Orthogonality catastrophe as a consequence of qubit embedding in an ultracold Fermi gas

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We investigate the behavior of a two-level atom coupled to a one-dimensional, ultracold Fermi gas. The *sudden* switching on of the scattering between the two entities leads to the loss of any coherence in the initial state of the impurity and we show that the exact dynamics of this process is strongly influenced by the effect of the *orthogonality catastrophe* within the gas. We highlight the relationship between the Loschmidt echo and the retarded Green's function—typically used to formulate the dynamical theory of the catastrophe—and demonstrate that the effect is reflected in the impurity dynamics. We show that the expected nonexponential decay of the spectral function can be observed using Ramsey interferometry on the two-level atom and comment on finite temperature effects.

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I. INTRODUCTION

In the past decade ultracold quantum gases have emerged as ideal candidates for designing controllable experiments to simulate effects in condensed-matter physics [1]. This success is likely to receive another boost due to the recent emergence of a fundamental new class of hybrid experimental systems. In these, two separate, ultracold atomic systems are combined in such a way that their coupling can be externally controlled and their states independently measured, thus allowing detailed investigations into the theory of quantum interactions and decoherence. Existing examples of such systems are single spin impurities embedded in ultracold Fermi gases [2,3] and the combination of neutral [4–8] or charged single atoms [9,10] with Bose-Einstein condensates. These experiments offer the possibility for controlled simulation of many different system-environment models synonymous with condensed-matter physics and nonequilibrium statistical physics [11].

Here we show that a fundamental and well-known quantum many-body effect, the Anderson orthogonality catastrophe (OC), can play an important role in ultracold, coupled systems [12]. We consider a single two-level system (impurity) embedded into a harmonically trapped, ultracold Fermi gas and demonstrate how the overlap between the many-body wave functions of the Fermi sea before and after a transition in the impurity vanishes as the number of particles and/or the scattering strength is increased. This signals the onset of OC and we show that it can be observed by looking at the dynamical dephasing features of the impurity alone. Our study identifies the OC as a significant effect even in mesoscopic systems and represents a prime example of how properties of out-of-equilibrium systems with many degrees of freedom can be inferred by looking at a simpler, auxiliary one.

II. ORTHOGONALITY CATASTROPHE

Let us briefly revisit the original idea of Anderson [12] by considering the ground state of a noninteracting, spin-polarized Fermi gas in a hard-wall, spherically symmetric box at zero temperature. The many-particle wave function of the gas is given by the Slater determinant of the radial single-particle eigenstates $\psi_n(k_j,x_j)$ as

$$\Psi(r_1,r_2,\dots,r_N)=\frac{1}{\sqrt{N!}}\det_{(n,j)=(0,1)}^{(N-1,N)}\psi_n(k_j,r_j),\qquad(1)$$

where r_j (k_j) is the coordinate (wave number) of the jth particle. For spherical symmetry ($l=0$) the eigenstates are given by the Bessel functions $\psi_n(k_j,r_j)=\frac{\sin(k_jr_j)}{k_jr_j}$. Consider now the same system, but in the presence of a static perturbation. Intuitively, the single-particle states are deformed and if the perturbation is highly localized, the new states can be written asymptotically as $\psi'_n(k_j,r_j)\sim\frac{\sin(k_jr_j+\delta)}{k_jr_j}(1-\frac{r_j}{R})$, where δ is an s-wave phase shift and R is the radius of the spherical box. In turn, this leads to a modified state of the Fermi sea $|\Psi'\rangle$ and the overlap between the perturbed and unperturbed many-body states is given by $\nu=\langle\Psi'|\Psi\rangle=\det[A_{nm}]$, where $A_{nm}=\int\psi_n(r)\psi'_m(r)dr$. When evaluating the overlap integral, one finds $\nu\propto N^{-\frac{\alpha}{2}}$, with $\alpha=\frac{2\delta^2}{\pi^2}$, so that $\nu\to0$ for N and/or δ sufficiently large [12].

While Anderson's original work involved stationary states, the creation of a perturbed many-body state is, in general, a time-dependent process. The dynamical theory of the OC was developed by Nozières and De Dominicis [13], who subsequently calculated the transient response of a Fermi sea after the *sudden* switching on of a core hole in a metal. A direct manifestation of the OC can then be observed in the single-particle spectrum of the Fermi gas:

$$A(\omega)=2\mathrm{Re}\int_{-\infty}^{\infty}dt\,e^{i(\omega-\omega_T)t}\nu(t),\qquad(2)$$

where $\nu(t)=\langle\Psi|e^{i\hat{H}t}e^{-i\hat{H}'t}|\Psi\rangle$ is the propagator of the core hole's retarded Green's function at zero temperature,

$$G(t)=-ie^{-it\omega_T}\Theta(t)\langle\Psi|e^{i\hat{H}t}e^{-i\hat{H}'t}|\Psi\rangle.\qquad(3)$$

Here $\Theta(t)$ is the Heaviside step function and $|\Psi\rangle$ is the initial equilibrium state of the Fermi system, governed by the Hamiltonian $\hat{H}$. The threshold frequency for the creation of the hole in the valence band of the metal is given by ω_T and the subsequent evolution of the Fermi sea in the presence of the impurity is given by $\hat{H}'$ [11]. In the absence of the hole, the single-particle spectrum of the homogeneous noninteracting Fermi gas is a Dirac δ function peaked at the Fermi energy. Experimentally, it was shown using x-ray spectroscopy that the occurrence of the OC broadens this spectrum, thus signaling a dramatic change in the fundamental excitations of the system.

The evaluation of the Green's function in Eq. (3) now amounts to calculating the overlap $\nu(t)$ between the perturbed and unperturbed time-dependent many-body wave functions. However, given that

$$\nu(t) = \langle\Psi|e^{i\hat{H}t}e^{-i\hat{H}'t}|\Psi\rangle = \det[A_{nm}(t)], \quad (4)$$

with $A_{nm}(t) = \int \psi_n'(x,t)\psi_m(x,t)dx$, this reduces to calculating the overlap of the time-dependent single-particle states.

III. SYSTEM-ENVIRONMENT MODEL

In what follows we demonstrate how the physics of the OC influences the dynamics of a single auxiliary two-level system which is coupled to the environment of a noninteracting Fermi gas in a harmonic trap. As our system we choose a highly localized neutral atom [14,15], whose relevant two levels, $|g\rangle$ and $|e\rangle$, are assumed to be separated by the energy $\hbar\Omega$, so that the free Hamiltonian reads $H_s = \frac{\hbar\Omega}{2}(|e\rangle\langle e| - |g\rangle\langle g|)$. The environment is described by

$$\hat{H} = \int \hat{\Psi}^\dagger(x)\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}m\omega^2x^2\right)\hat{\Psi}(x)dx, \quad (5)$$

where $\hat{\Psi}^\dagger(x)$ is the fermionic field creation operator. Our choice of a one-dimensional system is sufficient to demonstrate the fundamental effects we are interested in. At low enough temperatures, s-wave scattering is the dominating interaction process between the Fermi gas and the atom. For simplicity but without affecting the generality of our discussion, we assume that only $|e\rangle$ has a finite (positive) s-wave scattering length, while $|g\rangle$ does not interact with the environment. Assuming that a confining potential strongly localizes the impurity's wave function, so that its kinetic energy can be neglected, we are led to the following interaction Hamiltonian for the gas:

$$\hat{H}_I = \kappa \int \hat{\Psi}^\dagger(x)\delta(x-d)\hat{\Psi}(x)dx, \quad (6)$$

where we have used the standard pseudopotential approximation for the scattering interaction. The scattering potential only acts at position d in the gas, with a strength κ that can be related to the previously mentioned scattering phase shift δ [16]. In this work we specifically focus on the case $d = 0$. The analogy with the situation typically considered in the context of Anderson's OC theory should now be apparent: the localized spatial interaction of the impurity with the ultracold gas plays a role analogous to the interaction of the core hole with the rest of the electrons in a metal. A key point to stress is that here, in contrast to the case of a metal, we have typically a far smaller number of particles in the environment, which could in principle compromise the observability of the OC effects. However, ultracold atomic systems allow for the possibility to tune the s-wave scattering length to an arbitrarily large value by means of a Feshbach resonance, thus compensating for the lack of particles participating in such dynamics. We show that this offers the possibility of observing the OC effect even in the mesoscopic domain.

Let us start by assuming that, at time $t < 0$, the atom is prepared in $|g\rangle$ with the Fermi gas in its ground state $|\Psi\rangle$. The collective state of the hybrid system can be written as $|\Phi\rangle = |g\rangle \otimes |\Psi\rangle$. At $t = 0$, a properly set interaction between the atom and a classical laser field prepares the former in $(|g\rangle + |e\rangle)/\sqrt{2}$ and the perturbed Fermi sea evolves according to $\hat{H}_I$, driving the overall system into a correlated state of the form

$$|\Phi'\rangle = (|g\rangle \otimes e^{-i\hat{H}t}|\Psi\rangle + |e\rangle \otimes e^{-i(\hat{H}+\hat{H}_I)t}|\Psi\rangle)/\sqrt{2}. \quad (7)$$

The state of the environment now comprises the atomic states $|\Psi_g'(t)\rangle = e^{-i\hat{H}t}|\Psi\rangle$, associated with the noninteracting microscopic state $|g\rangle$, and $|\Psi_e'(t)\rangle = e^{-i(\hat{H}+\hat{H}_I)t}|\Psi\rangle$, which results from the scattering mechanism. The time-dependent density matrix of the impurity $\rho_s(t)$ can straightforwardly be evaluated by tracing out the environment and one immediately finds that the coherences of the reduced state are proportional to the scalar product:

$$\langle\Psi_g'(t)|\Psi_e'(t)\rangle = \langle\Psi|e^{i\hat{H}t}e^{-i(\hat{H}+\hat{H}_I)t}|\Psi\rangle = \nu(t). \quad (8)$$

The equivalence with the time propagator $\nu(t)$ highlighted in Eq. (4) proves a direct link between the decoherence of an impurity in a fermionic environment and the phenomenon of Anderson's OC. Furthermore, $|\nu(t)|^2$ is the so-called Loschmidt echo $L(t)$ [17,18], a widely used tool in the quantitative study of decoherence processes due to dynamical many-body environments [19,20].

The above argument holds for situations in which the Fermi gas is initially prepared in a pure state. However it is often the case that the gas has a thermal component and its quantum state is mixed. In this case, the state of the environment at $t = 0$ is described by a density matrix and we find the overlap to be

$$\nu(t) = \sum_{n,m} C_n|\Lambda_{m,n}|^2\, e^{-i\Delta_{m,n}t}, \quad (9)$$

with $\Delta_{m,n} = E_m' - E_n$ being the energy difference of two many-body states, $\Lambda_{m,n}$ being the overlap of two excited many-body states of the system, and $C_n = e^{-E_n/k_BT}/Z$ (here Z is the partition function, T is the temperature, and k_B the Boltzmann constant; see Appendix for further details). In the following, for the sake of clarity, our results are presented in natural units. Energies are correspondingly scaled in terms of $\hbar\omega$, lengths in terms of the harmonic trap length $l_0 = \sqrt{\hbar/m\omega}$, and time in units of the inverse trapping frequency ω^{-1}. From this, it follows that κ is scaled in units of $l_0/\hbar\omega$.

Given the formal connection between $\nu(t)$ and the impurity's dynamics, we can quantify the degree of entanglement within the state in Eq. (7) by means of the von Neumann entropy $S(t) = -\sum_i \lambda_i(t)\log_2\lambda_i(t)$, where $\lambda_i(t)$ are the time-dependent eigenvalues of $\rho_s(t)$, the reduced state of the impurity only. The time-dependent von Neumann entropy is shown in Fig. 1 for systems with different particle number and two different values of the interaction strengths. If the interaction energy is at or above the Fermi energy, as in Fig. 1(a), it can be seen that, after the interaction is switched on, the coupled system evolves into a fully entangled state ($S = 1$).

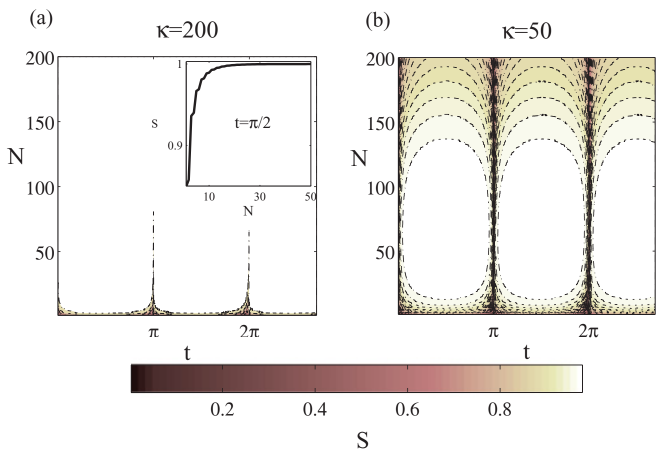

FIG. 1. (Color online) (a) Time-dependent von Neumann entropy as a function of the particle number for $\kappa = 200$ in units of $l_0/(\hbar\omega)$. The inset shows a time slice at $t = \pi/2$, in units of the inverse trapping frequency, ω^{-1}. (b) Time-dependent von Neumann entropy as a function of the particle number for $\kappa = 50$.

This indicates that the many-particle state, created after the disturbance is switched on, almost immediately becomes orthogonal to the initial equilibrium state, as demanded by the catastrophe effect. It is remarkable to note that, already for a small number of particles, the state of the atomic gas is not separable at any time following the quench. The inset of Fig. 1(a) shows the entropy at a fixed moment in time, clearly indicating that the orthogonal state is already reached for a mesoscopic number of particles [$N \approx 15$ in Fig. 1(a)]. An interesting point to make is that, provided one has the ability to tune the coupling to a large value, the qualitative features shown above remain similar for even smaller Fermi environments. This is in contrast to the case of a metal where large particle numbers and relatively weak scattering strengths are in order. Figure 1(b) shows the von Neumann entropy for the weaker value $\kappa = 50$ of the scattering strength. In this case, full orthogonality is established only for larger particle numbers and a maximally entangled state is achieved around the resonance at $N = 50$.

IV. RESULTS

Let us now show how the properties of our complex system-environment state can be directly inferred by looking at the system state only [21]. In particular, we suggest to use Ramsey interferometry on the atom to measure the time-dependent overlap $\nu(t)$ and, from it, the single-particle spectrum of the Fermi gas. As discussed previously, this spectrum is known to be strongly affected by the OC [11]. Spectral information will therefore provide a definite signature of the OC which can be easily compared to the original experiments in metals. Our scheme is based on a protocol put forward in Ref. [22]: after the creation of the entangled atom-environment state, we allow the hybrid system to freely evolve for a time t. During this time, a phase-shift gate is applied to the atom, such that $|g\rangle \rightarrow |g\rangle$ and $|e\rangle \rightarrow e^{i\phi}|e\rangle$, giving the state of the overall system as $|\Psi(t)\rangle = (|g\rangle \otimes e^{-i\hat{H}t}|\Psi\rangle + e^{i\phi}|e\rangle \otimes e^{-i(\hat{H}+H_I)t}|\Psi\rangle)/\sqrt{2}$. Using again a classical field, the state of the atom can be changed

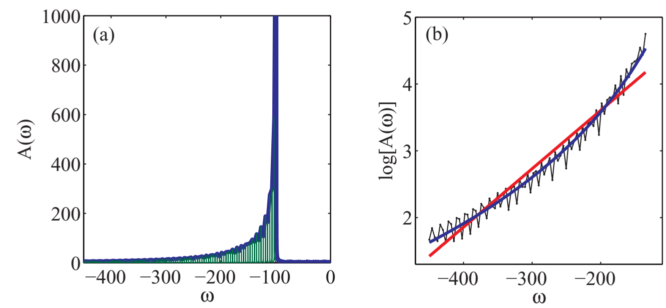

FIG. 2. (Color online) Spectral function. (a) The spectral function $A(\omega)$ for $N = 100$ and $\kappa = 100$ in units of $l_0/(\hbar\omega)$. The continuous blue curve is the envelop of the otherwise discrete spectrum. The asymmetry is clearly visible. (b) Comparison of the spectrum (black line) with an exponential (red line) and a power-law [continuous blue (gray) line] fit.

as $|g\rangle \rightarrow (|g\rangle + |e\rangle)/\sqrt{2}$ and $|e\rangle \rightarrow (|g\rangle - |e\rangle)/\sqrt{2}$, and we finally measure the probability for the atom to be found in $|g\rangle$, which reads

$$P_g(t,\phi) = [1 + \cos(\phi)\nu_R(t) - \sin(\phi)\nu_I(t)]/2, \qquad (10)$$

where $\nu(t) = \nu_R(t) + i\nu_I(t)$ is the overlap entering the OC theory in Eq. (8). Using, for example, resonance fluorescence techniques, $P_g(t,\phi)$ can be measured for various values of the phase ϕ and thus fitted to Eq. (10), from which the overlap function $\nu(t)$ can be extracted as a fitting parameter. The single-particle spectrum $A(\omega)$ can then be obtained from the Fourier transform on the time-dependent overlap $\nu(t)$, according to Eq. (2). A typical spectrum is shown in Fig. 2 and one can see that $A(\omega)$ exhibits features almost identical to those observed via x-ray absorption of metals [11]. First of all, the main peak has a finite height at the Fermi energy, which implies that the transition probability is not diverging anymore. Moreover, the spectrum is asymmetric with respect to the mean peak, showing that the "emission" and "absorption" rates are different at ω and $-\omega$, respectively. Physically, this means that the system is out of equilibrium and is trying to settle into a new state. Figure 2(b) shows that the tails of the spectrum decay following a power law [continuous blue (gray) line] instead of an exponential one (red line), as would be expected for a system at equilibrium.

Let us briefly discuss the influence of a finite temperature and a finite-sized impurity, which may lead to blurring of the OC effect. One can see from Eq. (9) that the effect of finite temperature is twofold: on one hand it introduces new frequencies to the system, since now $\Delta_{m,n} \neq 0$ even for $n \neq 0$. This is manifested in a broadening of the spectrum. On the other hand exponential factors are introduced, namely, $C_n = e^{-E_n/k_B T}/Z$, so that the heights of the peaks are exponentially suppressed (see Fig. 3). This leads to a loss of the characteristic power law for the spectrum tails and therefore requires us to work at temperatures which are well below the Fermi energy in order to observe the OC effects.

To investigate the effect of a finite size of the impurity one can replace the δ-like interaction in Eq. (6) with a Gaussian potential with a characteristic width σ. Since the δ-like interaction only affects the even-parity wave functions of the system, leaving the odd-parity ones unchanged [23], the

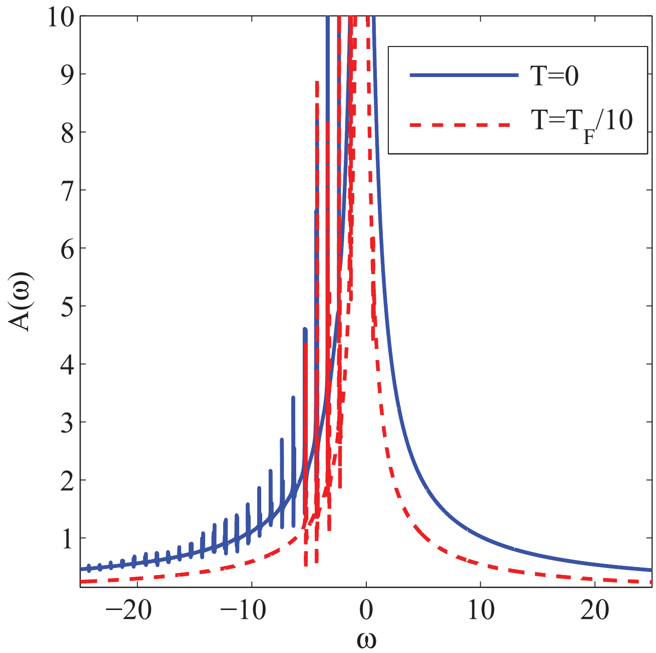

FIG. 3. (Color online) The effect of finite temperature for a system of $N=7$ particles is shown with $\kappa=5$ in units of $l_0/(\hbar\omega)$. The suppression given by the exponential coefficient appearing in Eq. (9) is evident as the spectrum for $T\neq 0$ has an amplitude lower than the one for $T=0$.

main effect of a finite width is a modification of all eigenstates of the system. However, this does not change any of the physics discussed above and we have numerically verified that the OC effects remain visible.

As remarked above, the OC also manifests itself in an intriguing way in the Loschmidt echo $L(t)$, which describes the environmental sensitivity to a generic perturbation. The echo corresponding to Fermi gases of various N and $\kappa=200$ is shown in Fig. 4. As expected from our previous considerations, it decreases rapidly once the system size is above a moderate number and Fig. 4(b) shows the behavior of $L(t)$ at finite time as a function of N, highlighting its decreasing trend as the OC conditions are approached. The revivals in the echo are located at the times corresponding to the inverse of

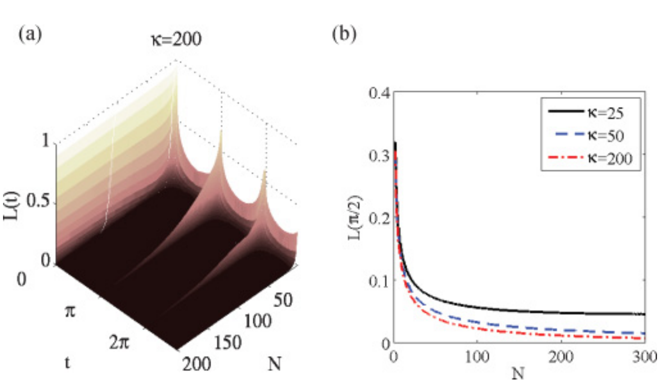

FIG. 4. (Color online) (a) Loschmidt echo $L(t)$ as a function of the particle number N for $\kappa=200$ in units of $l_0/(\hbar\omega)$. (b) Behavior of $L(t)$ at time $t=\pi/2$ for $\kappa=25$, 50, and 100. The time axis is in units of the inverse trap frequency ω^{-1}.

the particle-hole-like resonances $E'_n - E'_0$ in the (perturbed) Fermi gas. By measuring the dynamical overlap $\nu(t)$ one can thus probe the single-particle excitations of the system.

Let us finally comment on a possible experimental implementation of our model. A recent experiment has demonstrated a species-selective dipole potential to trap and tightly localize an individual impurity (^{40}K) in a quasi-one-dimensional gas of on average 180 atoms (^{87}Rb) [24]. Although the atoms used in this specific experiment are bosonic, there is no reason such a setup cannot be used for fermionic samples. Alternatively, one can use a confinement-induced resonance to drive the atoms into the fermionized Tonks-Girardeau regime, where the Loschmidt echo has recently been shown to be equivalent to that of noninteracting fermions [25,26]. A magnetic Feshbach resonance can then allow one to adjust the scattering length between the two species to drive the quench. Therefore the technology for direct experimental verification of our proposal is available.

V. CONCLUSION

In conclusion we have shown that the OC plays an important role in the dynamics of coupled systems consisting of an ultracold atomic gas interacting with a single two-level system. In this respect, we have quantitatively linked the OC to the mechanism of decoherence undergone by the two-level system and signaled by the Loschmidt echo. It should be stressed that, besides pointing out the exciting possibility to explore the OC in a realistic setup radically different from the one originally envisaged by Anderson, the scenario addressed here demonstrates that the measurement of a single impurity allows one to obtain highly nontrivial information about the behavior of a complex environment. Such information is invaluable for tasks of environmental characterization and interaction identification, thus suggesting an ideal probe for testing ultracold atomic gases. In this sense, our proposal stands as the ultracold counterpart of the hallmark experiments in the x-ray absorption spectrum of metals while demonstrating, at the same time, the appropriateness of auxiliary quantum systems as probes for ultracold quantum gases.

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APPENDIX A: THE STATIC ANDERSON OVERLAP FOR THE MANY-BODY EIGENSTATES OF NONINTERACTING FERMIONS

In order to calculate the dynamical overlap, $\nu(t)$, between two thermal states of noninteracting fermions, we first need to calculate the corresponding many-body overlaps of the excited many-body states. Let us consider the two single-particle Hamiltonians $\hat{h}$ and $\hat{h}'$, where $\hat{h}'$ is the system $\hat{h}$ with a static perturbation. The respective eigenproblems are

$$\hat{h}|\psi_i(x)\rangle = \epsilon_i|\psi_i(x)\rangle, \quad \hat{h}'|\psi'_j(x)\rangle = \epsilon'_j|\psi'_j(x)\rangle. \tag{A1}$$

The corresponding systems of N *noninteracting* fermions are described by many-body Hamiltonians which are simply sums of the single-particle ones, $\hat{H} = \sum_l^N \hat{h}_l$ and $\hat{H}' = \sum_l^N \hat{h}'_l$. The eigenproblems are

$$\hat{H}|\Psi_n(X)\rangle = E_n|\Psi_n(X)\rangle, \quad \hat{H}'|\Psi'_m(X)\rangle = E'_m|\Psi'_m(X)\rangle, \tag{A2}$$

where the sum is over the occupied single-particle states and $X = x_1,x_2,\ldots,x_N$ is the generalized position coordinate. The many-body energies are given by sums of the corresponding single-particle states $E_n = \sum_i \epsilon_i$ and $E'_n = \sum_j \epsilon'_j$. The many-body wave functions of the spectrum are easily obtained as Slater determinants:

$$\Psi_n(X) = \frac{1}{N}\sum_P \mathrm{sgn}(P)\ \psi_{P(1)}(x_1)\psi_{P(2)}(x_2)\cdots\psi_{P(N)}(x_N),$$
$$\Psi'_m(X) = \frac{1}{N}\sum_{P'} \mathrm{sgn}(P')\ \psi'_{P'(1)}(x_1)\psi'_{P'(2)}(x_2)\cdots\psi'_{P'(N)}(x_N), \tag{A3}$$

where $P(P')$ is the permutation over the indices labeling the occupied states. The general overlap between any two many-body wave functions is given by

$$\Lambda_{m,n} = \langle\Psi'_m(X)|\Psi_l(X)\rangle = \int dX[\Psi'_m(X)]^*\Psi_n(X) = \sum_{P,P'} \mathrm{sgn}(P)\mathrm{sgn}(P')\Pi_l A_{P(l),P'(l)}, \tag{A4}$$

where $A_{i,j} = \int dx[(\psi'_j(x)]^*\psi_i(x)$ is the overlap between two single-particle states. Assuming that only $N_1 \geqslant N$ and $N_2 \geqslant N$ single-particle states can be occupied among the two sets $\{|\psi_i(x)\rangle\}$ and $\{|\psi'_j(x)\rangle\}$, respectively, the elements of the matrix Λ are the minors of order N of the $N_2 \times N_1$ matrix A made of all the possible occupied single-particle states' overlaps. For the ground state this result is identical to that in Anderson's original paper [12]. All that is needed to construct the overlaps are the eigensolutions of the corresponding one-particle problems. In this work we are considering fermions in a one-dimensional harmonic trap which has a well-known eigenspectrum. The "perturbed" system that we are considering is that of the harmonic trap with a δ-function pseudopotential at the trap center. The solution for such a system is known and maybe found in Ref. [23] and references therein.

APPENDIX B: CALCULATION OF THE DYNAMICAL OVERLAP FOR MIXED STATES

We are interested in generalizing the dynamical overlap of a Fermi gas, following a perturbation induced by scattering from an impurity, to the case when initially the Fermi gas is in thermal equilibrium. The overlap for mixed states is given by

$$\nu(t) = \mathrm{Tr}[\hat{U}'(t)\hat{\rho}\hat{U}(-t)], \tag{B1}$$

where $U^{(\prime)}(t) = e^{-\iota H^{(\prime)}t}$ is the corresponding unitary evolution operator which generates dynamics in an unperturbed (perturbed) system. In the case of an initial pure state one recovers $\mathrm{Tr}[\hat{U}'(t)|\Psi\rangle\langle\Psi|\hat{U}(-t)] = \langle\Psi|\hat{U}(-t)\hat{U}'(t)|\Psi\rangle = \langle\Psi(t)|\Psi'(t)\rangle$. Using the generalized overlaps in the previous section one may derive a formula which holds for a general class of initial mixed states,

$$\begin{aligned}\mathrm{Tr}[\hat{U}'(t)\hat{\rho}(0)\hat{U}(-t)] &= \sum_n \langle\Psi_n(X)|\hat{U}'(t)\hat{\rho}(0)\hat{U}(-t)|\Psi_n(X)\rangle \\ &= \sum_n \langle\Psi_n(X)|\left(\sum_m |\Psi'_m(X)\rangle\langle\Psi'_m(X)|\right) \\ &\quad\times \hat{U}'(t)\hat{\rho}(0)\hat{U}(-t)|\Psi_n(X)\rangle \\ &= \sum_{n,m}\langle\Psi_n(X)|\Psi'_m(X)\rangle\langle\Psi'_m(X)|\hat{U}'(t) \\ &\quad\times \hat{\rho}(0)\hat{U}(-t)|\Psi_n(X)\rangle \\ &= \sum_{n,m} e^{-\iota(E'_m-E_n)t}\langle\Psi_n(X)|\Psi'_m(X)\rangle \\ &\quad\times \langle\Psi'_m(X)|\hat{\rho}(0)|\Psi_n(X)\rangle.\end{aligned} \tag{B2}$$

Let us assume that the initial state of the gas is in thermal equilibrium and can be described in the framework of the canonical ensemble, such that

$$\nu(t) = \mathrm{Tr}[\hat{U}'(t)\hat{\rho}\hat{U}(-t)] = \sum_{n,m} C_n|\Lambda_{m,n}|^2\, e^{-i\Delta_{m,n}t} \tag{B3}$$

with $C_n = e^{E_n/k_BT}/Z$ and we have set $\Delta_{m,n} = E'_m - E_n$. In the limit $k_B\,T \ll \mu_F$ we get $\nu(t) = \mathrm{Tr}[\hat{U}'(t)\hat{\rho}(0)\hat{U}(-t)] = \sum_m |\Lambda_{m0}|^2 e^{-\iota\Delta_{m0}t}$. We note that the Loschmidt echo is given by $|\nu(t)|^2 = \sum_{m,n}|\Lambda_{m0}|^2|\Lambda_{n0}|^2 e^{-\iota\Delta'_{mn}t}$, where $\Delta'_{mn} = E'_m - E'_n$ is the energy difference among the fermion-hole pairs in the two dispositions m and n as measured from the new Fermi energy E'_0.

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