Controlling resonant photonic transport along optical waveguides by two-level atoms

Cong-Hua Yan,[1,2] Lian-Fu Wei,[1,3,*] Wen-Zhi Jia,[1] and Jung-Tsung Shen[4]

[1] *Quantum Optoelectronics Laboratory, Southwest Jiaotong University, Chengdu 610031, China*

[2] *College of Physics and Electronic Engineering, Sichuan Normal University, Chengdu 610068, China*

[3] *State Key Laboratory of Optoelectronic Materials and Technologies, School of Physics and Engineering, Sun Yat-Sen University, Guangzhou 510275, China*

[4] *Department of Electrical and Systems Engineering, Washington University in St. Louis, St. Louis, Missouri 63130, USA*

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Recent works [Shen *et al.*, Phys. Rev. Lett. **95**, 213001 (2005); Zhou *et al.*, Phys. Rev. Lett. **101**, 100501 (2008)] showed that the incident photons cannot transmit along an optical waveguide containing a resonant two-level atom (TLA). Here we propose an approach to overcome such a difficulty by using asymmetric couplings between the photons and a TLA. Our numerical results show that the transmission spectrum of the photon depends on both the frequency of the incident photons and the photon-TLA couplings. Consequently, this system can serve as a controllable photon attenuator, by which the transmission probability of the resonantly incident photons can be changed from 0% to 100%. A possible application to explain the recent experimental observations [Astafiev *et al.*, Science **327**, 840 (2010)] is also discussed.

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I. INTRODUCTION

Since they are significantly more easily and more quickly integrated with the usual telecommunications networks than the conventional electronic components, photons are naturally considered to replace electrons in future computers [1]. In particular, due to the absence of direct interaction, photons can serve as ideal carriers of quantum information and transmit over a sufficiently large distance without perturbation, i.e., the free distance of photon is significantly long (compared with that of electron in solid). Therefore, the studies of the photons being transported along various nanostructures is of fundamental importance in both experiments [2–5] and theories [6–8]. For example, it is interesting to study how to control the flow of photons by using the interaction of light with matter at deep subwavelength scale [9]. In fact, the interaction between matter and light is always a fundamental topic in physics, and its most elementary unit is the coupling of a single photon to a single atom [10,11].

One of the basic topics in nanophotonics is how to realize all-optical devices such as switches at the single-photon level. Recent experiments [12–14] demonstrated that a superconducting circuit quantum electrodynamics (CQED) system, containing an artificial atom with controllable energy levels, can be utilized to realize the desirable all-optical devices operated at a single-photon level. However, an unavoidable difficulty, i.e., how to control the transport of a single resonant photon, exists in such a nanophotonic structure. Indeed, Shen *et al.* [15] have shown that the incident photon being transported along a one-dimensional waveguide would be completely reflected if the transition frequency of the contained single two-level atom (TLA) equals the transported photon. This implies that a single two-level atom can be served as a completely reflected mirror of the resonant photon. Such an argument has been further confirmed recently by Zhou *et al.* [16] by studying the photonic transport along a series of coupled cavities containing a TLA. Obviously, by manipulating the transition frequency of the contained TLA [16] the transmission of the single photon can be controlled: complete reflection at the resonant point and complete transmission at the off-resonant one. In this Brief Report, we discuss how to realize the transports of the resonant photons by introducing a new nanophotonic structure, wherein the TLA asymmetrically interacts with the left- and right-traveling photons. Our numerical results show that, once such a nanophotonic structure is realized, the resonant photons could be either perfectly reflected or transmitted by just controlling the asymmetric coupling strength (rather than manipulating the transition frequency of the TLA [16] or introducing additional control fields [17]). The possible realization of such a singular nanophotonic structure is also discussed.

This Brief Report is organized as follows: Sec. II introduces the effective real-space Hamiltonian and gives a generic model to describe the motion of the single photon moving along the one-dimensional (1D) structure with a single TLA. In the framework of the usual quantum waveguide theory, we solve the corresponding Schrödinger equation to obtain the transmission probabilities of the incident photon and the excited ones of the TLA. In Sec. III, we discuss how the transmission spectrum depends on the frequency of the incident photon and also on the asymmetric coupling between the atom-photon interactions. The main results and the discussion of the feasibility of the proposal are presented in Sec. IV.

II. SINGLE-PHOTON TRANSPORT ALONG THE QUANTUM WAVEGUIDE WITH A TWO-LEVEL ATOM

As a generic model sketched in Fig. 1, we consider the scattering problem of a photon being transported along a one-dimensional waveguide containing a TLA. If the atom-photon coupling is symmetric, i.e., $V_1 = V_2$, then the present model simplifies to that treated first in Ref. [18]. For simplicity, we assume that the photons have the usual linear dispersion relations for the entire frequency range, and the transition

*weilianfu@gmail.com, weilianf@mail.sysu.edu.cn

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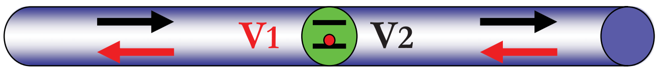


FIG. 1. (Color online) Schematics of an ideal quantum waveguide system: A single atom embedded in a one-dimensional quantum waveguide. Here the two-level atom (TLA) is indicated by a green ball with two short lines (representing the atomic energy levels). The arrows denote the transporting directions of the photons, and V_1 (V_2) represents the strength of the coupling between the atom and the left-traveling (right-traveling) photons.

frequency of the TLA is away from the cutoff frequency of the photon's energy dispersion. Therefore, it is safe to decompose the light field into two distinct contributions with positive and negative wave numbers, corresponding to the right- and left-traveling modes. As the typical dimension of an atom is significantly smaller than the wavelength of the transported photons, one can assume that the interaction between the photons and the atom occurs only at $x = 0$. As a consequence, the Hamiltonian of the system takes the following form (with $\hbar = 1$) [18–21]:

$$\begin{aligned} H = \int dx \Big\{ & -iV_g C_R^\dagger(x)\frac{\partial}{\partial x}C_R(x) + iV_g C_L^\dagger(x)\frac{\partial}{\partial x}C_L(x) \\ & + V_1\delta(x)[C_L^\dagger(x)a_g^\dagger a_e + C_L(x)a_e^\dagger a_g] \\ & + V_2\delta(x)[C_R^\dagger(x)a_g^\dagger a_e + C_R(x)a_e^\dagger a_g]\Big\} \\ & + \omega_e a_e^\dagger a_e + \omega_g a_g^\dagger a_g, \end{aligned} \tag{1}$$

with V_g being the group velocity of the photons. $C^\dagger_{L(R)}$ and $C_{L(R)}$ are the Fourier transforms of the bosonic creation and annihilation operators, describing the left- (L) and right-traveling (R) photons, respectively. Also, V_1 and V_2 are the couplings between the TLA and the left- and right-traveling photons, respectively. Note that the atom-photon coupling $V_{1(2)} = [2\pi\hbar/\omega_{1(2)}]^{1/2}\Omega\mathbf{D}\cdot\mathbf{E}_{1(2)}$ depends on the controllable parameters [22]: the dipole moment D of the TLA, the frequency $\omega_{1(2)}$, and the polarization $E_{1(2)}$ of the left-traveling (right-traveling) photons, respectively. $\delta(x)$ indicates that the interaction occurs at $x = 0$, and $a^\dagger_{e,g}(a_{e,g})$ is the creation (annihilation) operator of the atomic excited e and ground g states. Additionally, $\omega_e - \omega_g = \Omega$ is the energy difference between the atomic excited and the ground states.

Suppose that the atom is originally prepared at its ground state and the right-traveling photons are sent from the far left. After the interaction with the atom, the injected single photon may be absorbed by the atom or scattered into the left or the right direction. Therefore, the photonic state takes the following generic form:

$$|\Psi\rangle = \int dx\{\phi_R(x)C_R^\dagger(x) + \phi_L(x)C_L^\dagger(x)\}|0,g\rangle + c_e a_e^\dagger a_g|0,g\rangle, \tag{2}$$

where $|0\rangle$ indicates the photonic vacuum state and c_e is the probability amplitude of the atom in the excited state. After the interaction with the TLA, one could observe either the transmitted wave or the reflected wave. The corresponding wave function can be expressed as

$$\begin{aligned} \phi_R(x) &= te^{ikx}\theta(x) + e^{ikx}\theta(-x), \\ \phi_L(x) &= re^{-ikx}\theta(-x), \end{aligned} \tag{3}$$

where $t\,(r)$ is the relevant transmission (reflection) amplitude and $\theta(x)$ is a step function. From the eigenvalue equation, $H|\Psi\rangle = \omega|\Psi\rangle$, and the commutation relations, $[C_R(x'),C_R^\dagger(x)] = \delta(x'-x)$ and $[\frac{\partial}{\partial x'}C_R(x'),C_R^\dagger(x)] = \frac{\partial}{\partial x'}\delta(x'-x)$, we have

$$H|\Psi\rangle = (O_1 + O_2 + O_3 + O_4 + O_5)|0,g\rangle, \tag{4}$$

where

$$\begin{aligned} O_1 &= \int dx\Big\{-iV_g\Big[\frac{\partial}{\partial x}\phi_R(x)\Big]C_R^\dagger(x)\Big\} \\ &\quad + \int dx\Big\{iV_g\Big[\frac{\partial}{\partial x}\phi_L(x)\Big]C_L^\dagger(x)\Big\}, \\ O_2 &= V_1\int dx\phi_L(x)\delta(x)a_e^\dagger a_g + V_2\int dx\phi_R(x)\delta(x)a_e^\dagger a_g, \\ O_3 &= V_1\int dx\delta(x)c_e C_L^\dagger(x) + V_2\int dx\delta(x)c_e C_R^\dagger(x), \\ O_4 &= \omega_g\int dx\phi_R(x)C_R^\dagger(x) + \omega_g\int dx\phi_L(x)C_L^\dagger(x) \\ O_5 &= c_e\omega_e a_e^\dagger a_g. \end{aligned} \tag{5}$$

This directly yields the following equations:

$$\begin{aligned} -iV_g te^{ikx}\delta(x) + ie^{ikx}\delta(x) + V_2\delta(x)c_e &= 0, \\ -iV_g re^{-ikx}\delta(x) + V_1\delta(x)c_e &= 0, \\ Q_1 + Q_2 + \Omega c_e &= \omega c_e, \end{aligned} \tag{6}$$

with

$$\begin{aligned} Q_1 &= \int dx V_2\delta(x)[te^{ikx}\theta(x) + e^{ikx}\theta(-x)], \\ Q_2 &= \int dx V_1\delta(x)[re^{-ikx}\theta(-x)]. \end{aligned} \tag{7}$$

Solving these equations, we have

$$\begin{aligned} r &= \frac{-2V_2V_1}{V_2^2 + V_1^2 + i2V_g(\Omega-\omega)}, \\ t &= \frac{V_1^2 - V_2^2 + i2V_g(\Omega-\omega)}{V_2^2 + V_1^2 + i2V_g(\Omega-\omega)}, \\ c_e &= \frac{-i2V_2V_g}{V_2^2 + V_1^2 + i2V_g(\Omega-\omega)}. \end{aligned} \tag{8}$$

Finally, the transmission probability T is

$$T = |t|^2 = \cos^2\left[\arctan\left(\frac{2V_1V_2}{\sqrt{\left(V_1^2 - V_2^2\right)^2 + 4V_g^2(\omega-\Omega)^2}}\right)\right]. \tag{9}$$

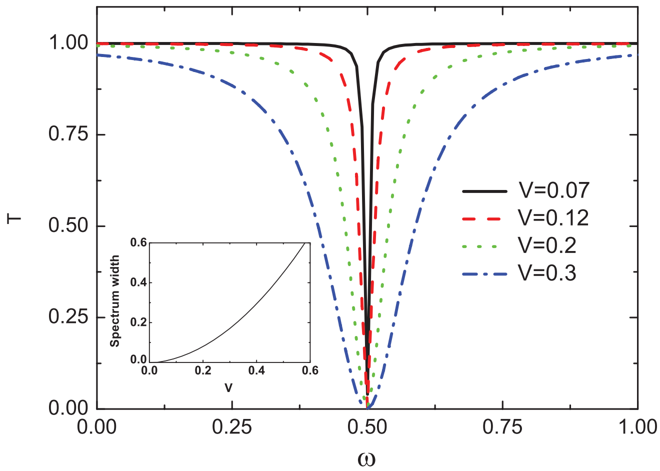
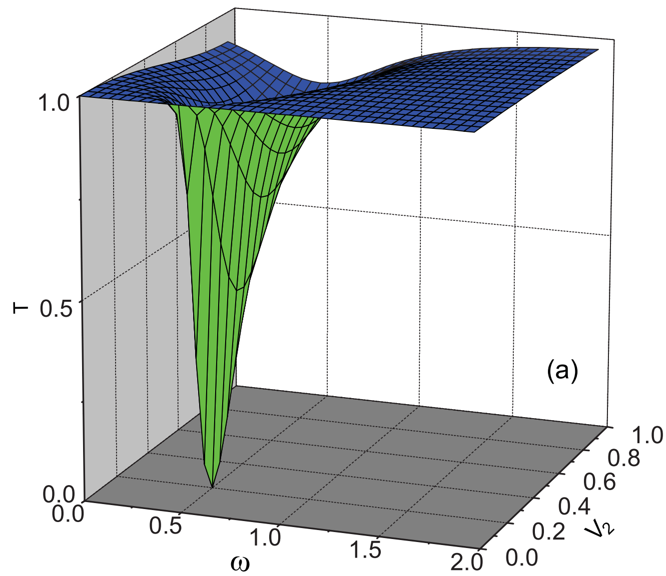


FIG. 2. (Color online) Transmission spectra of the transporting photons for different interactions with the TLA and $\Omega = 0.5$. Here, the energies ω, Ω, and the coupling strength V are in units of V_g. The inset shows the spectrum width of the transmission vs the coupling strength. It is clear that the symmetrical coupling only affects the spectrum width of the photonic transmission.

III. TRANSMISSION SPECTRA OF THE TRANSPORTING PHOTON

We now analyze numerically the transmission properties of the single photons being transported along the waveguide with a TLA. First, Fig. 2 clearly shows that, if the incident photon is resonance with the TLA and also $V_1 = V_2 = V$, then the photonic transmission probability $T \equiv 0$. This is consistent with the results given in Refs. [15,16,18] and indicates that the right-traveling photon is completely reflected. Therefore, a single TLA behaves really as a perfect mirror of the resonant photon. The inset in Fig. 2 shows how the coupling strength V affects the width of the transmission spectra. It is emphasized that, for any coupling strength V, the transporting photon is always reflected perfectly by the resonant TLA. Thus, the transmission of the incident photon cannot be controlled by adjusting the interaction with the resonant TLA.

In order to control the transmission of the resonant photon, we now consider another possible situation, in which $V_1 \neq V_2$. Without loss of generality, we fix one-direction coupling, e.g., let $V_1 = 0.2$, and adjust the coupling at another direction, i.e., V_2. Then, in Fig. 3 we show how the relevant transmission probability of the transporting photon depends on its frequency and the variable coupling V_2. The numerical results suggest that the transmission spectra of the incident photon depend on both the frequency of the photon and the symmetry of the two-side coupling strengths. One can see more clearly from Fig. 3(b) that (i) the transmission probability increases with the asymmetry of the couplings and the complete transmission can be achieved for either $V_1/V_2 \ll 1$ or $V_1/V_2 \gg 1$, (ii) in the situation of the weakly asymmetric coupling, i.e., V_1/V_2 slightly different from unity, the amplitude of the transmission probability is nonzero. This means that by adjusting the asymmetry of the couplings the transmission probability of the resonant photon can really be controlled from 1 to 0, and thus the system presented here could serve as an all-optical attenuator.



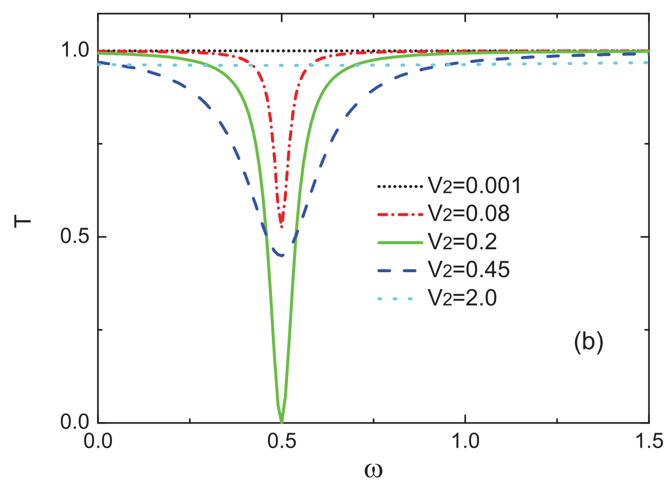


FIG. 3. (Color online) The transmission spectra of the transporting photons for asymmetric photon-TLA couplings for $V_1 = 0.2$ and $\Omega = 0.5$ vs (a) the parameters V_2 and photonic frequency ω and (b) the photonic frequency for various typical values of V_2. It is clear that the transmission probabilities of the resonant photons are controllable by adjusting the coupling strength $V_2 \neq V_1$.

IV. CONCLUSION AND DISCUSSION

In conclusion, we investigated the single-photon transport along a one-dimensional optical waveguide containing an asymmetrically coupled TLA. Differing from the symmetric coupling cases discussed previously, here we showed that the transmission probabilities of the incident single photons can vary from 0% to 100%, depending on the asymmetrical couplings with the TLA. Probably, this phenomenon is related to the redistribution of energy and momentum of photons after the scattering by the TLA, as the present photon-atom interaction loses the usual spatial symmetry. Especially, when the incident photon and the TLA are in resonance, the amplitude of the excited spectra can still be adjusted by the coupling strength. Therefore, the transporting properties of the single photons are related to both the photonic frequencies and the asymmetrical couplings between the photons and the TLA.

Note that a significantly strong coupling between an artificial atom and an open 1D transmission line has been achieved experimentally [12–14]. In this case, the size of the artificial atom is no longer negligible, compared with the wavelength of the traveling microwave, and thus the phase changing of the electromagnetic waves scattered by an artificial atom should be important. As a consequence, asymmetrical couplings between the photons (transported along different directions) and a TLA could be achieved. A possible example is that when a sufficiently big quantum dot couples with a nanowire with relatively small radii, its polaritonic emission is size dependent [23]. Therefore, the interactions between nanowire waveguides and the optically excited quantum emitters (i.e., the present quantum dots or artificial atoms) are beyond the usual dipole approximations.

Our proposal could be immediately utilized to explain an experimental result shown by Fig. 1(c) in Ref. [12]. There the maximal value of the extinction (i.e., reflected) probability of the resonant photon (scattered by an artificial TLA) is measured as 94% (not the desirable 100% reflection predicted by the previous theories [15–18,20]). Suppose that the environment- or measurement-induced departure is negligible; then such a 6% departure observed above could be explained by our numerical simulations for the parameters $V_2/V_1 = 0.7790$ or $V_2/V_1 = 1.2840$. This implies that asymmetrical couplings between the photon and a TLA could exist in various confined electromagnetic fields interacting with artificial atoms. Typically, the system introduced here may also be utilized to realize photonic attenuators at a single-photon level.

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