

Cooling of a mirror in cavity optomechanics with a chirped pulse

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We investigate the response of a harmonically confined mirror to an optical pulse in cavity optomechanics. We show that when the pulsed coupling strength takes the form of a chirped pulse, thermal fluctuations of the mirror can be significantly transferred to the cavity field. In addition, the frequency modulation of the pulse could enable a better cooling performance by suppressing the sensitivity of the dependence of detuning and pulse areas. Using numerical investigations, we find that the pulsed cooling is mainly limited by the cavity-field decay rate.

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I. INTRODUCTION

In cavity optomechanical systems [1–4], the cooling of a mechanical resonator is important to study the mechanical effects of light in the quantum domain [5–22]. In addition, the reduction of thermal noise generally is a requirement to implement various applications in quantum information relying on optomechanical couplings [23–26]. With the recent progress of cooling in cavity optomechanical systems [27–36], it is becoming possible to access quantum ground states. The cooling techniques, such as the resolved sideband cooling [27–29, 34–36], generally make use of a continuous light field that drives a mechanical resonator into a steady state of lower temperature. Recently, several authors have begun to explore the possibilities of cooling and manipulating the states of mirrors with optical pulses [37–40].

Motivated by the fact that chirped pulses can lead to an efficient population transfer in two-level systems [41–46], we examine how a chirped-pulse interaction can transfer thermal fluctuations from the mirror to the cavity field. As we shall discuss below, the correspondence between a two-level system and optomechanical systems [defined in Eq. (1)] can be established via the Heisenberg equations of motion of the linearized system under a rotating wave approximation (RWA). Therefore, analytical solutions known in two-level systems driven by chirped pulses can be applied here [46]. Owing to the frequency modulation in chirped pulses, population transfer can be made without the need for a high-precision control of detuning and pulse areas.

In this paper, we will first provide a formulation of the linearized quantum system driven by a general pulse. Then we will indicate how the mirror-field coupling can be shaped into a chirped form by using a proper time dependence of an external driving field. Specifically, we will study a class of chirped pulses proposed by Allen and Eberly for two-level systems [46]. Such a class of chirped pulses has analytic solutions for nondissipative systems, and depending on the chirped parameters, they describe both adiabatic and nonadiabatic transitions. Using numerical calculations, we include counter-rotating terms and dissipation effects, and we demonstrate that cooling can be achieved with the chirped-pulse coupling. In the case where the mechanical damping rate of the mirror is sufficiently small, the cooling performance is mainly limited by the cavity-field decay rate.

II. MODEL

Our model consists of a Fabry-Perot cavity formed by a fixed end mirror and a moving end mirror connected with a spring (Fig. 1). We consider a single cavity-field mode with a resonance frequency ω_c and creation (annihilation) operator a^\dagger (a). The moving mirror is treated as a quantum harmonic oscillator with a frequency ω_m and creation (annihilation) operator b^\dagger (b). Assuming the cavity is driven by an external field with a carrier frequency ω_L and a time-varying amplitude $\Omega(t)$, the Hamiltonian of the system (in a rotating frame with the frequency ω_L) is given by

$$H_S = \hbar \Delta_c a^\dagger a + \hbar \omega_m b^\dagger b - \hbar g a^\dagger a (b^\dagger + b) + \hbar \Omega(t) a^\dagger + \hbar \Omega^*(t) a, \quad (1)$$

where $\Delta_c = \omega_c - \omega_L$ is the detuning and g is the radiation-pressure coupling strength.

To treat the damping and noise in our model, we consider the system linearly coupled to oscillator baths. Under the Markovian approximation and neglecting counter-rotating terms in system-bath coupling, the quantum Langevin equations for the operators a and b are given by

$$\dot{a} = -i \Delta_c a + i g a (b^\dagger + b) - i \Omega(t) - \frac{\gamma_c}{2} a + a_{in}, \quad (2a)$$

$$\dot{b} = -i \omega_m b + i g a^\dagger a - \frac{\gamma_m}{2} b + b_{in}, \quad (2b)$$

where γ_c (γ_m) is the cavity-field (mirror-motion) decay rate, a_{in} is the vacuum radiation noise operator for the cavity, and b_{in} is the mechanical noise operator for the mirror. Both a_{in} and b_{in} have zero mean values and they are characterized by the correlation functions $\langle a_{in}(t) a_{in}^\dagger(t') \rangle = \gamma_c \delta(t - t')$, $\langle a_{in}^\dagger(t) a_{in}(t') \rangle = 0$, $\langle b_{in}(t) b_{in}^\dagger(t') \rangle = \gamma_m (\bar{n}_m + 1) \delta(t - t')$, and $\langle b_{in}^\dagger(t) b_{in}(t') \rangle = \gamma_m \bar{n}_m \delta(t - t')$, where $\bar{n}_m = [\exp(\hbar \omega_m / k_B T_m) - 1]^{-1}$ is the average thermal excitation number of the mirror at temperature T_m . In this paper, we will investigate the regime with $\omega_m \gg \gamma_m$. This is a regime where the Markovian approximation for the mirror noise can be justified [18].

III. LINEARIZED SYSTEM AND FORMAL SOLUTION

By writing $o = \langle o \rangle + \delta o$ ($o = a, b$) and assuming the fluctuations are small ($|\langle o \rangle|^2 \gg \langle \delta o^\dagger \delta o \rangle$) during the pulse

interaction, we may linearize Eq. (2) and obtain the equations of motion,

$$\delta\dot{a} = -i\Delta(t)\delta a + ig\langle a \rangle(\delta b^\dagger + \delta b) - \frac{\gamma_c}{2}\delta a + a_{in}, \quad (3a)$$

$$\delta\dot{b} = -i\omega_m\delta b + ig[\langle a^\dagger \rangle\delta a + \langle a \rangle\delta a^\dagger] - \frac{\gamma_m}{2}\delta b + b_{in}, \quad (3b)$$

with $\Delta(t) = \Delta_c - 2g\text{Re}[\langle b(t) \rangle]$. The $\langle a(t) \rangle$ and $\langle b(t) \rangle$ are governed by

$$\langle \dot{a} \rangle = -i\Delta(t)\langle a \rangle - i\Omega(t) - \frac{\gamma_c}{2}\langle a \rangle, \quad (4a)$$

$$\langle \dot{b} \rangle = -i\omega_m\langle b \rangle + ig|\langle a \rangle|^2 - \frac{\gamma_m}{2}\langle b \rangle. \quad (4b)$$

For the linear approximation made above, we have neglected nonlinear terms $ig\delta a(\delta b + \delta b^\dagger)$ in Eq. (3a) and $ig\delta a^\dagger\delta a$ in Eq. (3b) [47].

Equation (3) corresponds to a linear coupling described by the Hamiltonian $H_I = -\hbar g[\langle a^\dagger(t) \rangle\delta a + \langle a(t) \rangle\delta a^\dagger](\delta b^\dagger + \delta b)$. Here $\langle a(t) \rangle$ modulates the mirror-field coupling and its time dependence can be controlled by the driving amplitude $\Omega(t)$. We point out that any desirable $\langle a(t) \rangle$ as a function of time can, in principle, be achieved by a corresponding $\Omega(t)$, according to Eq. (4a). For convenience, we let

$$g\langle a^\dagger(t) \rangle \equiv \chi(t)e^{i\phi(t)}e^{-2ig\int_0^t \text{Re}[\langle b(\tau) \rangle]d\tau}, \quad (5)$$

where $\chi(t)$ and $\phi(t)$ are real functions, and the phase angle $-2g\int_0^t \text{Re}[\langle b(\tau) \rangle]d\tau$ is introduced in order to compensate for the phase shift induced by the dynamical cavity frequency shift in $\Delta(t)$.

By defining operators $\delta A(t) = \delta a e^{i[\phi(t) + \int_0^t \Delta(\tau)d\tau]}$ and $\delta B(t) = \delta b e^{i\omega_m t}$, Eq. (3) can be concisely written as $\dot{\mathbf{v}}(t) = \mathbf{M}(t)\mathbf{v}(t) + \mathbf{N}(t)$, where $\mathbf{v}(t) = [\delta A(t), \delta B(t), \delta A^\dagger(t), \delta B^\dagger(t)]^T$, and

$$\mathbf{M}(t) = \begin{bmatrix} -\frac{\gamma_c}{2} + i\dot{\phi}(t) & i\chi(t)e^{i(\Delta_c - \omega_m)t} & 0 & i\chi(t)e^{i(\Delta_c + \omega_m)t} \\ i\chi(t)e^{-i(\Delta_c - \omega_m)t} & -\frac{\gamma_m}{2} & i\chi(t)e^{i(\Delta_c + \omega_m)t} & 0 \\ 0 & -i\chi(t)e^{-i(\Delta_c + \omega_m)t} & -\frac{\gamma_c}{2} - i\dot{\phi}(t) & -i\chi(t)e^{-i(\Delta_c - \omega_m)t} \\ -i\chi(t)e^{-i(\Delta_c + \omega_m)t} & 0 & -i\chi(t)e^{i(\Delta_c - \omega_m)t} & -\frac{\gamma_m}{2} \end{bmatrix}, \quad (6)$$

and $\mathbf{N}(t) = [A_{in}(t), B_{in}(t), A_{in}^\dagger(t), B_{in}^\dagger(t)]^T$ with $A_{in}(t) = a_{in}e^{i[\phi(t) + \int_0^t \Delta(\tau)d\tau]}$ and $B_{in}(t) = b_{in}e^{i\omega_m t}$. The solution of $\mathbf{v}(t)$ is

$$\mathbf{v}(t) = \mathbf{G}(t)\mathbf{v}(0) + \mathbf{G}(t) \int_0^t \mathbf{G}^{-1}(\tau)\mathbf{N}(\tau)d\tau, \quad (7)$$

where $\mathbf{G}(t)$ is governed by

$$\dot{\mathbf{G}}(t) = \mathbf{M}(t)\mathbf{G}(t), \quad (8)$$

with $\mathbf{G}(0) = I$ being the identity matrix.

The state of the system can be conveniently described by a covariance matrix $\mathbf{R}(t)$ whose elements are $\mathbf{R}_{ll'}(t) = \langle \mathbf{v}_l(t)\mathbf{v}_{l'}(t) \rangle$ ($l, l' = 1, 2, 3, 4$). Therefore, $\mathbf{R}_{31}(t) = \langle \delta a^\dagger\delta a \rangle$ and

$\mathbf{R}_{42}(t) = \langle \delta b^\dagger\delta b \rangle$ are mean *displaced particle numbers* measuring the fluctuations. By Eq. (7), $\mathbf{R}(t)$ reads

$$\mathbf{R}(t) = \mathbf{G}(t)\mathbf{R}(0)\mathbf{G}^T(t) + \mathbf{G}(t)\mathbf{Z}(t)\mathbf{G}^T(t), \quad (9)$$

with

$$\mathbf{Z}(t) = \int_0^t \int_0^t \mathbf{G}^{-1}(\tau)\mathbf{C}(\tau, \tau')[\mathbf{G}^{-1}(\tau')]^T d\tau d\tau'. \quad (10)$$

The matrix $\mathbf{R}(0)$ is determined by the initial condition of the system, and $\mathbf{C}(\tau, \tau')$ is the two-time correlation function of noise operators, which is defined by the elements $\mathbf{C}_{ll'}(\tau, \tau') = \langle \mathbf{N}_l(\tau)\mathbf{N}_{l'}(\tau') \rangle$ ($l, l' = 1, 2, 3, 4$). Assuming that initially the cavity is in vacuum and the mirror is in a thermal equilibrium at the same temperature T_m as its bath, i.e., $\rho(0) = |0\rangle_c\langle 0|_c \otimes \rho_{th}(T_m)$ with $\rho_{th}(T_m) = \exp(-\hbar\omega_m b^\dagger b / k_B T_m) / \text{Tr}[\exp(-\hbar\omega_m b^\dagger b / k_B T_m)]$, then the matrix $\mathbf{R}(0)$ has three nonzero elements: $\mathbf{R}_{13}(0) = 1$, $\mathbf{R}_{24}(0) = \bar{n}_m + 1$, and $\mathbf{R}_{42}(0) = \bar{n}_m$. In addition, the Markovian baths imply $\mathbf{C}(\tau, \tau') = \mathbf{C}\delta(\tau - \tau')$, where \mathbf{C} is a constant matrix with three nonzero elements: $\mathbf{C}_{13} = \gamma_c$, $\mathbf{C}_{24} = \gamma_m(\bar{n}_m + 1)$, and $\mathbf{C}_{42} = \gamma_m\bar{n}_m$.

IV. COOLING OF THE MIRROR

We will employ the expectation value of displaced phonon number $\langle \delta b^\dagger\delta b \rangle$ as an indicator of cooling. This means that the idea of cooling in our scheme should be understood as a process of reducing excitations with respect to the mean amplitude $\langle b \rangle$ of the mirror.

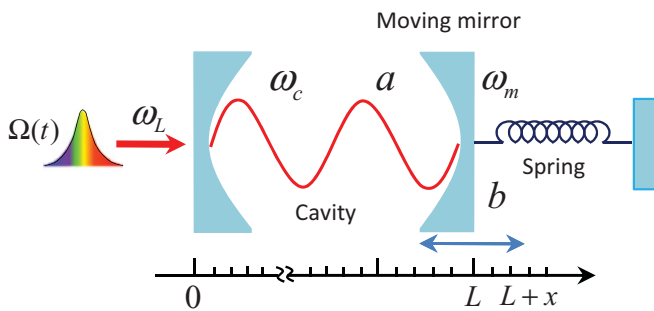


FIG. 1. (Color online) Schematic diagram of the cavity optomechanical system. A Fabry-Perot cavity, formed by a fixed end mirror and a harmonically bound end mirror, is driven by a pulse.

A. Chirped-pulse coupling

We now ask what $\Omega(t)$ is suitable for cooling the mirror. Guided by the fact that a chirped-pulse driving can efficiently realize population transfer in two-level systems [41–46], we consider the coupling strength in Eq. (5) taking the chirped form [46],

$$\chi(t) = \chi_0 \operatorname{sech}[\alpha(t - t_0)], \quad (11a)$$

$$\theta(t) = \dot{\phi}(t) = \beta \tanh[\alpha(t - t_0)]. \quad (11b)$$

Here t_0 determines the time of the pulse peak entering the cavity, α^{-1} measures the pulse duration, β controls the magnitude of the frequency modulation, and χ_0 is the strength of the pulse coupling.

The required driving amplitude $\Omega(t)$ for generating the above $\chi(t)$ and $\phi(t)$ can be found using Eqs. (4), (5), and (11) as

$$\Omega(t) = i \langle \dot{a}(t) \rangle - \left[\Delta(t) - i \frac{\gamma_c}{2} \right] \langle a(t) \rangle, \quad (12)$$

with

$$\langle a(t) \rangle = \frac{\chi_0}{g} \operatorname{sech}[\alpha(t - t_0)] e^{-i[\phi(t) - 2g \int_0^t \operatorname{Re}\langle b(\tau) \rangle d\tau]}, \quad (13a)$$

$$\langle b(t) \rangle = i \frac{\chi_0^2}{g} \int_0^t \operatorname{sech}^2[\alpha(\tau - t_0)] e^{-(\frac{\gamma_m}{2} + i\omega_m)(t-\tau)} d\tau, \quad (13b)$$

$$\phi(t) = \frac{\beta}{\alpha} \ln \left\{ \frac{\cosh[\alpha(t - t_0)]}{\cosh(\alpha t_0)} \right\}. \quad (13c)$$

An example illustrating the chirped-pulse coupling and the corresponding $\Omega(t)$ is given in Fig. 2.

B. Ideal case

To find the optimal relation for χ_0 , α , and β in Eq. (11) such that efficient cooling can be achieved, we first investigate the nondissipative case ($\gamma_c = 0$ and $\gamma_m = 0$) as a guide. Specifically, we consider the resonance case $\Delta_c = \omega_m$. Under the condition $(\Delta_c + \omega_m) \gg \chi_0$, we discard the terms $\pm i \chi(t) e^{\pm i(\Delta_c + \omega_m)t}$ in Eq. (6) by RWA, then by letting $u(t) = \langle \delta A^\dagger \delta B \rangle + \langle \delta B^\dagger \delta A \rangle$, $v(t) = i(\langle \delta B^\dagger \delta A \rangle - \langle \delta A^\dagger \delta B \rangle)$, and $w(t) = \langle \delta A^\dagger \delta A \rangle - \langle \delta B^\dagger \delta B \rangle$, we can obtain the Bloch

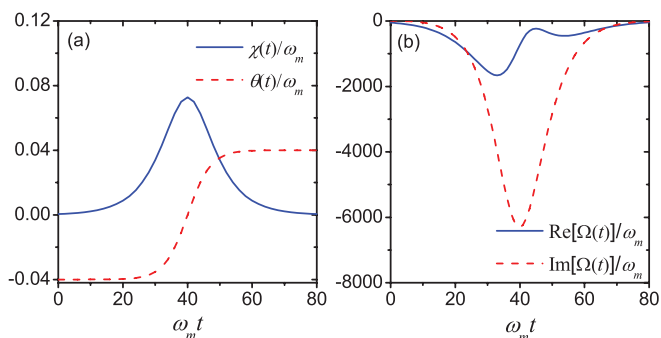


FIG. 2. (Color online) Plot of (a) the chirped pulse given in Eq. (11) and (b) the original driving amplitude $\Omega(t)$ vs the scaled time $\omega_m t$. The parameters are $\alpha/\omega_m = 0.14$, $\beta/\omega_m = 0.04$, $\omega_m t_0 = 40$, and $\chi_0 = \frac{1}{2}\sqrt{\alpha^2 + \beta^2}$.

equations

$$\dot{u}(t) = \dot{\phi}(t)v(t), \quad (14a)$$

$$\dot{v}(t) = -\dot{\phi}(t)u(t) + 2\chi(t)w(t), \quad (14b)$$

$$\dot{w}(t) = -2\chi(t)v(t). \quad (14c)$$

Under the initial condition $u_i = 0$, $v_i = 0$, and $w_i = -\bar{n}_m$, we find that when

$$\chi_0 = \frac{1}{2}\sqrt{\alpha^2 + \beta^2}, \quad (15)$$

the solution of the Bloch equations is [46]

$$u(t) = -\frac{\beta}{\alpha}v(t) = \frac{\bar{n}_m\beta}{2\chi_0} \operatorname{sech}[\alpha(t - t_0)], \quad (16a)$$

$$w(t) = \bar{n}_m \tanh[\alpha(t - t_0)]. \quad (16b)$$

Therefore, the average quasiphonon number evolves as

$$\langle \delta b^\dagger \delta b \rangle = \langle \delta B^\dagger \delta B \rangle = \frac{\bar{n}_m}{2} \{1 - \tanh[\alpha(t - t_0)]\}. \quad (17)$$

When $\alpha(t - t_0) \gg 1$, we have $\langle \delta b^\dagger \delta b \rangle \approx 0$ (for example, $\tanh 5 = 0.999909$), which implies that thermal noise in the mirror can be extracted almost completely. Thus, $\chi_0 = \frac{1}{2}\sqrt{\alpha^2 + \beta^2}$ is a relation to implement efficient cooling of the mirror in the absence of dissipation.

It is useful to note that for the constant-pulse coupling case [i.e., $\chi(t)$ is a constant], one can also extract energy from the mirror, but the corresponding solution is oscillatory at a Rabi frequency, i.e., the phonon number in the mirror is a cosine function of time. In this case, the timing of the constant pulse is crucial in order to locate the instant when the phonon number is minimum. However, our scheme does not have such a timing control issue because the solution (17) indicates that after the pulse duration (about $2t_0$), the residual quasiphonon number of the mirror changes slowly in time by the behavior of the $\tanh[\alpha(t - t_0)]$ function.

As a remark, we indicate that χ_0 should be much smaller than the mirror frequency ω_m in order to meet the condition of RWA. The violation of RWA would mean that the parametric interaction of the form $\delta a^\dagger \delta b^\dagger + \text{H.c.}$ becomes important, which generally leads to heating of the system. Therefore, by Eq. (15), the pulse duration characterized by α^{-1} cannot be arbitrarily short, i.e., $\alpha < 2\chi_0 \ll 4\omega_m$. Our numerical calculations (without RWA) indicate that the parameters used in Fig. 3 are quite sufficient for RWA to be valid.

C. Dissipative case

In realistic experiments, the interactions with environments will inevitably lead to dissipation of the system. In addition, the counter-rotating terms ($\delta a^\dagger \delta b^\dagger + \text{H.c.}$) ignored in RWA may modify the dynamic process. In what follows, we numerically study the cooling process in the dissipative case beyond RWA. We consider the realistically experimental parameters for the system [34]: $\omega_m \approx 2\pi \times 73.5$ MHz, $\gamma_m \approx 2\pi \times 1.3$ kHz, $\gamma_c \approx 2\pi \times 3.2$ MHz, and $g \approx 2\pi \times 843.1$ Hz. Namely, $\gamma_m/\omega_m \approx 1.768 \times 10^{-5}$, $\gamma_c/\omega_m \approx 0.0435$, and $g/\omega_m \approx 1.147 \times 10^{-5}$. With these parameters and the chirped pulse given in Fig. 2, we solve Eq. (9) numerically. In Fig. 3(a), we plot the time evolution of the mean displaced particle numbers. We see that the $\langle \delta b^\dagger \delta b \rangle$ decreases rapidly from its initial value ($\bar{n}_m = 1000$) to a relatively small number

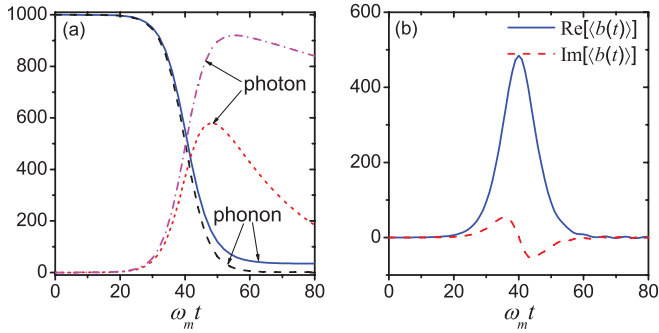


FIG. 3. (Color online) (a) Time evolution of the displaced phonon number $\langle \delta b^\dagger \delta b \rangle$ and displaced photon number $\langle \delta a^\dagger \delta a \rangle$ in the chirped-pulse coupling case for two different values for the γ_c . The solid and short dashed curves are for the case of $\gamma_c/\omega_m = 0.0435$, while the dashed and dash-dotted curves are for the case of $\gamma_c/\omega_m = 0.00435$. (b) Plot of the real and imaginal parts of $\langle b(t) \rangle$ vs the scaled time $\omega_m t$. Here $\Delta_c = \omega_m$.

(about 34) when the chirped pulse is applied. At the same time, $\langle \delta a^\dagger \delta a \rangle$ in the cavity increases rapidly from zero to a peak value and then decreases to zero gradually through the cavity decay channel. We note that the residual fluctuations of the mirror are limited by the noise of the system, mainly of the cavity-field damping. Our numerical investigations show that when the cavity-field decay rate is $\gamma_c/\omega_m \approx 0.00435$, the residual $\langle \delta b^\dagger \delta b \rangle$ can further be reduced to 1.08 [the dash line in Fig. 3(a)]. In addition, numerical calculations indicate the correction from the counter-rotating terms is negligible with these parameters.

We remark that during the pulse interaction, the mirror attains a nonzero coherent amplitude (i.e., $\langle b(t) \rangle \neq 0$) according to Eq. (13b). But such a coherent motion should not be confused with the fluctuations we aim to reduce in this paper. For the present parameters used in Fig. 3(a), we plot the time evolution of $\langle b(t) \rangle$ in Fig. 3(b). We see that there is a small amplitude $|\langle b(2t_0) \rangle| \approx 2$ near the end of the pulse interaction. Actually, when the thermal fluctuation of the mirror is completely transferred to the cavity, the mirror in the displaced representation will be in its ground state. Therefore, in the original representation, the mirror is prepared in a coherent state of the mechanical motion.

We also point out that although there are residual cavity photons after the pulse duration (around $2t_0$), the heating due to such photons is found to be negligible because of the weak coupling strength g . We can estimate that for a residual cavity photon number n_r at time $2t_0$, the mirror can be excited to have phonon number $(gn_r/\omega_m)^2$, which is of the order of 0.001 with the parameters used in Fig. 3 ($\gamma_c/\omega_m = 0.0435$). Therefore, the heating is mainly due to the heat bath of the mirror. For example, we find that the heat bath of the mirror would increase the phonon number from 34 to 38 when the time evolves from $\omega_m t = 80$ to 300.

D. Effects of the β parameter

The frequency modulation characterized by the parameter β is a main feature of the chirped coupling. In the case of $\beta = 0$, the coupling corresponds to a π pulse because $\int_{-\infty}^{\infty} \alpha \text{sech}[\alpha(t - t_0)] dt = \pi$ is the pulse area. However, such

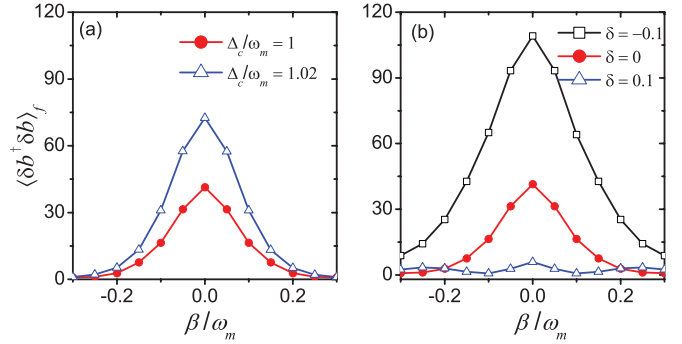


FIG. 4. (Color online) (a) Plot of the final mean displaced phonon number $\langle \delta b^\dagger \delta b \rangle_f$ vs the phase modulation amplitude β for various detunings $\Delta_c/\omega_m = 1$ and 1.02. (b) Plot of $\langle \delta b^\dagger \delta b \rangle_f$ vs β for various parameters $\delta = -0.1, 0, \text{ and } 0.1$.

a simple π pulse generally does not bring an optimal cooling when dissipation and counter-rotating terms are included. The parameter β therefore provides a way to adjust the pulse for a better cooling performance. In Fig. 4, we demonstrate this feature numerically by plotting the final mean displaced phonon number of the mirror (defined at time $t = 2t_0$) as a function of β . There are two situations [Figs. 4(a) and 4(b)] that we will discuss below, but in both figures, it is apparent that nonzero values of β can better reduce the displaced phonon number of the mirror.

We point out that the final displaced phonon number can become less sensitive to the detuning Δ_c when $|\beta|$ is increased. This is shown in Fig. 4(a), where we can compare the sideband resonance case $\Delta_c = \omega_m$ with a slightly off-resonance case. We see that although cooling with an off-resonance Δ_c is less effective, the dependence on Δ_c becomes weaker as $|\beta|$ increases. For the parameters used in Fig. 4(a), the final displaced phonon numbers are essentially the same when $|\beta| > 0.2\omega_m$. This shares a similar feature in two-level systems as a chirped pulse can make efficient population transfer in the presence of inhomogeneous broadening.

In Fig. 4(b), we illustrate the effect of β on cooling when there are uncertainties in controlling the pulse area. Such an error, for example, may come from an inaccurate value of the coupling strength g . Let us express $\chi(t)$ as

$$\chi(t) = (1 + \delta)\chi_0 \text{sech}[\alpha(t - t_0)], \quad (18)$$

with δ describing the deviation. At $\beta = 0$, Fig. 4(b) shows that a modest change of δ can affect the final displaced phonon number quite significantly. In fact, we notice that the case $\delta = 0.1$ in the figure actually corresponds to a better cooling. This indicates that χ_0 in Eq. (15) is no longer optimal for cooling because of dissipative effects. The search for optimal pulse parameters relies on numerical work, but Fig. 4(b) suggests that the frequency modulation with a suitable range of β may ease the sensitivity of δ and hence improve the cooling performance, even though the pulse parameters are not exactly optimal.

V. CONCLUSION AND REMARKS

To conclude, we have proposed a method to cool a moving mirror in cavity optomechanics by a chirped pulse.

Within the linearization framework, we have shown how a chirped-pulse coupling can be achieved by an external driving field, and numerically we have demonstrated that thermal fluctuations in the mirror can be significantly transferred to the cavity after the pulse. In particular, the frequency modulation plays a positive role in the cooling process, especially when there are uncertainties in controlling the detuning and pulse areas.

Finally, we remark that it would be difficult to present a general comparison of the cooling efficiency between our scheme and the resolved sideband cooling. This is because Eq. (8) has no analytic solution and so the residual phonon number can only be calculated numerically. Nevertheless, we notice that by decreasing the cavity decay rate, the residual phonon number can be lowered. As a specific example, with $\gamma_c/\omega_m = 0.001$ and the same other parameters as in Fig. 3, the

residual phonon number can reach 0.64. Therefore, the system under such parameters may effectively be considered as the ground state, although this residual phonon number is higher than the resolved sideband cooling limit $(\gamma_c/4\omega_m)^2$ [28]. The main purpose of this paper is to provide an alternative method of cooling based on pulsed interaction, which is a transient solution rather than a steady-state one. In other words, the process can occur in a finite duration of time, and this could be a useful feature for manipulating quantum states of the mirror.

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- [1] T. J. Kippenberg and K. J. Vahala, *Science* **321**, 1172 (2008).
 [2] I. Favero and K. Karrai, *Nature Photon.* **3**, 201 (2009).
 [3] F. Marquardt and S. M. Girvin, *Physics* **2**, 40 (2009).
 [4] M. Aspelmeyer, S. Gröblacher, K. Hammerer, and N. Kiesel, *J. Opt. Soc. Am. B* **27**, A189 (2010).
 [5] C. Fabre, M. Pinard, S. Bourzeix, A. Heidmann, E. Giacobino, and S. Reynaud, *Phys. Rev. A* **49**, 1337 (1994).
 [6] S. Mancini and P. Tombesi, *Phys. Rev. A* **49**, 4055 (1994).
 [7] A. Heidmann and S. Reynaud, *Phys. Rev. A* **50**, 4237 (1994).
 [8] S. Bose, K. Jacobs, and P. L. Knight, *Phys. Rev. A* **56**, 4175 (1997).
 [9] W. Marshall, C. Simon, R. Penrose, and D. Bouwmeester, *Phys. Rev. Lett.* **91**, 130401 (2003); D. Kleckner, I. Pikovski, E. Jeffrey, L. Ament, E. Eliel, J. van den Brink, and D. Bouwmeester, *New J. Phys.* **10**, 095020 (2008).
 [10] H. Ian, Z. R. Gong, Y. X. Liu, C. P. Sun, and F. Nori, *Phys. Rev. A* **78**, 013824 (2008); Z. R. Gong, H. Ian, Y. X. Liu, C. P. Sun, and F. Nori, *ibid.* **80**, 065801 (2009).
 [11] K. Jähne, C. Genes, K. Hammerer, M. Wallquist, E. S. Polzik, and P. Zoller, *Phys. Rev. A* **79**, 063819 (2009).
 [12] A. Mari and J. Eisert, *Phys. Rev. Lett.* **103**, 213603 (2009).
 [13] M. Wallquist, K. Hammerer, P. Zoller, C. Genes, M. Ludwig, F. Marquardt, P. Treutlein, J. Ye, and H. J. Kimble, *Phys. Rev. A* **81**, 023816 (2010).
 [14] A. Nunnenkamp, K. Børkje, J. G. E. Harris, and S. M. Girvin, *Phys. Rev. A* **82**, 021806 (2010).
 [15] J. Q. Liao and C. K. Law, *Phys. Rev. A* **83**, 033820 (2011).
 [16] S. Mancini, V. Giovannetti, D. Vitali, and P. Tombesi, *Phys. Rev. Lett.* **88**, 120401 (2002).
 [17] A. Ferreira, A. Guerreiro, and V. Vedral, *Phys. Rev. Lett.* **96**, 060407 (2006).
 [18] D. Vitali, S. Gigan, A. Ferreira, H. R. Böhm, P. Tombesi, A. Guerreiro, V. Vedral, A. Zeilinger, and M. Aspelmeyer, *Phys. Rev. Lett.* **98**, 030405 (2007).
 [19] M. Paternostro, D. Vitali, S. Gigan, M. S. Kim, C. Brukner, J. Eisert, and M. Aspelmeyer, *Phys. Rev. Lett.* **99**, 250401 (2007).
 [20] M. J. Hartmann and M. B. Plenio, *Phys. Rev. Lett.* **101**, 200503 (2008).
 [21] C. Genes, D. Vitali, and P. Tombesi, *Phys. Rev. A* **77**, 050307(R) (2008); C. Genes, A. Mari, P. Tombesi, and D. Vitali, *ibid.* **78**, 032316 (2008).
 [22] S. Gröblacher, K. Hammerer, M. R. Vanner, and M. Aspelmeyer, *Nature (London)* **460**, 724 (2009).
 [23] J. Zhang, K. Peng, and S. L. Braunstein, *Phys. Rev. A* **68**, 013808 (2003).
 [24] K. Stannigel, P. Rabl, A. S. Sørensen, P. Zoller, and M. D. Lukin, *Phys. Rev. Lett.* **105**, 220501 (2010).
 [25] L. Tian and H. Wang, *Phys. Rev. A* **82**, 053806 (2010).
 [26] M. Paternostro, *Phys. Rev. Lett.* **106**, 183601 (2011).
 [27] I. Wilson-Rae, N. Nooshi, W. Zwerger, and T. J. Kippenberg, *Phys. Rev. Lett.* **99**, 093901 (2007).
 [28] F. Marquardt, J. P. Chen, A. A. Clerk, and S. M. Girvin, *Phys. Rev. Lett.* **99**, 093902 (2007).
 [29] C. Genes, D. Vitali, P. Tombesi, S. Gigan, and M. Aspelmeyer, *Phys. Rev. A* **77**, 033804 (2008).
 [30] Y. Li, L. A. Wu, and Z. D. Wang, *Phys. Rev. A* **83**, 043804 (2011).
 [31] S. Gigan, H. R. Böhm, M. Paternostro, F. Blaser, G. Langer, J. B. Hertzberg, K. C. Schwab, D. Bäuerle, M. Aspelmeyer, and A. Zeilinger, *Nature (London)* **444**, 67 (2006).
 [32] O. Arcizet, P.-F. Cohadon, T. Briant, M. Pinard, and A. Heidmann, *Nature (London)* **444**, 71 (2006).
 [33] D. Kleckner and D. Bouwmeester, *Nature (London)* **444**, 75 (2006).
 [34] A. Schliesser, R. Rivière, G. Anetsberger, O. Arcizet, and T. J. Kippenberg, *Nature Phys.* **4**, 415 (2008).
 [35] A. Schliesser, O. Arcizet, R. Rivière, G. Anetsberger, and T. J. Kippenberg, *Nature Phys.* **5**, 509 (2009).
 [36] Y.-S. Park and H. Wang, *Nature Phys.* **5**, 489 (2009).
 [37] M. R. Vanner, I. Pikovski, G. D. Cole, M. S. Kim, Č. Brukner, K. Hammerer, G. J. Milburn, and M. Aspelmeyer, *Proc. Natl. Acad. Sci. USA* **108**, 16182 (2011).
 [38] J. Cerrillo, S. Machnes, M. Aspelmeyer, W. Wieczorek, M. B. Plenio, and A. Retzker, e-print arXiv:1104.5448.
 [39] S. G. Hofer, W. Wieczorek, M. Aspelmeyer, and K. Hammerer, e-print arXiv:1108.2586 (to be published).

- [40] V. Fiore, Y. Yang, M. C. Kuzyk, R. Barbour, L. Tian, and H. Wang, *Phys. Rev. Lett.* **107**, 133601 (2011).
- [41] D. Goswami, *Phys. Rep.* **374**, 385 (2003).
- [42] B. W. Shore, *Acta Phys. Slovaca* **58**, 243 (2008).
- [43] A. Bambini and P. R. Berman, *Phys. Rev. A* **23**, 2496 (1981).
- [44] J. Zakrzewski, *Phys. Rev. A* **32**, 3748 (1985).
- [45] F. T. Hioe, *Phys. Rev. A* **30**, 2100 (1984); F. T. Hioe and C. E. Carroll, *J. Opt. Soc. Am. B* **3**, 497 (1985).
- [46] L. Allen and J. H. Eberly, *Optical Resonance and Two-Level Atoms* (Dover, New York, 1987).
- [47] For the pulsed coupling discussed in Sec. IV, we have numerically checked that $\langle a \rangle^2 \gg \langle \delta a^\dagger \delta a \rangle$ and $\langle b \rangle^2 \gg \langle \delta b^\dagger \delta b \rangle$ during the pulse duration ($0 < t < 2t_0$).