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Frictionless quantum quenches in ultracold gases: A quantum-dynamical microscope

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In this Rapid Communication, a method is proposed to spatially scale up a trapped ultracold gas while conserving the quantum correlations of the initial many-body state. For systems supporting self-similar dynamics, this is achieved by implementing an engineered finite-time quench of the harmonic trap, which induces a frictionless expansion of the gas and acts as a quantum dynamical microscope.

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Research in ultracold strongly correlated states of matter has recently been spurred by the experimental realization of quantum gas microscopes, allowing detection, with single site resolution and nearly unit-efficiency, of individual atoms within a macroscopic sample in the strongly interacting regime optical lattice [1,2]. These experiments are based on high-resolution optical imaging. In a complementary way, the nonequilibrium dynamics following a quench of an external control parameter is often exploited to probe quantum correlations in many-body systems [3]. Here we propose a scheme to implement a quantum dynamical microscope, an engineered controlled expansion that allows an initial many-body state of an ultracold gas to be scaled up by a desired factor while preserving the quantum correlations of the initial state. The scheme rests on the possibility of driving a self-similar dynamics in certain systems, which is a powerful tool to understand the evolution of quantum correlations. Scaling laws can often be exploited to describe harmonically trapped ultracold gases, such as the Calogero-Sutherland model [4], the Tonks-Girardeau [5–7] and certain Lieb-Liniger states [8], Bose-Einstein condensates (BEC) [5,9,10], including dipolar interactions [11], strongly interacting ultracold gas mixtures [12], and more general many-body quantum systems [13]. Moreover, whenever the dynamics is not self-similar per se, it can often be assisted by tuning the interactions, either by means of Feschbach or confinement-induced resonances, or time-modulation of the transverse confinement in effectively low-dimensional Bose-Einstein condensates [14]. Nonetheless, in spite of the scaling laws, the expansion dynamics in these systems generally induces undamped breathing of the cloud and distorts the quantum correlations of the initial state [5–13]. The method proposed in this Rapid Communication suppresses these effects in a finite-time nonadiabatic expansion which acts as a lens to zoom-up the initial state of the system. This is achieved by carefully engineering the time modulation of the trapping frequency to induce a frictionless dynamics free from adiabaticity constraints.

Self-similar dynamics. Let us consider a D-dimensional many-body system composed of N indistinguishable particles confined in a time-dependent harmonic trap, described by a Hamiltonian,

$$\hat{\mathcal{H}}=\sum_{i=1}^{N}\left[-\frac{\hbar^2}{2m}\Delta_i^{(D)}+\frac{1}{2}m\omega^2(t)\mathbf{x}_i^2\right]+\epsilon\sum_{i<j}\mathrm{V}(\mathbf{x}_{ij}), \quad (1)$$

where $\mathbf{x}_i\in\mathbb{R}^D$, $\mathbf{x}_{ij}=\mathbf{x}_i-\mathbf{x}_j$, $\Delta_i^{(D)}$ is the D-dimensional Laplacian operator, and $\epsilon=\epsilon(t)$ is a dimensionless time-dependent coupling constant which reduces to the identity at $t=0$. We focus on systems with an interaction potential satisfying the relation

$$\mathrm{V}(\lambda\mathbf{x})=\lambda^{\alpha}\mathrm{V}(\mathbf{x}) \quad (2)$$

under scaling of the coordinates. An equilibrium state Φ of the system (1) at $t=0$ follows a self-similar evolution,

$$\Phi\left(\{\mathbf{x}_i\},t\right)=\frac{1}{b^{D/2}}e^{i\sum_{i=1}^{N}\frac{m\mathbf{x}_i^2\dot{b}}{2b\hbar}-i\mu\tau(t)/\hbar}\Phi\left(\left\{\frac{\mathbf{x}_i}{b}\right\},0\right), \quad (3)$$

where μ is the chemical potential and $\tau(t)=\int_0^t dt'/b^2(t')$, whenever the scaling factor $b=b(t)$ is the solution of the Ermakov differential equation

$$\ddot{b}+\omega^2(t)b=\omega_0^2/b^3, \quad (4)$$

with $\omega_0=\omega(0)$, satisfying the boundary conditions $b(0)=1$ and $\dot{b}(0)=0$. This dynamics further requires the coupling constant and the scaling factor to be related by $\epsilon(t)=b^{\alpha-2}$, which leads to the following cases: (a) $\alpha=2$, $\epsilon(t)=1$, with no need for auxiliary tuning of the interactions, for example, as happens in a quasi-one-dimensional BEC in the Thomas-Fermi limit or a two-dimensional (2D) Bose gas with contact interactions [10,15], which has recently been suggested as an instance of a quantum anomaly [16]. (b) $\alpha\neq 2$, $\epsilon(t)=b^{\alpha-2}$ which does require external tuning of the interactions as in one-dimensional (1D) and three-dimensional (3D) BEC to assist the self-similar dynamics. (Note that in a fast expansion the role of interactions might be disregarded, as usually done in time-of-flight experiments, and then the self-similar dynamics comes for free.) If the initial state Φ is not in equilibrium, it will then follow the evolution as if the trapping potential is kept constant in the scaled coordinates and picking the overall phase in Eq. (3). Scaling laws manifest in non-local correlations of the gas, such as the one-body reduced density matrix (OBRDM) given by $g_1(\mathbf{x},\mathbf{y};t)=N\int d\mathbf{x}_2\cdots d\mathbf{x}_N\Phi^*(\mathbf{x},\mathbf{x}_2,\ldots,\mathbf{x}_N;t)\Phi(\mathbf{y},\mathbf{x}_2,\ldots,\mathbf{x}_N;t)$, whose time evolution under self-similar dynamics can be conveniently written as [6,13]

$$g_1(\mathbf{x},\mathbf{y};t)=\frac{1}{b^D}g_1\left(\frac{\mathbf{x}}{b},\frac{\mathbf{y}}{b};0\right)\exp\left(-\frac{i}{b}\frac{\dot{b}}{\omega_0}\frac{\mathbf{x}^2-\mathbf{y}^2}{2l_0^2}\right) \quad (5)$$

where $l_0 = \sqrt{\hbar/m\omega_0}$, and its Fourier transform $n(\mathbf{k},t) = \int d\mathbf{x}d\mathbf{y}\, e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} g_1(\mathbf{x},\mathbf{y};t)$, the momentum distribution

$$n(\mathbf{k},t) = b^D \int d\mathbf{x}d\mathbf{y}\; g_1(\mathbf{x},\mathbf{y};0) \times \exp\left[-ib\left(\frac{\dot{b}}{\omega_0}\frac{\mathbf{x}^2-\mathbf{y}^2}{2l_0^2} - \mathbf{k}\cdot(\mathbf{x}-\mathbf{y})\right)\right], \quad (6)$$

as well as any higher-order correlation function, i.e., the n-body reduced density matrix $g_n(\{\mathbf{x}_i\}_{i=1}^n;\{\mathbf{x}'_i\}_{i=1}^n;t) = \frac{N!}{(N-n)!}\int\prod_{i=n+1}^N d\mathbf{x}_i\,\Phi^*(\{\mathbf{x}_i\}_{i=1}^N)\Phi(\{\mathbf{x}'_i\}_{i=1}^n,\{\mathbf{x}_i\}_{i=n+1}^N;t) = b^{-nD}\times \exp(-\frac{i}{b}\frac{\dot{b}}{\omega_0}\frac{\sum_{i=1}^n(\mathbf{x}_i^2-\mathbf{x}_i'^2)}{2l_0^2}) g_n(\{\frac{\mathbf{x}_i}{b}\}_{i=1}^n;\{\frac{\mathbf{x}'_i}{b}\}_{i=1}^n;0)$.

The Newton cradle experiment has singled out the 1D Bose gas in the strongly interacting limit as a paradigmatic example of an integrable system where relaxation to equilibrium is suppressed even when perturbed to breakdown integrability [17]. We shall illustrate our results with a 1D cloud of ultracold bosons in the limit of hard-core contact interactions, a Tonks-Girardeau (TG) gas confined in a harmonic trap of frequency ω_0, with single-particle eigenstates $\phi_n(x)$ and $n = 0,1,2,\ldots$ The many-body ground state of this system can be efficiently described using an auxiliary wave function, a normalized Slater determinant [18], $\Psi_F(x_1,\ldots,x_N) = \frac{1}{\sqrt{N!}}\det_{n,l=(0,1)}^{(N-1,N)}\phi_n(x_l)$, describing a spin-polarized Fermi gas in the ground-state of the trap. This wave function already includes the hard-core condition as a result of the Pauli exclusion principle encoded in the determinant structure. The bosonic symmetry can be enforced by applying the antisymmetric unit function $\mathcal{A}(\hat{x}_1,\ldots,\hat{x}_N) = \prod_{1\leqslant j<k\leqslant N}\epsilon(\hat{x}_k-\hat{x}_j)$, where $\epsilon(x) = 1(-1)$ if $x>0$ (<0) and $\epsilon(0)=0$. The Bose-Fermi mapping relating both dual systems reads $\Phi_{\rm TG}(x_1,\ldots,x_N) = \mathcal{A}(\hat{x}_1,\ldots,\hat{x}_N)\Psi_F(x_1,\ldots,x_N)$. This is a highly nonlocal mapping, but being involutive, it leaves invariant any local correlation function such as the density profile, i.e., those quantities derived from the probability density are shared by both dual systems. The computation of nonlocal correlations remains a nontrivial task, but elegant expressions are known after the work by Pezer and Buljan [19]. A many-body state in the TG regime obeys the self-similar scaling law in Eq. (3) whenever b is a solution of Eq. (4) [20]. The scaling is clearly exhibited by the density profile of a TG gas $n_{\rm TG}(x,t) = N\int dx_2\cdots dx_N|\Phi_{\rm TG}(x,x_2,\ldots,x_N;t)|^2 = N\int dx_2\cdots dx_N|\Psi_F(x,x_2,\ldots,x_N;t)|^2 = \frac{1}{b(t)}n_{\rm TG}(\frac{x}{b(t)},0)$, its width being governed by b.

Breathing and correlation dynamics. Consider the self-similar expansion of a many-body system after quenching the trapping potential between an initial ω_0 and final ω_f frequency in a finite quench time τ, to scale it up by a factor $\gamma = (\omega_0/\omega_f)^{1/2}$. Two main features of the dynamics are to be tailored to create a quantum dynamical microscope: the subsequent breathing of the density profile and the evolution of quantum correlations g_n. For illustrative purposes, we focus on those associated with the OBRDM. First we note that for the sudden expansion in free space $[\omega(t>0)=\omega_f=0]$, $b(t) = \sqrt{1+\omega_0^2t^2}$ and for $t\gg\omega_0^{-1}$, $b(t)\sim\omega_0 t$, $\dot{b}=\omega_0$. Using the method of the stationary phase, $n(\mathbf{k},t)\sim|2\pi\omega_0 l_0^2/\dot{b}|^D g_1(\omega_0\mathbf{k}l_0^2/\dot{b},\omega_0\mathbf{k}l_0^2/\dot{b})$, so the asymptotic momentum distribution is the scaled density profile of the initial state [13,19], which in the TG regime can be further related to the momentum distribution of the dual system, the ideal Fermi gas [6,7,21,22]. This mapping between local and nonlocal correlations is expected as the density decreases during expansion at a finite rate $\dot{b}$. Indeed, within the scheme of symplectic tomography, the evolution under quadratic Hamiltonians leads to a dynamical covering of correlations in phase space [23]. In an isolated system, a sudden quench of the trapping frequency between two given finite values also induces undamped breathing of the density profile. In the TG regime, this was shown by Minguzzi and Gangardt [6], who further illustrated the dynamics of the momentum distribution, periodically oscillating between that of the initial trapped state and the quasi-momentum distribution. Nonetheless, these two effects are ubiquitous in the family of systems supporting a dynamical scaling governed by the Ermakov equation. For a nonvanishing $\omega_f\neq 0$, increasing finite values of the quench time lead to a gradual suppression of the subsequent breathing, and one expects to recover the adiabatic limit whenever $\dot{\omega}(t)/\omega^2(t)\ll 1$. As a specific example let us consider the functional dependence of the trapping frequency to be given by

$$\omega(t) = \omega_0 + (\omega_f-\omega_0)\tanh\left(\frac{t}{\tau}\right) \quad (7)$$

with a characteristic quench time τ. The resulting $b(t)$ can be obtained by solving numerically the Ermakov equation subjected to the initial conditions $b(0)=1$, $\dot{b}(0)=0$. The effect of an increasing value of τ on the scaling factor $b(t)$ is shown in Fig. 1. Quenches of the potential in a finite time $\tau\sim\omega_0^{-1}$ distort the momentum distribution and induce breathing of the cloud. As it turns out, these quenches do lead to nonzero finite values of $\dot{b}(t_f\gg\tau)$ and the scaling factor $b(t_f)$ deviates from the desired target value γ, as shown in Fig. 2. As τ is increased and the adiabatic limit is approached, the variation of the scaling factor becomes slower and slower, and $\dot{b}(t_f)\to 0$. Eventually, the quench follows the adiabatic trajectory $b(t) = \sqrt{\omega(t)}$, for which the OBRDM reads

$$g_1(\mathbf{x},\mathbf{y};t) = \frac{1}{b^D(t)}g_1\left(\frac{\mathbf{x}}{b(t)},\frac{\mathbf{y}}{b(t)};0\right), \quad (8)$$

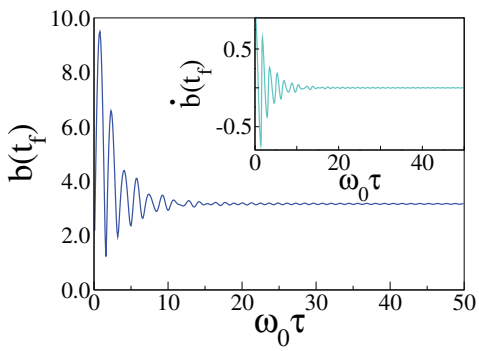

FIG. 1. (Color online) Adiabatic limit. The scaling factor following a quench for increasing quench time τ eventually approaches the adiabatic result $\gamma = \sqrt{(\omega_0/\omega_f)} = \sqrt{10}$. The inset shows how $\dot{b}(t_f)$ vanishes for large values of τ $(t_f = 20\tau)$.

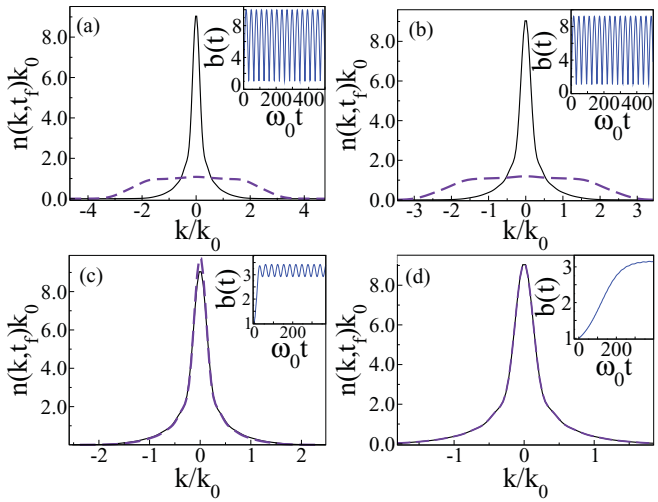

FIG. 2. (Color online) Adiabatic limit. Momentum distribution of a Tonks-Girardeau gas following a quench of the trapping potential [see Eq. (7)] with $N = 10$, $\omega(0) = \omega_0$, and $\omega(\tau) = \omega_0/10$ for increasing quench times τ =(A) 0.1, (B) 1, (C) 10, and (D) 100 in units of ω_0^{-1}. The dashed line represents the distribution following the quench, while the solid line corresponds to the desired adiabatic limit ($t_f = 500$, $k_0 = \sqrt{m\omega_0/\hbar}$). The insets show the evolution of the scaling function $b(t)$ along and after the quench. The adiabatic limit is approached for increasing values of $\tau \sim 100\omega_0$, leading to a suppression of the breathing dynamics of the cloud and scaling of quantum correlations.

which is the desired result of scaling the initial OBRDM. Similarly, the momentum distribution evolves according to

$$n(\mathbf{k},t) = b^D(t)\int d\mathbf{x}d\mathbf{y}\; g_1(\mathbf{x},\mathbf{y};0)\exp\left[i\gamma\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})\right] = b^D(t)n(b(t)\mathbf{k},0), \tag{9}$$

and coincides with the initial momentum distribution up to the scaling factor $b(t)$. Note that these expressions can be applied both for expansions [$b(t) > 1$] as well as compressions [$b(t) < 1$]. Nonetheless, the required adiabatic time scale can be exceedingly long making it unstable against perturbations (see below), and we next tackle the problem of achieving a final scaled state in a predetermined time of expansion τ.

Frictionless dynamics: Preserving quantum correlations during expansion. Following the theoretical proposals in [15,24], shortcuts to adiabaticity have been implemented in the laboratory both for thermal gases and Bose-Einstein condensates [25,26]. In the following, we show how shortcuts to adiabaticity can be exploited to control the dynamics of quantum correlations in the family of many-body systems given by Eq. (1). First we notice that the Ermakov equation can be inverted to design a many-body fast frictionless trajectory between an initial and a final trap in a given quench time τ (a time analog of the focal plane in an optical microscope), hence providing a shortcut to adiabaticity. To this aim we enforce the scaling law solution in Eq. (3) to reduce to the initial state Φ at $t = 0$ and its scaled-up form at $t = \tau$. This leads to the following boundary conditions for the scaling function: $b(0) = 1$, $\dot{b}(0) = 0$, $\ddot{b}(0) = 0$, $b(\tau) = \gamma = [\omega_0/\omega_f]^{1/2}$ being the scaling factor, and $\dot{b}(\tau) = 0$, $\ddot{b}(\tau) = 0$. This set of conditions can be used to fully determine the coefficients in an ansatz, say, of polynomial type $b(t) = \sum_{j=0}^{5} a_j t^j$. One finds that

$$b(t) = 6(\gamma-1)s^5 - 15(\gamma-1)s^4 + 10(\gamma-1)s^3 + 1 \tag{10}$$

with $s = t/\tau$, which is a solution of the Ermakov equation that drives the evolution from the initial trapped state $\Phi(\mathbf{x},0)$ to the final state $\Phi(\mathbf{x},t) = \gamma^{-D/2}\Phi(\mathbf{x}/\gamma,t=0)$ in a finite time τ, mimicking the adiabatic evolution.

The required modulation of the trapping frequency can be obtained as well from Eqs. (4) and (10), and might involve imaginary frequencies associated with a repulsive potential $\omega^2 < 0$ [24] whenever the demanded expansion time is small, $\tau \lesssim \omega_0^{-1}$. Should that be required, the trajectory can be implemented in the laboratory as in [27]. The upshot of the frictionless dynamics is that quantum correlations at the end of the quench ($t = \tau$, and only then) are those of the initial state scaled by a factor $b(\tau) = \gamma$. In particular,

$$g_1(\mathbf{x},\mathbf{y};\tau) = \frac{1}{\gamma^D}g_1\left(\frac{\mathbf{x}}{\gamma},\frac{\mathbf{y}}{\gamma};0\right),\quad n(\mathbf{k},\tau) = \gamma^D n(\gamma\mathbf{k},0). \tag{11}$$

Similar expressions hold for higher-order correlations, i.e., $g_n(\{\mathbf{x}_i\},\{\mathbf{y}_i\};\tau) = \gamma^{nD}g_n(\{\mathbf{x}_i/\gamma\},\{\mathbf{y}_i/\gamma\};0)$, with $\{\mathbf{x}_i\} = \{\mathbf{x}_i\}_{i=1}^n$. Moreover, as long as the initial state is an equilibrium state in the initial trap, so it is the state at τ with respect to the final trap, preventing any nontrivial dynamics after the quench, for $t > \tau$ if $\omega(t > \tau) = \omega_f$. Nonetheless, at intermediate times $t \in [0,\tau)$ the momentum distribution exhibits a rich nonequilibrium dynamics and can show, for instance, evolution toward the scaled density profile of the initial state. Note that this mapping is favored by a large scaling factor γ.

Implementing a frictionless dynamics allows a controlled expansion toward the scaled-up state to be performed, as illustrated in Fig. 3. For a given final time τ of expansion, the momentum distribution approaches most closely the

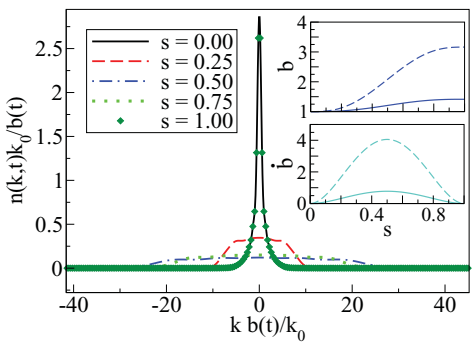

FIG. 3. (Color online) Frictionless quantum quench of a Tonks-Girardeau gas. Evolution of the momentum distribution exhibiting signs of dynamical fermionization at an intermediate stage of the expansion, before reaching the final time τ at which it reduces to the scaled-up distribution of the initial trapped state ($\gamma = \sqrt{10}$, $N = 10$). The inset shows the smooth evolution of the scaling factor in a frictionless expansion in an arbitrarily short quench time τ for both $\gamma = \sqrt{2}$ (solid line) and $\sqrt{10}$ (dashed line) expansion factors.

initial density around $t \approx \tau/2$. As shown in the inset, $\dot{b}$ has a maximum precisely at $t = \tau/2$, around which it is approximately constant as in the asymptotic free expansion. For $t > \tau/2$ the evolution proceeds so as to reconstruct the initial state, scaling it by the desired factor γ. We close noticing that the process is robust in the sense that within linear response, errors in the implementation of the trap modulation or a many-body perturbation represented by the operator $\varepsilon W(\mathbf{x},t)$ lead only to a quadratic decay of the fidelity between the target state $\Phi(\tau)$ and the resulting state at the end of the quench $\Phi'(\tau)$, $\mathcal{F}(\tau) = |\langle\Phi(\tau)|\Phi'(\tau)\rangle|^2 = 1 - \varepsilon^2(\tau/\tau_Z)^2 + O(\varepsilon^4)$, where the Zeno-like time $\tau_Z = \hbar/[\langle\widetilde{W}^2\rangle - \langle\widetilde{W}\rangle^2]^{1/2}$, with $\widetilde{W} = \frac{1}{\tau}\int_0^\tau [U_0(-t)W(t)U_0(t)]dt$ where U_0 is the time evolution operator. The upshot is that the effect of $\varepsilon W(\mathbf{x},t)$ is controlled by the ratio τ/τ_Z and can be suppressed in a nonadiabatic expansion. Hence, the adiabatic approach inevitably fails in the presence of perturbations, while the robustness of the engineered quench inducing a frictionless dynamics is warranted for low enough values of τ.

In conclusion, for many-body systems supporting self-similar dynamics, we have shown how to scale up the system by means of a controlled expansion without modifying the quantum correlations. While this goal can be achieved by slowing down the expansion, the effect of perturbations grows with the expansion time. As an alternative to the adiabatic dynamics, we propose implementation of a fast frictionless quench of the trapping potential, which acts as a quantum dynamical microscope, leading to the scaled-up initial state at the end of the quench.

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