Stability of a dipolar Bose-Einstein condensate in a one-dimensional lattice

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We show that in contrast with contact interacting gases, an optical lattice changes drastically the stability properties of a dipolar condensate, inducing a crossover from dipolar destabilization to dipolar stabilization for increasing lattice depths. Performing stability measurements on a ${}^{52}Cr$ Bose-Einstein condensate in an interactiondominated regime, repulsive dipolar interaction balances negative scattering lengths down to −17 Bohr radii. Our findings are in excellent agreement with mean-field calculations, revealing the important destabilizing role played by intersite dipolar interactions in deep lattices.

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Ultracold gases allow for a high level of experimental control, opening fascinating possibilities for the design and realization of novel quantum phases [\[1,2\]](#page-3-0). In particular, short-range isotropic interatomic interactions, which play a key role in the properties of quantum gases, can be tuned almost at will, e.g., using Feshbach resonances [\[3\]](#page-3-0). Interestingly, systems with significant or dominant dipolar interactions (DIs) open novel possibilities, due to the anisotropic and long-range character of the DI. Dipolar gases have indeed attracted major experimental and theoretical attention in the recent years [\[4,5\]](#page-3-0). In particular, quantum systems with strong DI have been realized in chromium Bose-Einstein condensates (BECs) [\[6,7\]](#page-3-0), ultracold heteronuclear molecules [\[8\]](#page-3-0), and BECs with Rydberg excitations [\[9\]](#page-3-0), while weaker dipolar effects were also reported in alkali-metal samples [\[10–12\]](#page-3-0).

The experimental realization of quantum phases in manybody systems crucially depends on their overall stability. Systems with dominant attractive short-range interactions are fundamentally unstable against collapse, and may only be stabilized away from the interaction-dominated regime, i.e., only for a limited number of particles at given external trapping potential [\[13\]](#page-3-0). The DI, being partially attractive and partially repulsive, induces a strong dependence of the stability of the system on the trapping geometry [\[4,5\]](#page-3-0), as recently observed experimentally [\[14\]](#page-3-0). Therein the regime with effective dipolar attraction has been explored with the realization of a purely dipolar interacting condensate in an oblate trap.

Dipolar gases in optical lattices have recently attracted particularly intense interest [\[5,15–17\]](#page-3-0), mainly motivated by the presence of intersite interactions, which are in general negligible in short-range interacting systems. So far, the influence of dipolar intersite interactions has only been observed in the dynamical properties of a very weakly interacting $39K$ BEC [\[11\]](#page-3-0). Beyond that, stronger dipolar interactions are expected to result in a wealth of novel ground-state phenomena, including supersolidity [\[18–20\]](#page-3-0) and intersite superfluids [\[21\]](#page-3-0). The observation of significant intersite effects demands, however, the careful determination of the stability of dipolar lattice gases, a fundamental question which has not been addressed so far.

In this work we investigate the stability of a dipolar ${}^{52}Cr$ condensate in a one-dimensional optical lattice. We identify the stability threshold for various lattice depths by measuring the BEC atom number when decreasing the *s-*wave scattering length via a magnetic Feshbach resonance. The stability threshold depends on the nontrivial interplay between intersite hopping, short-range interactions, and onsite and intersite dipolar interactions. From small to large lattice depths, we observe a continuous crossover from a dipolar destabilized to a dipolar stabilized regime, where the system can be considered as a stack of spatially separated highly oblate BECs. We, therefore, show that, in addition to the suppression of exothermic *two-body* chemistry in ultracold dipolar gases [\[22\]](#page-3-0), the DI can be used to stabilize otherwise unstable *many-body* states. Our measurements are in excellent agreement with mean-field calculations, revealing that intersite interactions play a significant destabilizing role in deep lattices.

Our experimental procedure is as follows: we produce a ⁵²Cr BEC in a crossed optical dipole trap (ODT, $v_{x,y,z}$ = $440,330,290$ Hz) at a magnetic field $B \simeq 600$ G, where the scattering length is large and positive ($a \simeq 90a_0$ with a_0 the Bohr radius). Dipoles are aligned along the *z* direction by the strong magnetic field. By changing the magnetic field strength *B* in the vicinity of a Feshbach resonance, we are able to tune the *s*-wave scattering length $a = a(B)$, as described in Ref. [\[6\]](#page-3-0). After reducing the scattering length to $a =$ $60a_0$, we load the BEC into a one-dimensional (1D) optical lattice oriented along the polarization direction *z*, populating approximately 15 lattice sites, with a maximum of 2000 atoms per site in the center of the trap. The lattice is produced by a $\lambda = 1064$ nm fiber laser in a nearly back-reflected geometry with a lattice spacing $d_{\text{lat}} = 534 \text{ nm}$, as illustrated in Fig. [1.](#page-1-0) The radial trapping frequencies $v_{x,y}$ are kept constant during the lattice ramp by adjusting the power of the ODT beam in the *z* direction. We then decrease the scattering length in 6 ms to reach its final value, where we hold the atoms for $t_{hold} = 2$ ms. Finally we switch off the optical trapping potential for a 6 ms time-of-flight (TOF) before taking an absorption image of the atomic cloud.

To extract the BEC atom number N_{BEC} after the TOF, the recorded two-dimensional (2D) density distribution is integrated along the *z* direction, and we perform a 1D bimodal fit. When the final scattering length is much larger than the critical value a_{crit} , we typically measure $N_{\text{BEC}} \simeq 15000$ $(N_{BEC} \simeq 10000)$ in a shallow (deep) lattice, while the atom number before loading the lattice is always $N_{\text{at}} \simeq 20000$. Getting close to the instability point, we observe a fast decrease in the atom number, as it is shown in Fig. [2](#page-1-0) for two different

FIG. 1. (Color online) Experimental setup. The measurements are performed in a 1D optical lattice (blue) with underlying crossed optical dipole trap (red). The magnetic field used to reach the Feshbach resonance is produced by two Helmholtz coils (black) and polarizes the dipoles along the lattice direction *z*. For deep lattices we obtain a stack of oblate dipolar BECs as depicted on the lower right.

values of the lattice depth [\[23\]](#page-3-0). We finally extract the critical scattering length from an empirically chosen function as described in Ref. [\[14\]](#page-3-0). Although atom losses are enhanced in a deep lattice, due to the larger mean trapping frequency, they do not affect the determination of the stability threshold.

Figure 3 shows the stability diagram of a dipolar ${}^{52}Cr$ BEC in a 1D optical lattice. The critical scattering length a_{crit} is measured for different lattice depths in the range from

FIG. 2. (Color online) Atom number vs scattering length for different lattice depths. For a moderate lattice depth of $U =$ $(6.2 \pm 0.6)E_R$ (open blue dots), the condensate becomes unstable at $a_{\text{crit}} = (6.5 \pm 1.9)a_0$, while in a deep lattice at $U = (37 \pm 4)E_R$ (filled red dots), we observe a stable BEC until $a = (-13.2 \pm 2.5)a_0$. The solid lines are fits to the data using the arbitrarily chosen form $N_{BEC} = \max\{0, N_0(a - a_{\text{crit}})^{\beta}\}\$, from which we extract the critical scattering length a_{crit} ($\beta \simeq 0.2$).

FIG. 3. (Color online) Stability diagram of the dipolar condensate in the 1D optical lattice. The critical scattering length a_{crit} is plotted vs the lattice depth *U* [\[23\]](#page-3-0). The lines are results of the numerical simulations for different atom numbers. We explore the full crossover from a dipolar destabilized ($a_{\text{crit}} > 0$) to a dipolar stabilized ($a_{\text{crit}} < 0$) regime. At $U = 50E_R$ we observe a stable condensate down to $a_{\text{crit}} = (-17 \pm 3)a_0$. For comparison, the dashed-dotted line (green) shows the simulated critical scattering length disregarding the dipolar interaction. We find $|a_{\text{crit}}| < 0.4a_0$ on the whole range, approaching $a_{\text{crit}} = 0$ (gray dotted line) for increasing lattice depth.

 $U = 0$ to 63 E_R [recoil energy, $E_R = \hbar^2 \pi^2/(2m d_{\text{lat}}^2)$, with *m* the atomic mass]. We find a *positive* a_{crit} until $U \simeq 10E_R$ and a *negative* a_{crit} down to $a_{\text{crit}} = (-17 \pm 3)a_0$ in the deep lattice regime.

Our experimental results are in excellent agreement with numerical simulations based on the nonlocal nonlinear Schrödinger equation

$$
i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = \left[-\frac{\hbar^2}{2m} \nabla^2 + V_{\text{ext}}(\mathbf{r}) + g N_{\text{at}} |\Psi(\mathbf{r}, t)|^2 + N_{\text{at}} \int d\mathbf{r}' \, V_{\text{dd}}(\mathbf{r} - \mathbf{r}') |\Psi(\mathbf{r}', t)|^2 \right] \Psi(\mathbf{r}, t),\tag{1}
$$

where $g = 4\pi \hbar^2 a(B)/m$. The potential $V_{ext}(\mathbf{r}) = U \sin^2 \theta$ $(\pi z/d_{\text{lat}}) + m \sum_{i=x,y,z} (2\pi v_i)^2 r_i^2/2$ results from the 1D optical lattice and the three-dimensional (3D) harmonic confinement given by the ODT. The DI potential is given by $V_{dd}(\mathbf{r}) =$ $\frac{\mu_0 \mu^2}{4\pi} \frac{1-3(\hat{r}\cdot\hat{z})^2}{r^3}$ (with $\hat{r} = \mathbf{r}/r$), where μ_0 is the vacuum permeability and μ the magnetic dipole moment ($\mu = 6\mu_\text{B}$ for ⁵²Cr with μ_B the Bohr magneton).

We determine the critical scattering length a_{crit} by integrating Eq. (1) in imaginary time, looking for the existence of a stable ground state. As no assumption is made on the condensate wave function in our three-dimensional calculations (i.e., separability or factorization in longitudinal and radial wave functions), we can determine in a consistent way a_{crit} for all lattice depths, ranging from $U = 0$ to very deep lattices. Alternatively, we simulate the whole experimental sequence by integrating Eq. (1) in real time. While in general the wave functions calculated by real and imaginary time evolution do not fully coincide, the stability threshold is basically unaffected. Hence, although the evolution is not fully adiabatic, the population of excitations does not influence appreciably the stability threshold. Our simulations show a weak dependence of the stability threshold on the initial BEC atom number *N*at (see Fig. [3\)](#page-1-0), being even weaker for deep lattices. Note the striking difference between the stability properties of dipolar and nondipolar lattice gases (dashed-dotted line in Fig. [3\)](#page-1-0). For a purely contact-interacting BEC in a shallow lattice, the ODT barely stabilizes the condensate, which becomes unstable for *a* < −0.4*a*₀. For deeper lattices *a*_{crit} approaches zero due to the highly oblate onsite trap geometry [\[24\]](#page-3-0), therefore showing also a qualitatively different behavior compared to the dipolar gas.

Three different regimes may be identified in Fig. [3.](#page-1-0) For $U \leqslant 3E_R$, the lattice has no relevant effect on the stability of the system, which is simply determined by the shape of the underlying ODT. Measuring $a_{\text{crit}} = (12 \pm 2)a_0$ at $U = 0$ in this essentially prolate trap, we recover the result of Ref. [\[14\]](#page-3-0). To generalize our results to other dipolar systems, we introduce a length scale associated with the DI, $a_{dd} = m\mu_0\mu^2/12\pi\hbar^2$ (\simeq 15*a*₀ for ⁵²Cr). In this regime of very shallow lattices, $a_{\rm crit} \simeq$ a_{dd} , i.e., the dipoles are maximally destabilizing the condensate due to the effectively attractive dipolar interaction in the prolate ODT.

By further increasing the lattice depth, a second regime is entered: in the range from $U = 3E_R$ to $15E_R$, the system undergoes the transition from weak to deep lattice $[25]$, the interlayer hopping being still significant. Even though we do not change the underlying trapping potential (*νx,y,z* are kept constant), we observe that the lattice radically modifies the stability properties. It induces a continuous crossover from dipolar destabilization ($a_{\text{crit}} > 0$) to dipolar stabilization $(a_{\text{crit}} < 0)$, which may be explained by the increasing role of onsite repulsive dipolar interactions. The precise form of the stability threshold in this regime depends, however, on a nontrivial interplay between contact and dipolar interactions and the tunneling between lattice sites.

Finally, for $U > 15E_R$ the tunneling is negligible and the system can be considered as a stack of highly oblate condensates interacting with each other only through attractive intersite DIs. This regime, which is well described within the tight-binding approximation, has recently attracted major theoretical attention [\[15,16\]](#page-3-0). Despite the destabilizing intersite attraction, the system proves to be stable down to $a_{\text{crit}} =$ $(-17 \pm 3)a_0$, where the effective dipolar interaction is mostly repulsive. To our knowledge, this is the first realization of a stable many-body system at *a <* 0 within an interactiondominated regime $(N|a|/\bar{a}_{\text{ho}} > 1$ with *N* the atom number in the central lattice site and \bar{a}_{ho} the mean onsite harmonic oscillator length). In this regime the external potential itself cannot stabilize the system, and the DI constitutes the key stabilization mechanism.

We quantitatively investigate the destabilizing effect of the dipolar intersite interactions by introducing a truncated DI potential, $V_{dd}^{box}(\mathbf{r}) = V_{dd}(\mathbf{r})$ $[\Theta((\mathbf{r} \cdot \hat{z}) + d_{\text{lat}}/2) - \Theta((\mathbf{r} \cdot \hat{z}) - d_{\text{lat}}/2)],$ where $\Theta(\xi)$ is the Heaviside function. Such cutoff is implemented following a procedure similar to that of Ref. [\[26\]](#page-3-0). For deep lattices the cutoff in $V_{dd}^{box}(\mathbf{r})$ effectively amounts to removing the intersite

FIG. 4. (Color online) Intersite coupling mediated by dipolar interactions: zoom on Fig. [3](#page-1-0) in the deep lattice regime. Intersite hopping is negligible on experimental time scales. Solid line: numerical simulation using the full dipolar potential $V_{dd}(\mathbf{r})$. Dashed-dotted line: simulation with the truncated dipolar potential $V_{dd}^{box}(\mathbf{r})$ (see text), for which intersite coupling is not taken into account. The deviation of the simulation with the truncated potential to the experimental data indicates significant mean-field energy contributions from long-range intersite interactions.

DI, while still taking into account the full short-range and long-range onsite interactions. In Fig. 4 we compare the experimental data to the stability threshold calculated with $V_{\text{dd}}^{\text{box}}(\mathbf{r})$ and $V_{\text{dd}}(\mathbf{r})$ in the deep lattice regime ($U \geq 15E_R$). The calculations with $V_{dd}^{box}(\mathbf{r})$ show a substantial deviation from the experimental data: at $U \simeq 20E_R$ the difference is $\Delta a_{\text{crit}} \simeq 8a_0$, which is more than three times our standard deviation. This discrepancy, in addition with the very good agreement of the measurements with the simulations with $V_{dd}(\mathbf{r})$, shows that dipolar intersite interactions have a strong impact on the stability of the system (e.g., for $U = 20E_R$, $|a_{\text{crit}}|$ is reduced to about 50%). Note, that in both the simulations (using the truncated and the nontruncated dipolar potential), the critical scattering length approaches the fundamental limit $a_{\text{crit}} = -2a_{\text{dd}}$ for very high lattice depths. This is due to the fact that two infinite homogeneous planes of dipoles would present zero averaged intersite DI [\[27\]](#page-3-0).

In conclusion, contrary to the case of contact interacting gases, an optical lattice changes drastically the stability properties of a dipolar condensate. For increasing lattice depths, a continuous crossover from dipolar destabilization to dipolar stabilization is observed. In particular in deep lattices, a condensate with attractive short-range interactions is stabilized by the effective dipolar repulsion, although attractive intersite interactions are responsible for a significant destabilization of the system.

The determination of the condensate stability is fundamental for the observation of both quantum phases and instability dynamics of dipolar condensates in lattices. In the highly oblate geometries that we obtained in the deep lattice regime, dipolar gases are expected to exhibit a roton-type excitation spectrum [\[28,29\]](#page-3-0), similar to the one observed

in liquid helium [30,31]. Additionally, intersite coupling in optical lattices is expected to enhance the roton character in the excitation spectrum $[32,33]$ and related self-organized structures [15,16]. Finally, the realization of a stack of stable mesoscopic ensembles interacting through long-range dipolar interactions, together with the possibility to tailor multi-well potentials at will [34,35], is particularly promising for the experimental realization of new quantum phases relying on the interplay between onsite and long-range intersite interactions, as predicted in [36].

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