

Sub-Doppler laser cooling of potassium atomsM. Landini,^{1,2,3,*} S. Roy,¹ L. Carcagní,¹ D. Trypogeorgos,^{1,†} M. Fattori,^{1,2} M. Inguscio,^{1,2} and G. Modugno^{1,2}¹*LENS and Dipartimento di Fisica e Astronomia, Università di Firenze, I-50019 Sesto Fiorentino, Italy*²*INFN, Sezione di Firenze, I-50019 Sesto Fiorentino, Italy*³*Dipartimento di fisica, Università di Trento, I-38123 Povo (Trento), Italy*

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We investigate the sub-Doppler laser cooling of bosonic potassium isotopes, whose small hyperfine splitting has so far prevented cooling below the Doppler temperature. We find instead that the combination of a dark optical molasses scheme that naturally arises in this kind of system and an adiabatic ramping of the laser parameters allows us to reach sub-Doppler temperatures for small laser detunings. We demonstrate temperatures as low as $25 \pm 3 \mu\text{K}$ and $47 \pm 5 \mu\text{K}$ in high-density samples of the two isotopes ^{39}K and ^{41}K , respectively. Our findings should find application to other atomic systems.

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I. INTRODUCTION

Sub-Doppler laser cooling of neutral atoms [1] is a key technique for the production of ultracold and quantum gases. It allows for atoms to be cooled to temperatures below the Doppler limit [2], not far from the single-photon recoil energy. This favors the application of further cooling techniques, such as Raman-sideband cooling or evaporative cooling, to reach quantum degeneracy. It also realizes a fast and effective cooling method for some classes of atomic interferometers and clocks [3]. The sub-Doppler cooling mechanism arises whenever the atomic ground state has an internal structure with state-dependent light shifts. Such a situation is typically accompanied by a hyperfine structure of the excited state [4]. The sub-Doppler cooling is efficient only if the excited state has a hyperfine splitting Δ either much larger than the natural linewidth Γ , like that for the alkali metals Na, Rb, and Cs, or smaller than Γ , as for example in Sr [5]. In the intermediate case of $\Delta \sim \Gamma$, it can instead be hindered by the presence of heating forces or by photon reabsorption [6]. The bosonic potassium isotopes fall into this latter category [7], and no efficient sub-Doppler cooling has been observed so far [8–13].

We now instead find that sub-Doppler cooling can take place also in atoms like K, by employing a near-detuned optical molasses and an appropriate strategy to tune the cooling laser parameters. We observe that the natural depumping toward dark states taking place in this kind of system allows us to reach low temperatures even in high-density samples. In experiments on the isotopes ^{39}K and ^{41}K , we achieve temperatures substantially lower than those previously achieved, with an efficiency similar to that of most other alkali-metal species.

II. SUB DOPPLER COOLING IN THE PRESENCE OF A NARROW HYPERFINE STRUCTURE

Before presenting the specific properties of K atoms, let us discuss the general aspects of laser cooling of atoms with a

hyperfine structure. In a two-level system, Doppler cooling arises when a laser is tuned below the atomic transition frequency, where the atoms experience a friction force: $F = -\alpha v$. If the presence of Zeeman sublevels is taken into account [14], a much larger friction arises for small velocities, leading to temperatures much lower than the Doppler limit $k_B T_D = \hbar\Gamma/2$. While in principle the lowest achievable sub-Doppler temperatures are independent of the laser detuning δ [15], the experiments with high-density samples are performed at large detunings $\delta \gg \Gamma$. This requirement arises from the need to keep the scattering rate of photons by individual atoms low in such a way that spontaneously emitted photons do not disturb the cooling process [6]. Most atomic systems cannot be modeled as simple two-level ones since they feature a hyperfine structure like the one in Fig. 1. This specific scheme applies only to the alkali-metal species with nuclear moment $I = 3/2$, namely, to ^{23}Na , ^{39}K , ^{41}K , and ^{87}Rb , but our discussion remains valid for a much larger class of atomic species. In the case of a hyperfine structure, it is commonly thought that δ must also be smaller than the main hyperfine splitting Δ , since otherwise the presence of the other excited states would turn the sub-Doppler mechanism into a heating one. As a matter of fact, in the case of the bosonic K isotopes, where $\Delta \approx 2\Gamma - 3\Gamma$, a clear sub-Doppler cooling has not been experimentally observed. This can be understood from the nature of the optical forces we have calculated for the level structure in Fig. 1. In general, the laser light provides cooling if red detuned from one of the hyperfine transitions, while it otherwise causes heating. Doppler and sub-Doppler forces, however, scale differently with the detuning. We therefore find that the Doppler part of the optical force provides cooling only in regions of types I and IV in Fig. 1, while sub-Doppler cooling is active only in regions of types I and II. Sub-Doppler cooling is therefore active either very close to resonance, where heating from photon reabsorption might be large, or for $\delta \gg \Delta$, where, however, the velocity capture range is very low. Note that in the case of $\Gamma > \Delta$, as for ^{87}Sr , the sub-Doppler cooling stays efficient for $\delta > \Delta$ also [5].

We have now realized that the presence of neighboring excited states also has a beneficial effect. Indeed, it causes a natural depumping of the atomic population into a dark state, such as the $F = 1$ ground state as shown in Fig. 1.

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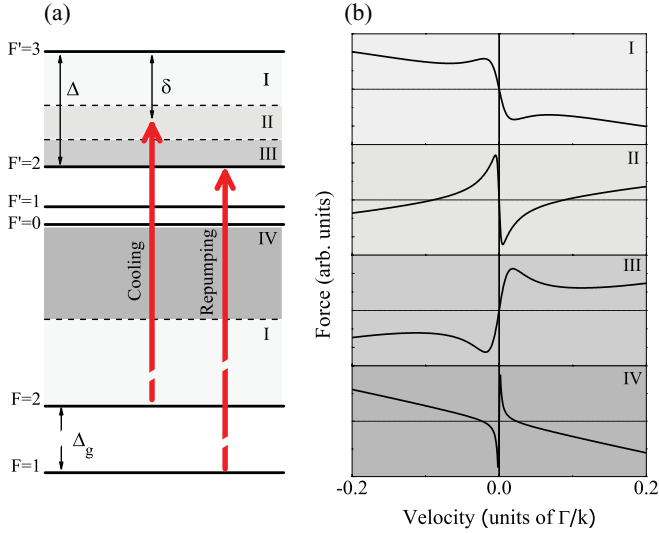


FIG. 1. (Color online) Working regions for sub-Doppler cooling of bosonic potassium. (a) Level scheme including the relevant hyperfine splitting Δ and the cooling-laser detuning δ ; the ground-state hyperfine splitting Δ_g is also indicated. (b) Calculated cooling forces vs the atomic velocity in the various regions of (a). Doppler cooling takes place only in regions I and IV, while sub-Doppler cooling is active only in regions I and II. The scheme does not show the detailed regions between the $F' = 2$ and 0 levels, and the ground-state splitting, not to scale, is typically much larger than that of the excited state.

The atoms can of course be moved back into the bright $F = 2$ state by the repumper laser but, unlike the situation in a pure two-level system, here this can be done in a controlled way. It is then possible to adjust the fraction of atoms in the state coupled to the cooling laser in order to optimize the cooling power, while keeping the reabsorption of spontaneously emitted photons under control. Note that the possibility of controlling the population of the bright state is absent if $\Delta \gg \Gamma$ unless an appropriate depumping laser is used [16]. This mechanism, which is widely used to trap atomic samples at high density in magneto-optical traps [17], turns out to be the first essential ingredient for sub-Doppler cooling when $\Delta \approx \Gamma$, since it allows low temperatures to be reached also when $\delta \approx \Gamma$. This is apparent from Fig. 2, which shows the minimum temperature we measured by time of flight for ^{39}K in near-resonant molasses with a very low intensity of the repumping light and a large atomic density (further details are given below). The measured temperature is well below the Doppler limit already for $\delta < \Gamma$, although a further decrease with increasing δ is apparent.

The second important observation is that the sub-Doppler cooling survives for detunings larger than does the Doppler cooling, as shown, for example, in the calculations of Fig. 1. The lowest temperatures can actually be reached only for a range of detunings where the Doppler force no longer provides an efficient cooling. This is in principle a problem in experiments, since the velocity capture range of the sub-Doppler cooling mechanism is usually smaller than the initial thermal velocity, for example at the end of the capture stage of a magneto-optical trap. Indeed, one normally needs to exploit both Doppler and sub-Doppler cooling to achieve low

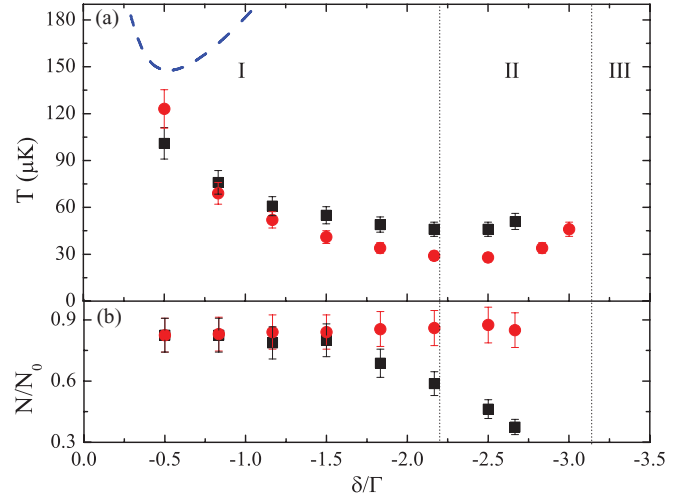


FIG. 2. (Color online) Optimal sub-Doppler temperatures for ^{39}K . (a) Measured temperature without (black squares) and with (red dots) the ramping strategy compared to the Doppler theory (dashed line). Temperatures are measured by time of flight. The dotted lines separate the various regions as in Fig. 1. (b) Fraction of atoms remaining in the colder component without (black squares) and with (red dots) the ramping strategy.

temperatures [14]. We now find that one can still combine an initial Doppler cooling with a final stage of optimal sub-Doppler cooling by using a proper dynamical variation of detuning and intensity of the cooling laser between the two regimes of operation. As a matter of fact, the combination of these two ingredients makes the cooling of K as efficient as in the other alkali-metal species.

III. EXPERIMENTAL STRATEGIES

We now discuss the experimental strategy in detail. The linewidth of the cooling transition for K is $\Gamma = 2\pi \times 6.0$ MHz, which corresponds to a Doppler temperature $T_D \approx 145$ μK . The excited-state hyperfine splitting Δ is about 3.5Γ and 2.2Γ for ^{39}K and ^{41}K , respectively. The ground-state hyperfine splitting Δ_g is about 77Γ and 42Γ for ^{39}K and ^{41}K , respectively. We perform cooling and trapping in a three-dimensional magneto-optical trap (MOT) on the D_2 transition around 767 nm. The trap is loaded with precooled atoms from a two-dimensional MOT. After 3 s of loading stage we have either about 2×10^{10} atoms of ^{39}K or 4×10^9 atoms of ^{41}K at temperatures in the 1 mK regime. We then compress the cloud via application of a compressed-MOT technique to densities around 1×10^{11} atoms/cm³. During this initial cooling stage we adopt the standard strategy used for bosonic potassium [9,13]. We use a detuning larger than the whole excited manifold (region IV) with total intensities as large as $20I_s$ for both the cooling (I_{cool}) and the repumping (I_{rep}) beams ($I_s = 1.75$ mW/cm²). This allows for a large Doppler capture velocity. Finally, we switch off the magnetic field and cool the cloud in a molasses scheme as described below.

We initially reduce I_{rep} to $(1/100)I_{\text{cool}}$ suddenly and set the repumper beam frequency on resonance with the $F = 1 \rightarrow F' = 2$ transition. If we then try to perform standard molasses cooling, i.e., by a sudden change of the laser parameters to

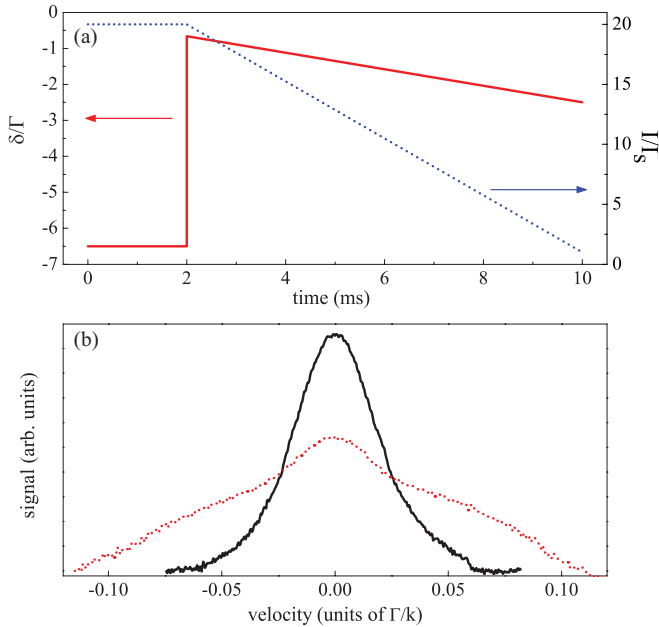


FIG. 3. (Color online) Sub-Doppler cooling strategy for ^{39}K . (a) Time evolution of I_{cool} and δ [$I_{\text{rep}} = (1/100)I_{\text{cool}}$]. (b) Resulting velocity distribution measured after a free expansion (black solid line) compared to that obtained without the ramp (red dashed line).

the optimal sub-Doppler cooling values, we observe moderate sub-Doppler cooling only for small δ . A larger δ results instead in a bimodal distribution of the atomic velocities. A typical instance of such distribution is shown in Fig. 3(b), as measured by fluorescence imaging after a free expansion of the cloud. The narrow peak corresponds to sub-Doppler temperatures, while the broader distribution can be attributed to inefficient Doppler cooling or even to Doppler heating. As shown in Fig. 2, the fraction of atoms in the central component decreases as δ is increased. A more effective strategy consists in first tuning the laser to $\delta \approx \Gamma/2$ to provide an initial Doppler cooling and then in slowly decreasing the intensity while increasing δ , as shown in Fig. 3(a). This method allows cooling of nearly 90% of the atoms to lower temperatures, as shown in Fig. 2. By minimizing the final temperature we find an optimal ramping time of about 10 ms, which corresponds to an adiabatic narrowing of the velocity distribution during the whole sequence.

The minimum temperature attained for ^{39}K is about $25 \mu\text{K}$ at $\delta \approx 2.5\Gamma$. It then rises again for larger δ , presumably because of the progressive reduction of the force. We have observed, as shown in Fig. 4, that an increase of the repumping power prevents the achievement of such low temperatures at high density, while the temperature does not depend on the repumper power at low density. This confirms the role of reabsorption of spontaneously emitted photons inside the cloud. Analogous measurements performed on ^{41}K demonstrate a similar behavior, as shown in Fig. 5. In this case a minimum temperature of about $50 \mu\text{K}$ is reached for a detuning $\delta \approx \Gamma$.

IV. ANALYSIS AND SIMULATIONS

We have compared the observations with a theoretical estimation of the temperatures achievable for our experimental

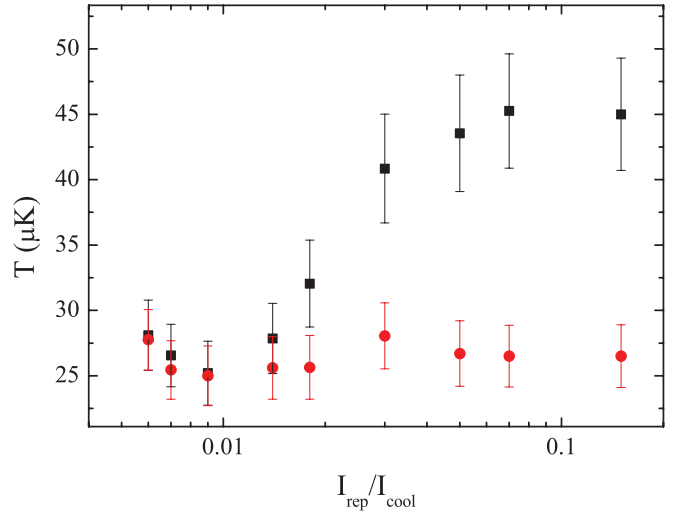


FIG. 4. (Color online) Measured temperatures for ^{39}K vs the intensity ratio of the repumping and cooling light, for densities of 4×10^{10} atoms/cm³ (black squares) and 8×10^8 atoms/cm³ (red dots). The heating arising from reabsorption effects at high density can be tuned by reducing the repumper intensity.

parameters. The optical force, shown in Fig. 1, is calculated from the solution of the optical Bloch equations in the semi-classical approximation, using the model developed in Ref. [7]. The calculations are worked out in one dimension (1D) and for $\sigma^+ - \sigma^-$ polarizations. This is only an approximation to the more complex polarization geometries arising in the 3D laser configuration we have in our experiment. In practical situations, in a magneto-optical trap, both Sisyphus cooling (lin \perp lin) and the $\sigma_+ - \sigma_-$ polarization gradient cooling play a role in the cooling process, with the former dominating at large and the latter at small detuning [14,18]. In our simulation, only the cooling beam is taken into account to find the cooling force. This approximation is justified by the very low repumper intensity we use in the experiment. The equilibrium temperature is then calculated as $k_B T = D_p/\alpha$,

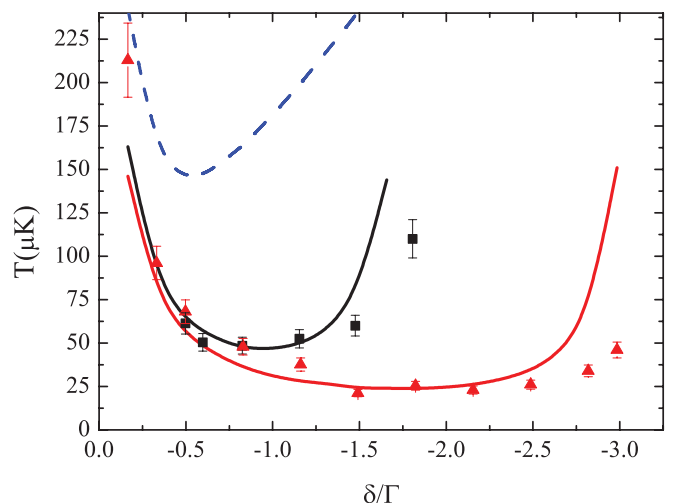


FIG. 5. (Color online) Measured temperatures for ^{39}K (red triangles) and ^{41}K (black squares) and calculated temperatures (lines) vs the cooling laser detuning. The dashed line is the predicted Doppler temperature.

where D_p is the momentum diffusion coefficient and α is the friction coefficient calculated as the slope of the force at $v = 0$. We estimate D_p by using a simple argument for the random step of the Brownian motion in momentum space in multilevel transitions described in Ref. [19]. The agreement with the experimental data shown in Fig. 5 is, however, rather good. To check the validity of these calculations we directly measured the spatial diffusion coefficient D_x in the optical molasses for ^{39}K . By a simultaneous measurement of the temperature, we estimated the friction coefficient as $\alpha = k_B T / D_x$ [20]. The magnitude of $\alpha \approx 10^{-3} \hbar k^2$ is in good agreement with the calculations. A simple estimation considering a two-level system would give a result about three orders of magnitude larger. We interpret this low friction as a result of the macroscopic occupation of the dark state. More details are provided in the Appendix.

The minimum temperatures measured in the experiment, shown in Fig. 5, were obtained essentially for a constant laser intensity. The data at small detunings are apparently well described by the scaling law observed in several other systems [5,21–24]:

$$T = C_{\sigma^+\sigma^-} \frac{\hbar\Gamma}{2k_B} \frac{\Gamma}{|\delta|} \frac{I}{I_s} + T_0. \quad (1)$$

From a combined fit we get $C_{\sigma^+\sigma^-} = 0.20(2)$ and $T_0 = 9(3) \mu\text{K}$. The $C_{\sigma^+\sigma^-}$ coefficient is smaller than those measured on the two species where sub-Doppler cooling has been observed for $\delta \lesssim \Gamma$, i.e., ^{87}Er [23] [0.38(2)] and ^{87}Sr [5] [1.3(3)], but larger than those measured in Rb and Cs at large detunings [22]. The observed scaling for K suggests that heating processes due to reabsorption are less efficiently suppressed when δ is small, as expected.

To characterize the robustness of the cooling process against stray magnetic fields we tried to keep the MOT magnetic field on during the cooling procedure. For gradients larger than 5 G/cm, we reached the Doppler temperature. This gradient

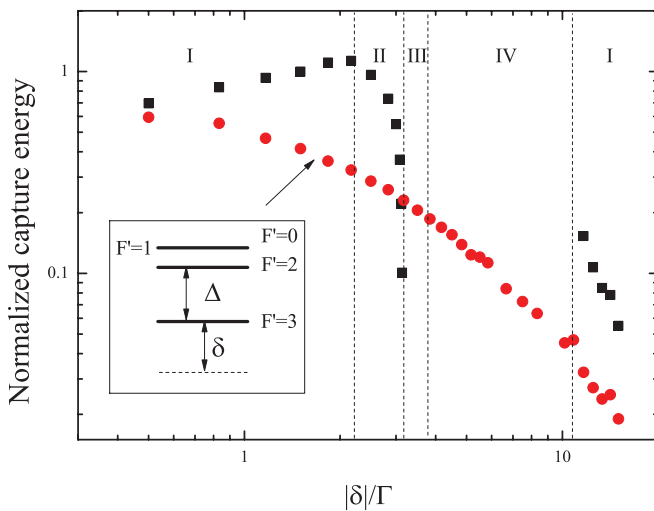


FIG. 6. (Color online) Calculated sub-Doppler capture energy for ^{39}K (black squares) and for a hypothetical atomic species with an inverted hyperfine structure as shown in the inset (red dots). Both quantities are normalized to the capture energy of an atomic species with $\Delta = 35\Gamma$. See text for more details.

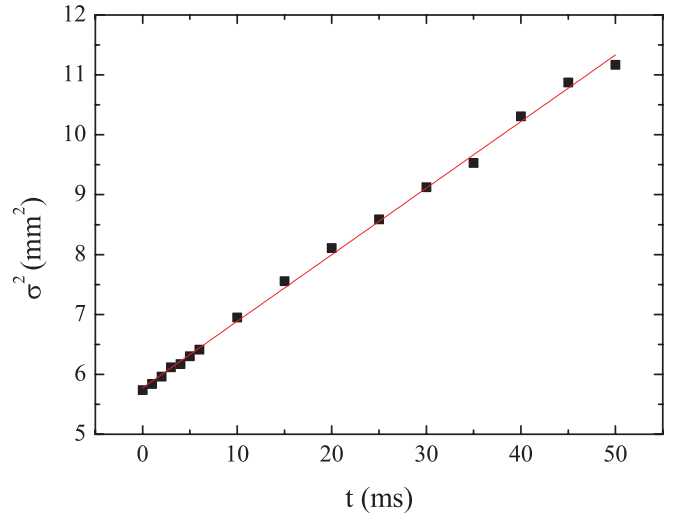


FIG. 7. (Color online) Typical data for the variation of the width of the atomic distribution as a function of the time spent inside the molasses cooling beams. The solid line is a fit by Eq. (A1).

corresponds to an average magnetic field of about 1 G, which is the same characteristic value found for the other alkali atoms.

The techniques described here might be applied to other systems, such as the $^1S \rightarrow ^1P$ transitions of ^{43}Ca [25] and ^{173}Yb [26], for which $\Delta \approx 3\Gamma$. Additionally, it would be interesting to apply our cooling strategy to Na, for which $\Delta \approx 6\Gamma$. Another interesting case is that of an inverted and narrow hyperfine structure as in ^{40}K . In this case, there are no interfering levels which can directly cause heating. However, an increase of δ to values of the order of Δ or more leads to a washing out of the sub-Doppler cooling mechanism itself, since the detuning from the various hyperfine levels becomes of the same order. We performed numerical simulations of the atomic force and found that this effect leads to a fast decrease of the capture velocity with increasing detuning, as shown in

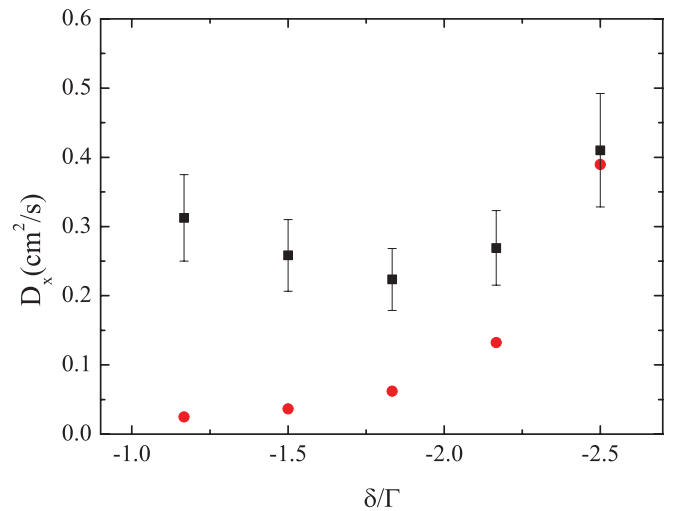


FIG. 8. (Color online) Measured diffusion coefficients in the optical molasses (black squares) compared to the numerically calculated values (red dots) as a function of the detuning of the cooling laser.

Fig. 6. In more detail, for this analysis we first calculate the velocity capture range v_c , defined as the velocity giving the first local maximum of force at low velocity. We do this in three cases: (a) a hyperfine level structure like that of ^{39}K , (b) an inverted hyperfine level structure of a hypothetical atomic species with the same total angular momentum, and (c) the same hyperfine structure of ^{39}K but with a tenfold increase in the hyperfine splitting, which reproduces an atomic species like ^{87}Rb . In Fig. 6 we plot the capture energy $E_c = \frac{1}{2}mv_c^2$ for the cases (a) and (b), normalized by that of case (c).

It is possible to see that the decay of the capture velocity for the inverted structure is faster than that for atoms with a large Δ (or with a large Γ/Δ as in the case of Sr). By increasing δ one rapidly reaches a regime in which the minimum sub-Doppler temperature $k_B T = D_p/\alpha$ exceeds the capture range. This is presumably the reason for the bimodal distribution seen in experiments with ^{40}K [27]. Weaker rates of natural depumping to dark states will possibly require forced depumping [16] in high-density samples, but further experimental and numerical investigations are needed [28].

V. CONCLUSIONS

We have shown how the limitations of sub-Doppler laser cooling in atomic species with small hyperfine splitting can be overcome by the natural control of the photon reabsorption and adiabatic ramping of the laser parameters. The laser-cooling techniques we have developed will be easily implemented in all existing experiments with potassium atoms. Finally, the direct application of laser-cooled potassium atoms to interferometric measurements might enable a new class of experiments [29].

Note added. Recently, we became aware of a recent experimental report of sub-Doppler temperatures in potassium isotopes [30].

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APPENDIX: DIFFUSION MEASUREMENT

We measure the spatial diffusion coefficient D_x for the Brownian atomic motion during the molasses cooling phase. This is a useful quantity since it connects to the other quantities characterizing the cooling process (namely, T , α , and D_p) via the simple formula

$$D_x = \frac{D_p}{\alpha^2} = \frac{k_B T}{\alpha}. \quad (\text{A1})$$

Hence, by measuring both T and D_x one can reconstruct the other useful quantities. The measurement is performed at different final detunings of the molasses cooling beams by allowing the atomic cloud to expand in the presence of the cooling light for a variable time, and then taking an image after 100 μs dark period. By recording the variation of the width with time in the molasses cooling beams, we find it to be in agreement with a diffusion process (Fig. 7). By fitting the evolution of the cloud size with the equation

$$\sigma(t)^2 = \sigma_0^2 + 2D_x t, \quad (\text{A2})$$

we are able to extract the spatial diffusion coefficient.

From the results in Fig. 8 we see that the agreement with the theory is good for our usual experimental parameters but the agreement becomes rather poor for smaller detunings. This might be due to the presence of additional heating from rescattered photons, since these measurements were taken at high densities (4×10^{10} atoms/cm³). These values for the diffusion coefficient are a factor of about 1000 times higher than the ones measured in [20]. The reason for this is the very low population in the $F = 2$ level caused by natural depumping and the use of a very weak repumping light.

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