} = \sum_{k=1}^{N} |i\rangle_{kk}\langle j|$ $(i,j = 1,2,3)$, which describe the atomic populations for $i = j$ and transitions for $i \neq j$ and obey the commutation relation $[S_{ij}, S_{i'j'}] = \delta_{ji'} S_{ij'} - \delta_{ij'} S_{i'j}$.

By taking into account the damping of the cavity field and the atoms, the density operator ρ of the system is governed by the following master equation ($\hbar = 1$):

$$\frac{d}{dt}\rho = -i[H,\rho] + L_f\rho + L_a\rho, \tag{4}$$

where

$$L_f\rho = \sum_{j=1,2} \kappa_j (2a_j\rho a_j^\dagger - a_j^\dagger a_j \rho - \rho a_j^\dagger a_j), \tag{5}$$

$$L_a\rho = \gamma_1[S_{31},\rho S_{13}] + \gamma_2[S_{32},\rho S_{23}] + \eta([S_{31},\rho S_{23}] + [S_{32},\rho S_{13}]) + \text{H.c.}, \tag{6}$$

where κ_j and γ_j are the damping rates of the cavity modes and the atom, respectively. Here the coefficient $\eta = p\sqrt{\gamma_1\gamma_2}$ is a measure of the amount of coherence, the so-called SGC, induced by dissipation between the $|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |3\rangle$ atomic transitions. The degree of the coherence, measured by the coefficient η, depends explicitly on the mutual polarization of the transition dipole moments with $p = \cos\theta$, where θ is the angle between the two dipole moments. Thus, $p = 0$ when the transition dipole moments are orthogonal to each other, and p attains its maximal value of $p = \pm 1$ when the dipole moments are parallel or antiparallel to each other.

Since the two dipole-allowed transitions are driven by two strong laser fields, it is convenient to work in the dressed-state picture [32]. We introduce collective dressed states, which are the eigenstates of the Hamiltonian (2):

$$\begin{aligned}
|\tilde{1}\rangle &= \frac{1+\sin\theta}{2}|1\rangle + \frac{1-\sin\theta}{2}|2\rangle + \frac{\cos\theta}{\sqrt{2}}|3\rangle,\\
|\tilde{2}\rangle &= -\frac{\cos\theta}{\sqrt{2}}|1\rangle + \frac{\cos\theta}{\sqrt{2}}|2\rangle + \sin\theta|3\rangle,\\
|\tilde{3}\rangle &= \frac{1-\sin\theta}{2}|1\rangle + \frac{1+\sin\theta}{2}|2\rangle - \frac{\cos\theta}{\sqrt{2}}|3\rangle,
\end{aligned} \tag{7}$$

where we assume $\Delta_1 = -\Delta_2 = \Delta$, $\Omega_1 = \Omega_2 = \Omega$ for simplicity and $\sin\theta \equiv s = \Delta/\Omega_0$, $\cos\theta \equiv c = \sqrt{2}\Omega/\Omega_0$ with $\Omega_0 = \sqrt{\Delta^2 + 2\Omega^2}$. We also apply the transformation $\tilde{S}_{j3} = S_{j3}e^{i\phi_j}$ and drop the tilde afterward. By defining the collective dressed operators $R_{\alpha\beta} = |\tilde{\alpha}\rangle\langle\tilde{\beta}|$ $(\alpha,\beta = 1,2,3)$, taking advantage of Eq. (7) in the master Eq. (4), and neglecting terms that oscillate with frequencies Ω_0 and larger in a secular approximation, one arrives at the following master equation in the dressed-state representation [33]:

$$\begin{aligned}
\frac{d}{dt}\tilde{\rho} = &-i\Omega_0[R_{11} - R_{33},\tilde{\rho}] - i[\tilde{V},\tilde{\rho}]\\
&- \{X_1([R_{11},R_{11}\tilde{\rho}] + [R_{13},R_{31}\tilde{\rho}])\\
&+ X_2([R_{33},R_{33}\tilde{\rho}] + [R_{31},R_{13}\tilde{\rho}])\\
&+ X_3([R_{23},R_{32}\tilde{\rho}] + [R_{21},R_{12}\tilde{\rho}])\\
&+ X_4[R_{22},R_{22}\tilde{\rho}] + X_5[R_{12},R_{21}\tilde{\rho}]\\
&+ X_6[R_{32},R_{23}\tilde{\rho}] + Y_1[R_{11},R_{22}\tilde{\rho}]\\
&+ Y_1^*[R_{22},R_{11}\tilde{\rho}] + Y_2[R_{22},R_{33}\tilde{\rho}]\\
&+ Y_2^*[R_{33},R_{22}\tilde{\rho}] + Y_3[R_{11},R_{33}\tilde{\rho}]\\
&+ Y_3^*[R_{33},R_{11}\tilde{\rho}] + \text{H.c.}\} + L_f\tilde{\rho},
\end{aligned} \tag{8}$$

where

$$\begin{aligned}
X_1 &= \alpha_+ c^2, \quad X_2 = \alpha_- c^2, \quad X_3 = f c^2,\\
X_4 &= 2 f s^2, \quad X_5 = 2\alpha_+ s^2, \quad X_6 = 2\alpha_- s^2,\\
Y_1 &= \beta_+ s c^2, \quad Y_2 = \beta_- s c^2, \quad Y_3 = h c^2,
\end{aligned} \tag{9}$$

and

$$\begin{aligned}
\alpha_\pm &= [\gamma_1(1 \pm s)^2 + \gamma_2(1 \mp s)^2 + 2c^2\eta\cos\Delta\phi]/8,\\
\beta_\pm &= [\mp\gamma_1(1 \pm s) \pm \gamma_2(1 \mp s)\\
&\quad + \eta(1+s)e^{-i\Delta\phi} - \eta(1-s)e^{i\Delta\phi}]/4,\\
f &= c^2(\gamma_1 + \gamma_2 - 2\eta\cos\Delta\phi)/4,\\
h &= [-c^2(\gamma_1 + \gamma_2)\\
&\quad - \eta(1+s)^2 e^{-i\Delta\phi} - \eta(1-s)^2 e^{i\Delta\phi}]/8,
\end{aligned} \tag{10}$$

with $\Delta\phi = \phi_1 - \phi_2$, and $\tilde{V}$ is the interaction of the dressed atom with the cavity modes, which can be easily obtained by taking advantage of Eq. (7) in Eq. (3).

We assume an intense pumping field, that is, $\Omega_0 \gg \{N\gamma_{1,2}, \sqrt{N}g_{1,2}\}$, and a high-quality cavity such that

$N\gamma_{1,2} \gg \kappa_{1,2}$, the atomic subsystem achieves its steady state on a time scale faster than the cavity field. Thus, the atomic variables can be eliminated to arrive at a master equation for the cavity field:

$$\frac{d}{dt}\tilde{\rho}_f = \sum_{j=1,2}\{-i\delta_{12}[a_j^\dagger a_j,\tilde{\rho}_f] + A_j(a_j^\dagger\tilde{\rho}_f a_j - \tilde{\rho}_f a_j a_j^\dagger) + (B_j+\kappa_j)(a_j\tilde{\rho}_f a_j^\dagger - a_j^\dagger a_j\tilde{\rho}_f)\} + \sum_{j\neq j'=1,2}\{C_j(a_j^\dagger a_{j'}^\dagger\tilde{\rho}_f - a_{j'}^\dagger\tilde{\rho}_f a_j^\dagger) + D_j(\tilde{\rho}_f a_{j'}^\dagger a_j^\dagger - a_j^\dagger\tilde{\rho}_f a_{j'}^\dagger)\} + \text{H.c.}, \tag{11}$$

where

$$\begin{aligned}
A_1 &= \frac{g_1^2}{8}\left[(1+s)^2\left(\frac{c^2D_{13}}{F_{13}(-\delta_1)} + \frac{2s^2D_{12}}{f_{12}(-\delta_1)}\right) + \frac{2c^4D_{21}}{f_{12}(\delta_1)} + (1-s)^2\left(\frac{c^2D_{31}}{F_{13}(\delta_1)} + \frac{2s^2D_{32}}{f_{23}(\delta_1)}\right) + \frac{2c^4D_{23}}{f_{23}(-\delta_1)}\right],\\
A_2 &= \frac{g_1^2}{8}\left[(1+s)^2\left(\frac{c^2D_{31}}{F_{13}^*(-\delta_2)} + \frac{2s^2D_{32}}{f_{23}^*(-\delta_2)}\right) + \frac{2c^4D_{23}}{f_{23}^*(\delta_2)} + (1-s)^2\left(\frac{c^2D_{13}}{F_{13}^*(\delta_2)} + \frac{2s^2D_{12}}{f_{12}^*(\delta_2)}\right) + \frac{2c^4D_{21}}{f_{12}^*(-\delta_2)}\right],\\
C_1 &= \frac{g_1g_2}{8}\left[(1-s)\left(\frac{2sc^2D_{23}}{f_{23}(\delta_2)} + \frac{2sc^2D_{12}}{f_{12}(\delta_2)}\right) - (1+s)\left(\frac{2sc^2D_{32}}{f_{23}(-\delta_2)} + \frac{2sc^2D_{21}}{f_{12}(-\delta_2)}\right) + \frac{(1+s)^2c^2D_{31}}{F_{13}(-\delta_2)} + \frac{(1-s)^2c^2D_{13}}{F_{13}(\delta_2)}\right],\\
C_2 &= \frac{g_1g_2}{8}\left[(1-s)\left(\frac{2sc^2D_{32}}{f_{23}^*(\delta_1)} + \frac{2sc^2D_{21}}{f_{12}^*(\delta_1)}\right) - (1+s)\left(\frac{2sc^2D_{23}}{f_{23}^*(-\delta_1)} + \frac{2sc^2D_{12}}{f_{12}^*(-\delta_1)}\right) + \frac{(1+s)^2c^2D_{13}}{F_{13}^*(-\delta_1)} + \frac{(1-s)^2c^2D_{31}}{F_{13}^*(\delta_1)}\right],\\
D_1^* &= C_2, \quad D_2^* = C_1,
\end{aligned} \tag{12}$$

with

$$\begin{aligned}
\delta_{12} &= (\delta_2-\delta_1)/2, \quad D_{\alpha\beta} = \langle R_{\alpha\beta}R_{\beta\alpha}\rangle_S,\\
F_{13}(\pm\delta_j) &= \Gamma_{13} \mp i(2\Omega_0 \pm \delta_j),\\
f_{12}(\pm\delta_j) &= \Gamma_{12} \mp i(\Omega_0 \pm \delta_j),\\
f_{23}(\pm\delta_j) &= \Gamma_{23} \mp i(\Omega_0 \pm \delta_j),
\end{aligned}$$

and

$$\begin{aligned}
\Gamma_{12} &= 3(2X_1+X_3) - (2X_1-X_3)(R_{11}^S - R_{22}^S) - (Y_1-Y_1^*)(R_{11}^S+R_{22}^S) - (3X_2-X_3-X_1)R_{33}^S - (Y_1+Y_1^*) - (Y_2^*-Y_2+Y_3-Y_3^*)R_{33}^S,\\
\Gamma_{23} &= (5X_3+2X_2) - (X_3-2X_2)(R_{22}^S-R_{33}^S) - (Y_2-Y_2^*)(R_{22}^S+R_{33}^S) - (3X_1-X_2-X_3)R_{11}^S - (Y_2+Y_2^*) - (Y_1^*-Y_1+Y_3-Y_3^*)R_{11}^S,\\
\Gamma_{13} &= (5X_1+3X_2) - (X_1-X_2)(R_{11}^S-R_{33}^S) - (Y_3-Y_3^*)(R_{11}^S+R_{33}^S) - 2(X_3-X_2-X_1)R_{22}^S - (Y_3+Y_3^*) - (Y_1-Y_1^*+Y_2-Y_2^*)R_{22}^S.
\end{aligned} \tag{13}$$

Further, B_1 (B_2) can be obtained from $A_1(A_2)$ by replacing $D_{\alpha\beta}$ with $D_{\beta\alpha}$, respectively. Here $R_{jj}^S \equiv \langle R_{jj}\rangle_S (j=1,2,3)$ are the steady-state collective atomic populations in the dressed states and we have chosen $\theta = \pi/4$ for simplicity in Eq. (13).

III. STEADY-STATE COLLECTIVE ATOMIC POPULATION IN THE DRESSED-STATE PICTURE

To determine the coefficients in Eq. (12) we should obtain the steady-state atomic population in the dressed-state picture. In the absence of the cavity modes, the solution for the steady-state density operator of the system can be written in the following form:

$$\rho_S = Z^{-1}\sum_{n=0}^{N}\sum_{m=n}^{N} P_1^nP_2^m|N,n,m\rangle\langle m,n,N|. \tag{14}$$

Here, $|N,m,n\rangle$ are eigenstates of the operators $R_{11}+R_{22}$, R_{11}, and $R_{11}+R_{22}+R_{33}$ with eigenvalues m, n, and N, respectively, and Z is a normalization constant which is obtained by the requirement $\text{Tr}\{\rho_S\} = 1$ [34]. The matrix elements of the collective operators can be obtained from the set of relations

$$\begin{aligned}
R_{11}|N,n,m\rangle &= n|N,n,m\rangle,\\
R_{22}|N,n,m\rangle &= (m-n)|N,n,m\rangle,\\
R_{12}|N,n,m\rangle &= \sqrt{(n+1)(m-n)}|N,n+1,m\rangle,\\
R_{13}|N,n,m\rangle &= \sqrt{(n+1)(N-m)}|N,n+1,m+1\rangle,\\
R_{23}|N,n,m\rangle &= \sqrt{(m-n+1)(N-m)}|N,n,m+1\rangle.
\end{aligned} \tag{15}$$

It is easy to determine Z, which is given by

$$Z = \frac{1-P_2^{N+1}(1-P_1^{N+2}) - (P_1P_2)^{N+2} - P_1(1-P_2^{N+2})}{(1-P_1)(1-P_2)(1-P_1P_2)}. \tag{16}$$

The dressed steady-state atomic populations then yield

$$\begin{aligned}
R_{11}^S &= Z^{-1}\sum_{n=0}^{N}\sum_{m=n}^{N} nP_1^nP_2^m,\\
R_{22}^S &= Z^{-1}\sum_{n=0}^{N}\sum_{m=n}^{N} (m-n)P_1^nP_2^m,\\
R_{33}^S &= N - R_{11}^S - R_{22}^S.
\end{aligned} \tag{17}$$

By using detailed balance [3], the coefficients P_1 and P_2 are given by

$$\begin{aligned}
P_1 &= \frac{c^4(\gamma_1+\gamma_2-2\eta\cos\Delta\phi)}{s^2[(1+s)^2\gamma_1+(1-s)^2\gamma_2+2c^2\eta\cos\Delta\phi]},\\
P_2 &= \frac{s^2[(1-s)^2\gamma_1+(1+s)^2\gamma_2+2c^2\eta\cos\Delta\phi]}{c^4(\gamma_1+\gamma_2-2\eta\cos\Delta\phi)}.
\end{aligned} \tag{18}$$

From Eq. (18), it is evident that the relative phase $\Delta\phi$ appears only in the terms related to the parameter η, so if SGC is absent ($p=0$) the dressed steady-state populations are independent of the relative phase $\Delta\phi$. Hence, without the SGC terms,

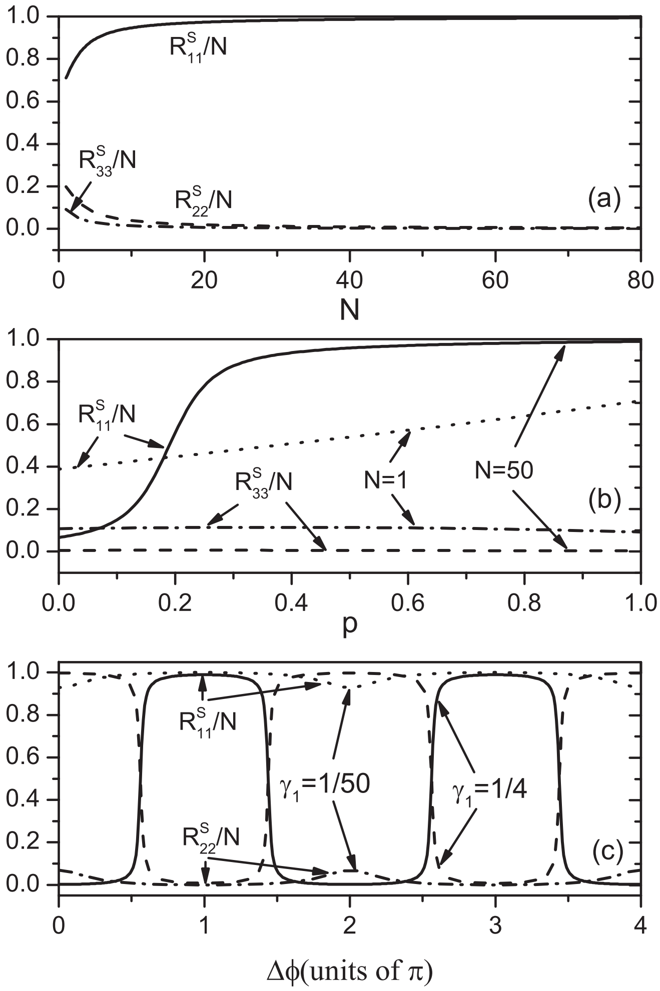

FIG. 2. The dressed steady-state populations (a) versus the number N of the atoms with $\gamma_1 = 1/4$ and $p = 1$, (b) versus the strength p of SGC with $\gamma_1 = 1/4$, and (c) versus the relative phase $\Delta\phi$ with $N = 50$ and $p = 1$. Here $\theta = \pi/4$ in (a)–(c). All parameters are normalized to γ_2.

phase control of the system dynamics is impossible. We decoupled the involved multiparticle correlators approximately as $D_{\alpha\beta} = R^S_{\alpha\alpha}(1 + R^S_{\beta\beta})$, which is valid for a large sample, that is, $N \gg 1$ [35,36]. For the single-atom case, the dressed steady-state atomic population reduces to $R^S_{11} = \frac{P_1 P_2}{1+P_2+P_1P_2}$, $R^S_{22} = \frac{P_2}{1+P_2+P_1P_2}$.

According to Eq. (17), the dressed steady-state atomic populations are shown in Fig. 2, which are influenced by the number N of the atoms, the strength p of SGC and the ratio of two spontaneous emission rates γ_1/γ_2. We can see from Fig. 2(a) that in a collection of atoms ($N \gg 1$), the dressed steady-state population R^S_{11}/N (R^S_{22}/N, R^S_{33}/N), contributed averagely per atom, increases (decreases) quickly with the increasing number N of the atoms. When $N \gg 1$ the population $R^S_{11}/N \approx 1$ ($R^S_{22}/N \approx 0$, $R^S_{33}/N \approx 0$), which means that the population is trapped in the collective dressed state $|\tilde{1}\rangle$. Likewise, from Fig. 2(b) it is revealed that in a collection of atoms the dressed steady-state population R^S_{11}/N increases with the increase of the strength p of SGC for both $N = 1$ and $N = 50$. On the contrary, R^S_{33}/N is small and decreases slowly. When $p \to 1$, however, R^S_{11}/N is trapped in the collective dressed state $|\tilde{1}\rangle$ for $N = 50$ but reaches only 0.7 for $N = 1$.

On the other hand, the dressed steady-state population depends strongly on the ratio γ_1/γ_2. Figure 2(c) shows that in the case of $\gamma_1/\gamma_2 = 1/4$ the dressed steady-state population against the relative phase $\Delta\phi$ exhibits jumps between the collective dressed states $|\tilde{1}\rangle$ and $|\tilde{2}\rangle$. Furthermore, the range of trapping in $|\tilde{1}\rangle$ and $|\tilde{2}\rangle$ becomes wider on increasing the number N of atoms. In the limit of $\gamma_1/\gamma_2 \ll 1$, such as $\gamma_1/\gamma_2 = 1/50$, however, we can see the population is almost trapped in the collective dressed state $|\tilde{1}\rangle$. Moreover, the range of trapping in $|\tilde{1}\rangle$ becomes wider on decreasing the ratio of γ_1/γ_2. In the opposite limit of $\gamma_1/\gamma_2 \gg 1$, the population is almost trapped in the collective dressed state $|\tilde{2}\rangle$. In a word, the relative phase $\Delta\phi$ leads to interference effects that are tempered by the relative decay rates for the two excited dressed states. Physically, when $\gamma_1 \ll \gamma_2$ the decay rates γ_{13} and γ_{31} for the dressed-state decay $|\tilde{1}\rangle \leftrightarrow |\tilde{3}\rangle$ are much smaller than γ_{23} and γ_{32} for the dressed-state decay $|\tilde{2}\rangle \leftrightarrow |\tilde{3}\rangle$. As a result, the atom is hardly populated in the dressed state $|\tilde{3}\rangle$.

IV. ENTANGLEMENT PROPERTIES OF THE CAVITY FIELD

Now we consider the properties of the entanglement of the cavity field generated in the system. The master equation (11) is of a form characteristic for a system composed of two field modes coupled to a multimode squeezed vacuum [37]. Thus, the cavity-field state should be a two-mode Gaussian state (TMGS) and the quantum statistical properties of a TMGS are determined by its correlation matrix [38], which takes the standard form:

$$\sigma = \begin{pmatrix} n_1 & 0 & c_+ & 0 \\ 0 & n_1 & 0 & c_- \\ c_+ & 0 & n_2 & 0 \\ 0 & c_- & 0 & n_2 \end{pmatrix}. \tag{19}$$

States whose standard form fulfills $n_1 = n_2$ are said to be symmetric. Here, for the cavity field governed by Eq. (11), it is not difficult to find that $n_1 = \langle a_1^\dagger a_1\rangle + 1/2$, $n_2 = \langle a_2^\dagger a_2\rangle + 1/2$, and $c_+ = -c_- = c = |\langle a_1 a_2\rangle|$.

For TMGS, to quantify entanglement between the modes, we use Duan's criterion [39] and logarithmic negativity [40,41]. The Duan's criterion states that

$$\Upsilon = \langle(\Delta\hat{u})^2\rangle + \langle(\Delta\hat{v})^2\rangle - a^2 - \frac{1}{a^2} < 0, \tag{20}$$

where the operators $\hat{u} = a\hat{X}_1 - \frac{1}{a}\hat{X}_2$, $\hat{v} = a\hat{Y}_1 + \frac{1}{a}\hat{Y}_2$, and a is a state-dependent real number. Here the quadrature operators of the two cavity modes are defined as $\hat{X}_l = (a_l e^{-i\theta_l} + a_l^\dagger e^{i\theta_l})/\sqrt{2}$ and $\hat{Y}_l = -i(a_l e^{-i\theta_l} - a_l^\dagger e^{i\theta_l})/\sqrt{2}$, with θ_l being the phase angles of the modes. On substituting the definition of $\hat{u}$ and $\hat{v}$ in Eq. (20), we obtain [25,39]

$$\Upsilon = 2n_1 a^2 + 2n_2/a^2 - 4c - a^2 - \frac{1}{a^2}, \tag{21}$$

where $a^2 = \sqrt{(2n_2 - 1)/(2n_1 - 1)}$. Plugging the expression of a^2 into Eq. (21), one can find that the entanglement condition $\Upsilon < 0$ is reduced to

$$\Upsilon = 4[\sqrt{(n_1 - 1/2)(n_2 - 1/2)} - c] < 0, \tag{22}$$

For a TMGS with the correlation matrix cast in the 2×2 block form

$$\sigma = \begin{pmatrix} N_1 & N_{12} \\ N_{12}^T & N_2 \end{pmatrix},$$

the logarithmic negativity E_N is given by [40,41]

$$E_N = \max[0, -\ln 2d], \tag{23}$$

where

$$d = \sqrt{\frac{\Delta\sigma - \sqrt{(\Delta\sigma)^2 - 4\mathrm{Det}\sigma}}{2}}, \tag{24}$$

with $\Delta\sigma = \mathrm{Det}N_1 + \mathrm{Det}N_2 - 2\mathrm{Det}N_{12}$. Therefore, a TMGS is entangled if and only if $d < 1/2$, which is equivalent to

$$[(n_1 - 1/2)(n_2 - 1/2) - c^2][(n_1 + 1/2)(n_2 + 1/2) - c^2] < 0. \tag{25}$$

Since $(n_1 + 1/2)(n_2 + 1/2) - c^2 > 0$, it is evident the entanglement condition of Eq. (25) is the same as Eq. (22). As a result, both Duan's criterion and logarithmic negativity are identical to determine inseparability of a TMGS.

Especially, if we choose $a = 1$, then the sum of the variances $\langle(\Delta\hat{u})^2\rangle + \langle(\Delta\hat{v})^2\rangle$ reduces to the variance

$$V = \langle(\hat{X}_1 - \hat{X}_2)^2\rangle = \langle(\hat{Y}_1 + \hat{Y}_2)^2\rangle = n_1 + n_2 - 2c, \tag{26}$$

which characterizes the normal two-mode squeezing and can be measured experimentally by using the technique of balanced homodyne detection. In this case, $n_1 = n_2 = \bar{n}$, the entanglement parameters Υ and E_N are simplified as

$$\begin{aligned} \Upsilon &= 4(\bar{n} - c) - 2, \\ E_N &= \max[0, -\ln 2(\bar{n} - c)]. \end{aligned} \tag{27}$$

Obviously, both of the parameters Υ and E_N are equivalent to quantify the entanglement between the two modes for $\langle a_1^\dagger a_1\rangle = \langle a_2^\dagger a_2\rangle$.

In order to calculate the parameters Υ and E_N, it is necessary to have available the cavity-field correlation functions n_1, n_2, and c. Using the master equation (11) we can derive

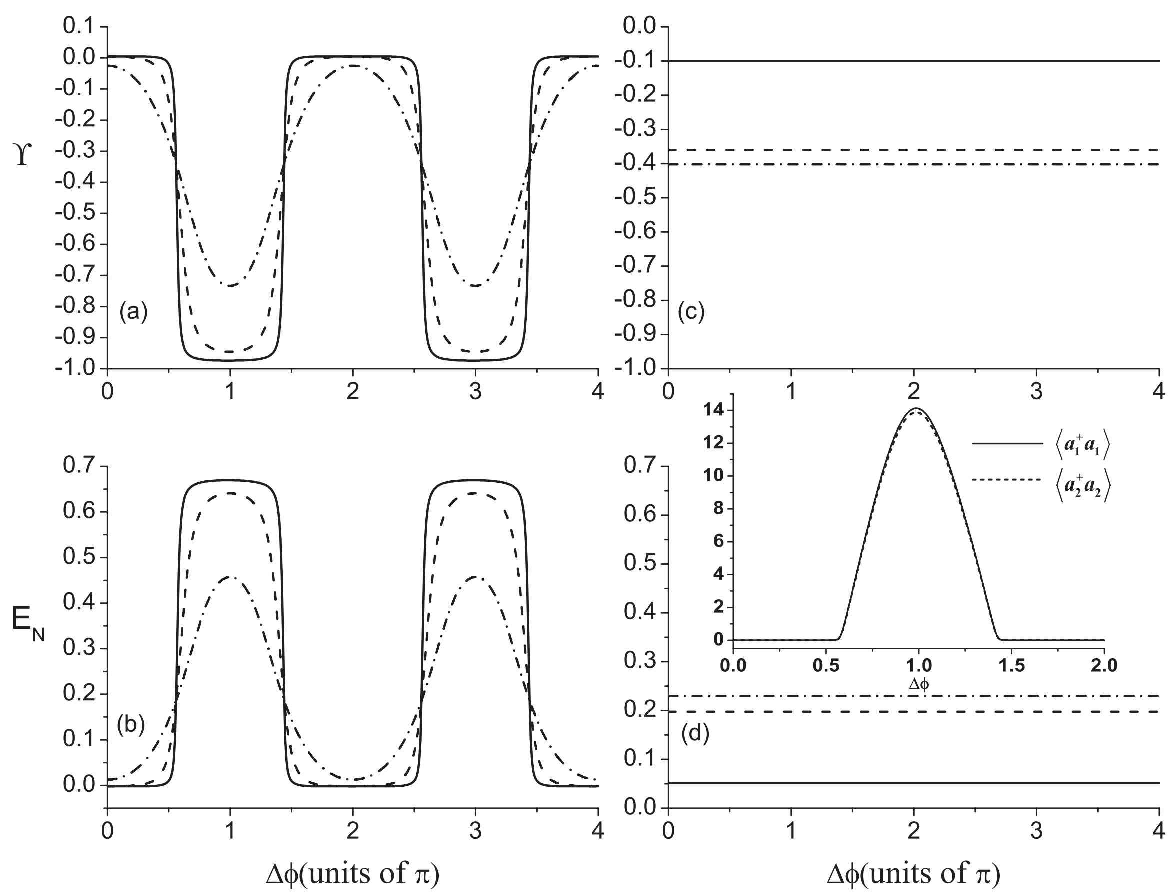

FIG. 3. The entanglement parameter Υ in (a) and (c) and logarithmic negativity E_N in (b) and (d) as a function of the relative phase $\Delta\phi$. Here $\gamma_1 = 1/4$, $\theta = \pi/4$, $\Omega = 500$, $\delta = 250$, $\kappa = 0.0325$, $g = 10$, and (a),(b) $p = 1$ and (c),(d) $p = 0$. The dash-dotted, dashed, and solid curves correspond to $N = 1$, 10, and 50, respectively. All parameters are normalized to γ_2. The inset in panel (d) is the mean photon number of the cavity modes with $N = 50$ and $p = 1$.

equations of motion for the correlation functions

$$\frac{d}{dt}\langle a_j^\dagger a_j\rangle = \frac{1}{2}(x_j + x_j^*)\langle a_j^\dagger a_j\rangle + y_j\langle a_1^\dagger a_2^\dagger\rangle + \frac{1}{2}e_j + \text{c.c.},$$
$$\frac{d}{dt}\langle a_1 a_2\rangle = y_2\langle a_1^\dagger a_1\rangle + y_1\langle a_2^\dagger a_2\rangle + (x_1 + x_2)\langle a_1 a_2\rangle + e_0, \quad (28)$$

where $x_j = A_j - B_j - \kappa_j - i\delta_{12}$, $y_j = C_j - D_j$, $e_j = A_j + A_j^*$, and $e_0 = C_1 + C_2$. The set of the differential equations (21) can be easily solved for arbitrary initial conditions.

Now we turn our attention to the generation of a stationary entanglement between the cavity modes. First, we illustrate the role of the SGC on entanglement creation. The source of SGC has an obvious interpretation that spontaneously emitted photons on one of the atomic transitions $|1\rangle \leftrightarrow |3\rangle$ drives the other transition $|2\rangle \leftrightarrow |3\rangle$. This coherence can occur for the case of nonorthogonal ($p \neq 0$) dipole moments of the atomic transitions. The degree of the coherence, measured by the value of p, will modify the dressed steady-state populations. By assuming $g_1 = g_2 = g$, $\kappa_1 = \kappa_2 = \kappa$, and $\delta_1 \approx \delta_2 = \delta$ for simplicity, we plot the the dependence of the entanglement with different number of atoms, which is measured by of the negativity Υ and the logarithmic negativity E_N, respectively, on the relative phase $\Delta\phi$ for both $p = 1$ [Figs. 3(a) and 3(b)] and $p = 0$ [Figs. 3(c) and 3(d)]. It is obvious the same properties of field entanglement are demonstrated in Figs. 3(a)–3(d), respectively. It can be seen that when SGC is absent ($p = 0$) the entanglement is independent of the relative phase $\Delta\phi$ and is very small. However, when SGC is maximal ($p = 1$) the entanglement is dependent on the relative phase $\Delta\phi$ so it can be conveniently controlled by the relative phase. As seen from Figs. 3(a) and 3(b), in the case of $\gamma_1/\gamma_2 = 1/4$ the entanglement maximizes at $\Delta\phi = (2k+1)\pi (k \in \{0,1,\ldots\})$, which is much bigger than the corresponding value when $p = 0$. That is to say, compared to the case without SGC, the entanglement can be considerably enhanced by the coherence. In fact, these parameters in our case satisfy the condition that $\{|\delta_l|, |\Omega_0 \pm \delta_l|\} \gg \{\delta_{12}, N\gamma_l\}$, then from the inset in panel (d) it is clear that the difference between the mean photon number of the cavity modes $\langle a_1^\dagger a_1\rangle$ and $\langle a_2^\dagger a_2\rangle$ is almost invisible. Since $\langle a_1^\dagger a_1\rangle \approx \langle a_2^\dagger a_2\rangle$, the entanglement parameters Υ and E_N will depict the same properties of field entanglement, which is in agreement with Eq. (27).

In order to show the influence of the SGC on entanglement between the cavity modes further, in Fig. 4(a) we plot the logarithmic negativity E_N as a function of the relative phase $\Delta\phi$ on increasing value of p for $N = 50$. It shows that the entanglement becomes bigger with the increase of the value of p. In Fig. 4(b) the logarithmic negativity E_N versus the strength p of SGC with different value of $\Delta\phi$ are plotted. By choosing appropriate relative phase $\Delta\phi$, it can be seen that the entanglement becomes larger quickly with the increase in the value of p.

Furthermore, we can demonstrate the influence of the number of atoms on entanglement between the cavity modes with maximum SGC from Figs. 3(a) and 3(b). In the case of $p = 1$, the entanglement can be enhanced significantly by increasing the number N of atoms. In the limit $N \to \infty$, the

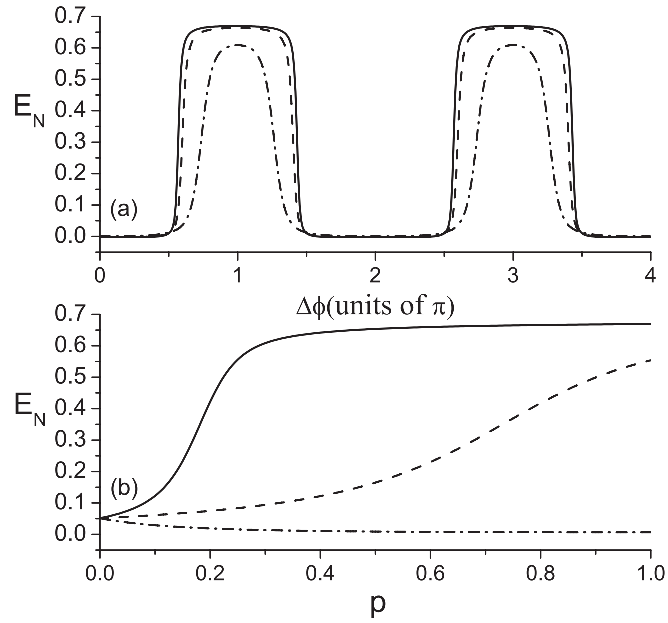

FIG. 4. The steady-state entanglement, quantified by logarithmic negativity E_N, as a function of (a) the relative phase $\Delta\phi$ and (b) the strength p of SGC with $N = 50$. The dash-dotted, dashed, and solid curves correspond to (a) $p = 0.3$, 0.7, and 1 and (b) $\Delta\phi = 0, 0.58\pi$, and π, respectively. The other parameters are the same as in Fig. 3.

system exhibits jumps between the maximally entangled state and unentangled state.

Finally, the amount of the generated entanglement depends also on the ratio of the spontaneous emission rates γ_1/γ_2. From Fig. 5, in the case of $N = 50$, the entanglement maximizes at $\Delta\phi = (2k+1)\pi$ and gradually is trapped in the maximally entangled state by decreasing the value of γ_1/γ_2.

Now we proceed to explain the physical origin of the process responsible for entanglement of the cavity modes. As we shall see, the physics of the process can be quantitatively explained by the level of the stationary population of the atomic system. In the present collective system, under the situation

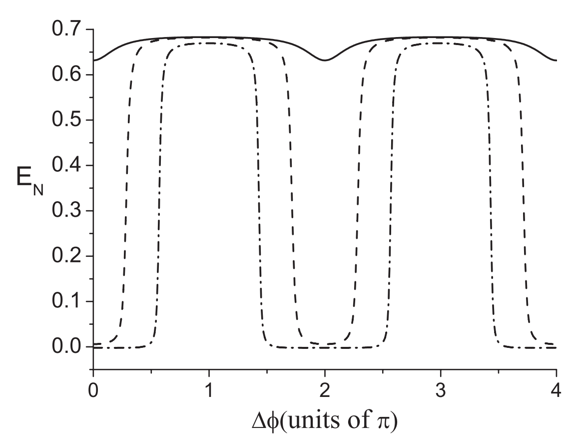

FIG. 5. The steady-state entanglement, measured by logarithmic negativity E_N, as a function of the relative phase $\Delta\phi$. The dash-dotted, dashed, and solid curves correspond to $\gamma_1 = 1/4$, 1/20, and 1/50, respectively. Here $N = 50$ and $p = 1$. The other parameters are the same as in Fig. 3.

that $\{|\delta_l|,|\Omega_0 \pm \delta_l|\} \gg \{\delta_{12}, N\gamma_l\}$, the master equation (11) is reduced to

$$\frac{d}{dt}\tilde{\rho}_f \approx -i\{\delta_{12} + d_1(R_{11}^S - R_{33}^S)\}[a_1^\dagger a_1 + a_2^\dagger a_2, \tilde{\rho}_f] + id_2(R_{11}^S - R_{33}^S)[a_1^\dagger a_2^\dagger + a_1 a_2, \tilde{\rho}_f] + L_f\tilde{\rho}_f, \quad (29)$$

where

$$d_1 = \frac{1}{8}\left(\frac{(1+s)^2c^2}{2\Omega_0 - \delta} + \frac{(1-s)^2c^2}{2\Omega_0 + \delta} + \frac{(1+s)^2s^2}{\Omega_0 - \delta} + \frac{(1-s)^2s^2}{\Omega_0 + \delta} + \frac{2c^4\Omega_0}{\Omega_0^2 - \delta^2}\right), \quad (30)$$

$$d_2 = \frac{c^2}{8}\left(\frac{(1+s)^2}{2\Omega_0 - \delta} + \frac{(1-s)^2}{2\Omega_0 + \delta} + \frac{2s(1-s)}{\Omega_0 + \delta} - \frac{2s(1+s)}{\Omega_0 - \delta}\right).$$

A choice of $\delta_{12} = -d_1(R_{11}^S - R_{33}^S)$ simplifies the master equation further and leaves only the parametric amplifying term, which is responsible for entanglement between the modes. Then the system reduces to a nondegenerate parametric down-conversion (NPDC) with its Hamiltonian

$$V_{\text{eff}} = -d_2(R_{11}^S - R_{33}^S)(a_1^\dagger a_2^\dagger + a_1 a_2). \quad (31)$$

Physically, the entanglement of the cavity modes in the effective NPDC process arises from the two-photon couplings between the dressed atom and the cavity modes. It is evident that the magnitude of entanglement attains maximal value when the dressed steady-state population difference $R_{11}^S - R_{33}^S$ maximizes. In order to show explicitly how the population difference $R_{11}^S - R_{33}^S$ affects the amount of the steady-state entanglement, one can easily derive the the correlation functions $\langle a_1^\dagger a_1\rangle$, $\langle a_2^\dagger a_2\rangle$, and $\langle a_1 a_2\rangle$ from master equation (29) in such an effective NPDC process, which takes the form

$$\langle a_1^\dagger a_1\rangle = \langle a_2^\dagger a_2\rangle = \frac{F^2}{2(\kappa^2 - F^2)}, \quad \langle a_1 a_2\rangle = \frac{F\kappa}{2(\kappa^2 - F^2)}, \quad (32)$$

where $F = d_2(R_{11}^S - R_{33}^S)$ and we assume $\kappa > F$.

It is easy to verify that the logarithmic negativity E_N reads

$$E_N = -\ln\frac{\kappa}{\kappa + |F|} = \ln\left[1 + \frac{|d_2(R_{11}^S - R_{33}^S)|}{\kappa}\right], \quad (33)$$

from which one can easily check that E_N is greatest when the population difference $R_{11}^S - R_{33}^S$ approaches its maximum.

From Fig. 2 we can see that in a collection of atoms ($N \gg 1$), the dressed steady-state population difference $(R_{11}^S - R_{33}^S)/N \approx 1$, even at very big detuning ($\Delta = \sqrt{2}\Omega$), in contrast to the independent atoms case with $(R_{11}^S - R_{33}^S)/N \approx 0.6$ [see Fig. 2(a)]. The reason is that, with the help of the atomic collective interactions, the atomic collectivity can increase $(R_{11}^S - R_{33}^S)/N$, which leads to the significant enhancement of the entanglement. That is to say, the growth of the atomic number N leads to the increase of the effective atom-cavity coupling which, in turn, results in the increase of the two-mode entanglement. Similarly, from Fig. 2(b), with the increase of the value of p, the dressed population difference $R_{11}^S - R_{33}^S$ becomes lager for both $N = 50$ and $N = 1$. Specifically, by setting successively $p = 0,0.4,0.8$, we obtain $(R_{11}^S - R_{33}^S)/N \approx 0.06,0.93,0.98$ in the case of $N = 50$ and $(R_{11}^S - R_{33}^S)/N \approx 0.28,0.39,0.53$ in the case of $N = 1$, respectively. Evidently, the strength of SGC can modify $R_{11}^S - R_{33}^S$ dramatically for $N = 50$. By contrast, the increase trend of $R_{11}^S - R_{33}^S$ is not significant for $N = 1$. Therefore, the entanglement of the cavity field can be enhanced greatly by adopting collective atomic system with strong SGC.

On the other hand, in a collection of atoms the dressed steady-state population against the relative phase $\Delta\phi$ exhibits jumps between the collective dressed states $|\tilde{1}\rangle$ and $|\tilde{2}\rangle$ in the case of $\gamma_1/\gamma_2 = 1/4$ and the dressed population difference $(R_{11}^S - R_{33}^S)/N$ maximizes at $\Delta\phi = (2k+1)\pi$ [see Fig. 2(c)]. In the case of $\gamma_1/\gamma_2 = 1/50$ ($\gamma_2/\gamma_1 = 1/50$), the population is almost trapped in one of the collective dressed states $|\tilde{1}\rangle$ ($|\tilde{2}\rangle$). Therefore, the dressed population difference $(R_{11}^S - R_{33}^S)/N$ can remain approximately the value of $+1$ for $\gamma_1 \ll \gamma_2$ so the entanglement maximizes at $\Delta\phi = (2k+1)\pi$ and is gradually trapped in the maximally entangled state and the trapping range becomes wider on decreasing the ratio of γ_1/γ_2, which coincides with Fig. 5.

V. CONCLUSIONS

In conclusion, the generation of the entanglement of two-mode field in a collective three-level atomic system is investigated by taking into account the SGC. Under the condition that the couplings between the cavity modes and the dressed atoms are far from resonance, the system can reduce to an effective NPDC which is responsible for the entanglement of the cavity modes. It is found that the entanglement between the cavity modes can be significantly enhanced by the collectivity of the atoms compared to the case of independent atoms when the relative phase $\Delta\phi = \pi$. On the other hand, the SGC can also greatly enhance the entanglement in comparison to the case in the absence of SGC.

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