

Collective modes, stability, and superfluid transition of a quasi-two-dimensional dipolar Fermi gasL. M. Sieberer^{1,2} and M. A. Baranov^{1,2,3}¹*Institute for Theoretical Physics, University of Innsbruck, A-6020 Innsbruck, Austria*²*Institute for Quantum Optics and Quantum Information of the Austrian Academy of Sciences, A-6020 Innsbruck, Austria*³*RRC “Kurchatov Institute,” Kurchatov Square 1, RU-123182 Moscow, Russia*

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We examine collective modes, stability, and BCS pairing in a quasi-two-dimensional gas of dipolar fermions aligned by an external field. By using the (conserving) Hartree-Fock approximation, which treats direct and exchange interactions on an equal footing, we obtain the spectrum of single-particle excitations and long-wavelength collective modes (zero sound) in the normal phase. It appears that exchange interactions result in strong damping of zero sound when the tilting angle between the dipoles and the normal to the plane of confinement is below some critical value. In particular, zero sound cannot propagate if the dipoles are perpendicular to the plane of confinement. At intermediate coupling, we find unstable modes that can lead either to collapse of the system or to the formation of a density wave. The BCS transition to a superfluid phase, on the other hand, occurs at arbitrarily weak strengths of the dipole-dipole interaction, provided the tilting angle exceeds a critical value. We determine the critical temperature of the transition, taking into account many-body effects as well as virtual transitions to higher excited states in the confining potential, and discuss prospects of experimental observations.

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I. INTRODUCTION

In recent years, experimentalists have achieved major breakthroughs in preparing samples of diatomic molecules in the rovibrational ground state and cooling them toward quantum degeneracy [1–3]. With heteronuclear molecules, in particular, rotational degrees of freedom can be excited in a controlled way by applying external electric fields and are associated with large electric dipole moments [4–7]. This possibility of inducing strong and anisotropic dipole-dipole interactions between molecules opens fascinating prospects for the observation of various many-body effects and novel quantum phases [4,6–19].

The above-mentioned experimental studies of heteronuclear polar molecules [1,2] suffer from losses due to chemical reactions such as $\text{KRb} + \text{KRb} \rightarrow \text{K}_2 + \text{Rb}_2$ [20,21], which place severe limitations on the achievable densities in three-dimensional samples. These reactions are significantly suppressed if one confines the molecules to a quasi-two-dimensional (quasi-2D) geometry and orients their dipole moments perpendicular to the plane of the 2D translational motion [22–24], as has been verified experimentally by de Miranda *et al.* [25]. Moreover, in some of the polar molecules that consist of alkali-metal atoms, atom-exchanging reactions are endothermic and, therefore, do not occur [26]. Thus, it seems most promising for future investigations of dipolar molecules to either focus on species that do not undergo chemical reactions or consider samples that are strongly confined to a quasi-2D regime.

In this paper, we consider a quasi-2D gas of fermionic dipoles, aligned by an external field (see Fig. 1). The simplest case corresponds to the dipoles being oriented perpendicular to the plane of confinement, so that the pairwise dipole-dipole interaction is isotropic and repulsive. At nonzero values of the tilting angle θ_0 it becomes anisotropic and, for $\theta_0 > \arcsin(1/\sqrt{3})$, it becomes partially attractive. A discussion of this physical setup within the framework of Fermi liquid theory was given in Ref. [27] where, for example, single-particle

properties such as the anisotropic self-energy and the resulting deformation of the Fermi surface from the spherical shape corresponding to the noninteracting case were calculated to first order in perturbation theory. While this perturbative approach provides reliable answers in the weak-coupling regime, moderate interaction strengths require more sophisticated methods such as the Hartree-Fock approximation (HFA), which was used in Ref. [28] to obtain the spectrum of single-particle excitations at zero and finite temperature. At zero temperature, the results were found to agree very well with the outcome of a variational approach that was initially used to study Fermi surface deformations in the three-dimensional (3D) case [29] and adapted to the 2D case by the authors of Ref. [12].

Collective modes in single-, bi- and multilayered structures of dipolar Fermi gases were studied in Ref. [30] using the random phase approximation (RPA), which neglects exchange interactions. As a result and in particular for the single-layer setup, these studies predict the spectrum of long-wavelength collective excitations to be sensitive to microscopic details of the two-body interaction potential in the form of a short-distance cutoff, which is needed to handle the singular behavior of a dipole-dipole interaction. In a quasi-2D setting, the characteristic length of the harmonic confinement plays the role of the cutoff, resulting in a confinement-dependent value of the RPA speed of zero sound [31,32]. The authors of Ref. [33], however, correctly remark that a cutoff-dependent constant term in the momentum space representation of the interparticle potential corresponds to a short-range contact interaction in real space and, therefore, must not have an effect in a single-component Fermi gas. Thus, the existence of the RPA zero-sound mode is questionable. In this paper, we study zero-sound collective modes on the basis of the so-called conserving HFA developed in Refs. [34,35]. The advantage of this method is that it provides a way to fully include exchange contributions in a given order of perturbation theory, such that the results are consistent with conservation of particle number, energy, and momentum, as well as with fermionic statistics of

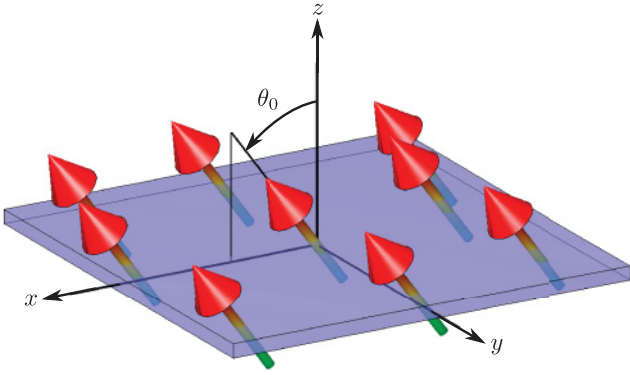


FIG. 1. (Color online) Fermionic dipoles confined to the xy plane. The dipoles are aligned in the xz plane and form an angle θ_0 with the z axis.

particles. We show that the existence of a zero-sound collective mode in a quasi-2D dipolar Fermi gas for small values of the tilting angle θ_0 and the dependence of the sound velocity on a short-distance cutoff are artifacts of the RPA. In particular, we find that the propagation of zero sound is not possible if the dipoles are aligned perpendicular to the plane of confinement or if they are tilted only slightly—which is consistent with the homogeneous 3D setting [27,36] in which there is no propagating zero sound in the directions perpendicular (or close to perpendicular) to the direction of dipole polarization.

The issue of stability of the normal phase of the system against collapse was addressed by Chan *et al.* [27] following Pomeranchuk's approach [37]: The normal phase is thermodynamically stable if an arbitrary distortion of the Fermi surface results in an increase of the ground-state energy. In Ref. [27], however, distortions do not refer to the deformed Fermi surface, but rather to the circular one of the noninteracting system; that is, the authors of this reference are performing the stability analysis around a configuration that does not extremize the ground-state energy. Hence their expression for the change in the ground-state energy contains a term that is linear in the distortion [Eq. (81) in Ref. [27]], which is absent if one takes the deformed Fermi surface as reference center [see Eq. (46) below]. An alternative approach by Bruun and Taylor [12] uses a variational ansatz for the shape of the Fermi surface, on the basis of which the compressibility is calculated. The collapse instability is then identified with a negative value of this quantity.

The stability of the normal phase against density fluctuations with a finite momentum (density wave instability) was investigated in the RPA in Refs. [28,33], with the former extending the discussion to finite temperatures and taking into account the deformation of the Fermi surface. It was found that a density wave transition takes place in a broad region in the parameter space (the coupling strength and the tilting angle θ_0) where the system is stable against collapse. Correlations beyond the RPA can be treated approximately by including a local field factor in the density-density response function. This method was first used in Ref. [38] with an *ad hoc* choice of the local field factor that served to gain qualitative insight into the effects of exchange interactions. In Ref. [39], the local field factor was calculated by using a self-consistent scheme. As a

result, the density wave instability is expected at higher values of the interaction strength than predicted by the RPA.

We address the issue of stability on the basis of the conserving HFA that provides a consistent (and, within this approximation scheme, exact) treatment of exchange contributions. For the case $\theta_0 = 0$, the same approach was used in Ref. [40] to study the density wave instability in dipolar mono- and multilayer systems. Our result (see Sec. VII B) for the critical value of the coupling strength for $\theta_0 = 0$ agrees very well with that from Ref. [40]. As concerns the collapse instability, our findings are in good agreement with the results of the above-mentioned variational approach [12,28].

The critical temperature T_c of the transition to the superfluid phase for the quasi-2D dipolar Fermi gas was obtained by Bruun and Taylor [12] in the BCS approach with the dipole-dipole interaction restricted to the dominant p -wave channel. We extend this work by taking into account the full angular dependence of the dipole-dipole interaction, as well as by calculating the preexponential factor in the expression for the critical temperature. The latter requires taking into account both the many-body contributions to the interparticle interaction [the so-called Gor'kov–Melik-Barkhudarov (GM) corrections [41]] and virtual transitions to excited states in the trapping potential, which ultimately result in a nontrivial dependence of the critical temperature on the trapping frequency, the gas density, and the tilting angle.

This paper is organized as follows: In Sec. II we present the microscopic model for the quasi-2D dipolar Fermi gas and identify the relevant parameters and parameter regimes. We review key quantities in the many-body problem and the equations that relate them and discuss our strategy to solve these equations in the intermediate-coupling regime in Secs. III and IV. An investigation of the single-particle excitation spectrum in the normal phase is given in Sec. V and is a prerequisite for the study of collective modes in Sec. VI. We address the issue of instability toward collapse and the formation of a density wave in Secs. VII A and VII B, respectively. Results for the critical temperature of the transition to the superfluid phase are given in Sec. VIII. Finally, Sec. IX is devoted to a summary of our findings and a discussion of the prospects of observing the described phenomena in experiments. Details of our numerical methods and analytical expressions for the matrix elements of the dipole-dipole interaction are given in the Appendices.

II. SYSTEM

We consider a gas of dipolar fermions of mass m with dipole moments $\mathbf{d} = d\hat{\mathbf{d}}$, which are polarized along the direction $\hat{\mathbf{d}} = (\sin \theta_0, 0, \cos \theta_0)$; that is, θ_0 is the angle between the orientation of dipoles and the z axis (see Fig. 1). The gas is strongly confined to the xy plane by a harmonic trapping potential $V(z) = m\omega_0^2 z^2/2$. Here, strong confinement means that the transverse extension of the gas cloud, which is on the order of $l_0 = \sqrt{\hbar/(m\omega_0)}$, is small compared with the mean interparticle separation in the xy plane. The latter quantity is proportional to the inverse Fermi momentum $p_F^{(0)}$, which, in turn, is determined by the area density n_{2D} , $p_F^{(0)} = \hbar\sqrt{4\pi n_{2D}}$. Thus, the small parameter characterizing the tight-confinement

or quasi-2D limit is $\eta \equiv p_F^{(0)} l_0 / \hbar \ll 1$. This condition implies that the Fermi energy $\varepsilon_F^{(0)}$ is much smaller than the trapping potential level spacing: $\varepsilon_F^{(0)} / \hbar \omega_0 \ll 1$.

We also assume ultracold temperatures, $T \ll \varepsilon_F^{(0)}$, such that the average kinetic energy of particles is given by the Fermi energy $\varepsilon_F^{(0)}$.

The Hamiltonian of this system reads

$$H = \int d\mathbf{r} \hat{\psi}^\dagger(\mathbf{r}) \left[-\frac{\hbar^2}{2m} \Delta + \frac{1}{2} m \omega_0^2 z^2 - \mu' \right] \hat{\psi}(\mathbf{r}) + \frac{1}{2} \int d\mathbf{r} d\mathbf{r}' \hat{\psi}^\dagger(\mathbf{r}) \hat{\psi}^\dagger(\mathbf{r}') V_d(\mathbf{r} - \mathbf{r}') \hat{\psi}(\mathbf{r}') \hat{\psi}(\mathbf{r}), \quad (1)$$

where $\hat{\psi}(\mathbf{r})$ is the fermionic field operator, μ' denotes the chemical potential, and $V_d(\mathbf{r}) = (d^2/r^3)[1 - 3(\hat{\mathbf{r}} \cdot \hat{\mathbf{d}})^2]$ is the dipole-dipole interaction. Here we omit the contribution of the short-range part of the interparticle interaction: In the considered case of a single-component Fermi gas, it results only in p -wave scattering, which is small assuming that $p_F^{(0)} r_0 / \hbar \ll 1$, where r_0 is the radius of the short-range part of the interparticle interaction.

A characteristic length of the dipole-dipole interaction is given by $r_d = m d^2 / \hbar^2 \gg r_0$. This is the length scale below which the dipole-dipole interaction substantially influences the relative wave function of two particles. We assume $r_d \ll l_0$, such that interparticle collisions are essentially three-dimensional. This gives us another small parameter $g \equiv p_F^{(0)} r_d / \hbar \ll 1$.

Under the above conditions, the motion of particles in the z direction is limited to the ground state of the confining potential $\phi_0(z)$ —the lowest harmonic oscillator (HO) level, and a single-particle wave function is $\psi(\mathbf{r}) = \varphi(\boldsymbol{\rho}) \phi_0(z)$, where $\varphi(\boldsymbol{\rho})$ describes the in-plane motion [$\boldsymbol{\rho} = (x, y)$].

As a result, first-order interaction effects (see Secs. V, VI, and VII) can be described by an effective interaction

$$V_0(\boldsymbol{\rho}) \equiv \int dz dz' \phi_0(z)^2 V_d(\boldsymbol{\rho}, z - z') \phi_0(z')^2, \quad (2)$$

with the Fourier transform [$\mathbf{p} = (p_x, p_y)$] [31]:

$$V_0(\mathbf{p}) = \sqrt{2\pi} \frac{d^2}{l_0} w \left(\frac{pl_0}{\sqrt{2}\hbar} \right) [u(\mathbf{p}) \sin^2 \theta_0 - 2P_2(\cos \theta_0)], \quad (3)$$

where $u(\mathbf{p}) \equiv (p_x^2 - p_y^2) / p^2$ and $w(x) \equiv x e^{x^2} \operatorname{erfc}(x)$.¹ When $p \approx p_F^{(0)}$, the argument of $w(\frac{pl_0}{\sqrt{2}\hbar})$ is as small as $pl_0 / \hbar \approx \eta \ll 1$ and we have

$$V_0(\mathbf{p}) \approx \pi \frac{d^2 p}{\hbar} [u(\mathbf{p}) \sin^2 \theta_0 - 2P_2(\cos \theta_0)]. \quad (4)$$

We note that, in this limit, the effective interaction is independent of the confinement length l_0 .

¹This expression is unique up to a momentum-independent additive constant, which depends on the regularization of the Fourier integral at the origin. Such a constant, however, corresponds to a short-range interaction and has no physical effect in a single-component Fermi gas because its contributions vanish upon proper antisymmetrization. Therefore, we set this constant to zero.

Processes of second order in the interaction (see Sec. VIII), on the other hand, involve virtual transitions to excited states $\phi_n(z)$ with $n > 0$ of the harmonic confining potential. The matrix elements of the interaction for these transitions, in momentum representation for the in-plane motion, are

$$V_{n_1, n_2, n_3, n_4}(\mathbf{p}) \equiv \int d\boldsymbol{\rho} dz dz' V_d(\boldsymbol{\rho}, z - z') \times e^{-i\mathbf{p} \cdot \boldsymbol{\rho} / \hbar} \phi_{n_1}(z) \phi_{n_2}(z') \phi_{n_3}(z) \phi_{n_4}(z'). \quad (5)$$

For $n_1 = n_2 = n_3 = n_4 = 0$ we obtain the effective 2D interaction (3). Note that, although the relative magnitude of a single virtual excitation is as small as $g\eta^2 \ll 1$ (see Appendix C), the totality of these processes is essential for the correct description of the contribution of short distances $\lesssim l_0$ (or virtual energies $\gtrsim \hbar\omega_0$) to the interparticle scattering. We mention also that, within second order the p -wave contribution due to the short-range part of the interparticle interaction can still be neglected (see Ref. [18] for discussion).

III. APPROACHING INTERMEDIATE COUPLING REGIME

The problem considered is characterized by two parameters: $g = p_F^{(0)} r_d / \hbar$ and $\eta = p_F^{(0)} l_0 / \hbar$. The first parameter describes the strength of the interparticle interaction with respect to the mean kinetic energy: $g \ll 1$ characterizes the weakly interacting regime, while we have $g \gtrsim 1$ in the regime of intermediate and strong interactions. The second parameter η describes the strength of the confinement: $\eta \ll 1$ corresponds to strong confinement. When g and η are small ($g, \eta \ll 1$), one can use perturbation theory to calculate various quantities. Many physical effects, however, occur at intermediate values of g (we will always assume strong confinement with $\eta \ll 1$)—see Fig. 9—for which one cannot limit oneself to lowest-order diagrams or to a specific sequence of diagrams (ladder diagrams in a dilute system). To obtain analytic expressions in this case, one can use analyticity arguments to extrapolate expressions obtained in the weak-coupling regime to intermediate coupling strength. Of course, the accuracy of such expressions cannot be estimated. However, (provided the relevant physics is present in the weak-coupling regime) they can be used to make qualitative statements on the behavior of the system and often obtain reasonable quantitative estimates. Following this strategy, one has to select a “reasonable” set of Feynman diagrams, which allows one to write down a closed set of integral equations for relevant physical quantities. This sequence should, of course, catch the relevant physics and be consistent with general physical principles such as conservation laws and particle statistics. For the purposes of this paper we will use the (conserving) HFA (see Refs. [34,35] and discussion below), which describes the motion of particles in an average potential with exchange effects taken into account and, therefore, can be used to describe phenomena for which interparticle collisions are not important (zero sound in our case). Note that taking exchange contributions into account is crucial: They guarantee the cancellation of all interaction contributions in the case of a short-range interparticle interaction [when $V_0(\mathbf{p})$ is momentum-independent,

$V_0(\mathbf{p}) = \text{const.}$], as it should be in a single-component Fermi gas.

IV. QUASI-2D DIPOLAR FERMION LIQUID

Properties of a normal (nonsuperfluid) Fermi system at $T = 0$ are conveniently described in terms of Green's functions. The single-particle Green's function (see, for example, Ref. [42]) for a two-dimensional system (in the following we shall be using units in which $\hbar = 1$),

$$G(p) = -i \int dt d\boldsymbol{\rho} e^{i(\omega t - \mathbf{p} \cdot \boldsymbol{\rho})} \langle T \{ \hat{\psi}(t, \boldsymbol{\rho}) \hat{\psi}^\dagger(0, \mathbf{0}) \} \rangle, \quad (6)$$

where T stands for the time-ordering operator (T product) and $p = (\omega, \mathbf{p})$, carries information on single-particle excitations, while collective behavior resulting from two-particle correlations is described by the two-particle Green's function

$$\begin{aligned} G_2(p_1, p_2; p_3, p_4) &= \int \prod_{j=1}^4 dt_j d\boldsymbol{\rho}_j e^{i(\omega_j t_j - \mathbf{p}_j \cdot \boldsymbol{\rho}_j)} \langle T \{ \hat{\psi}(t_1, \boldsymbol{\rho}_1) \hat{\psi}(t_2, \boldsymbol{\rho}_2) \\ &\quad \times \hat{\psi}^\dagger(t_4, \boldsymbol{\rho}_4) \hat{\psi}^\dagger(t_3, \boldsymbol{\rho}_3) \} \rangle \\ &= G(p_1)G(p_2)[\delta(p_1 - p_3)\delta(p_2 - p_4) \\ &\quad - \delta(p_1 - p_4)\delta(p_2 - p_3)] \\ &\quad + G(p_1)G(p_2)\Gamma(p_1, p_2; p_3, p_4)G(p_3)G(p_4), \quad (7) \end{aligned}$$

or the closely related vertex function $\Gamma(p_1, p_2; p_3, p_4)$. Single-particle excitations correspond to poles of $G(p)$, collective modes and instabilities of the many-body system are encoded in poles of the vertex function $\Gamma(p_1, p_2; p_3, p_4)$. In a homogeneous system, this quantity depends only on three independent momenta, and it is convenient to introduce $\Gamma(p_1, p_2; q) \equiv \Gamma(p_1, p_2; p_1 + q, p_2 - q)$ such that q is the transferred momentum which satisfies the Bethe-Salpeter equation in the particle-hole channel [42]:

$$\begin{aligned} \Gamma(p_1, p_2; q) &= \tilde{\Gamma}_{\text{ph}}(p_1, p_2; q) - i \int \tilde{\Gamma}_{\text{ph}}(p_1, p' + q; q) \\ &\quad \times G(p' + q)G(p')\Gamma(p', p_2; q) \frac{dp'}{(2\pi)^3}, \quad (8) \end{aligned}$$

where $\tilde{\Gamma}_{\text{ph}}$ denotes the particle-hole irreducible vertex; that is, the sum of connected vertex diagrams with two incoming and two outgoing fermionic lines which cannot be divided into two parts by cutting two fermion lines of opposite direction.

The single-particle Green's functions can be expressed in terms of the self-energy function $\Sigma(p)$ through the Dyson equation [42]

$$G(p)^{-1} = G^{(0)}(p)^{-1} - \Sigma(p), \quad (9)$$

where the noninteracting Green's function is

$$G^{(0)}(p) = \frac{1}{\omega - \xi(\mathbf{p}) + i \text{sgn}[\xi(\mathbf{p})]}, \quad (10)$$

with $\xi(\mathbf{p}) = p^2/(2m) - \mu$ and the shifted chemical potential $\mu = \mu' - \omega_0/2$. Σ , in turn, is connected with the

vertex function by the equation of motion (Schwinger-Dyson equation) [42]:

$$\begin{aligned} \Sigma(p) &= i \int [V_0(\mathbf{p} - \mathbf{p}_1) - V_0(\mathbf{0})] G(p_1) \frac{dp_1}{(2\pi)^3} \\ &\quad + \int \Gamma(p_1 + p_2 - p, p; p_1, p_2) G(p_1) G(p_2) \\ &\quad \times G(p_1 + p_2 - p) V_0(\mathbf{p} - \mathbf{p}_2) \frac{dp_1}{(2\pi)^3} \frac{dp_2}{(2\pi)^3}. \quad (11) \end{aligned}$$

The solution of the coupled system of equations (8), (9), and (11) is specified by the irreducible vertex $\tilde{\Gamma}_{\text{ph}}$, which is the sum of an infinite set of Feynman diagrams. As a result, one cannot write the irreducible vertex $\tilde{\Gamma}_{\text{ph}}$ in a closed form in terms of the Green's function G and the vertex Γ , and some approximation procedure of choosing a subset of contributions is needed. This procedure should be consistent with conservation laws and statistics of the system. A prescription for generating such a conserving approximation is to replace the right-hand side (RHS) of Eq. (11) for Σ by a functional of G and V_0 [34,35]. The one-particle propagator is then to be obtained self-consistently from the approximate equation for Σ and (9), and the approximate irreducible vertex $\tilde{\Gamma}_{\text{ph}}$, which determines Γ via (8), can be found by suitable functional differentiation. In other words, by fixing an appropriate expression for the self-energy Σ , one uniquely determines the expression for $\tilde{\Gamma}_{\text{ph}}$ in order to make the approximation conserving.

As discussed in Refs. [34,35], the simplest example of a conserving approximation taking exchange effects into account is the HFA, which we will use in this paper. In this approximation, the self-energy is given diagrammatically as

$$\text{---} \circ \Sigma \text{---} = \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \quad (12)$$

or analytically by

$$\Sigma(\mathbf{p}) = i \int [V_0(\mathbf{p} - \mathbf{p}') - V_0(\mathbf{0})] G(p') \frac{dp'}{(2\pi)^3}, \quad (13)$$

and it is frequency independent. The corresponding particle-hole irreducible vertex is

$$\text{---} \circ \tilde{\Gamma}_{\text{ph}} \text{---} = \begin{array}{c} p_1 \quad p_2 - q \quad p_1 \quad p_2 - q \\ \text{---} \text{---} \text{---} \quad \text{---} \text{---} \text{---} \\ p_1 + q \quad p_2 \quad p_1 + q \quad p_2 \end{array}, \quad (14)$$

or

$$\begin{aligned} \tilde{\Gamma}_{\text{ph}}(p_1, p_2; q) &= V_0(\mathbf{q}) - V_0(\mathbf{p}_1 - \mathbf{p}_2 + \mathbf{q}) \\ &\equiv \tilde{\Gamma}_{\text{ph}}(\mathbf{p}_1 - \mathbf{p}_2 + \mathbf{q}, \mathbf{q}), \quad (15) \end{aligned}$$

which is also frequency independent. Equation (8) then shows that $\Gamma(p_1, p_2; q)$ does not depend on the frequencies ω_1 and ω_2 [i.e., $\Gamma(p_1, p_2; q) = \Gamma(\mathbf{p}_1, \mathbf{p}_2; q)$], and we have

$$\begin{aligned} \Gamma(\mathbf{p}_1, \mathbf{p}_2; q) &= \tilde{\Gamma}_{\text{ph}}(\mathbf{p}_1 - \mathbf{p}_2 + \mathbf{q}, \mathbf{q}) \\ &\quad - i \int \tilde{\Gamma}_{\text{ph}}(\mathbf{p}_1 - \mathbf{p}', \mathbf{q}) G(p' + q) G(p') \\ &\quad \times \Gamma(\mathbf{p}', \mathbf{p}_2; q) \frac{dp'}{(2\pi)^3}. \quad (16) \end{aligned}$$

V. SINGLE-PARTICLE EXCITATIONS IN HFA

From Eqs. (9), (10), and (13) we obtain the single-particle Green's function in momentum space representation:

$$G(\omega, \mathbf{p}) = \frac{1}{\omega - \varepsilon(\mathbf{p}) + i0\text{sgn}[\varepsilon(\mathbf{p})]}, \quad (17)$$

where the quasiparticle dispersion relation is

$$\varepsilon(\mathbf{p}) = \xi(\mathbf{p}) + \Sigma(\mathbf{p}), \quad (18)$$

and the self-energy Σ , after performing the integration over ω' in Eq. (13), can be written as [note that the direct term does not contribute, because $V_0(\mathbf{0}) = 0$ for the effective dipole-dipole interaction (3)]

$$\Sigma(\mathbf{p}) = - \int V_0(\mathbf{p} - \mathbf{p}') n(\mathbf{p}') \frac{d\mathbf{p}'}{(2\pi)^2}. \quad (19)$$

In this expression $n(\mathbf{p}) = \theta[-\varepsilon(\mathbf{p})]$ is the Fermi-Dirac distribution at zero temperature. Equation (19) has to be solved self-consistently together with the particle number equation

$$n_{2D} = \int \frac{d\mathbf{p}}{(2\pi)^2} n(\mathbf{p}). \quad (20)$$

The problem of finding the solution to Eqs. (19) and (20) is simplified considerably by noting that the evaluation of the RHS of both equations does not require full knowledge of the quasiparticle dispersion relation $\varepsilon(\mathbf{p})$ but only of the Fermi momentum p_F , which is determined by the requirement that the quasiparticle energy (18) is equal to zero for $\mathbf{p} = \mathbf{p}_F \equiv p_F \hat{\mathbf{p}}$:

$$\varepsilon(\mathbf{p}_F) = p_F^2/(2m) - \mu + \Sigma(\mathbf{p}_F) = 0. \quad (21)$$

The solution p_F to this equation actually depends on the direction $\hat{\mathbf{p}} = (\cos \phi, \sin \phi)$. However, to shorten the notation, in the following we will often write p_F and \mathbf{p}_F instead of $p_F(\phi)$ and $\mathbf{p}_F(\phi)$, respectively.

We proceed by specializing (19) to the Fermi surface $\mathbf{p} = \mathbf{p}_F$ and inserting the resulting expression for $\Sigma(\mathbf{p}_F)$ in Eq. (21),

$$\frac{p_F^2}{2m} = \mu + \frac{1}{m} \int \frac{d\phi'}{2\pi} \int_0^{p'_F} dp' p' v V_0(\mathbf{p}_F - \mathbf{p}'), \quad (22)$$

where $p'_F \equiv p_F(\phi')$ and $v = m/(2\pi)$ is the density of states. An expression for the chemical potential μ can be obtained by taking the integral of this formula over the angle ϕ and making use of the fact that the particle number equation (20) is equivalent to the condition

$$\int \frac{d\phi}{2\pi} p_F^2 = p_F^{(0)2}, \quad (23)$$

if we express the density as $n_{2D} = p_F^{(0)2}/(4\pi)$. We find

$$\mu = \varepsilon_F^{(0)} - \frac{1}{m} \int \frac{d\phi}{2\pi} \frac{d\phi'}{2\pi} \int_0^{p'_F} dp' p' v V_0(\mathbf{p}_F - \mathbf{p}'). \quad (24)$$

Equations (22) and (24) form a closed system for the deformed Fermi surface p_F and we obtain the joint solution to these equations numerically by means of an iterative scheme which is described in detail in Appendix A. The resulting

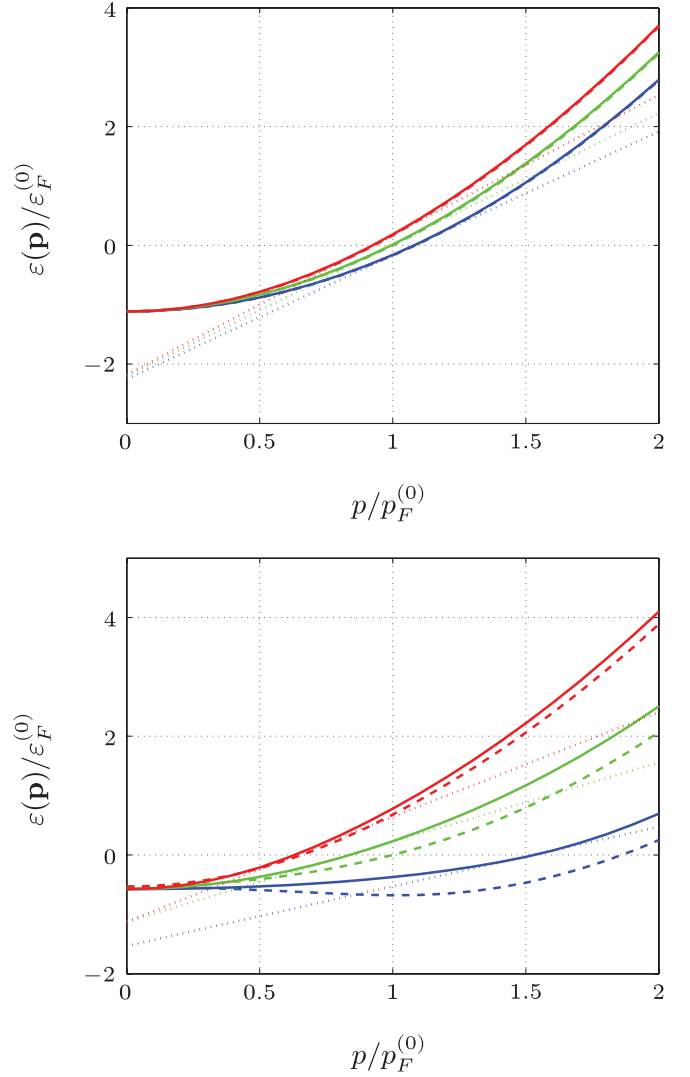


FIG. 2. (Color online) Quasiparticle dispersion relation for $g = 1, \theta_0 = \pi/4$ (top) and $g = 2, \theta_0 = \pi/2$ (bottom) in HFA (solid lines) and first-order perturbation theory (dashed lines) for some values of ϕ_p [$\mathbf{p} = p(\cos \phi_p, \sin \phi_p)$]. The lowermost blue curves are for $\phi_p = 0$, the middle green curves are for $\phi_p = \pi/4$, and the topmost red curves are for $\phi_p = \pi/2$. The linear approximation (25) at the Fermi surface is shown as dotted lines. Note that the results obtained in the HFA and perturbation theory agree very well for $g = 1, \theta_0 = \pi/4$, whereas deviations are clearly visible for $g = 2, \theta_0 = \pi/2$.

quasiparticle dispersion relation (18) is shown in Fig. 2, along with the linear approximation at the Fermi surface:

$$\varepsilon(\mathbf{p}) \approx v_F(\phi)[p - p_F(\phi)]. \quad (25)$$

Here, $v_F(\phi)$ is the radial component of the Fermi velocity:

$$\mathbf{v}_F(\phi) \equiv \nabla \varepsilon[\mathbf{p}_F(\phi)] = \hat{\mathbf{p}} v_F(\phi) + \hat{\mathbf{e}}_\phi v_F^\perp(\phi). \quad (26)$$

For future reference we summarize the corresponding results obtained in perturbation theory [27]. To first order in the dipole-dipole interaction, on the RHS of Eq. (19) we insert the distribution function of a noninteracting Fermi gas, $n(\mathbf{p}) = \theta(p_F^{(0)} - p)$. The self-energy function can then be

expressed in terms of complete elliptic integrals. We omit the cumbersome analytical expression and content ourselves with stating the result for $\mathbf{p} = \mathbf{p}_F^{(0)} = p_F^{(0)} \hat{\mathbf{p}}$:

$$\Sigma^{(1)}(\mathbf{p}_F^{(0)}) = -\frac{16}{9\pi} g \varepsilon_F^{(0)} \left[\frac{3}{5} \sin^2 \theta_0 \cos(2\phi) - 2P_2(\cos \theta_0) \right]. \quad (27)$$

We write the chemical potential as $\mu = \varepsilon_F^{(0)} + \delta\mu$. The first-order correction $\delta\mu$ is given by Eq. (24) [with p'_F on the RHS replaced by $p_F^{(0)}$]

$$\delta\mu = \frac{32}{9\pi} g \varepsilon_F^{(0)} P_2(\cos \theta_0). \quad (28)$$

Combining these results with Eq. (21) we find the equilibrium deformation of the Fermi surface $\delta p_F(\phi) = p_F(\phi) - p_F^{(0)}$:

$$\begin{aligned} \delta p_F(\phi) &= \frac{m}{p_F^{(0)}} [\delta\mu - \Sigma^{(1)}(\mathbf{p}_F^{(0)})] \\ &= \frac{8}{15\pi} g p_F^{(0)} \sin^2 \theta_0 \cos(2\phi). \end{aligned} \quad (29)$$

From its definition in Eq. (26), the radial component of the Fermi velocity is then

$$v_F(\phi) = v_F^{(0)} \left[1 + \frac{4}{3\pi} g P_2(\cos \theta_0) - \frac{2}{5\pi} g \sin^2 \theta_0 \cos(2\phi) \right], \quad (30)$$

Finally, the (radial) effective mass is defined as $m^*(\phi) \equiv p_F(\phi)/v_F(\phi)$. Then, for the deviation $\delta m(\phi) = m^*(\phi) - m$ of the effective mass from the bare mass we have

$$\begin{aligned} \frac{\delta m(\phi)}{m} &= -\frac{m}{p_F^{(0)}} \frac{\partial \Sigma^{(1)}(\mathbf{p}_F^{(0)})}{\partial p} \\ &= -\frac{4}{3\pi} g P_2(\cos \theta_0) + \frac{14}{15\pi} g \sin^2 \theta_0 \cos(2\phi). \end{aligned} \quad (31)$$

VI. COLLECTIVE EXCITATIONS IN HFA: ZERO SOUND

Collective modes $\omega = \omega(\mathbf{q})$ of the system correspond to poles of $\Gamma(\mathbf{p}_1, \mathbf{p}_2; q)$ with respect to the variable ω , which is the frequency component of q . In the vicinity of a pole we have $\Gamma \gg \tilde{\Gamma}_{\text{ph}}$ and, therefore, we may neglect the first term on the RHS of Eq. (16). In the resulting homogeneous equation the second argument \mathbf{p}_2 of Γ acts as a parameter. Hence, near its pole, the function Γ can be represented as a product $\chi(\mathbf{p}_1; \mathbf{q})\chi'(\mathbf{p}_2; \mathbf{q})$ of two functions. After canceling $\chi'(\mathbf{p}_2; \mathbf{q})$ on both sides of Eq. (16) and integrating over ω' , we obtain

$$\begin{aligned} \chi(\mathbf{p}; \mathbf{q}) &= \int \frac{d\mathbf{p}'}{(2\pi)^2} \tilde{\Gamma}_{\text{ph}}(\mathbf{p} - \mathbf{p}', \mathbf{q}) \\ &\quad \times \frac{n(\mathbf{p}') - n(\mathbf{p}' + \mathbf{q})}{\omega + \varepsilon(\mathbf{p}') - \varepsilon(\mathbf{p}' + \mathbf{q}) + i0 \text{sgn}(\omega)} \chi(\mathbf{p}'; \mathbf{q}). \end{aligned} \quad (32)$$

The quantity $\varepsilon(\mathbf{p}' + \mathbf{q}) - \varepsilon(\mathbf{p}')$ on the RHS of the last equation is just the energy cost of creating a particle-hole pair by exciting a particle from a state \mathbf{p}' within the Fermi surface to a state $\mathbf{p}' + \mathbf{q}$ outside the Fermi surface. Therefore, a stable collective mode is possible only when the energy (frequency)

ω of the mode lies outside the particle-hole continuum (PHC) (so we can omit the imaginary term in the denominator). Otherwise, the integrand has a pole at $\omega = \varepsilon(\mathbf{p}' + \mathbf{q}) - \varepsilon(\mathbf{p}')$, which ultimately leads to strong Landau damping of the collective mode [42].

In the long-wavelength limit $|\mathbf{q}| \rightarrow 0$, the main contribution to the integral (32) comes from states in the vicinity of the Fermi surface, and we can rewrite this equation as

$$\chi(\mathbf{p}) = \int \frac{d\phi'}{2\pi} v \tilde{\Gamma}_{\text{ph}}(\mathbf{p} - \mathbf{p}'_F, \mathbf{0}) \frac{p'_F/m}{\hat{\mathbf{p}}' \cdot \mathbf{v}'_F} \frac{\mathbf{q} \cdot \mathbf{v}'_F}{\omega - \mathbf{q} \cdot \mathbf{v}'_F} \chi(\mathbf{p}'_F), \quad (33)$$

where $\chi(\mathbf{p}) \equiv \chi(\mathbf{p}; \mathbf{0})$, $p'_F \equiv p_F(\phi')$, and $\mathbf{v}'_F \equiv \mathbf{v}_F(\phi')$. The function $\chi(\mathbf{p})$ is thus completely determined by its values on the Fermi surface and by setting $\mathbf{p} = \mathbf{p}_F$ we can obtain an equation for the restriction of $\chi(\mathbf{p})$ to the Fermi surface,

$$\chi(\mathbf{p}_F) = \int \frac{d\phi'}{2\pi} F(\phi, \phi') \frac{p'_F/m}{\hat{\mathbf{p}}' \cdot \mathbf{v}'_F} \frac{\mathbf{q} \cdot \mathbf{v}'_F}{\omega - \mathbf{q} \cdot \mathbf{v}'_F} \chi(\mathbf{p}'_F), \quad (34)$$

where we define the dimensionless quasiparticle interaction function (f function) as

$$\begin{aligned} F(\phi, \phi') &\equiv v \tilde{\Gamma}_{\text{ph}}(\mathbf{p}_F - \mathbf{p}'_F, \mathbf{0}) = v [V_0(\mathbf{0}) - V_0(\mathbf{p}_F - \mathbf{p}'_F)] \\ &= g \frac{|\mathbf{p}_F - \mathbf{p}'_F|}{p_F^{(0)}} \\ &\quad \times \left[P_2(\cos \theta_0) - \frac{1}{2} u(\mathbf{p}_F - \mathbf{p}'_F) \sin^2 \theta_0 \right]. \end{aligned} \quad (35)$$

Note that only the exchange interaction contributes to the f function. For $\theta_0 = 0$, that is, when the dipoles are perpendicular to the xy plane and the system is symmetric with respect to rotations around the z axis, the Fermi momentum is isotropic and equal to $p_F^{(0)}$ [see Eq. (23)], and (35) simplifies to

$$F(\phi, \phi') = 2g \sin(|\phi - \phi'|/2), \quad \theta_0 = 0. \quad (36)$$

Equation (34) shows that ω depends linearly on $q = |\mathbf{q}|$:

$$\omega = v_F^{(0)} s q. \quad (37)$$

Due to the anisotropy of the dipole-dipole interaction, s will in general be a function of the propagation direction $\phi_{\mathbf{q}}$ [$\mathbf{q} = q(\cos \phi_{\mathbf{q}}, \sin \phi_{\mathbf{q}})$]. The symmetry of the problem, however, requires the dependence of s on $\phi_{\mathbf{q}}$ to be π periodic and even; hence, it is sufficient to restrict ourselves to the range $0 \leq \phi_{\mathbf{q}} \leq \pi/2$.

As has already been pointed out, the excitation energy ω of the collective mode has to be separated from the PHC. Equation (34) shows that, in the long-wavelength limit $q \rightarrow 0$, this requirement reduces to

$$v_F^{(0)} s > v_{\text{ph}}, \quad (38)$$

where the quantity v_{ph} is the slope of the upper boundary of the PHC in the direction $\hat{\mathbf{q}}$ at $q = 0$ and can be computed by taking the maximum over all values of the angle ϕ' :

$$v_{\text{ph}} \equiv \max_{\phi'} \{\hat{\mathbf{q}} \cdot \mathbf{v}'_F\}. \quad (39)$$

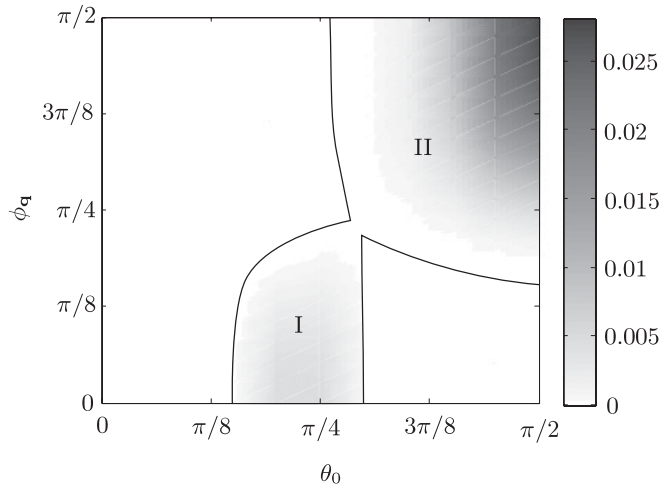


FIG. 3. Condition (38) for the propagation of zero sound is met in regions I and II, where the gray levels encode the difference $v_F^{(0)s} - v_{\text{ph}}$ for the largest eigenvalue s of Eq. (41).

For the purpose of solving Eq. (34) numerically, it is convenient to replace χ by another function

$$v(\phi) \equiv \frac{1}{\hat{\mathbf{p}} \cdot \mathbf{v}_F} \frac{\hat{\mathbf{q}} \cdot \mathbf{v}_F}{v_F^{(0)s} - \hat{\mathbf{q}} \cdot \mathbf{v}_F} \chi(\mathbf{p}_F). \quad (40)$$

Equation (34) then becomes

$$(v_F^{(0)s} - \hat{\mathbf{q}} \cdot \mathbf{v}_F)v(\phi) = \frac{1}{m} \frac{\hat{\mathbf{q}} \cdot \mathbf{v}_F}{\hat{\mathbf{p}} \cdot \mathbf{v}_F} \int \frac{d\phi'}{2\pi} F(\phi, \phi') p'_F v(\phi'). \quad (41)$$

One should note that Eq. (41)—in contrast to Eq. (34)—allows for nontrivial solutions even for $g = 0$ [or $F(\phi, \phi') = 0$]. These solutions with $s \in [-1, 1]$ take the form of a δ function in ϕ and correspond to single particle-hole pairs from the PHC.

We solve Eq. (41) numerically by discretizing it in the variable ϕ . Our results for $g = 1$ are shown in Figs. 3, 4, and 5. We see that the existence of zero sound and the value of the sound velocity strongly depend on the propagation direction, on the tilting angle, and on the strength of the interaction. There is no dissipationless zero-sound mode if the tilting angle is smaller than some critical value (see Fig. 3), because then all numerically calculated eigenvalues s are below or equal to the limiting velocity v_{ph} of the PHC, thus violating the propagation criterion (38). This is to some extent similar to the propagation of zero sound in the homogeneous three-dimensional case, which was studied in Refs. [27,36]: The authors of these works found that there is no undamped propagation of zero sound for a wide range of angles perpendicular to the direction of the polarization of dipoles. The reason for this at-first-glance counterintuitive statement is that the contribution of the direct interaction (which is repulsive in coordinate space) to $\tilde{\Gamma}_{\text{ph}}$ [or to $F(\phi, \phi')$] vanishes [see Eqs. (4) and (15)], such that the long-wavelength collective behavior of the dipolar gas is governed by the exchange interaction (the f function contains only the exchange contribution). [The collective modes without the exchange contribution were considered in Ref. [32] and, as a consequence, their result is determined by the momentum-independent term in the Fourier transform $V_0(\mathbf{q})$ of the dipole-

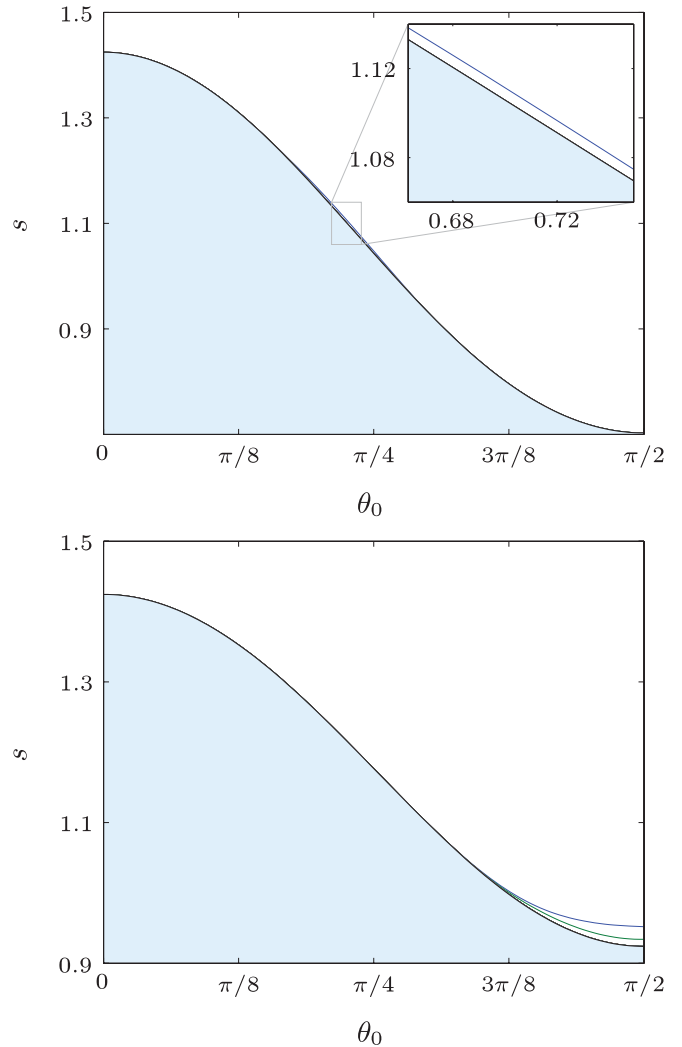


FIG. 4. (Color online) Speed of zero sound as a function of tilting angle θ_0 for $g = 1$ and $\phi_{\mathbf{q}} = 0, \pi/2$ (top, bottom). Note that, for $\phi_{\mathbf{q}} = \pi/2$ and θ_0 close to $\pi/2$, we have two zero-sound modes. The region that is forbidden by condition (38) is shaded.

dipole interaction which is omitted in our paper because it is canceled by the corresponding exchange contribution.]

Within our approach, we first find a zero-sound mode that satisfies the propagation criterion (38) (see Figs. 3 and 4, top) for $\theta_0 \approx 0.5$ and $\phi_{\mathbf{q}} = 0$ (i.e., in the direction of the projection of the dipoles \mathbf{d} on the xy plane). It is a longitudinal mode that is concentrated symmetrically around its propagation direction. As θ_0 increases, this mode can propagate in a broader range of angles up to $\phi_{\mathbf{q}} \approx \pi/4$ (region I in Fig. 3; see also Fig. 5, top). However, the corresponding sound velocity drops below the propagation boundary v_{ph} at $\theta_0 \approx 0.9$.

A different mode emerges from the continuum at $\theta_0 \approx \pi/4$ and $\phi_{\mathbf{q}} = \pi/2$ (region II in Fig. 3). This mode is antisymmetrically peaked around the direction of propagation. At even higher values of θ_0 we find more than one mode that satisfies (38) (see Figs. 4, bottom, and 5, bottom).

This behavior of collective modes remains the same when we increase the interaction strength to $g \approx 2$. In particular, the angles $\theta_0 \approx 0.5$ and $\theta_0 \approx \pi/4$ at which the symmetric and antisymmetric modes appear, respectively, are practically

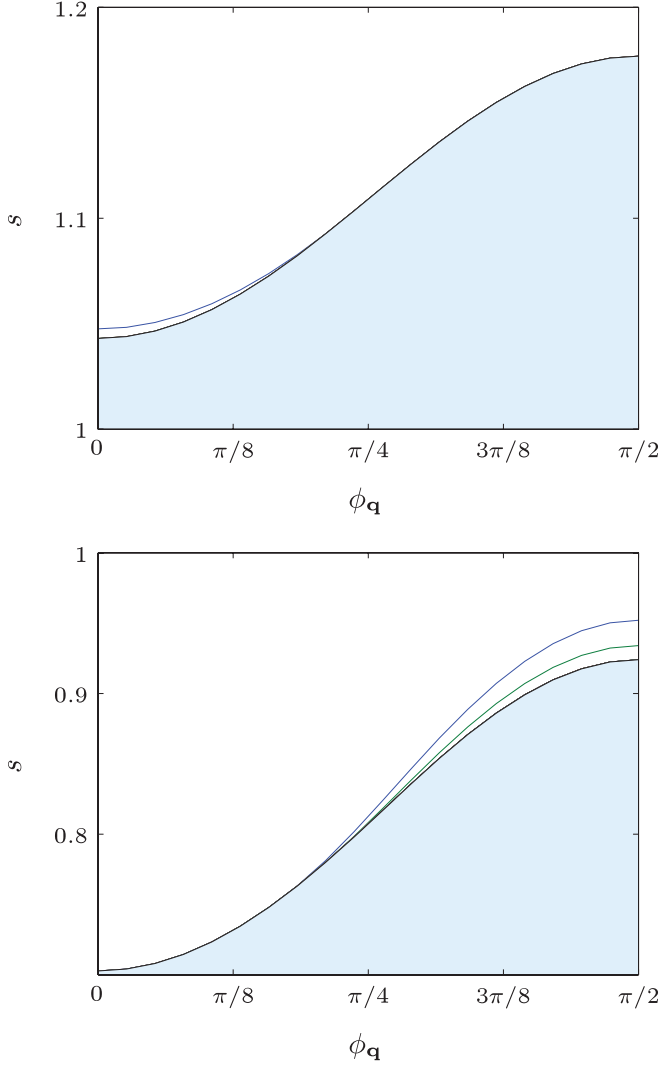


FIG. 5. (Color online) Speed of zero sound as a function of propagation direction $\phi_{\mathbf{q}}$ for $g = 1$ and $\theta_0 = \pi/4, \pi/2$ (top, bottom). Note that, for $\theta_0 = \pi/2$ and $\phi_{\mathbf{q}} \gtrsim \pi/4$, we have two zero-sound modes. As in Fig. 4, the region that is forbidden by condition (38) is shaded.

left unchanged. However, we find that the zero-sound modes become more “distinct,” (i.e., the curves in Figs. 4 and 5 are separated further from the PHC).

For $g < 1$ the above-mentioned peaking of modes around the forward direction is even more pronounced, and a high number of grid points is required to properly resolve these modes, making it impossible to go to very small values $g \ll 1$ with our numerical method. At $0.1 \leq g < 1$ it becomes increasingly difficult to make quantitative statements with regard to the region where the condition (38) met, since the quantity of interest $v_F^{(0)}s - v_{\text{ph}}$ is of the same order of magnitude as its estimated error. Qualitatively, however, we find the same behavior as described above for $1 \leq g$.

We also solve Eq. (32) with finite values of \mathbf{q} . Some results are shown in Fig. 6 and demonstrate that the dispersion $\omega = \omega(\mathbf{q})$ of the collective mode obtained in this manner agrees well with the linear approximation from Eq. (37). For details of the numerical procedure see Appendix B.

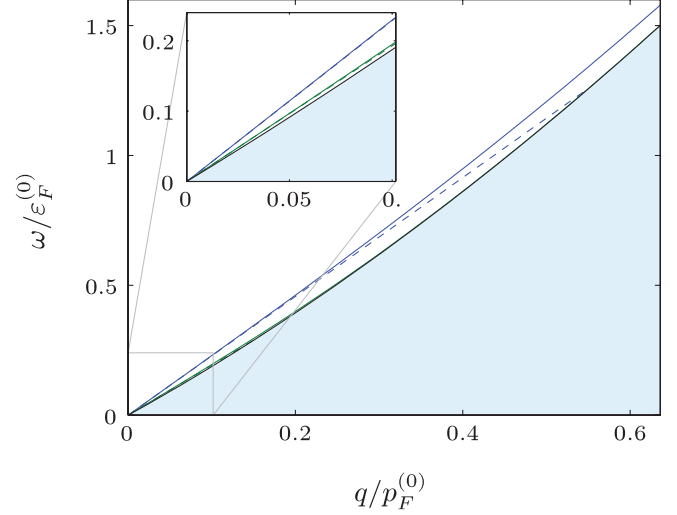


FIG. 6. (Color online) Collective mode dispersion relation for $\phi_{\mathbf{q}} = \pi/2$, $g = 2$, and $\theta_0 = \pi/2$ (solid lines). For comparison we also plot the linear approximation (37) as dashed lines. The inset shows that, at long wavelengths, we have two zero-sound modes.

VII. INSTABILITY OF SPATIALLY HOMOGENEOUS SYSTEM

Apart from poles on the real axis corresponding to excitation frequencies of collective modes, Eq. (32) can also have purely imaginary eigenvalues that indicate the existence of unstable modes growing exponentially with time. At the onset of an instability we have an eigenvalue $\omega = 0$ and, depending on the corresponding value of the wave vector $q = |\mathbf{q}|$, one has a long-wavelength instability ($q = 0$) resulting in local collapse of the system, or finite wavelength instability ($q \neq 0$) leading to breaking of translational invariance and formation of a periodic spatial structure (density waves).

A. Long-wavelength instability

Equation (41) shows that, if the instability occurs at $q \rightarrow 0$, the boundary of the instability region is determined by

$$-v(\phi) = \frac{1}{m v_F(\phi)} \int \frac{d\phi'}{2\pi} F(\phi, \phi') p_F(\phi') v(\phi'), \quad (42)$$

and, hence, the instabilities could occur only in the regime of intermediate coupling, $g \approx 1$.²

Equation (42) is equivalent to the Pomeranchuk criterion [37] on the Landau f function to ensure stability of a three-dimensional isotropic Fermi liquid [42]. We briefly review this method and its generalization to the two-dimensional anisotropic case.

²In Eq. (41), on the other hand, the prefactor on the LHS can become arbitrarily small when $v_F^{(0)}s$ gets close to the limiting velocity of the PHC v_{ph} , and there is no generic restriction on the size of g for this equation to have a solution (i.e., for zero sound to occur).

In the framework of Fermi liquid theory [43], the change of the quasiparticle momentum distribution function $\delta n(\mathbf{p})$ results in the change in the energy density

$$\delta\mathcal{E} = \int \frac{d\mathbf{p}}{(2\pi)^2} [\varepsilon(\mathbf{p}) - \mu] \delta n(\mathbf{p}) + \frac{1}{2} \int \frac{d\mathbf{p}}{(2\pi)^2} \frac{d\mathbf{p}'}{(2\pi)^2} f(\mathbf{p}, \mathbf{p}') \delta n(\mathbf{p}) \delta n(\mathbf{p}'), \quad (43)$$

where $f(\mathbf{p}, \mathbf{p}')$ is the quasiparticle interaction function (f function). A slight distortion $\delta p_F(\phi)$ [note that in this section, δp_F is used another way than in Secs. V and VIII] of the anisotropic Fermi surface $p_F(\phi)$ corresponds to a distribution function of the form

$$\delta n(\mathbf{p}) = \theta[p_F(\phi) + \delta p_F(\phi) - p] - \theta[p_F(\phi) - p]. \quad (44)$$

Following Pomeranchuk [37], we expand $\delta\mathcal{E}$ in δp_F : The first-order term vanishes due to the fact that the energy of the Fermi liquid, considered as a functional of δp_F , is stationary at $\delta p_F \equiv 0$. To second order we have

$$\delta\mathcal{E} = \frac{1}{4\pi m^2} \left[\int \frac{d\phi}{2\pi} p_F(\phi) m v_F(\phi) \delta p_F(\phi)^2 + \int \frac{d\phi}{2\pi} \frac{d\phi'}{2\pi} p_F(\phi) p_F(\phi') F(\phi, \phi') \delta p_F(\phi) \delta p_F(\phi') \right], \quad (45)$$

where $F(\phi, \phi') \equiv v f(\mathbf{p}_F, \mathbf{p}'_F)$ is the dimensionless quasiparticle interaction function already introduced in Eq. (35).

Thermodynamic stability requires that the variation of the energy density be positive for any choice of δp_F . This condition can conveniently be formulated by expanding the ϕ -dependent functions in Fourier (double) series $f(\phi) = \sum_m f_m e^{im\phi}$, where $f(\phi)$ is one of $p_F(\phi)$, $\delta p_F(\phi)$, and $v_F(\phi)$, and $F(\phi, \phi') = \sum_{m, m'} F_{m, m'} e^{i(m\phi - m'\phi')}$, where the coefficients $F_{m, m'}$ take the role of generalized Landau parameters.

Replacing the functions in the expression for $\delta\mathcal{E}$ by their respective Fourier series expansions, we obtain

$$\delta\mathcal{E} = \frac{1}{4\pi m} \sum_{m, m'} \delta p_m^* M_{m, m'} \delta p_{m'}, \quad (46)$$

where the entries of the (self-adjoint) matrix M are

$$M_{m, m'} \equiv m \sum_k p_{m-k} v_{m'-k}^* + \sum_{k, l} p_{m-k} F_{k, l} p_{m'-l}^*. \quad (47)$$

A 2D anisotropic Fermi liquid, therefore, is thermodynamically stable if and only if all eigenvalues of M are positive. In the case of isotropic interactions, we have $M_{m, m'} = \delta_{m, m'} M_m$, and the requirement of stability reduces to the usual Pomeranchuk criterion (i.e., $M_m > 0$ for all $m \in \mathbb{Z}$).

At the onset of an instability there exists an eigenvalue that is equal to zero; that is, there is a nontrivial solution to the equation

$$\sum_{m'} M_{m, m'} \delta p_{m'} = 0. \quad (48)$$

In terms of the original Fermi surface deformation δp_F this equation reads

$$\delta p_F(\phi) + \frac{1}{m v_F(\phi)} \int \frac{d\phi'}{2\pi} F(\phi, \phi') p_F(\phi') \delta p_F(\phi') = 0, \quad (49)$$

and if we identify δp_F with v we are led back to Eq. (42). The line in the $g\theta_0$ plane on which this equation has a nontrivial solution determines the boundary to the red-shaded region that is labeled LWI in Fig. 9. In the interior of this region, which agrees very well with the ‘‘collapse’’ regions in the phase diagrams of Refs. [12,28], there exists an unstable long-wavelength mode.

B. Finite wavelength (density wave) instability

An instability at finite momentum $q = |\mathbf{q}|$ drives the system toward a state with stationary fluctuations in the density. We address this problem numerically (see details in Appendix B). After solving Eq. (B4) [which is equivalent to Eq. (32)] numerically for $q \neq 0$, we find such an instability with $\phi_{\mathbf{q}} = \pi/2$ (i.e., \mathbf{q} is perpendicular to the x axis—the projection of the dipoles \mathbf{d} on the xy plane) and $q \approx 2p_F(\phi_{\mathbf{q}})$ in the blue-shaded region in Fig. 9. As can be seen from Eq. (B4), the eigenvectors $v_{\mathbf{q}}(\mathbf{p}) \propto \langle a_{\mathbf{p}+\mathbf{q}/2}^\dagger a_{\mathbf{p}-\mathbf{q}/2} \rangle$ which correspond to an instability, signal the formation of a density modulation with momentum \mathbf{q} (density wave). On the other hand, the dependence of $v_{\mathbf{q}}(\mathbf{p})$ on \mathbf{p} excludes a description of the transition in terms of a simple local order parameter ($\langle \hat{\psi}^\dagger(\boldsymbol{\rho}) \hat{\psi}(\boldsymbol{\rho}) \rangle = \rho(\mathbf{q}) \cos(\mathbf{q} \cdot \boldsymbol{\rho})$). In the isotropic case with $\theta_0 = 0$, where there is no preferred direction and the system is invariant with respect to rotations around the z axis, the instability occurs at $g \approx 1.45$ and is independent of the angle $\phi_{\mathbf{q}}$. This result is in agreement with the value $g \approx 1.42$ which was found by the authors of Ref. [40]. Previous studies within the RPA [28,33] predicted a considerably smaller value of $g = 0.5$, which is due to the fact that the RPA overestimates the effects of the interparticle interaction because it neglects the exchange contribution.

Note that, for $\tilde{\Gamma}_{\text{ph}}(\mathbf{p} - \mathbf{p}', \mathbf{q}) = V_0(\mathbf{q})$ (i.e., when only the direct interaction is taken into account and the exchange one is neglected), $\chi(\mathbf{p}; \mathbf{q})$ is \mathbf{p} independent and Eq. (32) reduces to

$$1 - V_0(\mathbf{q}) \Pi^{(0)}(\omega, \mathbf{q}) = 0, \quad (50)$$

where

$$\begin{aligned} \Pi^{(0)}(\omega, \mathbf{q}) &= \text{Diagram with two vertices labeled } p' \text{ and } p'+q \text{ connected by two arcs.} \\ &= \int \frac{d\mathbf{p}'}{(2\pi)^2} \frac{n(\mathbf{p}') - n(\mathbf{p}' + \mathbf{q})}{\omega + \varepsilon(\mathbf{p}') - \varepsilon(\mathbf{p}' + \mathbf{q}) + i0 \text{sgn}(\omega)} \end{aligned} \quad (51)$$

is the 2D polarization operator. Equation (50) is used to study long-wavelength ($q \rightarrow 0$) plasmon oscillations in electrically charged systems (see, for example, [44]). Although keeping only the direct interaction in the long-wavelength limit is legitimate for Coulomb systems (because of the divergence of the Coulomb interaction for a small transferred momentum $q \rightarrow 0$, while the exchange one is finite due to a nonzero momentum transfer, $|\mathbf{p} - \mathbf{p}'| \sim p_F$), this approximation gives physically incorrect results in a Fermi system with a finite Fourier transform of the interparticle interaction for small momentum transfer (like in the considered case of a dipolar monolayer). In this case, the direct and the exchange contributions are of the same order and keeping only the former yields unphysical results. For a short range interparticle interaction

(with a momentum-independent Fourier transform), the two contributions have to cancel each other, resulting in no interaction effects in a single-component Fermi gas with a short-range interaction. Similar considerations also apply to the analysis of instabilities in a dipolar system on the basis of Eq. (32): Keeping the exchange contribution in this equation is essential in order to obtain correct results consistent with fermionic statistics of particles.

In order to take the exchange contribution into account in Eq. (50), one can modify the polarization propagator by including an entire interaction ladder in the polarization bubble (this is equivalent to including the exchange interaction in $\tilde{\Gamma}_{\text{ph}}$; see also Ref. [40]:

$$\Pi(\omega, \mathbf{q}) = \Pi^{(0)}(\omega, \mathbf{q}) + \text{diagram} + \text{diagram} + \dots \quad (52)$$

(Another possibility is to include a local field correction [38,39], similar to the consideration of the density-density response in Coulomb systems.) With the modified polarization operator $\Pi(\omega, \mathbf{q})$, the density-density correlation function (in frequency-momentum space) reads

$$\chi(\omega, \mathbf{q}) = \frac{\Pi(\omega, \mathbf{q})}{1 - V_0(\mathbf{q})\Pi(\omega, \mathbf{q})}, \quad (53)$$

and Eq. (50) with $\Pi^{(0)}(\omega, \mathbf{q})$ replaced with $\Pi(\omega, \mathbf{q})$ corresponds to the instability in the density-density correlation function. Note that, when $V_0(\mathbf{q})$ is replaced by a momentum-independent constant V_0 (that corresponds to a short-range interaction), the modified polarization operator is $\Pi(\omega, \mathbf{q}) = \Pi^{(0)}(\omega, \mathbf{q})[1 + V_0\Pi^{(0)}(\omega, \mathbf{q})]^{-1}$, and the density-density correlation function reduces to the polarization operator of a noninteracting gas, $\chi(\omega, \mathbf{q}) = \Pi^{(0)}(\omega, \mathbf{q})$, as it should be in a single-component Fermi gas.

VIII. SUPERFLUID TRANSITION

We now discuss the superfluid instability in a dipolar monolayer at finite temperature $T > 0$. As we will show, this instability is sensitive to the details of two-particle scattering and, as was already pointed out at the end of Sec. II, this requires us to take into account contributions of short distances $\lesssim l_0$ and high energies $\gtrsim \omega_0$. Therefore, in the following we will not limit ourselves *a priori* to configurations with all particles residing in the ground state of the trapping potential but rather allow for virtual transitions to arbitrarily highly excited states.

A. Gap equation

The superfluid transition is characterized by the order parameter (gap)

$$\Delta_{n,n'}(\mathbf{p}) = \int d\rho dz dz' e^{-i\mathbf{p}\cdot\rho} \phi_n(z) \phi_{n'}(z') \times V_d(\rho, z) \langle \hat{\psi}(\rho, z) \hat{\psi}(\mathbf{0}, z') \rangle, \quad (54)$$

which attains nonzero values for temperatures below the critical temperature T_c . Note that in two dimensions at finite temperatures long-range order is actually destroyed by phase fluctuations and the mean-field order parameter is zero. The superfluid density, however, remains finite and the transition

to the superfluid phase follows the Berezinskii-Kosterlitz-Thouless scenario [45–47] and occurs at a temperature $T_c^{(\text{BKT})}$. Nevertheless, we may consider the mean-field critical temperature T_c as a reliable estimate of the value of $T_c^{(\text{BKT})}$, as the difference between the two of them is small in the weak-coupling regime [48].

For $0 < T_c - T \ll T_c$, the order parameter is a solution to the homogeneous Bethe-Salpeter equation in the Cooper channel or linearized gap equation [42],

$$\text{diagram} = \text{diagram}, \quad (55)$$

where $\tilde{\Gamma}_{\text{pp}}$ is the particle-particle irreducible vertex, which is the sum of connected diagrams with two incoming and two outgoing lines which cannot be divided into two parts by cutting two fermion lines of the same direction. In the above diagrams, thick lines correspond to interacting Matsubara Green's functions in the presence of the transverse trapping potential (see, for example, Ref. [42]),

$$\mathcal{G}_{n,n'}(\omega_s, \mathbf{p}) = - \int d\tau d\rho dz dz' e^{i(\omega_s \tau - \mathbf{p}\cdot\rho)} \times \phi_n(z) \phi_{n'}(z') \langle T_\tau \{ \hat{\psi}(\tau, \rho, z) \hat{\psi}(0, \mathbf{0}, z') \} \rangle, \quad (56)$$

where $\hat{\psi}(\tau, \rho, z)$ and $\hat{\psi}(\tau, \rho, z)$ are Matsubara field operators, T_τ orders these operators chronologically according to their time argument $\tau \in [0, 1/T]$, and fermionic Matsubara frequencies are $\omega_s = (2s + 1)\pi T$ for integer s . Denoting the matrix elements of the particle-particle irreducible vertex by $\tilde{\Gamma}_{n_1, n_2, n_3, n_4}$ [cf. Eq. (5)], the analytical expression corresponding to (55) reads³

$$\Delta_{n_1, n_2}(\mathbf{p}) = -\frac{1}{2} \int \sum_{n_3, n_4, n_5, n_6} \tilde{\Gamma}_{n_1, n_2, n_3, n_4}(\mathbf{p}, \mathbf{p}') \times T \sum_s \mathcal{G}_{n_3, n_5}(\omega_s, \mathbf{p}') \mathcal{G}_{n_4, n_6}(-\omega_s, -\mathbf{p}') \Delta_{n_5, n_6}(\mathbf{p}'). \quad (57)$$

Following Gor'kov and Melik-Barkhudarov [41] we expand the RHS of Eq. (55) to second order in the dipole-dipole interaction V_d ,

$$\text{diagram} = \text{diagram} + \text{diagram} + \text{diagram}. \quad (58)$$

³Actually, the quantity $\tilde{\Gamma}_{\text{pp}}$ depends on the Matsubara frequencies of incoming and outgoing particles, ω_s and $\omega_{s'}$, respectively. This dependence, however, appears first in the second-order correction δV_d to the bare dipole-dipole interaction, where we need only take into account the logarithmic contribution (see discussion below and Ref. [41]), which is obtained for $\omega_s = \omega_{s'} = 0$. Therefore, for our purposes it is sufficient to consider $\tilde{\Gamma}_{\text{pp}}$ with frequency components set to zero.

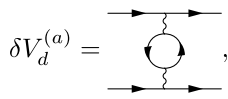
Here, with each thin line is associated a noninteracting Matsubara Green's function, which is diagonal in the HO quantum numbers and can be written in the form [43]

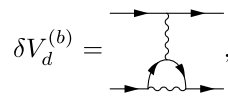
$$\mathcal{G}_n^{(0)}(\omega_s, \mathbf{p}) = \frac{1}{i\omega_s - \xi(\mathbf{p}) - \omega_0 n}, \quad (59)$$

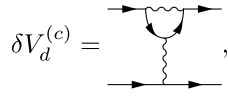
where $\xi(\mathbf{p}) = p^2/(2m) - \mu$. The chemical potential is $\mu = \varepsilon_F^{(0)} + \delta\mu$, where the first-order correction $\delta\mu$ is given by Eq. (28) [due to the exponential smallness of T_c (see discussion below) we may use the zero temperature value of μ]. Thus, strictly speaking (58) contains terms of infinite order. It is, however, convenient to perform the expansion in $\delta\mu$ at a later stage.

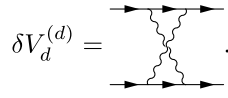
Of the diagrams on the RHS of Eq. (58) the first one gives the leading (first-order) contribution, the others correspond to (second-order) corrections: The second diagram is obtained from the first one by inserting an exchange self-energy part (note that the direct term is absent for the dipole-dipole interaction, see Sec. V). This diagram comes with a factor of 2, as we could have equally well inserted the self-energy part in the upper particle line. The quantity δV_d represents second-order corrections to the bare dipole-dipole interaction and is given by the set of diagrams

$$\delta V_d = \delta V_d^{(a)} + \delta V_d^{(b)} + \delta V_d^{(c)} + \delta V_d^{(d)}, \quad (60)$$


 $\delta V_d^{(a)} =$


 $\delta V_d^{(b)} =$


 $\delta V_d^{(c)} =$


 $\delta V_d^{(d)} =$

These diagrams describe processes in which one of the incoming particles polarizes the medium by exciting a virtual particle-hole pair. In diagram (a) the particle and hole annihilate each other while interacting with the other incoming particle, whereas in (b), (c), and (d) the hole is annihilated by one of the incoming particles: by the second incoming particle in (b) and (c), and by the very same particle that created the particle-hole pair in the first place in (d).

We denote the matrix elements of δV_d by $\delta V_{n_1, n_2, n_3, n_4}$ [cf. Eq. (5)]. Then, the diagrams on the RHS of Eq. (58) can be written as

$$\begin{aligned} \text{Diagram 1} &= - \int \sum_{n_3, n_4} V_{n_1, n_2, n_3, n_4}(\mathbf{p} - \mathbf{p}') T \sum_s \mathcal{G}_{n_3}^{(0)}(\omega_s, \mathbf{p}') \\ &\times \mathcal{G}_{n_4}^{(0)}(-\omega_s, -\mathbf{p}') \Delta_{n_3, n_4}(\mathbf{p}') \frac{d\mathbf{p}'}{(2\pi)^2}, \end{aligned} \quad (62)$$

$$\begin{aligned} \text{Diagram 2} &= -2 \int \sum_{n_3, n_4, n_5} V_{n_1, n_2, n_3, n_4}(\mathbf{p} - \mathbf{p}') T \\ &\times \sum_s \mathcal{G}_{n_3}^{(0)}(\omega_s, \mathbf{p}') \mathcal{G}_{n_4}^{(0)}(-\omega_s, -\mathbf{p}') \\ &\times \Sigma_{n_4, n_5}^{(1)}(\mathbf{p}') \mathcal{G}_{n_5}^{(0)}(-\omega_s, -\mathbf{p}') \Delta_{n_3, n_5}(\mathbf{p}') \frac{d\mathbf{p}'}{(2\pi)^2}, \end{aligned} \quad (63)$$

$$\Sigma_{n_1, n_2}^{(1)}(\mathbf{p}) = \sum_{n_3} \int T \sum_s V_{n_1, n_2, n_3, n_3}(\mathbf{p} - \mathbf{p}') \mathcal{G}_{n_3}^{(0)}(\omega_s, \mathbf{p}') \frac{d\mathbf{p}'}{(2\pi)^2}, \quad (64)$$

$$\begin{aligned} \text{Diagram 3} &= - \int \sum_{n_3, n_4} \delta V_{n_1, n_2, n_3, n_4}(\mathbf{p} - \mathbf{p}') \\ &\times T \sum_s \mathcal{G}_{n_3}^{(0)}(\omega_s, \mathbf{p}') \\ &\times \mathcal{G}_{n_4}^{(0)}(-\omega_s, -\mathbf{p}') \Delta_{n_3, n_4}(\mathbf{p}') \frac{d\mathbf{p}'}{(2\pi)^2}, \end{aligned} \quad (65)$$

[Analytical expressions corresponding to the diagrams (62) that make up δV_d will be given below; see Eqs. (114) to (117).]

The sum over Matsubara frequencies in Eqs. (62) and (65) can be evaluated by rewriting it as a contour integral [49] and gives

$$\begin{aligned} \mathcal{K}_{n, n'}(\mathbf{p}) &\equiv T \sum_s \mathcal{G}_n^{(0)}(\omega_s, \mathbf{p}) \mathcal{G}_{n'}^{(0)}(-\omega_s, -\mathbf{p}) \\ &= \frac{\tanh\left[\frac{\xi(\mathbf{p}) + \omega_0 n}{2T}\right] + \tanh\left[\frac{\xi(\mathbf{p}) + \omega_0 n'}{2T}\right]}{2[2\xi(\mathbf{p}) + \omega_0(n + n')]}. \end{aligned} \quad (66)$$

For $n, n' \neq 0$ and in the quasi-2D limit $\omega_0 \gg \mu$ the denominator in Eq. (66) is positive for all values of \mathbf{p} and the arguments of the hyperbolic tangent functions are as large as ω_0/T . Then, to within exponential accuracy in this ratio, we may approximate

$$\mathcal{K}_{n, n'}(\mathbf{p}) \approx \frac{1}{2\xi(\mathbf{p}) + \omega_0(n + n')}. \quad (67)$$

If only one of n, n' is different from zero, the denominator in Eq. (66) is still positive for all \mathbf{p} . For concreteness, let us assume that $n \neq 0$ and $n' = 0$. Then we set

$$\mathcal{K}_{n, 0}(\mathbf{p}) \approx \frac{1 + \text{sgn}[\xi(\mathbf{p})]}{2[2\xi(\mathbf{p}) + \omega_0 n]}, \quad (68)$$

thereby omitting a subdominant contribution to the integral (62) from a narrow shell in momentum space $|\xi(\mathbf{p}')| \lesssim T$ where $\tanh[\xi(\mathbf{p}')/(2T)]$ deviates appreciably from $\text{sgn}[\xi(\mathbf{p})]$.

Finally, in the case $n = n' = 0$, (66) reduces to

$$\mathcal{K}[\xi(\mathbf{p})] \equiv \mathcal{K}_{0, 0}(\mathbf{p}) = \tanh[\xi(\mathbf{p})/(2T)]/[2\xi(\mathbf{p})]. \quad (69)$$

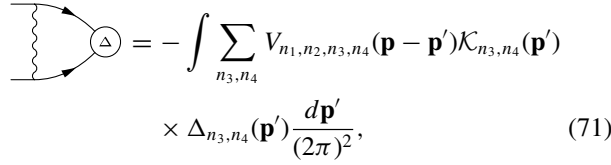
In this expression, the denominator vanishes on the Fermi surface [i.e., for $\xi(\mathbf{p}) = 0$]. This leads in the limit $T/\mu \ll 1$ to a logarithmically divergent integral $\propto \ln(T/\mu)$ on the RHS of Eq. (62) when $n_3 = n_4 = 0$.

We have to keep contributions of second order in the gap equation only if they are multiplied by such a large logarithm [41]. In Eq. (63) this is the term with $n_3 = n_4 = n_5 = 0$ which contains the factor

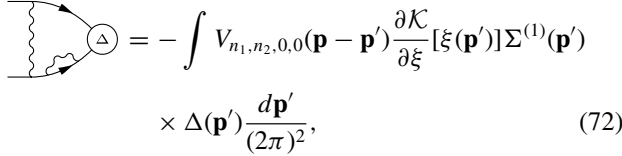
$$T \sum_s \mathcal{G}_0^{(0)}(\omega_s, \mathbf{p}) \mathcal{G}_0^{(0)}(-\omega_s, -\mathbf{p})^2 = \frac{1}{2} \frac{\partial \mathcal{K}}{\partial \xi}[\xi(\mathbf{p})], \quad (70)$$

and in Eq. (65) it is the summand with $n_3 = n_4 = 0$. These simplifications allow us to rewrite Eqs. (62), (63), and (65) as

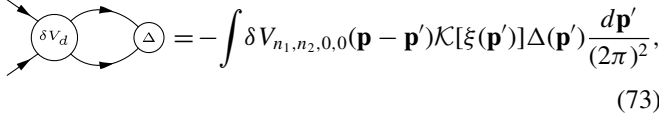
$$[\Delta(\mathbf{p}) \equiv \Delta_{0,0}(\mathbf{p}), \Sigma^{(1)}(\mathbf{p}) \equiv \Sigma_{0,0}^{(1)}(\mathbf{p})]$$



$$\Delta(\mathbf{p}) = - \int \sum_{n_3, n_4} V_{n_1, n_2, n_3, n_4}(\mathbf{p} - \mathbf{p}') \mathcal{K}_{n_3, n_4}(\mathbf{p}') \times \Delta_{n_3, n_4}(\mathbf{p}') \frac{d\mathbf{p}'}{(2\pi)^2}, \quad (71)$$



$$\Delta(\mathbf{p}) = - \int V_{n_1, n_2, 0, 0}(\mathbf{p} - \mathbf{p}') \frac{\partial \mathcal{K}}{\partial \xi}[\xi(\mathbf{p}')] \Sigma^{(1)}(\mathbf{p}') \times \Delta(\mathbf{p}') \frac{d\mathbf{p}'}{(2\pi)^2}, \quad (72)$$

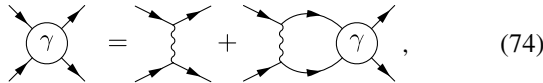


$$\Delta(\mathbf{p}) = - \int \delta V_{n_1, n_2, 0, 0}(\mathbf{p} - \mathbf{p}') \mathcal{K}[\xi(\mathbf{p}')] \Delta(\mathbf{p}') \frac{d\mathbf{p}'}{(2\pi)^2}, \quad (73)$$

where $\mathcal{K}_{n, n'}(\mathbf{p})$ and $\mathcal{K}[\xi(\mathbf{p})]$ are given by Eqs. (67)–(69), respectively.

B. Renormalization

Apart from the region $p' \approx p_F^{(0)}$, major contributions to the integral in Eq. (71) that are actually divergent and need to be cutoff come also from high momenta. This region, however, is related to short interparticle distances, at which the presence of other particles becomes irrelevant and the dynamics corresponds to the scattering of just two particles in vacuum. In the gap equation this two-body physics can be taken into account by expressing the bare dipole-dipole interaction in terms of the vertex function $\gamma_{n_1, n_2, n_3, n_4}(E, \mathbf{p}, \mathbf{p}')$ for two particles in vacuum with a total energy E in their center-of-mass reference frame. The Lippmann-Schwinger equation for the vertex function [50] can be represented diagrammatically as



$$\gamma(\mathbf{p}) = \text{diagram 1} + \text{diagram 2}, \quad (74)$$

where particle lines correspond to noninteracting zero-temperature Green's functions,

$$G_n^{(0)}(\omega, \mathbf{p}) = \frac{1}{\omega - p^2/(2m) - \omega_0 n + i0}. \quad (75)$$

For our purposes it is convenient to rearrange the order of terms on the RHS of Eq. (74) as

$$\begin{aligned} \gamma_{n_1, n_2, n_3, n_4}(E, \mathbf{p}, \mathbf{p}') &= V_{n_1, n_2, n_3, n_4}(\mathbf{p} - \mathbf{p}') \\ &\quad - \int \sum_{n_5, n_6} \gamma_{n_1, n_2, n_5, n_6}(E, \mathbf{p}, \mathbf{q}) K_{n_5, n_6}(E, \mathbf{q}) \\ &\quad \times V_{n_5, n_6, n_3, n_4}(\mathbf{q} - \mathbf{p}') \frac{d\mathbf{q}}{(2\pi)^2}, \end{aligned} \quad (76)$$

where the kernel $K_{n, n'}(E, \mathbf{p})$ is given by the integral over frequencies,

$$\begin{aligned} K_{n, n'}(E, \mathbf{p}) &= -i \int G_n^{(0)}(E + \omega, \mathbf{p}) G_{n'}^{(0)}(-\omega, -\mathbf{p}) \frac{d\omega}{2\pi} \\ &= \frac{1}{p^2/m + \omega_0(n + n') - E - i0}. \end{aligned} \quad (77)$$

Below we shall choose $E = 2\mu$, in which case (77) becomes

$$K_{n, n'}(2\mu, \mathbf{p}) = \frac{1}{2\xi(\mathbf{p}) + \omega_0(n + n') - i0}. \quad (78)$$

In order to carry out the renormalization in the most transparent way we rewrite the gap equation (58) and the Lippmann-Schwinger equation (76) schematically as

$$\Delta = -V\mathcal{K}\Delta - V\partial\mathcal{K}\Sigma^{(1)}\Delta - \delta V\mathcal{K}\Delta, \quad (79)$$

$$\gamma = (1 - \gamma\mathcal{K})V. \quad (80)$$

We “multiply” (79) from the left by $1 - \gamma\mathcal{K}$. Then, using (80) and neglecting terms that contain $\gamma V\Sigma^{(1)}$ and $\gamma\delta V$ (those are contributions of third order), we obtain the renormalized gap equation

$$\Delta = -\gamma(\mathcal{K} - K)\Delta - V\partial\mathcal{K}\Sigma^{(1)}\Delta - \delta V\mathcal{K}\Delta. \quad (81)$$

In the renormalized gap equation, the kernel $\mathcal{K}_{n, n'}(\mathbf{p})$ has been replaced by the difference $\mathcal{K}_{n, n'}(\mathbf{p}) - K_{n, n'}(2\mu, \mathbf{p})$ which converges rapidly at high energies (i.e., high p and large HO quantum numbers n and n'). To wit, for $n = n' = 0$, from Eq. (69) we have

$$\mathcal{K}_{0,0}(\mathbf{p}) - K_{0,0}(2\mu, \mathbf{p}) = \frac{\tanh[\xi(\mathbf{p})/(2T)]}{2\xi(\mathbf{p})} - \frac{1}{2\xi(\mathbf{p}) - i0}, \quad (82)$$

which decays as $1/p^4$ for $p \rightarrow \infty$, ensuring the convergence of the integral over the momentum in the first term on the RHS of Eq. (81) without the need to introduce an additional cutoff. If either n or n' is nonzero, with Eq. (68) we have

$$\mathcal{K}_{n,0}(\mathbf{p}) - K_{n,0}(2\mu, \mathbf{p}) \approx -\frac{\theta[-\xi(\mathbf{p})]}{2\xi(\mathbf{p}) + \omega_0 n}. \quad (83)$$

In Eq. (81) this results in a term that is $O(\mu/\omega_0)$ and may safely be neglected. Finally, for both n and n' not equal to zero, with the aid of Eq. (67) we obtain

$$\mathcal{K}_{n, n'}(\mathbf{p}) - K_{n, n'}(2\mu, \mathbf{p}) \approx 0. \quad (84)$$

Therefore, in the propagators in Eq. (81), we may restrict ourselves to the HO quantum numbers being equal to zero, and we obtain a closed equation for $\Delta(\mathbf{p}) \equiv \Delta_{0,0}(\mathbf{p})$. With $\gamma(E, \mathbf{p}, \mathbf{p}') \equiv \gamma_{0,0,0,0}(E, \mathbf{p}, \mathbf{p}')$ and $\delta V_0(\mathbf{p}, \mathbf{p}') \equiv \delta V_{0,0,0,0}(E, \mathbf{p}, \mathbf{p}')$, we have

$$\begin{aligned} \Delta(\mathbf{p}) &= - \int \left\{ \gamma(2\mu, \mathbf{p}, \mathbf{p}') \left[\mathcal{K}(\xi') - \frac{1}{2\xi' - i0} \right] \right. \\ &\quad + V_0(\mathbf{p} - \mathbf{p}') \Sigma^{(1)}(\mathbf{p}') \frac{\partial \mathcal{K}}{\partial \xi}(\xi') \\ &\quad \left. + \delta V_0(\mathbf{p}, \mathbf{p}') \mathcal{K}(\xi') \right\} \Delta(\mathbf{p}') \frac{d\mathbf{p}'}{(2\pi)^2}. \end{aligned} \quad (85)$$

To proceed, we need to find an expression for the vertex function $\gamma(2\mu, \mathbf{p}, \mathbf{p}')$.

C. Two-body vertex function

Iteration of Eq. (76) yields the familiar Born series. Terminating this series at second order we obtain for $n_1 = n_2 = n_3 = n_4 = 0$:

$$\gamma(2\mu, \mathbf{p}, \mathbf{p}') = \gamma^{(1)}(2\mu, \mathbf{p}, \mathbf{p}') + \gamma^{(2)}(2\mu, \mathbf{p}, \mathbf{p}'), \quad (86)$$

where the first-order contribution is just the Fourier transform of the effective 2D dipole-dipole interaction

$$\gamma^{(1)}(2\mu, \mathbf{p}, \mathbf{p}') = V_0(\mathbf{p} - \mathbf{p}'), \quad (87)$$

and to second order we have to include virtual excitations to higher HO levels at the intermediate stage of the interaction [i.e., we have to take the sum over $n, n' \in \mathbb{N}_0$ (non-negative integers)],

$$\gamma^{(2)}(2\mu, \mathbf{p}, \mathbf{p}') = - \sum_{n, n'} \int \frac{V_{n, n'}(\mathbf{p} - \mathbf{q}) V_{n, n'}(\mathbf{q} - \mathbf{p}')}{2\xi(\mathbf{q}) + \omega_0(n + n') - i0} \frac{d\mathbf{q}}{(2\pi)^2}, \quad (88)$$

where $V_{n, n'}(\mathbf{p}) \equiv V_{0, 0, n, n'}(\mathbf{p}) = V_{n, n', 0, 0}(\mathbf{p})$. Explicit expressions for these matrix elements—calculated exactly as well as in WKB approximation—are given in Appendix C.

D. Asymptotic gap equation

In the integral on the RHS of Eq. (85), we perform the change of variables

$$\mathbf{p} = \mathbf{p}(\xi, \phi) = \sqrt{2m(\xi + \varepsilon_F^{(0)})} \hat{\mathbf{p}}, \quad (89)$$

where $\hat{\mathbf{p}} = (\cos \phi, \sin \phi)$. We shall simplify the notation by introducing the functions (avoiding to explicitly state the dependence on ξ and ϕ)

$$\begin{aligned} f(\xi') &\equiv \int \frac{d\phi'}{2\pi} v[\gamma(2\mu, \mathbf{p}, \mathbf{p}') + \delta V_0(\mathbf{p}, \mathbf{p}')] \Delta(\xi', \phi'), \\ g(\xi') &\equiv \int \frac{d\phi'}{2\pi} v\gamma(2\mu, \mathbf{p}, \mathbf{p}') \Delta(\xi', \phi'), \\ h(\xi') &\equiv \int \frac{d\phi'}{2\pi} vV_0(\mathbf{p} - \mathbf{p}') \Sigma^{(1)}(\mathbf{p}') \Delta(\xi', \phi'). \end{aligned} \quad (90)$$

With these definitions we can write the gap equation (85) in a very compact form:

$$\Delta = - \int_{-\mu}^{\infty} d\xi' \left[\mathcal{K}(\xi') f(\xi') + \mathcal{K}'(\xi') h(\xi') - \frac{g(\xi')}{2\xi' - i0} \right]. \quad (91)$$

In the second term on the RHS we integrate by parts. Neglecting boundary terms that are of second order in the

dipole-dipole interaction and are not multiplied by the large logarithm $\ln(T/\mu)$, we obtain

$$\Delta = - \int_{-\mu}^{\infty} d\xi' \left\{ \mathcal{K}(\xi') [f(\xi') - h'(\xi')] - \frac{g(\xi')}{2\xi' - i0} \right\}. \quad (92)$$

In the limit $T/\mu \ll 1$ the function $\mathcal{K}(\xi')$ behaves as $\mathcal{K}(\xi') \sim 1/(2|\xi'|)$. Hence, the main contribution to the integral over the first term on the RHS of Eq. (92), as has already been discussed above, is logarithmic in T/μ and comes from states near the Fermi surface where $|\xi'| \ll \mu$. In order to single out this contribution we divide the integral over ξ' into two parts: (a) the integration of $\mathcal{K}(\xi') [f(\xi') - h'(\xi')]$ from $-\mu$ to μ , and (b) the sum of the integrals of $\mathcal{K}(\xi') [f(\xi') - h'(\xi') - f(0) + h'(0)]$ over $[-\mu, \mu]$ and of $\mathcal{K}(\xi') [f(\xi') - h'(\xi')]$ over (μ, ∞) . In part (a) we use the asymptotic formula

$$\int_{-\mu}^{\mu} d\xi' \frac{\tanh[\xi'/(2T)]}{2\xi'} \sim \ln\left(\frac{2e^\gamma \mu}{\pi T}\right), \quad (93)$$

where $\gamma \approx 0.5772$ is the Euler constant, and in part (b) we once more [cf. Eq. (68)] replace $\tanh[\xi'/(2T)]$ by $\text{sgn}(\xi')$ and integrate by parts. Consequently, keeping terms that are $O(g^2)$ only when they are multiplied by the large logarithm $\ln(T/\mu)$, Eq. (92) takes the form

$$\begin{aligned} \Delta &= - \ln\left(\frac{2e^\gamma \mu}{\pi T}\right) [f(0) - h'(0)] \\ &+ \frac{1}{2} \int_{-\mu}^{\infty} d\xi' \ln\left|\frac{\xi'}{\mu}\right| \partial_{\xi'} [f(\xi') - h'(\xi') - g(\xi')] \\ &- \int_{-\mu}^0 d\xi' \ln\left|\frac{\xi'}{\mu}\right| \partial_{\xi'} [f(\xi') - h'(\xi')] + i\frac{\pi}{2} g(0). \end{aligned} \quad (94)$$

The value $\xi = 0$ corresponds to the momentum

$$p = \sqrt{2m\mu} = \sqrt{2m(\varepsilon_F^{(0)} + \delta\mu)} \approx p_F^{(0)} + \frac{m}{p_F^{(0)}} \delta\mu, \quad (95)$$

where $\delta\mu$ is the first-order correction to the chemical potential (28). Inserting this expansion as well as the explicit expressions (90) for f , g , and h in the gap equation (94), we obtain

$$\begin{aligned} \Delta(\xi, \phi) &= - \ln\left(\frac{2e^\gamma \varepsilon_F^{(0)}}{\pi T}\right) \int \frac{d\phi'}{2\pi} v \left\{ \left[V_0(\mathbf{p} - \mathbf{p}_F^{(0)'}) + \frac{m}{p_F^{(0)}} \frac{\partial V_0(\mathbf{p} - \mathbf{p}_F^{(0)'})}{\partial p'} \delta\mu + \gamma^{(2)}(2\varepsilon_F^{(0)}, \mathbf{p}, \mathbf{p}_F^{(0)'}) + \delta V_0(\mathbf{p}, \mathbf{p}_F^{(0)'}) \right] \Delta(0, \phi') \right. \\ &- \left. \frac{\partial}{\partial \xi'} [V_0(\mathbf{p} - \mathbf{p}') \Sigma^{(1)}(\mathbf{p}') \Delta(\xi', \phi')]_{\xi'=0} \right\} - \int_{-\varepsilon_F^{(0)}}^0 d\xi' \ln\left|\frac{\xi'}{\varepsilon_F^{(0)}}\right| \frac{\partial}{\partial \xi'} \left[\int \frac{d\phi'}{2\pi} v V_0(\mathbf{p} - \mathbf{p}') \Delta(\xi', \phi') \right] \\ &+ i\frac{\pi}{2} \int \frac{d\phi'}{2\pi} v V_0(\mathbf{p} - \mathbf{p}_F^{(0)'}) \Delta(0, \phi'). \end{aligned} \quad (96)$$

Our calculation of the critical temperature is carried out in two steps: First we omit all terms on the RHS of Eq. (96) but the first. This is equivalent to the commonly used BCS approach and allows us to obtain the controlling factor for the dependence of T_c on g as well as the leading

behavior of $\Delta(\xi, \phi)$ in the limit $g \ll 1$. We will find that $T_c \propto \varepsilon_F^{(0)} \exp(1/\lambda_0)$, where $\lambda_0 \propto g$. The calculation of the prefactor of $\exp(1/\lambda_0)$ in the second step requires us to take account of all terms on the RHS of Eq. (96), which is referred to as the GM approach.

E. Critical temperature in BCS approach

Keeping only the dominant contribution on the RHS of Eq. (96), we have

$$\Delta(\xi, \phi) = \ln\left(\frac{T}{\varepsilon_F^{(0)}}\right) \int \frac{d\phi'}{2\pi} v V_0(\mathbf{p} - \mathbf{p}_F^{(0)'}) \Delta(0, \phi'). \quad (97)$$

We expand $\Delta(0, \phi)$ in terms of a complete set of eigenfunctions $\Phi_s(\phi)$, $s \in \mathbb{N}_0$ of the integral operator with the kernel $V_0(\mathbf{p}_F^{(0)} - \mathbf{p}_F^{(0)'})$, which for $p_F^{(0)} l_0 \ll 1$ can be written as [cf. Eq. (4)]

$$V_0(\mathbf{p}_F^{(0)} - \mathbf{p}_F^{(0)'}) \approx -(g/v) \sin(|\phi - \phi'|/2) \times [\cos(\phi + \phi') \sin^2 \theta_0 + 2P_2(\cos \theta_0)]. \quad (98)$$

The expansion of $\Delta(0, \phi)$ reads

$$\Delta(0, \phi) = \sum_{s=0}^{\infty} \Delta_s \Phi_s(\phi), \quad (99)$$

where the functions $\Phi_s(\phi)$ satisfy the eigenvalue equation

$$\int \frac{d\phi'}{2\pi} v V_0(\mathbf{p}_F^{(0)} - \mathbf{p}_F^{(0)'}) \Phi_s(\phi') = \lambda_s \Phi_s(\phi), \quad (100)$$

and are normalized to unity according to $\int [d\phi/(2\pi)] \Phi_s(\phi)^2 = 1$. We label the eigenvalues λ_s such that $\lambda_s < \lambda_{s'}$ for $s < s'$. Inserting the expansion (99) in Eq. (97) and specifying the resulting equation to $\xi = 0$ we obtain the set of equations, for $s \in \mathbb{N}_0$,

$$[1 - \lambda_s \ln(T/\varepsilon_F^{(0)})] \Delta_s = 0. \quad (101)$$

Thus, the existence of a nontrivial solution for $\Delta(0, \phi)$ requires at least the smallest eigenvalue λ_0 to be negative [note that $\ln(T/\varepsilon_F^{(0)}) < 0$ since $T/\varepsilon_F^{(0)} \ll 1$]. Then the controlling factor of the critical temperature follows immediately from $1 - \lambda_0 \ln(T_c/\varepsilon_F^{(0)}) = 0$, and we have $\Delta_0 \neq 0$ whereas $\Delta_s = 0$ for $s \neq 0$. Solving the eigenvalue equation (100) numerically reveals that, as the tilting angle θ_0 of the dipoles is increased, λ_0 becomes negative at $\theta_c \approx 0.72$, and for θ_0 not too close to θ_c the corresponding eigenfunction is well approximated by

$$\Phi_0(\phi) \approx \sqrt{2} \cos(\phi) \quad (102)$$

(i.e., the order parameter has p -wave symmetry). Contributions from higher partial waves are important only in the immediate vicinity of the critical angle and may safely be neglected otherwise (see Fig. 7). Within the approximation (102), the eigenvalue λ_0 equals the diagonal matrix element

$$\begin{aligned} \lambda_0 &\approx 2 \int \frac{d\phi}{2\pi} \cos(\phi) \int \frac{d\phi'}{2\pi} v V_0(\mathbf{p}_F^{(0)} - \mathbf{p}_F^{(0)'}) \cos(\phi') \\ &= (g/\pi)(4/3 - 3 \sin^2 \theta_0). \end{aligned} \quad (103)$$

In the BCS approach, therefore, for $\theta_0 \gtrsim \theta_c$ the critical temperature is given by

$$T_c^{(\text{BCS})} = \varepsilon_F^{(0)} e^{-\pi/(g|4/3 - 3 \sin^2 \theta_0|)}, \quad (104)$$

and Eq. (97) determines the order parameter as

$$\Delta(\xi, \phi) = \frac{1}{\lambda_0} \int \frac{d\phi'}{2\pi} v V_0(\mathbf{p} - \mathbf{p}_F^{(0)'}) \Phi_0(\phi'). \quad (105)$$

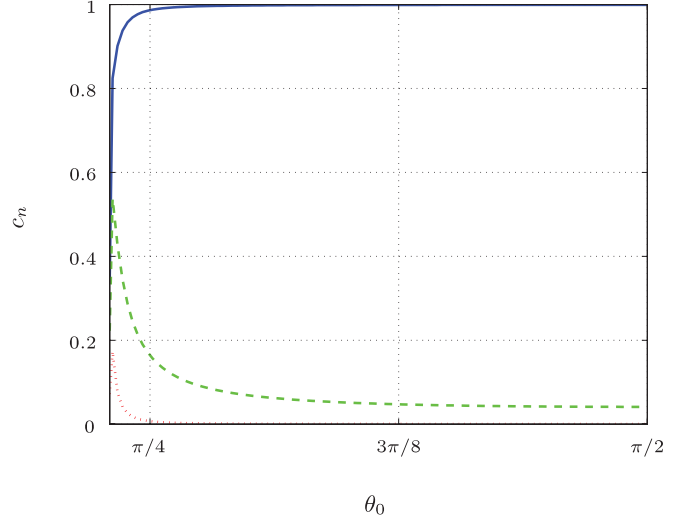


FIG. 7. (Color online) To solve (100) numerically we expand $\Phi_0(\phi)$ in a Fourier cosine series [the integral operator in Eq. (100) does not couple cosine and sine series and it turns out that $\Phi_0(\phi)$ can be represented as a cosine series; note that in this series expansion we retain only those terms that ensure the correct parity of the order parameter, $\Delta(\xi, \phi + \pi) = -\Delta(\xi, \phi)$] $\Phi_0(\phi) = \sqrt{2} \sum_{n=1}^N c_n \cos[(2n-1)\phi]$ with sufficiently large N and solve the resulting linear system for the coefficients c_n . The above plot shows the coefficients c_1 (solid blue line), c_2 (dashed green line), and c_3 (dotted red line) as functions of θ_0 for $\theta_0 \geq \theta_c$.

We note that within the BCS mean-field theory with only the p -wave contribution to the order parameter [12], the same exponent appears in the expression for the gap at $T = 0$ as in Eq. (104).

F. Critical temperature in GM approach

As we have pointed out earlier, within the BCS approach it is possible to obtain only the controlling factor [the exponential in Eq. (104)] of the dependence of T_c on g . In order to find the correct prefactor we proceed by substituting the ansatz

$$T_c = (\varpi/\varepsilon_F^{(0)}) T_c^{(\text{BCS})} \quad (106)$$

into Eq. (96). We set $\xi = 0$ in the resulting equation, multiply it by $\Phi_0(\phi)$, and take the integral over the angle ϕ . Using the normalization condition of the eigenfunction Φ_0 we obtain, to leading order in g ,

$$\begin{aligned} \ln\left(\frac{\varpi}{\varepsilon_F^{(0)}}\right) &= -\frac{\delta\lambda_0}{\lambda_0^2} - \ln\left(\frac{i\pi}{2e^\gamma}\right) \\ &+ \frac{1}{\lambda_0} \int \frac{d\phi}{2\pi} \Phi_0(\phi) \int_{-\varepsilon_F^{(0)}}^0 d\xi' \ln\left|\frac{\xi'}{\varepsilon_F^{(0)}}\right| \\ &\times \frac{\partial}{\partial \xi'} \left[\int \frac{d\phi'}{2\pi} v V_0(\mathbf{p}_F^{(0)} - \mathbf{p}') \Delta(\xi', \phi') \right], \end{aligned} \quad (107)$$

where the correction $\delta\lambda_0$ to the eigenvalue λ_0 is composed of four contributions:

$$\delta\lambda_0 = \delta\lambda_0^{(\delta p_F)} + \delta\lambda_0^{(\delta m)} + \delta\lambda_0^{(\text{mb})} + \delta\lambda_0^{(B)}. \quad (108)$$

Physically, $\delta\lambda_0^{(\delta p_F)}$ and $\delta\lambda_0^{(\delta m)}$ are due to the fact that the pairing occurs between quasiparticles and not bare particles [these corrections contain the Fermi surface deformation (29) and the effective mass (31), respectively, see discussion below], $\delta\lambda_0^{(\text{mb})}$ has its origin in the many-body corrections δV_d to the

bare interaction, and $\delta\lambda_0^{(B)}$ incorporates the second-order Born correction to the two-body vertex function.

(a) *Self-energy corrections.* The terms in Eq. (108) that involve the self-energy $\Sigma^{(1)}$ and the correction to the chemical potential $\delta\mu$ are

$$\delta\lambda_0^{(\delta p_F)} + \delta\lambda_0^{(\delta m)} = \int \frac{d\phi}{2\pi} \Phi_0(\phi) \int \frac{d\phi'}{2\pi} v \left\{ \frac{m}{p_F^{(0)}} \left[\frac{\partial V_0(\mathbf{p}_F^{(0)} - \mathbf{p}_F^{(0)'})}{\partial p} + \frac{\partial V_0(\mathbf{p}_F^{(0)} - \mathbf{p}_F^{(0)'})}{\partial p'} \right] \delta\mu \Phi_0(\phi') \right. \\ \left. - \frac{\partial}{\partial \xi'} [V_0(\mathbf{p}_F^{(0)} - \mathbf{p}') \Sigma^{(1)}(\mathbf{p}') \Delta(\xi', \phi')] \Big|_{\xi'=0} \right\}. \quad (109)$$

With the aid of Eqs. (100) and (105), the expression for $\delta\lambda_0^{(\delta p_F)}$ can be put in the form

$$\delta\lambda_0^{(\delta p_F)} = 2 \int \frac{d\phi}{2\pi} \Phi_0(\phi) \int \frac{d\phi'}{2\pi} v \delta p_F(\phi) \\ \times \hat{\mathbf{p}} \cdot \nabla V_0(\mathbf{p}_F^{(0)} - \mathbf{p}_F^{(0)'}) \Phi_0(\phi'), \quad (110)$$

where the Fermi surface deformation δp_F is given by Eq. (29) [the exponential smallness of T_c allows us to use the zero temperature expression (27) for the self-energy].

For $\delta\lambda_0^{(\delta m)}$ we find

$$\delta\lambda_0^{(\delta m)} = \int \frac{d\phi}{2\pi} \Phi_0(\phi) \int \frac{d\phi'}{2\pi} v \frac{\delta m(\phi')}{m} \\ \times V_0(\mathbf{p}_F^{(0)} - \mathbf{p}_F^{(0)'}) \Phi_0(\phi'), \quad (111)$$

with $\delta m(\phi)$ given by Eq. (31). Performing the angular integrals in Eqs. (109) and (111), we find

$$\delta\lambda_0^{(\delta p_F)} = \frac{16}{5\pi^2} g^2 \sin^2 \theta_0 \left(\frac{1}{9} - \frac{1}{5} \sin^2 \theta_0 \right), \\ \delta\lambda_0^{(\delta m)} = -\frac{2}{225\pi^2} g^2 (200 - 820 \sin^2 \theta_0 + 829 \sin^4 \theta_0). \quad (112)$$

(b) *Many-body corrections.* The leading many-body corrections are given by

$$\delta\lambda_0^{(\text{mb})} = v \int \frac{d\phi}{2\pi} \Phi_0(\phi) \int \frac{d\phi'}{2\pi} \delta V_0(\mathbf{p}_F^{(0)}, \mathbf{p}_F^{(0)'}) \Phi_0(\phi'), \quad (113)$$

where the analytical expressions corresponding to the diagrams (60) that make up δV_0 read

$$\delta V_0^{(a)}(\mathbf{p}, \mathbf{p}') = [V_0(\mathbf{p}_-)]^2 \\ \times \int \frac{n(\mathbf{q} + \mathbf{p}_-/2) - n(\mathbf{q} - \mathbf{p}_-/2)}{\xi(\mathbf{q} + \mathbf{p}_-/2) - \xi(\mathbf{q} - \mathbf{p}_-/2) - i0} \frac{d\mathbf{q}}{(2\pi)^2}, \quad (114)$$

$$\delta V_0^{(b)}(\mathbf{p}, \mathbf{p}') = -V_0(\mathbf{p}_-) \int V_0(\mathbf{q} - \mathbf{p}_+/2) \\ \times \frac{n(\mathbf{q} + \mathbf{p}_-/2) - n(\mathbf{q} - \mathbf{p}_-/2)}{\xi(\mathbf{q} + \mathbf{p}_-/2) - \xi(\mathbf{q} - \mathbf{p}_-/2) - i0} \frac{d\mathbf{q}}{(2\pi)^2}, \quad (115)$$

$$\delta V_0^{(c)}(\mathbf{p}, \mathbf{p}') = -V_0(\mathbf{p}_-) \int V_0(\mathbf{q} + \mathbf{p}_+/2) \\ \times \frac{n(\mathbf{q} + \mathbf{p}_-/2) - n(\mathbf{q} - \mathbf{p}_-/2)}{\xi(\mathbf{q} + \mathbf{p}_-/2) - \xi(\mathbf{q} - \mathbf{p}_-/2) - i0} \frac{d\mathbf{q}}{(2\pi)^2}, \quad (116)$$

$$\delta V_0^{(d)}(\mathbf{p}, \mathbf{p}') = - \int V_0(\mathbf{q} - \mathbf{p}_-/2) V_0(\mathbf{q} + \mathbf{p}_-/2) \\ \times \frac{n(\mathbf{q} + \mathbf{p}_+/2) - n(\mathbf{q} - \mathbf{p}_+/2)}{\xi(\mathbf{q} + \mathbf{p}_+/2) - \xi(\mathbf{q} - \mathbf{p}_+/2) - i0} \frac{d\mathbf{q}}{(2\pi)^2} \quad (117)$$

(we are neglecting contributions that involve excited states of the transverse trapping potential because they contain an additional factor of $\varepsilon_F^{(0)}/\omega_0 \ll 1$). Here $\mathbf{p}_\pm = \mathbf{p} \pm \mathbf{p}'$, and $n(\mathbf{p}) = \theta(p_F^{(0)} - p)$ is the Fermi-Dirac distribution at zero temperature [the use of $n(\mathbf{p})$ at zero temperature is justified by the exponential smallness of T_c]. Performing a numerical integration we find

$$\delta\lambda_0^{(\text{mb})} = g^2 (0.37 - 1.67 \sin^2 \theta_0 + 1.82 \sin^4 \theta_0). \quad (118)$$

(c) *Second-order Born correction.* The second-order Born correction (88) to the vertex function results in the contribution

$$\delta\lambda_0^{(B)} = \int \frac{d\phi}{2\pi} \Phi_0(\phi) \int \frac{d\phi'}{2\pi} v \gamma^{(2)}(2\varepsilon_F^{(0)}, \mathbf{p}_F^{(0)}, \mathbf{p}_F^{(0)'}) \Phi_0(\phi') \quad (119)$$

to $\delta\lambda_0$. We decompose (119) as a sum of contributions with fixed HO quantum numbers at the intermediate state of the second-order scattering process, $\delta\lambda_0^{(B)} = \sum_{n,n'} \delta\lambda_{n,n'}^{(B)}$, where

$$\delta\lambda_{n,n'}^{(B)} = -v \int \frac{d\phi}{2\pi} \int \frac{d\phi'}{2\pi} \Phi_0(\phi) \Phi_0(\phi') \\ \times \frac{V_{n,n'}(\mathbf{p}_F^{(0)} - \mathbf{q}) V_{n,n'}(\mathbf{q} - \mathbf{p}_F^{(0)'})}{2\xi(\mathbf{q}) + \omega_0(n + n') - i0} \frac{d\mathbf{q}}{(2\pi)^2}. \quad (120)$$

For $n = n' = 0$ we find the asymptotic expression

$$\delta\lambda_{0,0}^{(B)} \stackrel{\eta \rightarrow 0}{\sim} g^2 (0.35 - 1.56 \sin^2 \theta_0 + 1.79 \sin^4 \theta_0) \\ + g^2 \ln(\eta) \left(\frac{1}{4} - \frac{9}{8} \sin^2 \theta_0 + \frac{41}{32} \sin^4 \theta_0 \right) - i \frac{\pi}{2} \lambda_0. \quad (121)$$

Note that, despite the smallness of $\eta = p_F l_0$, terms which are proportional to $g^2 \ln(\eta)$ represent a small correction to $\lambda_0 = O(g)$ in the limit where $g/\eta = r_d/l_0 \ll 1$.

We are left with the calculation of the contribution to the second-order Born correction that involves excited states of the transverse trapping potential, $\delta\lambda_*^{(B)} = \delta\lambda_0^{(B)} - \delta\lambda_{0,0}^{(B)}$. In terms of new summation indices that may be interpreted as “relative” and “center-of-mass” HO quantum numbers, we put it in the form

$$\delta\lambda_*^{(B)} = \sum_{N=1}^{\infty} \sum_{n=-N}^N \delta\lambda_{N+n,N-n}^{(B)} + \sum_{N=1}^{\infty} \sum_{n=-N+1}^N \delta\lambda_{N+n-1,N-n}^{(B)}; \quad (122)$$

that is, we separate parts in which the sum of HO quantum numbers is even and odd, respectively.

For the present purpose we rewrite the WKB matrix elements (C8) and (C9) as

$$V_{N+n,N-n}(\mathbf{p}) = \sqrt{\frac{1}{2N}} \frac{g}{\pi\eta\nu} e^{-n^2/(2N)} \times [u(\mathbf{p}) \sin^2 \theta_0 - 2P_2(\cos \theta_0)] \frac{(pl_0)^2}{(pl_0)^2 + 2N}, \quad (123)$$

and

$$V_{N+n-1,N-n}(\mathbf{p}) = 2i \frac{g}{\pi\eta\nu} (-1)^{1+n} e^{-(n-1/2)^2/(2N)} \times \sin(2\theta_0) v(\mathbf{p}) \frac{pl_0}{(pl_0)^2 + 2N}. \quad (124)$$

Inserting these expressions in Eq. (120) and performing the integrals we find, in the limit $\eta \rightarrow 0$,

$$\delta\lambda_{N+n,N-n}^{(B)} = -\left(\frac{g}{\pi N}\right)^2 e^{-n^2/N} \times \left(\frac{2}{3} - 4 \sin^2 \theta_0 + \frac{43}{12} \sin^4 \theta_0\right), \quad (125)$$

$$\delta\lambda_{N+n-1,N-n}^{(B)} = -\frac{7}{6} \left(\frac{g}{4\pi N}\right)^2 e^{-(n-1/2)^2/N} \sin(2\theta_0)^2. \quad (126)$$

In the second line we are actually restricting ourselves to the asymptotic behavior of $\delta\lambda_{N+n-1,N-n}^{(B)}$ for $N \rightarrow \infty$, which, however, gives a sufficiently accurate approximation even for $N = 1$. We insert Eqs. (125) and (126) into (122) to obtain

$$\delta\lambda_*^{(B)} = -\frac{1}{16\pi^2} g^2 \left[S_1 \left(\frac{2}{3} - 4 \sin^2 \theta_0 + \frac{43}{12} \sin^4 \theta_0 \right) + \frac{7}{6} S_2 \sin(2\theta_0)^2 \right], \quad (127)$$

where the sums S_α for $\alpha = 1, 2$ are given by

$$S_\alpha = \sum_{N=1}^{\infty} S_\alpha(N)/N^2, \quad (128)$$

with

$$S_1(N) = \sum_{n=-N}^N e^{-n^2/N}, \quad S_2(N) = \sum_{n=-N+1}^N e^{-(n-1/2)^2/N}. \quad (129)$$

These sums can be calculated semianalytically with the result

$$\delta\lambda_*^{(B)} = g^2(-0.02 - 0.01 \sin^2 \theta_0 + 0.03 \sin^4 \theta_0). \quad (130)$$

Adding the contributions of Eqs. (112), (118), (121), and (130) we obtain the correction to the eigenvalue λ_0 :

$$\delta\lambda_0 = g^2(0.52 - 2.47 \sin^2 \theta_0 + 2.83 \sin^4 \theta_0) + g^2 \ln(\eta)(0.25 - 1.13 \sin^2 \theta_0 + 1.28 \sin^4 \theta_0). \quad (131)$$

In the third term on the RHS of Eq. (107) we express the gap $\Delta(\xi', \phi')$ via (105) and perform a numerical integration. We find that this term gives a contribution

$$(g/\lambda_0)^2(-0.14 + 0.63 \sin^2 \theta_0 - 0.71 \sin^4 \theta_0) \quad (132)$$

to $\ln(\varpi/\varepsilon_F^{(0)})$.

The combination of Eqs. (107), (131), and (132) yields the final expression for the critical temperature in the GM approach ($\theta_0 \gtrsim \theta_c$):

$$T_c \approx \frac{2e^\gamma \varepsilon_F^{(0)}}{\pi} f(\theta_0) \eta^{g(\theta_0)} \exp(-\pi/g|4/3 - 3 \sin^2 \theta_0|), \quad (133)$$

where

$$f(\theta_0) = \exp\left(\frac{0.52 - 2.47 \sin^2 \theta_0 + 2.83 \sin^4 \theta_0}{0.18 - 0.81 \sin^2 \theta_0 + 0.91 \sin^4 \theta_0}\right), \quad (134)$$

and

$$g(\theta_0) = \frac{0.25 - 1.13 \sin^2 \theta_0 + 1.28 \sin^4 \theta_0}{0.18 - 0.81 \sin^2 \theta_0 + 0.91 \sin^4 \theta_0}. \quad (135)$$

Figure 8 shows the critical temperature as a function of the tilting angle for values of g and η that correspond to a gas of polar KRb molecules. The corresponding eigenfunction $\Phi_0(\phi)$ for the order parameter is given by Eq. (102).

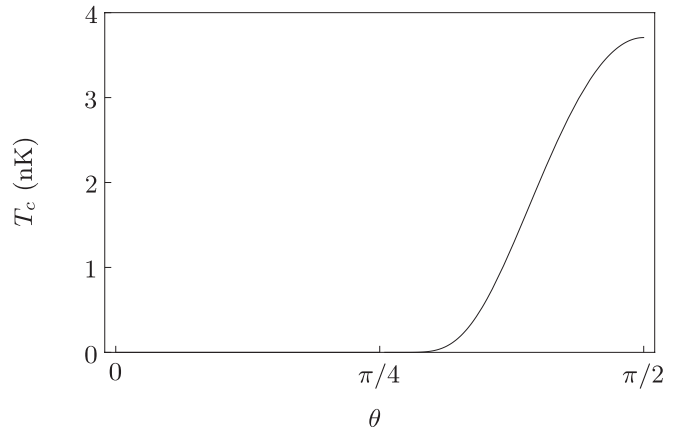


FIG. 8. Critical temperature as a function of θ_0 for $g \approx 1.2$ and $\eta \approx 0.2$. These values correspond to a gas of KRb molecules, with $n_{2D} = 4 \times 10^8 \text{ cm}^{-2}$ and $\omega_0 = 2\pi \times 100 \text{ kHz}$; see discussion in Sec. IX.

IX. SUMMARY AND CONCLUSION

In this paper, we studied various properties of a quasi-2D dipolar Fermi gas. Using the (conserving) Hartree-Fock approximation at $T = 0$, we found that the normal phase is characterized by an anisotropic Fermi surface, which—in order to minimize the interaction energy—is elongated along the projection of the dipoles on the plane of confinement (here, the x axis). Consequently, the dispersion relation of single-particle excitations is anisotropic, too, with an increased effective mass in the x direction and a decreased effective mass in the y direction (see Fig. 2).

Long-wavelength collective excitations (zero sound), corresponding to deformations of the equilibrium Fermi surface, can propagate through the medium only when the tilting angle exceeds a value of $\theta_0 \approx 0.5$, which depends only weakly on g for (at least) $0.1 \leq g \leq 2$. In this regime we found two distinct modes, symmetric and antisymmetric about the propagation direction, respectively.

The experimental observation of zero sound requires T so low that the attenuation is negligible. Note, however, that for $\theta_0 > \theta_c \approx 0.7$ (the critical angle for the superfluid transition) we also require $T > T_c$ for the observation of zero sound, although under certain conditions, zero sound can still persist below T_c (see discussion in Ref. [36] and references given therein).

At higher values of g and for large tilting angles, we found complex eigenfrequencies of the long-wavelength collective modes indicating the instability of the system toward collapse in this part of the phase diagram; see Fig. 9. Instabilities with finite momentum that drive the system toward a state with periodic density modulation, which breaks translational symmetry, appear at small tilting angles. This phase can be detected experimentally using Bragg spectroscopy. In the isotropic case with $\theta_0 = 0$ (dipoles are aligned perpendicular to

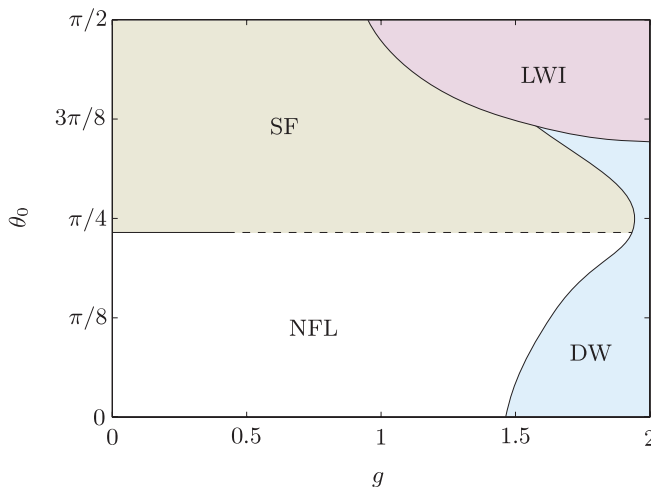


FIG. 9. (Color online) Phase diagram of the quasi-2D dipolar Fermi gas at $T = 0$. For $0 \leq g \lesssim 1.45$ and small tilting angles the system is a normal Fermi liquid (NFL). The transition to the superfluid phase (SF) occurs at the critical angle $\theta_c \approx 0.7217$. In our perturbative approach, which is—strictly speaking—valid only for $g \ll 1$, this value is independent of g . At moderately strong interactions there appears a density wave phase (DW) in the lower part of the phase diagram and a long-wavelength instability (LWI) in the upper part.

the plane of confinement) also rotational symmetry is broken, and it is believed that three density waves form a triangular lattice to minimize the breaking of rotational symmetry [33].

Finally, let us discuss prospects for observing the superfluid transitions in the quasi-2D dipolar Fermi gas at finite temperature. In current experiments with KRb at the Joint Institute for Laboratory Astrophysics (JILA), electric dipole moments of $d \approx 0.3D$ are available. Then, assuming an area density of $n_{2D} \approx 4 \times 10^8 \text{ cm}^{-2}$ corresponding to $\varepsilon_F^{(0)} \approx 100 \text{ nK}$, the dimensionless coupling constant takes the value $g \approx 1.2$. Note that, strictly speaking, this value of g lies beyond the weak-coupling regime in which Eq. (133) is valid. However, this formula still provides an estimate for the onset of superfluidity in the intermediate-coupling regime.

The strong confinement condition is realized for a transverse trapping frequency of $\omega_0 = 2\pi \times 100 \text{ kHz}$, so that $\eta \approx 0.2$. For the above value of g the system is stable up to a tilting angle of $\theta_0 \approx 1.3$ (see Fig. 9). Then we have $T_c \approx 0.04\varepsilon_F^{(0)} \approx 4 \text{ nK}$, which is comparable to the values of T_c that are to be expected in bilayer systems (see Refs. [14,18,19]).

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APPENDIX A: CALCULATION OF HF SELF-ENERGY

In order to solve Eqs. (22) and (24) for p_F it is convenient to introduce another unknown function $f(\phi)$ according to

$$f(\phi) \equiv p_F(\phi)^2 / p_F^{(0)2} - 1. \quad (\text{A1})$$

The expansion of f in a Fourier series takes the form

$$f(\phi) = \sum_{n \in \mathbb{N}} c_n \cos(2n\phi); \quad (\text{A2})$$

that is, the constant term is absent, as is immediately apparent by inserting the definition (A1) into Eq. (23). Moreover, due to the symmetry of the problem, terms which are proportional to $\sin(n\phi)$ or $\cos(n\phi)$ with odd n do not appear.

We solve for the Fourier coefficients c_n by iteration. As initial values we take the results from first-order perturbation theory (see Sec. V):

$$c_n^{(1)} = \begin{cases} \frac{16}{15\pi} g \sin^2 \theta_0 & \text{for } n = 1 \\ 0 & \text{for } n = 2, 3, \dots \end{cases} \quad (\text{A3})$$

The iteration scheme consists of repeatedly carrying out the integrals [which result from the combination of Eqs. (22), (24), (A1), and (A2)]

$$c_n^{(i+1)} = \frac{1}{\pi^2} \int_0^1 dx \int d\phi d\phi' \cos(2n\phi) x [1 + f^{(i)}(\phi')] \times v V_0 (p_F^{(0)} \sqrt{1 + f^{(i)}(\phi)} \hat{\mathbf{p}} - p_F^{(0)} x \sqrt{1 + f^{(i)}(\phi')} \hat{\mathbf{p}}), \quad (\text{A4})$$

for given values of g and θ_0 , where we keep coefficients up to $n = 4$ [i.e., terms in the series (A2) up to $\cos(8\phi)$] and $f^{(i)}(\phi)$ is obtained by replacing c_n in Eq. (A2) by $c_n^{(i)}$. A measure for the convergence of this procedure is the relative change in f in one step of the iteration process,

$$\frac{\|f^{(i+1)} - f^{(i)}\|}{\|f^{(i)}\|} = \sqrt{\frac{\sum_n (c_n^{(i+1)} - c_n^{(i)})^2}{\sum_n c_n^{(i)}}}, \quad (\text{A5})$$

and we terminate the iteration when this quantity drops below 10^{-3} .

From the final result for f we immediately obtain p_F by inverting Eq. (A1). Then, Σ and ε follow from Eq. (19), and by taking the gradient in Eq. (19) we obtain the Fermi velocity (26) as

$$\mathbf{v}_F = \frac{1}{m} \left[\mathbf{p}_F - \int \frac{d\phi'}{2\pi} \int_0^{p_F} dp' p' v \nabla V_0(\mathbf{p}_F - \mathbf{p}') \right]. \quad (\text{A6})$$

APPENDIX B: NUMERICAL SOLUTION TO BETHE-SALPETER EQUATION

To solve Eq. (32) numerically it is advantageous to rewrite it in a form that is symmetrized with respect to the transferred momentum \mathbf{q} . Toward this end, we introduce the shifted function $\tilde{\chi}_{\mathbf{q}}(\mathbf{p}) \equiv \chi(\mathbf{p} - \mathbf{q}/2; \mathbf{q})$. Then we have

$$\tilde{\chi}_{\mathbf{q}}(\mathbf{p}) = \int \frac{d\mathbf{p}'}{(2\pi)^2} \tilde{\Gamma}_{\text{ph}}(\mathbf{p} - \mathbf{p}', \mathbf{q}) \quad (\text{B1})$$

$$\times \frac{n(\mathbf{p}' - \mathbf{q}/2) - n(\mathbf{p}' + \mathbf{q}/2)}{\omega + \varepsilon(\mathbf{p}' - \mathbf{q}/2) - \varepsilon(\mathbf{p}' + \mathbf{q}/2)} \tilde{\chi}_{\mathbf{q}}(\mathbf{p}'). \quad (\text{B2})$$

The factor $n(\mathbf{p}' - \mathbf{q}/2) - n(\mathbf{p}' + \mathbf{q}/2)$ in the numerator on the RHS restricts the area of integration as depicted in Fig. 10. Rewriting the integral in polar coordinates $\mathbf{p}' = p'(\cos \phi', \sin \phi')$ and in terms of a new function

$$v_{\mathbf{q}}(\mathbf{p}) \equiv \frac{\tilde{\chi}_{\mathbf{q}}(\mathbf{p})}{\omega + \varepsilon(\mathbf{p} - \mathbf{q}/2) - \varepsilon(\mathbf{p} + \mathbf{q}/2)}, \quad (\text{B3})$$

Eq. (B1) becomes [for the definition of the angles ϕ_{\pm} that limit the ϕ' integration see Fig. 10; $p_{\pm}(\phi')$ are the angle-dependent upper and lower boundaries for the integration over p']

$$\begin{aligned} & [\omega + \varepsilon(\mathbf{p} - \mathbf{q}/2) - \varepsilon(\mathbf{p} + \mathbf{q}/2)] v_{\mathbf{q}}(\mathbf{p}) \\ &= \frac{1}{2\pi} \int_{\phi_-}^{\phi_+} \frac{d\phi'}{2\pi} \int_{p_-(\phi')}^{p_+(\phi')} dp' p' \tilde{\Gamma}_{\text{ph}}(\mathbf{p} - \mathbf{p}', \mathbf{q}) v_{\mathbf{q}}(\mathbf{p}') \\ & - \frac{1}{2\pi} \int_{\pi+\phi_-}^{\pi+\phi_+} \frac{d\phi'}{2\pi} \int_{p_-(\phi')}^{p_+(\phi')} dp' p' \tilde{\Gamma}_{\text{ph}}(\mathbf{p} - \mathbf{p}', \mathbf{q}) v_{\mathbf{q}}(\mathbf{p}'). \end{aligned} \quad (\text{B4})$$

We introduce a new variable $x \in [0, 1]$ that parametrizes the momentum as $p(x, \phi) = p_-(\phi) + [p_+(\phi) - p_-(\phi)]x$ and discretize the resulting integrals over x' and ϕ' .

APPENDIX C: MATRIX ELEMENTS $V_{n,n'}$

In this Appendix we present explicit expressions for the matrix elements (5) with $n_3 = n_4 = 0$, which appear in the second-order term (88) in the Born series for the two-body vertex function. Omitting the momentum-independent

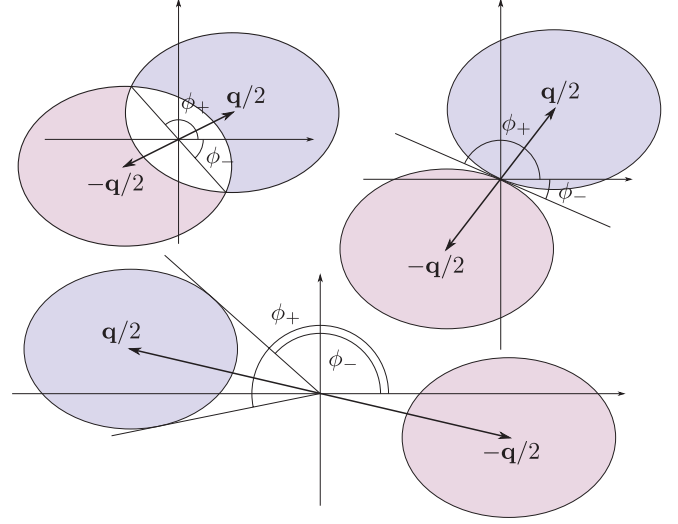


FIG. 10. (Color online) Area of integration in Eq. (B1) and definition of the limiting angles ϕ_{\pm} , for three different values of \mathbf{q} , with $|\mathbf{q}| < 2p_F(\phi_q)$ (top left), $|\mathbf{q}| = 2p_F(\phi_q)$ (top right), and $|\mathbf{q}| > 2p_F(\phi_q)$ (bottom).

contribution (see footnote 1), with the aid of the integral table [51] we find, for $n + n' \in 2\mathbb{N}_0$,

$$\begin{aligned} V_{n,n'}(\mathbf{p}) &= (2^{n+n'} n! n'!)^{-1/2} \Gamma\left(\frac{1+n+n'}{2}\right) \\ & \times (d^2/l_0) [u(\mathbf{p}) \sin^2 \theta_0 - 2P_2(\cos \theta_0)] \\ & \times (pl_0)^{1+n+n'} e^{p^2 l_0^2/2} \Gamma\left(\frac{1-n-n'}{2}, p^2 l_0^2/2\right), \end{aligned} \quad (\text{C1})$$

where $u(\mathbf{p}) \equiv (p_x^2 - p_y^2)/p^2$, and $\Gamma(z)$ and $\Gamma(v, z)$ are the complete and incomplete Gamma functions, respectively. For $n + n' \in 2\mathbb{N}_0 + 1$, with $v(\mathbf{p}) \equiv p_1/p$, we have

$$\begin{aligned} V_{n,n'}(\mathbf{p}) &= [(-1)^{n-n'} / 2^{n+n'} n! n'!]^{1/2} \Gamma\left(1 + \frac{n+n'}{2}\right) \\ & \times (2d^2/l_0) v(\mathbf{p}) \sin(2\theta_0) \\ & \times (pl_0)^{1+n+n'} e^{p^2 l_0^2/2} \Gamma\left(-\frac{n+n'}{2}, p^2 l_0^2/2\right). \end{aligned} \quad (\text{C2})$$

The relatively complicated functional dependence of the matrix elements $V_{n,n'}$ on the momentum \mathbf{p} prevents us from calculating the integrals in Eq. (120) analytically. We avoid this problem by using WKB matrix elements [52] instead of the exact ones.

The WKB eigenfunctions of the one-dimensional HO, which give good approximations to the exact eigenfunctions for $n \geq 1$, are [9]

$$\phi_n(z) = (-1)^n \sqrt{2m\omega_0/[\pi p_n(z)]} \cos[\Psi_n(z) - \pi/4], \quad (\text{C3})$$

where the position-dependent momentum $p_n(z)$ and the phase $\Psi_n(z)$ are given by

$$p_n(z) = \sqrt{2m[E_n - V(z)]}, \quad (\text{C4})$$

$$\Psi_n(z) = \int_{-z_n}^z dz' p_n(z'), \quad (\text{C5})$$

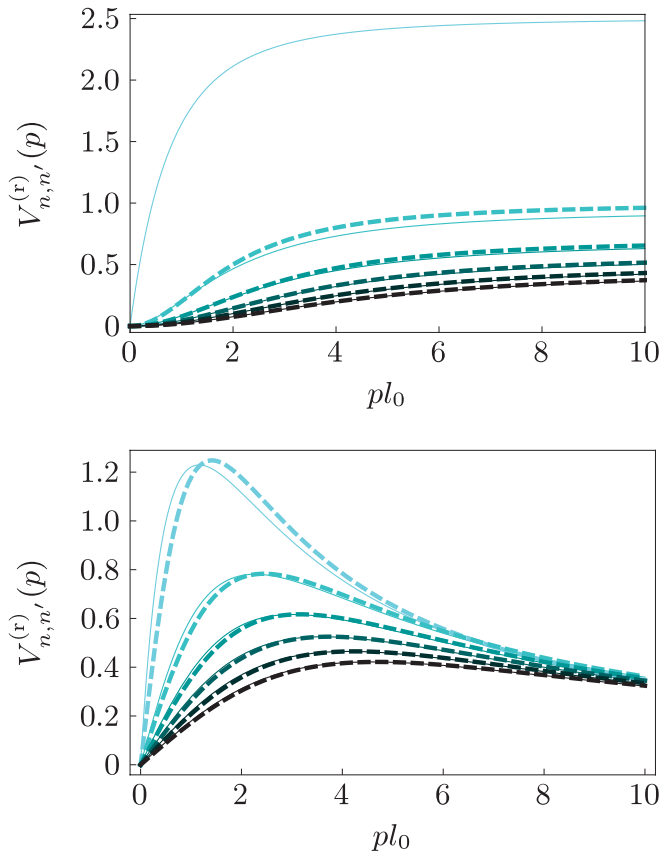


FIG. 11. (Color online) Top: Solid and dashed lines correspond to $V_{n,n}^{(r)}(p)$ with $n = 0, 2, \dots, 10$, calculated exactly and in WKB approximation, respectively. (We do not use the WKB approximation in the case $n = 0$, hence there is no dashed line corresponding to the uppermost solid line.) Bottom: Same as above, for $V_{n,n+1}^{(r)}(p)$ with $n = 0, 2, \dots, 10$. In both figures higher values of n correspond to darker colors.

with the potential $V(z) = m\omega_0^2 z^2/2$ and the HO energy levels $E_n = \omega_0(n + 1/2)$. Above expression for $\phi_n(z)$ is valid in the classically allowed region [i.e., for values of z such that $E_n > V(z)$] and far away from the two turning points at $\pm z_n = \pm l_0 \sqrt{2n + 1}$, at which the kinetic energy is zero, $p_n(\pm z_n) = 0$ or, equivalently, $E_n = V(\pm z_n)$.

The HO ground-state wave function decays exponentially on a scale that is set by l_0 . Therefore, the integration in the matrix elements $V_{n,n'}$ is essentially restricted to the interval $|z| \lesssim l_0$. For these values of z we may use the approximations

$$p_n(z) \approx p_n(0) \equiv p_n, \quad \Psi_n(z) \approx p_n z + (\pi/4)(2n + 1). \quad (\text{C6})$$

The WKB wave function (C3) then becomes

$$\phi_n(z) \approx (-1)^n \sqrt{2m\omega_0/(\pi p_n)} \cos(p_n z + n\pi/2). \quad (\text{C7})$$

Consequently, for the matrix elements $V_{n,n'}$ in WKB approximation we find

$$V_{n,n'}(\mathbf{p}) \approx 2(n + n')^{-1/2} e^{-(n-n')^2/[4(n+n')]} (d^2/l_0) \times [u(\mathbf{p}) \sin^2 \theta_0 - 2P_2(\cos \theta_0)] \frac{(pl_0)^2}{(pl_0)^2 + n + n'} \quad (\text{C8})$$

for even $n + n'$, and

$$V_{n,n'}(\mathbf{p}) \approx 4i^{n-n'} e^{-(n-n')^2/[4(1+n+n')]} \times \frac{d^2}{l_0} \sin(2\theta_0) v(\mathbf{p}) \frac{pl_0}{(pl_0)^2 + 1 + n + n'} \quad (\text{C9})$$

for odd $n + n'$.

In order to conveniently compare the exact results for $V_{n,n'}$ with those obtained in WKB approximation we introduce reduced matrix elements which are real, dimensionless, and depend only on the magnitude p of the momentum. For even $n + n'$ the reduced matrix element $V_{n,n'}^{(r)}(p)$ is defined via the relation

$$V_{n,n'}(\mathbf{p}) = \frac{d^2}{l_0} [u(\mathbf{p}) \sin^2 \theta_0 - 2P_2(\cos \theta_0)] V_{n,n'}^{(r)}(p), \quad (\text{C10})$$

and, for odd $n + n'$, we set

$$V_{n,n'}(\mathbf{p}) = i \frac{d^2}{l_0} v(\mathbf{p}) V_{n,n'}^{(r)}(p). \quad (\text{C11})$$

Figure 11 compares the reduced matrix elements in the WKB approximation with the exact ones. Quantitative agreement improves with increasing HO quantum numbers (corresponding to darker colors in Fig. 11) and is, however, satisfactory even for $n, n' \geq 1$.

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