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Optical vortex interaction and generation via nonlinear wave mixing

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Optical vortex beams are made to interact via degenerate two-wave mixing in a Kerr-like nonlinear medium. Vortex mixing is shown to occur inside the medium, leading to exchange of topological charge and cascaded generation of vortex beams. A mean-field model is developed and is shown to account for the selection rules of the topological charges observed after the wave-mixing process. Fractional charges are demonstrated to follow the same rules as for integer charges.

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Optical vortices, or wave-front dislocations, have been identified as the singular points where the field goes to zero and around which the phase screws up as an n armed spiral, with n the topological charge [1–3]. When appearing in a large number, as after the propagation through a distorting medium, such phase singularities, also called topological defects, have been seen as disturbances imposing severe limitations to aberration correction systems [4]. Their statistical properties have been investigated also in nonlinear optical systems and shown to provide the scaling laws associated with the route toward space-time chaos [5]. More recently, the presence of single, or a few, optical vortices, appearing as phase singularities in low-order Gauss-Laguerre beams, has been revisited in view of useful applications, as the exchange of angular momentum between light and matter [6], realization of optical tweezers [7,8], quantum computation [9], and improvement of astronomical imaging [10].

Up to now, the controlled generation of optical vortex beams has been mainly realized by using linear methods, including the synthesis through holographic masks [11,12], the deformation of segmented adaptive mirrors [13], spiral glass plates [14] or pre-imposed director orientation in liquid-crystal samples, so-called, q-plates [15], or from micron-sized liquid-crystal droplets [16]. On the other hand, nonlinear vortex interaction has been investigated in a few experimental situations, namely, second-harmonic generation (SHG) [17] and parametric down-conversion in solid-state crystals [18], Raman-resonant four-wave mixing (4WM) in hydrogen gas [19], and nondegenerate 4WM in Rb atomic vapors [20]. Except for the parametric down-conversion, angular momentum conservation has been shown to be satisfied in the other cases, leading to selection rules of the type $l_{\mathrm{SHG}} = 2l_{\mathrm{in}}$ for SHG, where l_{SHG} is the charge of the frequency doubled beam and l_{in} that of the input beam; $l_A = 2l_P - l_S$ for resonant 4WM, where l_A is the charge of the anti-Stokes beam, l_P that of the pump, and l_S that of the signal; and $l_S = l_F + l_B - l_P$, where l_F and l_B are the charges of, respectively, the forward and the backward beam, for the nondegenerate 4WM. From the theoretical point of view, nonlinear vortex interaction in wave-mixing processes has been predicted to induce a cascaded generation of vortices, leading to fundamental effects such as the generation of helical soliton beams in nonlinear optics [21] or Bose-Einstein condensation in two-dimensional wave turbulence [22].

Here, we show that optical vortex beams can be created and controlled by realizing degenerate two-wave mixing processes in a Kerr-like nonlinear (NL) medium. As a NL medium we use a liquid-crystal light valve (LCLV), for which a high Kerr-like nonlinearity and efficient wave-mixing processes have been already demonstrated [24,25]. The principle scheme of the vortex beam interaction is shown in Fig. 1(a), whereas Fig. 1(b) displays the corresponding experimental snapshots. Two optical vortex beams, with respective topological charges l_a and l_b and propagation wave vectors $\vec{k}_a$ and $\vec{k}_b$, are synthesized via a holographic mask and, then, made to interact in the NL medium. There, they mix up by inducing a dynamical refractive index grating, from which they are themselves diffracted. The NL medium is thin, $d = 15\ \mu$m is the thickness of the liquid-crystal layer, so that several diffracted order beams are obtained at the output, each one carrying a new topological charge. We observe that while the charge is the same along the respective propagation directions of the input beams (0 and −1 output orders), it changes on the outer orders. In particular, on the +1 and −2 orders, the charge satisfies the selection rules $l_{+1} = 2l_a - l_b$ and $l_{-2} = 2l_b - l_a$, respectively. From the theoretical side, we model the wave mixing between the beams carrying the phase singularities in the mean-field approximation, providing the selection rules for the topological charge on each m output order beam. The theoretical predictions are confirmed by the experimental observations, both for integer and fractional charges of the interacting beams.

The whole experimental setup is represented in Fig. 2. An input laser beam, $\lambda_0 = 532$ nm and intensity $I = 3$ mW/cm^2, is divided in two beams of equal intensity, both sent to a spatial light modulator (SLM), where the input vortex beams are synthesized. Polarizers and half-wave plates are used to select the polarization of the input beams to be linear and parallel to the liquid-crystal nematic director. A lens system is used to image the beams at the output of the SLM onto the LCLV, and another lens imaging system allows the recording of the output beam onto a CCD camera, as displayed in the bottom inset of Fig. 2. Optical vortex beams of desired topological charge $l_{a,b}$ are obtained by programing the SLM with suitable holographic masks [11]. The incoming beams are enlarged to a transverse size diameter of 1.3 cm, whereas the SLM transmission function is programed to be the interference pattern of a plane wave with a helical phase structure of

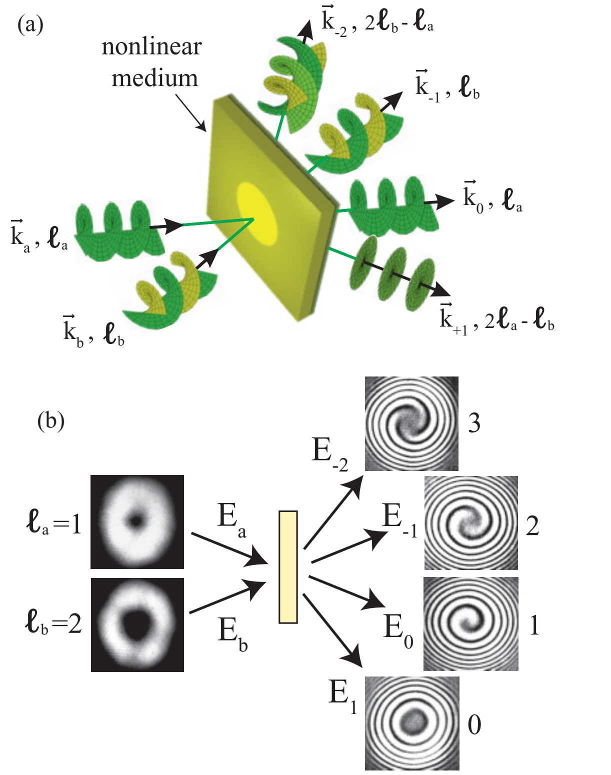

FIG. 1. (Color online) (a) Principle scheme of the vortex interaction for $l_a = 1$ and $l_b = 2$, and (b) experimental snapshots showing the intensity of the input beams and the phase profiles of the corresponding output beams.

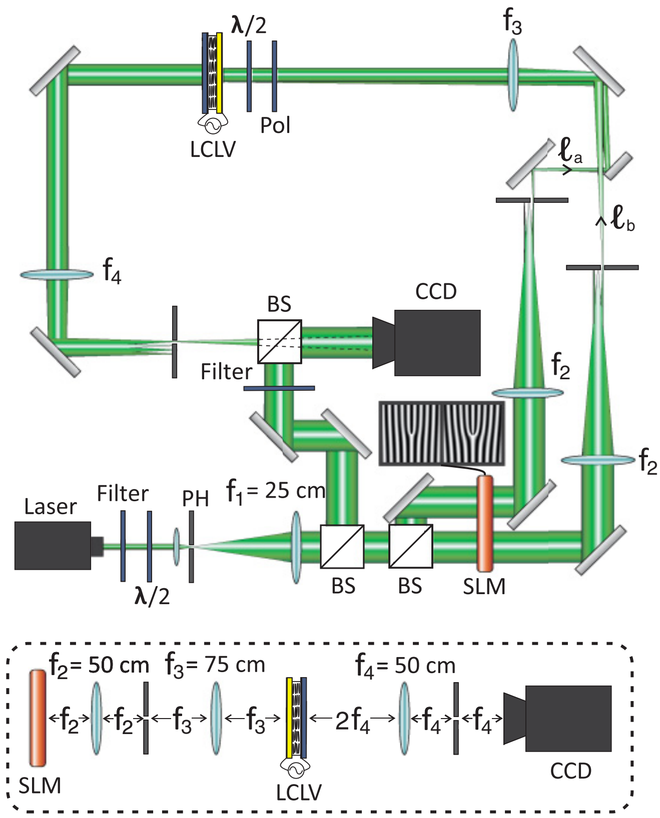

FIG. 2. (Color online) Experimental setup: the input laser is divided into two beams, each passing through the spatial light modulator (SLM) where the input vortex beams E_a and E_b are synthesized through holographic masks; the two beams are then sent to interact in the LCLV; PH, pinhole; BS, beam splitter. The bottom inset shows the self-imaging system before and after the LCLV.

the type $e^{il_{\mathrm{hol}}\varphi}$, that is, $f(x,\varphi) = C[1 + \cos(l_{\mathrm{hol}}\varphi - 2\pi x/\Lambda)]$, where C is a normalization constant and Λ the fringe spacing of the hologram. In correspondence with the phase singularity, the hologram has a pitchfork structure from which the diffraction of a plane wave provides a series of scattered orders, the mth order being at an angle $\sin\theta_m = m\lambda/\Lambda$ and carrying a topological charge ml_{hol}, with λ the wavelength of the input beam. Moreover, in order to get a better diffraction efficiency on the $m > 0$ orders, we have used the equivalent function of a blazed grating [23], that is, $f(r,\varphi) = \mathrm{mod}\ [l_{\mathrm{hol}}\varphi - (2\pi N_f/L)x, 2\pi]$, where L is the length of the grid of the SLM (in pixels) and N_f is the total number of fringes. The self-imaging system before the LCLV is used to minimize diffraction effects. Indeed, topological charges with $|l| > 1$ are structurally unstable upon propagation, tending to separate in $|l|$ unitary topological charges of sign sign(l) [26]. Once the vortex beams E_a and E_b are prepared, the wave mixing is performed by sending them to interact in the LCLV. E_a and E_b have diameters of 3 mm and intensities of (116 ± 3) μW/cm^2 and, respectively, (103 ± 3) μW/cm^2.

Examples of vortex interaction in the case of integer input charges are shown in Fig. 1(b) and in Fig. 3. In Fig. 3, the displayed snapshots, cases $l_a = 0$, $l_b = 1$ [Fig. 3(a)] and $l_a = 1$, $l_b = -2$ [Fig. 3(b)], represent the input beams (left panels) together with the corresponding -2 and $+1$ output order beams (right panels). The phase profiles are obtained by interfering the vortex beam with a homogeneous reference beam that has passed through a long focal length (50 cm) lens, hence, it has a slightly curved wave front. The resulting concentric fringe pattern allows an easy visualization of the optical vortex as an l-armed spiral starting from the center, l being the topological charge and the direction of the spiral rotation being the sign. From the interferometric measurements it appears clearly that the topological charges mix up during the wave-mixing process. On each output order, new charges are observed that follow the selection rules fixed by the vortex interaction inside the medium. On the examples shown in Fig. 3 it can be easily verified that for $l_a = 0$, $l_b = 1$ we obtain $l_{-2} = 2$, $l_{+1} = -1$; while for $l_a = 1$, $l_b = -2$ we get $l_{-2} = -5$, $l_{+1} = 4$.

Similar selection rules are observed also for input beams with fractional charges. A fractional charge can be constructed in the near field by imposing a half-fringe phase slippage over one half of the interferometric hologram. However, it is unstable upon propagation in free space, because it transforms into a charge with a value equal to its nearest integer plus an infinite array of unitary charges of alternate sign [26,27]. Correspondingly, the field amplitude goes to zero, generating a characteristic black line in the intensity profile. Examples of the interaction between vortex beams with fractional charges are shown in Fig. 4 for $l_a = 0$, $l_b = +1/2$ in Fig. 4(a) and $l_a = +1$, $l_b = +1/2$ in Fig. 4(b). The output charges are $l_{-2} = 1$, $l_{+1} = -1/2$ and $l_{-2} = 0$, $l_{+1} = +3/2$,

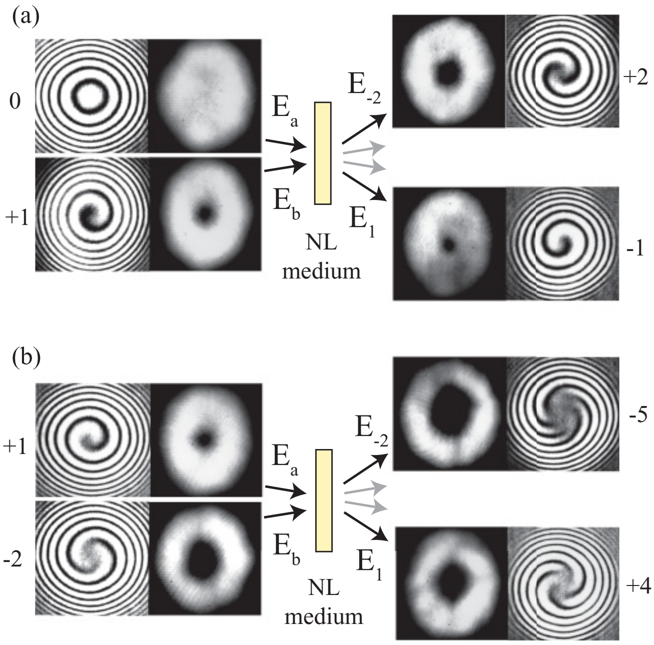

FIG. 3. (Color online) Vortex interaction for integer input charges. Left: input beams E_a and E_b (phase, intensity); right: output beams E_1 and E_{-2} (intensity, phase) with new topological charges; (a) $l_a = 0$, $l_b = +1$ and $l_{-2} = -1$, $l_{+1} = +2$; (b) $l_a = +1$, $l_b = -2$ and $l_{-2} = -5$, $l_{+1} = +4$.

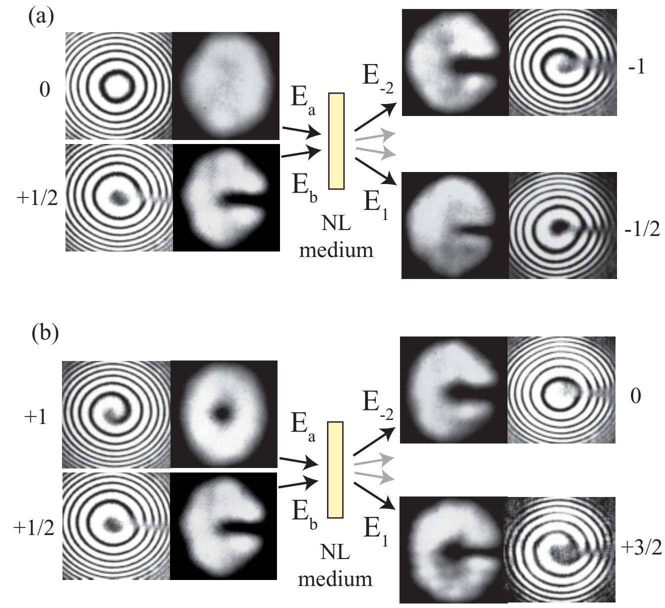

FIG. 4. (Color online) Vortex interaction for fractional input charges. Left: input beams E_a and E_b (phase, intensity); right: output beams E_1 and E_{-2} (intensity, phase) with new topological charges; (a) $l_a = 0$, $l_b = +1/2$ and $l_{-2} = +1$, $l_{+1} = -1/2$; (b) $l_a = +1$, $l_b = +1/2$ and $l_{-2} = 0$, $l_{+1} = +3/2$.

respectively. In the second example, on the -2 order we obtain the cancellation of the topological charge. Even though this cancellation results clearly from the interference image, the near-field intensity profile still conserves the zeros of the field. These zeros disappear upon propagation and are totally absent in the far field.

Note that two-wave mixing in the LCLV can lead to optical amplification, when the intensity of one of the two beams, usually called the pump, is much higher than the intensity of the other beam, the signal to be amplified [25]. In the present set of experiments, for the purpose of better revealing the output selection rules, vortex mixing was performed with two beams of almost equal intensity. However, it is worth noting that vortex mixing can in general be performed in the classical pump-signal scheme, with the pump intensity much higher than the signal intensity. In particular, a zero topological charge pump could be used to amplify a weak vortex beam, as previously synthesized through a holographic mask or other methods often introducing intensity losses.

The theoretical description can be developed by considering the nonlinear Schrödinger equation [28]

$$\imath \frac{\partial A}{\partial z} = -\left(\frac{1}{2k_0 n_0} \nabla_\perp^2 + n_2 k |A|^2 \right) A, \tag{1}$$

where $k_0 = 2\pi/\lambda_0$ is the optical wave vector, A is the slowly varying amplitude of the optical field $E = Ae^{\imath(k_0 z - \omega t)}$, $n_0 = 1.6$ the constant part of the refractive index, and $n_2 = -7\ \text{cm}^2/\text{W}$ the Kerr-like coefficient of the LCLV [25]. The amplitude of the optical field before the interaction is the sum of the two vortex beams A_a and A_b, carrying, respectively, the topological charges l_a and l_b, that is, $|A(\vec{r}, z = 0)|^2 = |A_a|^2 + |A_b|^2 + 2A_a A_b \cos[\vec{K}_g \cdot \vec{r} + (\ell_b - \ell_a)\phi]$, the two beams induce a phase grating with spatial period $\Lambda = 2\pi/K_g$ and with a dislocation given by the difference $\ell_b - \ell_a$. In the usual experimental conditions, $\Lambda \gg d$, so the beam coupling can be treated in the Raman-Nath regime of diffraction; hence, the field at the exit of the NL medium can be analytically calculated and is given by [25]

$$E(\vec{r}, z = d) = \sum_{m=-\infty}^{+\infty} A_m e^{\imath(\vec{k}_m \cdot \vec{r} + \ell_m \phi - \omega t)} + \text{c.c.}, \tag{2}$$

that is, it is made up of a series of diffracted orders, the amplitude of each is given by $A_m = \imath^m [A_a J_m(\rho) + \imath A_b J_{m+1}(\rho)] e^{\imath k_0 d (n_0 + n_2 |A_a|^2 + n_2 |A_b|^2)}$, with J_m the Bessel function of the first kind and of order m and $\rho = 2k_0 d n_2 A_a A_b$, and each is characterized by the wave vector

$$\vec{k}_m = \vec{k}_a - m\vec{K}_g, \tag{3}$$

which expresses the momentum conservation, and by the topological charge

$$\ell_m = \ell_a - m(\ell_b - \ell_a), \tag{4}$$

which expresses the angular momentum conservation.

Equation (4) gives the selection rules on each output order, nicely accounting for the experimental observations. The 0 and -1 orders, propagating parallel to E_a and, respectively, E_b, maintain the same topological charge. On the other hand, the topological charge of the $+1$ and -2 orders is, respectively, $(2\ell_a - \ell_b)$ and $(2\ell_b - \ell_a)$, that is, during the vortex mixing the topological charge is exchanged on the diffracted beams. Experimentally, we have verified that the charge observed on the outer orders up to $m = \pm 3$ coincides with the theoretical predictions. Note, also, that the vortex interaction generates a cascade of topological charges, with multiple vortex beams at the output. For instance, if we consider the $+1$ and -2 orders, we have the same $l_a + l_b$ topological charge as at the input, but taking also the -2 and $+2$ orders, we get $2(l_a + l_b)$, that is, the double of the input charge.

Finally, we have verified that the diffusive term characterizing the NL medium response, and neglected in the mean-field approximation, does not influence the topological charge of the output orders. Indeed Eq. (1) should be coupled with a spatial relaxation equation for the refractive index $(1 - l_d^2 \nabla_\perp^2) n = n_0 + n_2 |E|^2$, where $l_d \sim 5\ \mu$m is the transverse diffusion length in the NL medium [25]. When inserted into Eq. (1), the above expression allows us to arrive at the same result as before, except that the shape of the vortex is slightly smoothed by a weak correction provided by the diffusion term. However, the output charge remains unaltered for all the output orders.

In conclusion, we have shown that vortex mixing can be performed in a Kerr-like nonlinear medium. The selection rules for the output charges are theoretically identified and experimentally demonstrated. The obtained cascade of topological charges is particularly interesting for vortex control applications, such as multiplication of the topological charge and generation of new vortex beams, whereas vortex beam amplification could be obtained by exploiting the gain feature of the wave-mixing process.

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