Comment on "Temperature dependence of the Casimir force for lossy bulk media"

G. Bimonte,[1] G. L. Klimchitskaya,[2,3,*] and V. M. Mostepanenko[3,4]

[1]*Dipartimento di Scienze Fisiche, Università di Napoli Federico II, Complesso Universitario MSA, Via Cintia I, I-80126 Napoli, and INFN, Sezione di Napoli, Napoli, Italy*

[2]*North-West Technical University, Millionnaya St. 5, St. Petersburg 191065, Russia*

[3]*Department of Physics, Federal University of Paraíba, C.P. 5008, CEP 58059-900, João Pessoa, Brazil*

[4]*Noncommercial Partnership "Scientific Instruments," Tverskaya St. 11, Moscow 103905, Russia*

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Recently Yampol'skii *et al.* [Phys. Rev. A **82**, 032511 (2010)] advocated that Lifshitz theory is not applicable when the characteristic wavelength of the fluctuating electromagnetic field, responsible for the thermal correction to the Casimir force, is larger than the size of the metal test bodies. It was claimed that this is the case in experiments which exclude Lifshitz theory combined with the Drude model. We calculate the wavelengths of the evanescent waves making the dominant contribution to the thermal correction and we find that they are much smaller than the sizes of the test bodies. The opposite conclusion obtained by the authors arose from confusion between propagating and evanescent waves.

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It is the subject of a considerable body of literature that theoretical predictions for the thermal Casimir force between lossy metal plates described by the Drude model, based on Lifshitz theory, are in disagreement with experimental data (see, e.g., review [1]). In Ref. [2] an attempt was made to explain this contradiction by arguing that Lifshitz theory is indeed inapplicable to test bodies of finite size, such as those used in the experiments. According to Ref. [2], the thermal electromagnetic fluctuations responsible for the predicted large thermal correction [3], excluded by several experiments, have a characteristic wavelength which is larger than the size of the test bodies used in the experiments. On this basis, the conclusion is made that the predicted correction can be observed experimentally only for sufficiently large metal bodies. Here we show that the wavelengths of the fluctuations contributing to the large thermal correction engendered by the Drude model are in fact much less than the sizes of test bodies used in related experiments. Because of this, the purported explanation of the contradiction between experiment and theory in Ref. [2] is in error. We argue that the considerable overestimate made in Ref. [2] of the wavelengths of the contributing fluctuations was the result of confusion between traveling (propagating) and evanescent waves.

The frequencies and wave vectors of a fluctuating electromagnetic field making a major contribution to the thermal correction to the Casimir force can be found using the Lifshitz formula written in terms of real frequencies. In modern notation, the thermal correction to the Casimir force per unit area, between two parallel semispaces at temperature T separated by a gap of width l, can be represented in the form [4,5]

$$F_{\rm rad}(l) = -\frac{\hbar}{\pi^2}\int_0^\infty k_\perp dk_\perp \int_0^\infty \frac{d\omega}{e^{\hbar\omega/k_BT}-1} \times {\rm Im}\left\{ q \sum_{\alpha={\rm TM,TE}} \left[r_\alpha^{-2}(\omega,k_\perp)e^{2lq}-1\right]^{-1}\right\}. \quad (1)$$

Here, $k_\perp = |\boldsymbol{k}_\perp|$ is the magnitude of the projection of the wave vector onto the boundary planes, ω is the wave frequency, k_B is the Boltzmann constant, and $q^2 \equiv q^2(\omega,k_\perp) = k_\perp^2 - \omega^2/c^2$. The reflection coefficients for two independent polarizations of the electromagnetic field (transverse magnetic, $\alpha = {\rm TM}$, and transverse electric, $\alpha = {\rm TE}$) are given by

$$r_{\rm TM}(\omega,k_\perp) = \frac{\varepsilon(\omega)\,q-k}{\varepsilon(\omega)\,q+k}, \quad r_{\rm TE}(\omega,k_\perp) = \frac{q-k}{q+k}, \quad (2)$$

where

$$k^2 \equiv k^2(\omega,k_\perp) = k_\perp^2 - \varepsilon(\omega)\frac{\omega^2}{c^2}, \quad (3)$$

and $\varepsilon(\omega)$ is the frequency-dependent dielectric permittivity of the material of the semispaces. Equation (1) coincides with Eq. (3) in Ref. [2], after correcting one misprint contained there (in the exponent in the Boltzmann factor on the right-hand side of Eq. (3), the factor of 2 should be erased; an analogous misprint should be corrected in Eq. (5) in Ref. [2]).

In Ref. [6] it was shown that if the material of the semispaces is described by the Drude model,

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega+i\nu)}, \quad (4)$$

where ω_p is the plasma frequency and ν is the relaxation parameter, the major contribution to $F_{\rm rad}$ is made by TE *evanescent* waves. At room temperature $T = 300$ K this holds at all separations $l \ll \hbar c/k_BT \approx 7.6\ \mu$m. For example, for Au semispaces with $\hbar\omega_p = 9$ eV and $\nu = 5.32\times 10^{13}$ rad/s at a separation $l = 162$ nm, TE evanescent waves contribute about 99.7% of the thermal correction. This contribution can be denoted $F_{\rm rad,\,TE}^{\rm evan}$. In Eq. (1), the quantity $F_{\rm rad,\,TE}^{\rm evan}$ is obtained by taking the term with $\alpha = {\rm TE}$ for frequencies ω varying in the interval from 0 to $ck_\perp$, for which the quantity q is real. Even though the concept of evanescent waves is never mentioned in Ref. [2], the contribution of TE evanescent waves is actually reproduced by the quantity in the first pair of square brackets in Eq. (3) in Ref. [2], integrated over imaginary values of p ranging from $i0$ to $i\infty$. The same contribution

*galina.klimchitskaya@itp.uni-leipzig.de

can be physically interpreted in terms of interaction of eddy currents [7,8].

As also shown in Ref. [6], at short separations between two semispaces described by Eq. (4), the frequencies ω making a dominant contribution to $F^{\rm evan}_{\rm rad,\,TE}$ satisfy the inequality $\omega \lesssim \nu(\omega_c/\omega_p)^2$, where $\omega_c = c/(2l)$ is the characteristic frequency. This result was qualitatively confirmed in Ref. [2], where the frequencies contributing to the quantity $F^{\rm evan}_{\rm rad,\,TE} - F^{\rm evan}_{\rm rad,\,TE}|_{\nu=0}$ were found to satisfy the inequality $\omega \lesssim \nu$ (at $l = 100$ nm, it holds that $\omega_c \approx \omega_p/9$). We note that the term $F^{\rm evan}_{\rm rad,\,TE}|_{\nu=0}$, which is subtracted from $F^{\rm evan}_{\rm rad,\,TE}$ in Ref. [2], represents a negligibly small thermal effect that results once the material for the semispaces is described by the plasma model, and it does not influence any of the obtained conclusions.

It is important to realize that the characteristic wavelength of the evanescent waves making the largest contribution to the thermal correction is determined, however, not by the frequency spectrum of $F^{\rm evan}_{\rm rad,\,TE}$ but, rather, by its *wave-vector spectrum*. In order to determine the latter spectrum numerically, we have recast the quantity $F^{\rm evan}_{\rm rad,\,TE}$ in the following equivalent form in terms of dimensionless variables:

$$F^{\rm evan}_{\rm rad,\,TE}(l) = \frac{\hbar\,\nu\,c^2}{\pi^2\omega_p^2\,l^5}\int_0^\infty dv v^2 g(v), \tag{5}$$

where

$$g(v) = \int_0^\infty \frac{du}{\exp\left(\frac{\hbar\nu}{k_BT}\frac{c^2}{\omega_p^2 l^2}u\right) - 1}\,{\rm Im}\left[1 - \frac{e^{2v}}{r^2_{\rm TE}(u,v)}\right]^{-1}. \tag{6}$$

Here, the new variables are defined as

$$u = \frac{\omega_p^2 l^2}{\nu c^2}\omega, \quad v = lq. \tag{7}$$

In terms of these variables the TE reflection coefficient is given by

$$r_{\rm TE}(u,v) = \frac{v - \sqrt{v^2 + \frac{\omega_p^2 l^2 u}{i\omega_p^2 l^2 + c^2 u}}}{v + \sqrt{v^2 + \frac{\omega_p^2 l^2 u}{i\omega_p^2 l^2 + c^2 u}}}. \tag{8}$$

We have computed the range $v_1 \leqslant v \leqslant v_2$ of the variable v which contributes 90% of $F^{\rm evan}_{\rm rad,\,TE}$ at the experimental separation $l = 162$ nm. For $\nu = 5.32 \times 10^{13}$ rad/s, we found $v_1 = 0.26$ and $v_2 = 3$, while for $\nu = 10^{10}$ rad/s, we obtained $v_1 = 0.28$ and $v_2 = 3$. Thus, independently of the values of the relaxation parameter ν, the dimensionless quantity v contributing to $F^{\rm evan}_{\rm rad,\,TE}$ is always of order 1, and therefore q is always of order $1/l$.

The wave vector of an evanescent wave is given by the expression

$$\boldsymbol{k} = (k_x,k_y,k_z), \quad k_z = \sqrt{\frac{\omega^2}{c^2} - k_\perp^2} = iq, \tag{9}$$

and its wavelength is determined as

$$\lambda = \frac{2\pi}{k_\perp} = \frac{2\pi}{\sqrt{k_x^2 + k_y^2}}. \tag{10}$$

Keeping in mind the definition of q, we then obtain that for the most contributing wave vectors, it holds that $k_\perp^2 > q^2 \sim 1/l^2$, in such a way that the corresponding wavelengths satisfy the inequality

$$\lambda \lesssim 2\pi\,l. \tag{11}$$

In experiments aiming at measuring the Casimir force between a sphere and a plate, these wavelengths are always much smaller than the characteristic size L of the part of the sphere surface,

$$L \approx 2\sqrt{R^2 - (R-l)^2} \approx 2\sqrt{2Rl}, \tag{12}$$

which contributes to the force. For example, in the experiment in Ref. [9] the sphere radius is $R = 150\ \mu$m, and the separation distances vary from $l = 162$ nm to $l = 750$ nm. For such values of R and l, the inequality $\lambda_{\max} = 2\pi l < L$, i.e., $l < 2R/\pi^2 \approx 30\ \mu$m is satisfied with large safety margins, for all separations considered. In fact, the relevant contributing wavelengths are smaller than the sizes of the bodies in all other experiments measuring the Casimir force performed up to date as well [1].

The opposite conclusion obtained in Ref. [2] is caused by the confusion between propagating and evanescent waves. Starting from a qualitatively correct inequality for the contributing frequencies $\omega \lesssim \nu$, the authors of Ref. [2] used the relation between the frequency and the period

$$\omega = \frac{2\pi c}{\lambda} \tag{13}$$

to obtain the estimate $\lambda \gtrsim 2\pi c/\nu$ for the wavelengths of the fluctuations contributing to $F^{\rm evan}_{\rm rad,\,TE}$. Thereafter, it was concluded that Lifshitz theory is only applicable if the size of test bodies $L \gg 2\pi c/\nu$, i.e., $\nu \gg 2\pi c/L$ [Eq. (9) in Ref. [2]]. The problem with this argument, though, is that Eq. (13) is valid only for traveling (propagating) waves in vacuum. In this case the two definitions of the wavelength $\lambda = 2\pi c/\omega = 2\pi/|\boldsymbol{k}|$ coincide. Unfortunately, in the case of evanescent waves, which do not propagate and are more similar to standing waves, Eq. (13) does not hold, and the wavelength has no relation to the frequency. If instead of using Eq. (13), the authors of Ref. [2] had considered the characteristic values of their parameter $x = 2ip\omega l/c$ (where in our notation $p = -iqc/\omega$) to determine the most contributing wavelengths, our result $\lambda \lesssim 2\pi l$ would have been reproduced. Indeed, as shown in Ref. [2], $x = 2lq \sim 1$, leading to $q \sim 1/(2l)$, in qualitative agreement with our estimate, (11). Bearing in mind that for evanescent waves the frequency is unrelated to the wavelength, the second inequality, $\omega \lesssim k_BT/\hbar$, considered in Ref. [2] does not lead to any constraint on the size of bodies L. For the same reason, the results of numerical computations presented in Figs. 1 and 2 in Ref. [2] do not contain any information concerning the role of finite sizes of the test bodies in calculations of the thermal Casimir forces.

To conclude, the problem of the disagreement between the experimental data of several experiments and the theoretical prediction of the thermal effect in the Casimir force, obtained by using Lifshitz theory in combination with the Drude model, remains unsolved.

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