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Entanglement purification and concentration of electron-spin entangled states using quantum-dot spins in optical microcavities

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We present an entanglement purification protocol and an entanglement concentration protocol for electron-spin entangled states, resorting to quantum-dot spin and optical-microcavity-coupled systems. The parity-check gates (PCGs) constructed by the cavity-spin-coupling system provide a different method for the entanglement purification of electron-spin entangled states. This protocol can efficiently purify an electron ensemble in a mixed entangled state. The PCGs can also concentrate electron-spin pairs in less-entangled pure states efficiently. The proposed methods are more flexible as only single-photon detection and single-electron detection are needed.

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I. INTRODUCTION

Quantum entanglement is a key ingredient of quantum information processing (QIP). It can speed up the computation [1–3]. Also, entanglement provides some different methods for quantum communication, such as quantum key distribution [4–8], quantum secret sharing [9–12], quantum teleportation [13,14], quantum dense coding [15–17], quantum secure direction communication [18–21], and so on. Long-distance quantum communication requires quantum repeaters [22,23] in which entangled photon pairs are necessary for linking the two remote-location nodes. However, in a practical transmission of photons, inevitably, they will interact with their environment, which will decrease the entanglement of the entangled photon pairs. The decoherence of entanglement will make quantum-communication protocols [4–12] insecure, or it will decrease the fidelity of teleportation [13,14]. In order to improve the fidelity of entanglement of entangled quantum systems transmitted over a noisy channel, the parties in quantum communication can recur to entanglement purification and entanglement concentration.

Entanglement purification is used to obtain a subset of quantum systems in a maximally entangled state from an ensemble in a mixed entangled state. In 1996, Bennett *et al.* [24] proposed an entanglement purification protocol (EPP) for a Werner state with controlled NOT (CNOT) gates and bilateral rotations. Subsequently, Deutsch *et al.* [25] proposed an optimal EPP by using quantum privacy amplification with two CNOT operations and two special unitary transformations. In 2001, Pan *et al.* [26] introduced an EPP, resorting to polarization beam splitters and single-photon detectors. In 2002, Simon and Pan proposed an EPP using spatial entanglement [27], which was demonstrated by Pan *et al.* [28] in 2003. In 2008, Sheng *et al.* [29] presented an efficient EPP based on a parametric down-conversion source and cross-Kerr nonlinearity. In 2008, Xiao *et al.* [30] proposed an EPP with frequency entanglement. In 2010, Sheng and Deng proposed the concept of deterministic entanglement purification [31] for two-photon entangled systems, and they presented a two-step protocol for polarization entanglement purification with the hyperentanglement in both the spatial mode and the frequency degrees of freedom of photon pairs. Subsequently, they proposed a one-step protocol [32] for polarization entanglement purification with only the spatial entanglement of photon pairs. Simultaneously, Li [33] independently presented an interesting deterministic EPP using spatial entanglement. In 2011, Deng [34] extended the deterministic entanglement purification to multipartite entanglement with the spatial entanglement or the frequency entanglement of photon systems. Now, multipartite entanglement purification was also discussed with CNOT gates [35], XOR gates [36,37], or nonlinear optics [38].

Entanglement concentration is another way to distill a subset system in a maximally entangled state from a set of systems in a less-entangled pure state. In 1996, Bennett *et al.* proposed an entanglement concentration protocol (ECP) [39] in which the two communication parties obtained information about the coefficients by performing a collective and nondestructive measurement. Later, Shi *et al.* [40] designed an ECP by exploiting a two-particle collective unitary evaluation. In 2001, Yamamoto *et al.* [41] and Zhao *et al.* [42] independently proposed an ECP with polarization beam splitters, and they also experimentally demonstrated their ECP using linear optics [43]. In 2008, Sheng *et al.* [44] proposed an efficient ECP using cross-Kerr nonlinearity. In 2010, they designed an ECP for single-photon entangled systems [45].

Entanglement purification for electron-spin systems is an essential problem in quantum-communication- and quantum-computation-based electrons [46–49]. Although there are some interesting EPPs and ECPs for photon-entangled systems, there are only a few EPPs [47,48] and ECPs [49] for electronic systems. In 2005, Feng *et al.* [47] proposed an electronic EPP by using charge detection [46], following some ideas in the original EPP proposed by Bennett *et al.* [24] for photon pairs in a Werner state. In 2011, Sheng *et al.* [48] presented an interesting multipartite electronic entanglement purification protocol with charge detection. In 2009, an efficient multipartite electronic entanglement concentration

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protocol was proposed by Sheng *et al.* [49] with charge detection. As pointed out in Ref. [46], the time-resolved detection required for the operation as a logical gate has not yet been realized, which will improve the difficulty of the implementation of the EPPs [47,48] and the ECP [49] based on charge detection at present.

Recently, Waks and Vuckovic discussed the interaction of a cavity coupled with a dipole where the vacuum Rabi frequency is less than the cavity decay rate, and they showed that, even in the bad cavity limit, the cavity can be switched perfectly. They proposed the method for designing quantum repeaters in a weak-coupling regime [50]. The potential application of this system has also been discussed, such as photon entangler [51], entanglement beam splitter [52], and optical Faraday rotation [53]. As discussed in Ref. [54], a single-electron-charged quantum dot (QD) in a resonator exhibited a good interaction between a photon and an electron spin. Exploiting this regime, a hybrid entanglement between a photon and an electron spin can be generated with a QD coupled to a microcavity. The realization of CNOT gates and Bell-state analyzer processes are discussed. Recently, Hu and Rarity presented the protocol of the state teleportation and entanglement swapping using a QD spin in an optical microcavity [55].

In this paper, we proposed an entanglement purification protocol and an entanglement concentration protocol for electronic systems by exploiting a weak-coupling regime. We first construct a parity-check gate (PCG) based on the QD and cavity-coupling systems and then exploit it to complete our EPP and ECP for electron systems. In our EPP, as the ancillary photons are used as the parity-checking index at the two remote parties, the spatial modes of the output photons can indicate the parity of the electrons. By locally performing single-photon measurements, the two remote parities in quantum communication, say Alice and Bob, compare their outcomes and can recover the original entangled state with high fidelity. The PCG setups can also be used in the entanglement concentration protocol, in which the two remote parties can distill one pair of maximally entangled electron states from two pairs of less-entangled states with high efficiency. The proposed methods are more flexible as only single-photon detection and single-electron detection are required.

This paper is organized as follows. In Sec. II, the construction of a PCG is demonstrated, and the protocol of entanglement purification based on PCGs is discussed. In Sec. III, we propose an entanglement concentration protocol of the electron entangled state by using the coupling cavity. A discussion and summary are given in Sec. II A.

II. ENTANGLEMENT PURIFICATION USING THE QD SPIN-CAVITY-COUPLING SYSTEM

A. PCG based on photon- and electron-coupled systems

When a singly charged QD is embedded in a microcavity, an exciton with negative charges can be created by the optical excitation of the system. The charged exciton consists of two electrons bound in one hole. As illustrated by Pauli's exclusion principle, when a photon passes through the cavity and interacts with the electron in a weak-coupling cavity, the electron-spin-cavity system behaves like a beam splitter in the limit of a weak incoming field [56]: The left circularly polarized photon only couples the electron in the spin-up state $|\uparrow\rangle$ to the exciton X^- in state $|\uparrow\downarrow\Uparrow\rangle$; the right circularly polarized photon only couples with the electron of the spin-down state $|\downarrow\rangle$ in state $|\downarrow\uparrow\Downarrow\rangle$. Here ,$|\Uparrow\rangle$ and $|\Downarrow\rangle$ represent the spin directions of the heavy-hole spin state. As discussed in Ref. [54], the interface between the spin of a photon and the spin of an electron, confined in a QD in a cavity, shows good features of spin-photon interaction. For example, if the spin lies in the up direction $|\uparrow\rangle$, the left-circular-polarized photon feels a coupled cavity, and the right-circular-polarized photon feels an uncoupled cavity. This is called the giant circular birefringence [53].

Consider a photon in state $s_z = +1$ and the spin of the electron in state $|\uparrow\rangle$. As the polarization of the photon is defined according to the direction of propagation, the circularly polarized light might change its polarization upon reflection. The photon that passes through the cavity will be reflected by the cavity, and both the photon polarization and the propagation direction will be flipped. Otherwise, the photon will be transmitted. The rules of state change under the interaction of the photons with $s_z = \pm 1$, and the cavity is described as follows:

$$\begin{aligned} &|R^{\uparrow},\uparrow\rangle \to\ \ |L^{\downarrow},\uparrow\rangle, && |L^{\uparrow},\uparrow\rangle \to -|L^{\uparrow},\uparrow\rangle,\\ &|R^{\downarrow},\uparrow\rangle \to -|R^{\downarrow},\uparrow\rangle, && |L^{\downarrow},\uparrow\rangle \to |R^{\uparrow},\uparrow\rangle,\\ &|R^{\uparrow},\downarrow\rangle \to -|R^{\uparrow},\downarrow\rangle, && |L^{\uparrow},\downarrow\rangle \to |R^{\downarrow},\downarrow\rangle,\\ &|R^{\downarrow},\downarrow\rangle \to\ \ |L^{\uparrow},\downarrow\rangle, && |L^{\downarrow},\downarrow\rangle \to -|L^{\downarrow},\downarrow\rangle. \end{aligned} \tag{1}$$

Here, $|L\rangle$ and $|R\rangle$ represent the states of the left- and right-circular-polarized photons, respectively. The superscript arrow in the photon state indicates the propagation direction along the z axis, and the arrows represent the direction of the electrons.

Based on the rules discussed above, we can construct a PCG for electron systems. Its principle is shown in Fig. 1. An input photon in state $|L^{\downarrow}\rangle$ is injected into the first optical microcavity from the upper input port (i.e., Input), and it interacts with the first QD spin (Spin 1). After transmission from the first optical microcavity, the photon enters the second optical microcavity and interacts with the second QD spin (Spin 2). If the two electron spins in the two optical microcavities are in the same state (both in states $|\uparrow\rangle_1|\uparrow\rangle_2$ or $|\downarrow\rangle_1|\downarrow\rangle_2$), the state of the input photon will be changed and will trigger the detector in the lower mode (D2), and the evolution of the photon-electron state can be written as

$$\begin{aligned} &|L^{\downarrow}\rangle|\uparrow\rangle_1|\uparrow\rangle_2 \Rightarrow -|R^{\downarrow}\rangle|\uparrow\rangle_1|\uparrow\rangle_2,\\ &|L^{\downarrow}\rangle|\downarrow\rangle_1|\downarrow\rangle_2 \Rightarrow -|R^{\downarrow}\rangle|\downarrow\rangle_1|\downarrow\rangle_2. \end{aligned} \tag{2}$$

Otherwise, the state of the photon will remain unchanged and will be detected by the detector in the upper mode (D1),

$$\begin{aligned} &|L^{\downarrow}\rangle|\uparrow\rangle_1|\downarrow\rangle_2 \Rightarrow |L^{\uparrow}\rangle|\uparrow\rangle_1|\downarrow\rangle_2,\\ &|L^{\downarrow}\rangle|\downarrow\rangle_1|\uparrow\rangle_2 \Rightarrow |L^{\uparrow}\rangle|\downarrow\rangle_1|\uparrow\rangle_2. \end{aligned} \tag{3}$$

By detecting the output of the in photon, one can distinguish the spin states of electron systems $\{|\uparrow\rangle_1|\uparrow\rangle_2, |\downarrow\rangle_1|\downarrow\rangle_2\}$ from $\{|\uparrow\rangle_1|\downarrow\rangle_2, |\downarrow\rangle_1|\uparrow\rangle_2\}$. That is, the setup shown in Fig. 1 acts as a PCG.

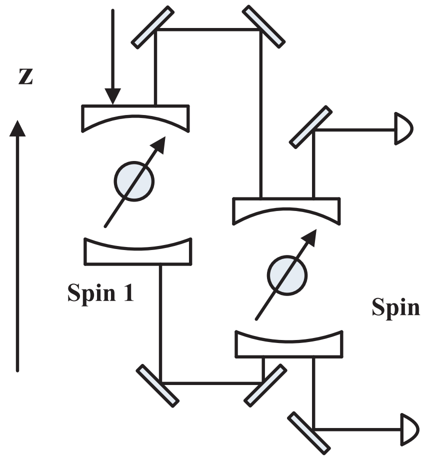

FIG. 1. (Color online) Schematic showing the principle of our PCG based on photon- and electron-coupled systems. Spin 1 and Spin 2 represent the two QD spins coupled in two optical microcavities, respectively. D1 and D2 represent two single-photon detectors. Input represents the input port of a photon.

B. Entanglement purification using QD spin- and cavity-coupled systems

In the realization of QIP using the electron-spin entangled state, the environment will inevitably affect the spins of the electron entangled state. This process will transform a pure entangled state ensemble into a mixed one. Here, we introduce the application of PCG in the entanglement purification process and purify the unwanted electron-spin entangled state.

Suppose that the electron entangled state, which is desired is described as

$$|\phi^+\rangle_{1,2} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\uparrow\rangle_2 + |\downarrow\rangle_1|\downarrow\rangle_2), \quad (4)$$

where $|\uparrow\rangle$ and $|\downarrow\rangle$ represent the states of the electron spins in the up direction and in the down direction, respectively. And the environment noise will flip the directions of the spins and will cause the initial state to become the following state:

$$|\psi^+\rangle_{1,2} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2 + |\downarrow\rangle_1|\uparrow\rangle_2). \quad (5)$$

Also, the phases will be changed and the spin entangled state can be transformed to the following states:

$$|\phi^-\rangle_{1,2} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\uparrow\rangle_2 - |\downarrow\rangle_1|\downarrow\rangle_2), \quad (6)$$

$$|\psi^-\rangle_{1,2} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2). \quad (7)$$

We first discuss the purification of bit-flip errors in the entangled state ensemble. After entanglement distribution, entangled electrons are shared between Alice and Bob. Suppose that the initial mixed entangled state ensemble shared by the two parties in quantum communication, say Alice and Bob, is described by the density matrix,

$$\rho = F|\phi^+\rangle\langle\phi^+| + (1-F)|\psi^+\rangle\langle\psi^+|. \quad (8)$$

Here, $F = \langle\phi^+|\rho|\phi^+\rangle$ describes the fidelity of state $|\phi^+\rangle$, and $(1-F) = \langle\psi^+|\rho|\psi^+\rangle$ describes the fidelity of state $|\psi^+\rangle$ affected by the environment noise with which a bit-flip error takes place on one of the two electrons. At the first step of purification, Alice and Bob select two pairs of entangled electrons randomly, the four particles selected randomly are in state $|\phi^+\rangle_{1,2}|\phi^+\rangle_{3,4}$ with a probability of F^2, in states $|\phi^+\rangle_{1,2}|\psi^+\rangle_{3,4}$ and $|\psi^+\rangle_{1,2}|\phi^+\rangle_{3,4}$ with a probability of $F(1-F)$, and in state $|\psi^+\rangle_{1,2}|\psi^+\rangle_{3,4}$ with a probability of $(1-F)^2$. Here, the electrons with subscripts 1 and 3 belong to Alice and the other two electrons with subscripts 2 and 4 belong to Bob.

Suppose that the initial two electron pairs are in state $|\phi^+\rangle_{1,2}|\phi^+\rangle_{3,4}$. Alice and Bob produce two input photons in the left-circular-polarized state $|L\rangle$. The setup shown in Fig. 2 will evolve the composite system composed of the four electron spins and the two input photons to be

$$\begin{aligned}
&\tfrac{1}{2}(|\uparrow\rangle_1|\uparrow\rangle_2 + |\downarrow\rangle_1|\downarrow\rangle_2)(|\uparrow\rangle_3|\uparrow\rangle_4 \\
&\quad +|\downarrow\rangle_3|\downarrow\rangle_4)|L^{\downarrow}\rangle_A|L^{\downarrow}\rangle_B \\
\Rightarrow\ &\tfrac{1}{2}[(|\uparrow\rangle_1|\uparrow\rangle_2|\uparrow\rangle_3|\uparrow\rangle_4 \\
&\quad +|\downarrow\rangle_1|\downarrow\rangle_2|\downarrow\rangle_3|\downarrow\rangle_4)|R^{\downarrow}\rangle_A|R^{\downarrow}\rangle_B \\
&\quad +(|\uparrow\rangle_1|\uparrow\rangle_2|\downarrow\rangle 3|\downarrow\rangle_4 \\
&\quad +|\downarrow\rangle_1|\downarrow\rangle_2|\uparrow\rangle_3|\uparrow\rangle_4)|L^{\uparrow}\rangle_A|L^{\uparrow}\rangle_B].
\end{aligned} \quad (9)$$

Here, subscripts A and B represent the circular-polarized photons at the input ports on Alice's and Bob's sides, respectively. If Alice and Bob get the results of the photons in state $|R^{\downarrow}\rangle_A|R^{\downarrow}\rangle_B$, which means that the two photons trigger the detectors in the lower modes (detector 2 and detector 4), the entangled electron spins collapse to state $|\Phi^+\rangle_{1,2,3,4} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\uparrow\rangle_2|\uparrow\rangle_3|\uparrow\rangle_4 + |\downarrow\rangle_1|\downarrow\rangle_2|\downarrow\rangle_3|\downarrow\rangle_4)$. The state of the composite system composed of the four electron spins can be written as

$$\begin{aligned}
|\Phi^+\rangle_{1,2,3,4} = \tfrac{1}{2}[&|\phi^+\rangle_{1,2}(|+\rangle_3|+\rangle_4 + |-\rangle_3|-\rangle_4) \\
&+|\phi^-\rangle_{1,2}(|+\rangle_3|-\rangle_4 + |-\rangle_3|+\rangle_4)].
\end{aligned} \quad (10)$$

Here, $|\pm\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle \pm |\downarrow\rangle)$ are the two eigenvectors of basis X for the spin of a single electron. That is, Alice and Bob perform the measurement with basis X on both electrons 3 and 4, and they obtain a maximally entangled state for the electron-spin system. In detail, if the results on both Alice's and Bob's sides are $|+\rangle$ or $|-\rangle$, they get the maximally entangled state $|\phi^+\rangle_{1,2}$. Otherwise, they will obtain the maximally entangled state $|\phi^-\rangle_{1,2}$, and they can perform a phase-flip operation on the first electron to obtain the maximally entangled state $|\phi^+\rangle_{1,2}$.

If Alice and Bob get the results of the photons in state $|L^{\uparrow}\rangle_A|L^{\uparrow}\rangle_B$, which means that the two photons trigger the detectors in the upper modes (detector 1 and detector 3), the entangled electron spins collapse to state $|\Psi^+\rangle_{1,2,3,4} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\uparrow\rangle_2|\downarrow\rangle_3|\downarrow\rangle_4 + |\downarrow\rangle_1|\downarrow\rangle_2|\uparrow\rangle_3|\uparrow\rangle_4)$,

$$\begin{aligned}
|\Psi^+\rangle_{1,2,3,4} = \tfrac{1}{2}[&|\psi^+\rangle_{1,2}(|+\rangle_3|+\rangle_4 - |-\rangle_3|-\rangle_4) \\
&-|\psi^-\rangle_{1,2}(|+\rangle_3|-\rangle_4 - |-\rangle_3|+\rangle_4)].
\end{aligned} \quad (11)$$

If Alice and Bob obtain the same outcome for their single-electron measurements on spins 3 and 4 as each other, they can perform a bit-flip operation on the first electron spin to obtain the maximally entangled state $|\phi^+\rangle_{1,2}$. Otherwise, they can perform both a bit-flip operation and a phase-flip operation

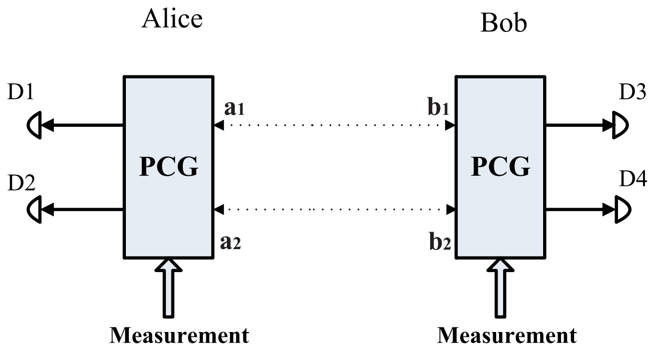

FIG. 2. (Color online) Schematic showing the principle of entanglement purification using PCGs. PCG represents the PCG shown in Fig. 1. The dashed line between a_1b_1 and a_2b_2 represents the entanglement between the electrons. D1, D2, D3, and D4 represent the single-photon detectors. Alice and Bob perform the single-electron measurement on spins a_2 and b_b when they obtain the even parity, respectively.

on the first electron spin to obtain the maximally entangled state $|\phi^+\rangle_{1,2}$.

Similarly, the evolution of the other two terms $|\phi^+\rangle_{1,2}|\psi^+\rangle_{3,4}$ and $|\psi^+\rangle_{1,2}|\phi^+\rangle_{3,4}$ is described by

$$\begin{aligned}&|\phi^+\rangle_{1,2}|\psi^+\rangle_{3,4}|L^\downarrow\rangle_A|L^\downarrow\rangle_B\\&\Rightarrow -(|\uparrow\rangle_1|\uparrow\rangle_2|\uparrow\rangle_3|\downarrow\rangle_4\\&\quad+|\downarrow\rangle_1|\downarrow\rangle_2|\downarrow\rangle_3|\uparrow\rangle_4)|R^\downarrow\rangle_A|L^\uparrow\rangle_B-(|\uparrow\rangle_1|\uparrow\rangle_2|\downarrow\rangle_3|\uparrow\rangle_4\\&\quad+|\downarrow\rangle_1|\downarrow\rangle_2|\uparrow\rangle_3|\downarrow\rangle_4)|L^\uparrow\rangle_A|R^\downarrow\rangle_B. \qquad (12)\end{aligned}$$

$$\begin{aligned}&|\psi^+\rangle_{1,2}|\phi^+\rangle_{3,4}|L^\downarrow\rangle_A|L^\downarrow\rangle_B\\&\Rightarrow -(|\uparrow\rangle_1|\downarrow\rangle_2|\uparrow\rangle_3|\uparrow\rangle_4\\&\quad+|\downarrow\rangle_1|\uparrow\rangle_2|\downarrow\rangle_3|\downarrow\rangle_4)|R^\downarrow\rangle_A|L^\uparrow\rangle_B-(|\uparrow\rangle_1|\downarrow\rangle_2|\downarrow\rangle_3|\downarrow\rangle_4\\&\quad+|\downarrow\rangle_1|\uparrow\rangle_2|\uparrow\rangle_3|\uparrow\rangle_4)|L^\uparrow\rangle_A|R^\downarrow\rangle_B. \qquad (13)\end{aligned}$$

Alice and Bob can distinguish these two cases with single-photon measurements on the output ports. If Alice and Bob cannot obtain the same outcome as each other when they measure their photons, they discard their two electron pairs.

If the two electron-spin pairs are in state $|\psi^+\rangle_{1,2}|\psi^+\rangle_{3,4}$, the evolution of the composite system can be described by

$$\begin{aligned}&|\psi^+\rangle_{1,2}|\psi^+\rangle_{3,4}|L\rangle_A|L\rangle_B\\&\Rightarrow (|\uparrow\rangle_1|\downarrow\rangle_2|\uparrow\rangle_3|\downarrow\rangle_4\\&\quad+|\downarrow\rangle_1|\uparrow\rangle_2|\downarrow\rangle_3|\uparrow\rangle_4)|R^\downarrow\rangle_A|R^\downarrow\rangle_B+(|\downarrow\rangle_1|\uparrow\rangle_2|\uparrow\rangle_3|\downarrow\rangle_4\\&\quad+|\uparrow\rangle_1|\downarrow\rangle_2|\downarrow\rangle_3|\uparrow\rangle_4)|L^\uparrow\rangle_A|L^\uparrow\rangle_B. \qquad (14)\end{aligned}$$

Under this condition, the outcomes of the single-photon measurements by Alice and Bob cannot be distinguished with the case of $|\phi^+\rangle_{1,2}|\phi^+\rangle_{3,4}$. So, the two electron-spin pairs are preserved.

Alice and Bob discard the electrons if their single-photon measurement results are $|R^\downarrow\rangle_A|L^\uparrow\rangle_B$ or $|L^\uparrow\rangle_A|R^\downarrow\rangle_B$. If the photons are in state $|L^\uparrow\rangle_A|L^\uparrow\rangle_B$ or $|R^\downarrow\rangle_A|R^\downarrow\rangle_B$, the two cases $|\phi^+\rangle_{1,2}|\phi^+\rangle_{3,4}$ and $|\psi^+\rangle_{1,2}|\psi^+\rangle_{3,4}$ are preserved with probabilities F^2 and $(1-F)^2$, respectively. With this postselection process, according to the detection of the photons, Alice and Bob only keep the first electron pair in the instances in which they get the photons in the same spatial mode. After this entanglement purification process, the new fidelity of the electron pairs that was kept becomes $F'=F^2/[F^2+(1-F)^2]$. If the initial fidelity of entanglement F is larger than $1/2$, the entanglement fidelity after purification F' is larger than F, which means that the probability of state $|\phi^+\rangle_{1,2}$ in the ensemble is increased.

Here, we discussed the principle of primary entanglement purification based on bit-flip errors using the QD and an optical-cavity system. The two parties can purify the entangled electrons by performing the single-photon measurements. However, under the affection of environment noise, the relative phase between the entangled electrons also is induced. This phase-flip error cannot be purified directly, but it can be converted into bit-flip errors. As discussed in Ref. [26], a phase-flip error for a two-particle system can be transformed into a bit-flip error using a bilateral local operation. So, we only discuss bit-flip error entanglement purification in this paper.

III. ENTANGLEMENT CONCENTRATION OF THE ELECTRON-SPIN ENTANGLED STATE USING THE QD SPIN-CAVITY-COUPLING SYSTEM

Entanglement concentration is used to distill maximally entangled states from a less-entangled ensemble in a pure state by linear or nonlinear operations. Consider that the two electrons in the two cavities are in the partially entangled state $|\varphi^+\rangle_{A,B}=\alpha|\uparrow\rangle_A|\uparrow\rangle_B+\beta|\downarrow\rangle_A|\downarrow\rangle_B$, which is shared by Alice and Bob. The ultimate goal is to generate the maximally entangled state $|\phi^+\rangle_{A,B}=\frac{1}{\sqrt{2}}(|\uparrow\rangle_A|\uparrow\rangle_B+|\downarrow\rangle_A|\downarrow\rangle_B)$. Usually, entanglement concentration protocols do not require that the parties know the accurate information about the partially entangled states, i.e., coefficients α and β of the states.

The principle of our ECP is illustrated in Fig. 3. The entanglement of electrons is denoted between spin 1 and spin 2 and that between spin 3 and spin 4. Suppose that the environment noise will affect the entanglement of the electrons, and the two pairs of less-entangled pure states can be described by

$$\begin{aligned}|\varphi^+\rangle_{1,2}&=\alpha|\uparrow\rangle_1|\uparrow\rangle_2+\beta|\downarrow\rangle_1|\downarrow\rangle_2,\\|\varphi^+\rangle_{3,4}&=\alpha|\uparrow\rangle_3|\uparrow\rangle_4+\beta|\downarrow\rangle_3|\downarrow\rangle_4.\end{aligned} \qquad (15)$$

Here, coefficients α and β satisfy the relation $|\alpha|^2+|\beta|^2=1$.

In order to distill maximally entangled electrons between the two remote parties, an ECP process is needed. Here, we assume that spin 1 and spin 3 are on Alice's side and spin 2 and spin 4 are on Bob's side after entanglement distribution. Bob produces a single photon in the left-circular-polarized state $|L\rangle$ and sends through the input port of the cavity that contains spin 2 and detects the output port of the cavity that contains spin 4 using single-photon detectors (detector 1 and detector 2).

The evolution of the whole spin-photon system can be described as follows:

$$\begin{aligned}&(\alpha|\uparrow\rangle_1|\uparrow\rangle_2+\beta|\downarrow\rangle_1|\downarrow\rangle_2)(\alpha|\uparrow\rangle_3|\uparrow\rangle_4+\beta|\downarrow\rangle_3|\downarrow\rangle_4)|L\rangle\\&\Rightarrow -\alpha^2|\uparrow\rangle_1|\uparrow\rangle_2|\uparrow\rangle_3|\uparrow\rangle_4|R^\downarrow\rangle-\beta^2|\downarrow\rangle_1|\downarrow\rangle_2|\downarrow\rangle_3|\downarrow\rangle_4|R^\downarrow\rangle\\&\quad+\alpha\beta(|\uparrow\rangle_1|\uparrow\rangle_2|\downarrow\rangle_3|\downarrow\rangle_4+|\downarrow\rangle_1|\downarrow\rangle_2|\uparrow\rangle_3|\uparrow\rangle_4)|L^\uparrow\rangle.\end{aligned} \qquad (16)$$

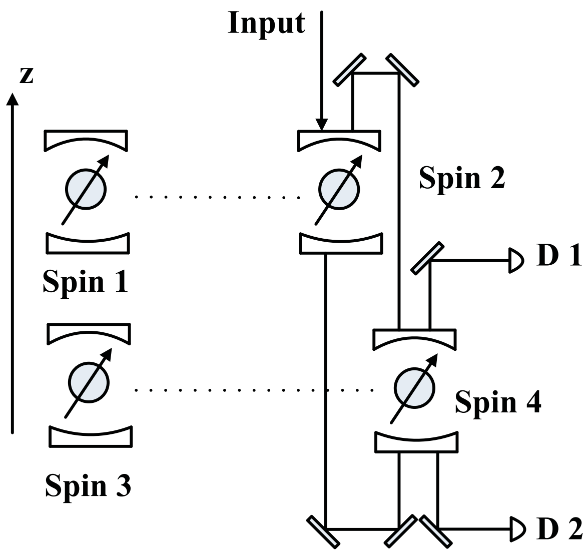

FIG. 3. (Color online) Schematic showing the principle of entanglement concentration using QD spins and optical-cavity systems. D1 and D2 are single-photon detectors.

Then, the two parties, Alice and Bob, rotate the spins of their second electron pairs, say spin 3 and spin 4, by 90° with the magnetic field, and the spin directions are flipped. The state after the rotation process can be described as

$$-\alpha^2|\uparrow\rangle_1|\uparrow\rangle_2|\downarrow\rangle_3|\downarrow\rangle_4|R^\downarrow\rangle-\beta^2|\downarrow\rangle_1|\downarrow\rangle_2|\uparrow\rangle_3|\uparrow\rangle_4|R^\downarrow\rangle \\ +\alpha\beta(|\uparrow\rangle_1|\uparrow\rangle_2|\uparrow\rangle_3|\uparrow\rangle_4+|\downarrow\rangle_1|\downarrow\rangle_2|\downarrow\rangle_3|\downarrow\rangle_4)|L^\uparrow\rangle. \tag{17}$$

Based on the equations shown above, one can see that terms $|\uparrow\rangle_1|\uparrow\rangle_2|\uparrow\rangle_3|\uparrow\rangle_4$ and $|\downarrow\rangle_1|\downarrow\rangle_2|\downarrow\rangle_3|\downarrow\rangle_4$ have the same coefficient $\alpha\beta$. We can conclude that, if the measurement on Bob's side reveals that the photon is in state $|L^\uparrow\rangle$, then they can determine that the four electrons are in state $\frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\uparrow\rangle_2|\uparrow\rangle_3|\uparrow\rangle_4+|\downarrow\rangle_1|\downarrow\rangle_2|\downarrow\rangle_3|\downarrow\rangle_4)$. Otherwise, they discard the two electron pairs. For the remaining two pairs, Alice and Bob both perform single-electron measurements on spin 3 and spin 4 in the measuring basis $|\pm\rangle=\frac{1}{\sqrt{2}}(|\uparrow\rangle\pm|\downarrow\rangle)$, respectively. The state of the composite system becomes

$$\frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\uparrow\rangle_2|\uparrow\rangle_3|\uparrow\rangle_4+|\downarrow\rangle_1|\downarrow\rangle_2|\downarrow\rangle_3|\downarrow\rangle_4) \\ =|\phi^+\rangle_{1,2}(|+\rangle|+\rangle_3|+\rangle_4+|-\rangle_3|-\rangle_4) \\ +|\phi^-\rangle_{1,2}(|+\rangle_3|-\rangle_4+|-\rangle_3|+\rangle_4). \tag{18}$$

If the measurement results on Alice's and Bob's sides are $|+\rangle_A|+\rangle_B$ or $|-\rangle_A|-\rangle_B$, spins 1 and 2 are in the maximally entangled state $|\phi^+\rangle_{1,2}$. However, if the results are antiparallel, one of them performs a phase-flip operation on the electron and recovers the original state $|\phi^+\rangle_{1,2}$.

Here, the preparation of electron-spin superposition states $|+\rangle$ and $|-\rangle$ is a crucial aspect. Spin manipulation of a single spin requires Zeeman splitting of the spin ground state. In this scheme, we only need to perform a measurement on the electron spins, which requires a magnetic field and electron detection.

In our ECP, in order to reconstruct some maximally entangled states, the two parties are not required to know the coefficients of the less-entangled states beforehand. Only one PCG is used to detect the parity of the two electrons. By performing the parity-check operations and single-electron measurements, the maximally entangled state can be recovered. The efficiency of the proposed entanglement concentration protocol has the same efficiency as that for entangled photon pairs based on linear optics [41,42]. The yield of maximally entangled states Y is $|\alpha\beta|^2$, which is defined as the ratio of the number of maximally entangled photon pairs and the number of originally less-entangled photon pairs.

IV. DISCUSSION AND SUMMARY

In our EPP process, we consider a singly charged QD inside an optical cavity, and the efficiency of our EPP relies on the coupling between the QD and its cavity system. The reflection and transmission coefficients of this cavity system can be investigated by solving the Heisenberg equations of motion for the cavity-field operator and the trion dipole operator in weak excitation approximation. Here, we denote the frequencies of the input photon, cavity mode, and the spin-dependent optical transition by ω_0, ω_c, and ω_{X^-}, respectively. As discussed in Ref. [53], the reflection and transmission coefficients in the system can be described by

$$r(\omega)=1+t(\omega), \\ t(\omega)=\frac{-\kappa\left[i(\omega_{X^-}-\omega)+\frac{\gamma}{2}\right]}{\left[i(\omega_{X^-}-\omega)+\frac{\gamma}{2}\right]\left[i(\omega_c-\omega)+\kappa+\frac{\kappa_s}{2}\right]+g^2}, \tag{19}$$

where g represents the coupling constant and κ and κ_s are the cavity decay rate and the leaky rate, respectively. Considering the resonant interaction with $\omega_c=\omega_{X^-}=\omega_0$, by taking $g=0$, the reflection and transmission coefficients r and t for the uncoupled cavity system can be written as

$$r_0(\omega)=\frac{i(\omega_0-\omega)+\frac{\kappa_s}{2}}{i(\omega_0-\omega)+\frac{\kappa_s}{2}+\kappa}, \\ t_0(\omega)=\frac{-\kappa}{i(\omega_0-\omega)+\frac{\kappa_s}{2}+\kappa}. \tag{20}$$

For the case of $\omega_0=\omega$, the reflection coefficient $|r(\omega)|$ and transmission coefficient $|t_0(\omega)|$ approach 1. For example, the spin is in state $|\uparrow\rangle$, the left-polarized photon feels the coupled cavity with the reflected coefficient, and the transmitted coefficients are $|r(\omega)|$ and $|t(\omega)|$, respectively. However, the right-polarized photon feels the uncoupled cavity with the reflected coefficient, and the transmitted coefficients are $|r_0(\omega)|$ and $|t_0(\omega)|$, respectively.

The performance of the system relative to the frequency detuning and the normalized coupling strength is discussed by Hu *et al.* [52]. To test the performance of our EPP, we employed the coupling constant to calculate the fidelity in the coupled system. Figure 4 presents the entanglement fidelities versus the coupling strength in our EPP.

We then calculate the fidelity of the entangled state for different κ_s values in the coupled system. The computed entanglement fidelities are shown in Fig. 5. Here, coefficient κ_s describes the cavity side leakage of the transmission process,

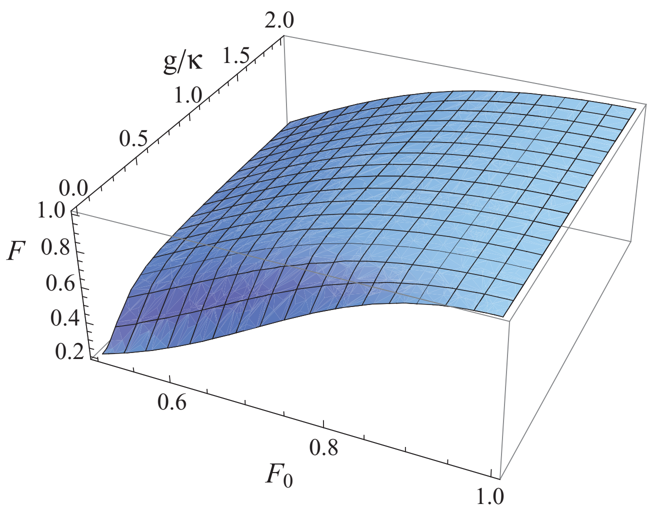

FIG. 4. (Color online) The fidelity of entanglement purification versus the initial fidelity F_0 and the normalized coupling strength g/κ. Here, we assume that $\gamma = 0.1\kappa$, $\kappa_s = 0$.

which may affect the efficiency of our EPP process. After purification, the fidelities are compared in the ideal conditions without side leakage and with leakage $\kappa_s = 0.05\kappa$.

Note that the system is composed of a singly charged QD, e.g., a self-assembled GaAs QD or InAs interface QD in micropillar microcavities. The core techniques for realizing our EPP and ECP processes in the system are the long coherence time of QDs and the strong coupling of the QD with the cavity. Recent experiments have shown that the coherence time of the GaAs- or InAs-based QDs is long enough [57]. In current experiments, it is easy to achieve the weak coupling of QDs and microcavities. Also, strong coupling has been observed in various systems [58–61]. In Ref. [58], the coupling strength has been reported to be $g = 0.5$ in a microcavity with a diameter of $d = 1.5$ μm with the cavity leakage acceptable. As g was determined by the trion oscillator strength and the cavity modal volume, the quality factors for the micropillars of the same size were increased to 4×10^4 for pillar diameters below $d = 2$ μm, corresponding to $g/(\kappa + \kappa_s) \approx 2.4$ by improving the sample designs and fabrication [62]. However, the spin

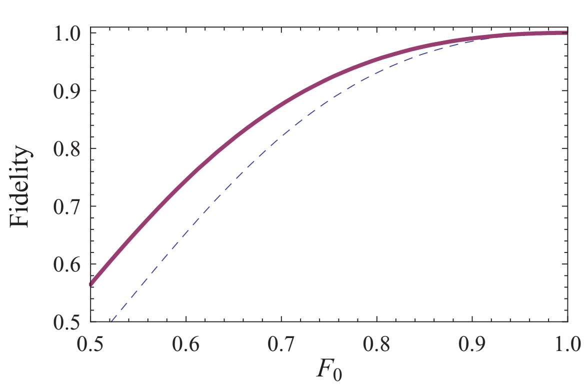

FIG. 5. (Color online) The fidelity of entanglement purification with and without side leakage. Here, the solid line represents the fidelity with no leakage, and the dashed line represents the fidelity with leakage $\kappa_s = 0.05\kappa$. Here, $g/\kappa = 0.5$ and $\gamma = 0.1\kappa$.

decoherence may also reduce the fidelity of entanglement in EPP and ECP [55]. The fidelity of our EPP may decrease by a factor $[1 + \exp(-\frac{t}{T^e})]/2$ due to the spin decoherence; here, T^e is the electron-spin coherence time, which could be extended to microseconds using spin-echo techniques [63,64], and t is the cavity photon lifetime. Also, the fidelity of entanglement relies on the coefficient of the cavity photon lifetime over trion coherence time. It is reported that the optical coherence time in QDs can be as long as several hundred picoseconds, which is ten times larger than the cavity photon lifetime [65,66]. Recently, some progress has been made on optical spin manipulation in QDs [67–69], which provides us with a useful method for controlling the spin state and measurement. This implies the feasibility of our EPP and ECP using QDs in the microcavity system. Therefore, the proposed schemes could be implemented with current technology.

To summarize, we have proposed an entanglement purification protocol and an entanglement concentration protocol based on a QD spin and an optical cavity. As the ancillary photons are used as the parity-checking index for the two remote parties, the spatial modes of the output photons can indicate the parity of the electrons. By locally performing a measurement on the photons, the two remote parities, Alice and Bob, compare their outcomes and can recover the original entangled state with high fidelity. The PCG setups can also be used in the entanglement concentration protocol in which the two remote parties can distill one pair of maximally entangled electron states from two pairs of less-entangled states with high efficiency.

The setup developed in this paper can be employed to study the general cases. One can apply the PCG to study the multiparticle entanglement purification and entanglement concentration by extending the QD and microcavity systems to N QDs in the cavities. Also, the proposed setup provides a method for realizing the entanglement transfer between photons and electron spins. Based on these ideas, we can build quantum repeaters between remote QDs [70] in which the QD in the cavity unit plays the role of a quantum node in long-distance quantum communication.

Compared with the previous entanglement purification and entanglement concentration schemes for photon systems, our protocols are used for realizing the electron entanglement purification and concentration by using the QD spin and optical-cavity systems with the same efficiency. The entangled state can be transformed to the spatial mode of the ancillary photons, the electrons do not need to move in the whole process, and the CNOT operations are not needed. Our proposed schemes are more flexible in realization with current technology.

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