Unreliability of the autocorrelation function as evidence for Ericson fluctuations

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In a recent combined experimental and theoretical paper for the total photoionization cross section of helium up to the single-ionization threshold I_{13}, we had shown that Ericson *fluctuations* were absent, although the Ericson *regime* had already been reached. In the present paper, we study the autocorrelation functions of the total photoionization cross section below I_9–I_{13} and find that these are essentially identical to that of an artificial spectrum with Ericson fluctuations. Consequently, we have to conclude that the autocorrelation function alone—although its Lorentzian-like shape for small displacements ε has been used by some authors—is a necessary but is not a sufficient proof for the existence of Ericson fluctuations. In the present case of He, the absence of Ericson fluctuations is a consequence of a strong hierarchy between the transitions belonging to different series with approximate quantum numbers that still exist in this region; this causes the spectra to be dominated by the principal series.

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I. INTRODUCTION

In quantum chaos, the properties of quantum systems, whose classical counterparts behave in a chaotic way, are studied. For bound systems, standard signatures of quantum chaos are the statistical properties of energy levels, which can be described by a Poissonian model for regular systems and random matrix theory for chaotic systems [1]. For open systems, bound states no longer exist, but rather, resonances exist. One possible signature of quantum chaos for open systems is the appearance of Ericson *fluctuations*, i.e., randomlike fluctuations in the scattering cross section. Ericson fluctuations can occur in the so-called Ericson *regime*, where consecutive resonances overlap strongly, a situation that renders the assignment of features in the cross section to individual resonances impossible.

Originally, it was assumed that strongly overlapping resonances level out causing a smooth cross section. However, already half a century ago, several studies revealed that nuclear cross sections with strongly overlapping resonances can be anything but smooth exhibiting strong fluctuations. In 1960, Ericson [2] had shown that spectra, composed of a large number of overlapping individual resonances with Lorentzian-like or Fano-like line shapes, lead, in fact, to Lorentzian-like spectral features, the so-called Ericson fluctuations. Specifically, Ericson had predicted that spectra, consisting of these fluctuations, display an autocorrelation function $C(\varepsilon)$,

$$C(\varepsilon) = \frac{1}{\Delta\sigma^2(E_2 - E_1)} \times \int_{E_1}^{E_2} [\sigma(E+\varepsilon) - \bar{\sigma}]\,[\sigma(E) - \bar{\sigma}]\,dE, \qquad (1)$$

that is characterized by a Lorentzian shape around $\varepsilon = 0$ [2]. Here, $\bar{\sigma}$ is the average cross section, $\Delta\sigma^2 = \overline{\sigma^2} - \bar{\sigma}^2$ is its variance in the energy interval $E_1 \leqslant E \leqslant E_2$, and ε is the displacement. In addition, one can estimate the average linewidth of the strongly overlapping resonances that contribute to Ericson fluctuations from the width of the Lorentzian shape around $\varepsilon = 0$, even though individual resonances are not resolved.

In the meantime, it was realized that Ericson fluctuations are a general fingerprint for quantum chaotic scattering rather than being only a nuclear phenomenon [3]. The autocorrelation function was commonly applied to analyze fluctuating spectra not only in the context of nuclear reactions [4], but also in atomic scattering [5]. More recently, Rydberg states of ^{85}Rb in crossed electric and magnetic fields were studied both experimentally by Stania and Walther [6] and theoretically by Madroñero and Buchleitner [7], with the aim of observing Ericson fluctuations in an atomic system. Stania and Walther used the autocorrelation function of the spectrum to prove the presence of Ericson fluctuations on the basis of its Lorentzian shape around $\varepsilon = 0$. Subsequently, Xu *et al.* [8] studied the relevance of the autocorrelation function with regard to Ericson fluctuations by the so-called s^2 model for doubly excited helium. The authors came to the conclusion that, in their simplified model, the autocorrelation function was not capable of distinguishing regular from irregular spectral regions in doubly excited helium.

It is the aim of this paper, where we combine our extensive experimental results for doubly excited helium up to the single-ionization threshold (SIT) I_{13} with the results of calculations using the complex-rotation method up to I_{16}, to further clarify the discrepancy between the conclusions in Refs. [6–8]. Our main result is that both our experimental results, as well as the theoretical results from the complex-rotation method, which is much more adequate to describe the real three-dimensional helium as compared to the s^2 model, confirm the previous conclusion of Xu *et al.* [8]. However, we came to a different conclusion than Xu *et al.* for the threshold above which Ericson

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fluctuations are expected to occur, a fact that might be due to the different models applied.

There are two prerequisites for Ericson fluctuations to occur: (i) The Ericson regime, i.e., strongly overlapping resonances with an average decay width $\bar{\Gamma}$ that is much larger than the average energy spacing between neighboring resonances $\bar{S}$. Introducing the Ericson parameter $\Lambda = \bar{\Gamma}/\bar{S}$, the Ericson regime is defined by $\Lambda = \bar{\Gamma}/\bar{S} \gg 1$. (ii) The intensities of the various resonances have to be comparable. This is actually expected from random matrix theory for a chaotic system, where a typical matrix element displays a relatively narrow Gaussian distribution. Alternatively, equal intensities of all lines or—more generally—any distribution, where very small and very large values of the intensity are rare, will lead to Ericson fluctuations. We will call such distributions quasiuniform. In contrast to this, in a regular or a mixed regular-chaotic system, characterized by good or approximate quantum numbers, various series of resonances with widely varying intensities can exist that contribute to the cross section. Such a hierarchy in the intensities of resonances—corresponding to a broad distribution of intensities—is not taken into account in the usual Ericson description, a fact that has important consequences, as shown in the following.

Since the classical counterpart of the helium atom, the three-body Coulomb system, is well known to be nonintegrable and mainly chaotic, high doubly excited states (actually resonances) of helium close to the double-ionization threshold, where they overlap strongly, are expected to display Ericson fluctuations [3]. As shown by calculations of Grémaud and Delande [9], the Ericson regime was already reached below the SIT I_9, i.e., when the average linewidth of the resonances became larger than the average spacing. Statistical studies of the nearest-neighbor spacings of the energy levels by Püttner *et al.* [10] revealed a transition toward a distribution typical for quantum chaos. Recently, however, for three-dimensional (3D) helium, the same group proved that the transition to randomly fluctuating energy positions was not completed before the SIT I_{17} was reached [11,12].

In order to assign the doubly excited $^1P^o$ states of helium, we introduce the labeling N,K_n [13,14]. In this labeling, N (n) is the principal quantum number for the inner (outer) electron, and K is the angular-correlation quantum number that reflects angular correlations between the two electrons. Recently, as pointed out in Refs. [11,12], although the individual quantum numbers—with increasing energy—were progressively less favorable quantum numbers, $F = N - K$ remained an approximate quantum number at least up to the SIT I_{17}. The existence of such a quantum number had been proposed before on the basis of a group-theoretical description by Herrick and Kellman [15] (these authors used $F = N - K - 1$ instead of $F = N - K$), and, subsequently, it was discussed by Rau for states on the Wannier ridge as a quantum number of the electron pair [16]. Later on, Nicolaides *et al.* confirmed this quantum number theoretically for low-n states ($n \leqslant N + 4$) below SITs up to I_{25}, however, without considering autoionization [17].

Moreover, the photoionization cross section is such that the intensities of the various resonances observe a strong hierarchy, with $F = 2$ states being largely dominant [11,12]. The first prerequisite for Ericson fluctuations—average width larger than the mean-level spacing, i.e., the Ericson regime—is fulfilled; nevertheless, *inside* the dominant $F = 2$ series, the mean-level spacing is much larger than the average width. This essentially makes the cross section the sum of nonoverlapping or weakly overlapping resonances of the $F = 2$ series, with the consequence that it will not display Ericson fluctuations. Very recently, analogous observations have been reported in two theoretical studies of the photoionization cross section of doubly excited triplet P states [18] and singlet P states [19] in planar helium up to I_{20}. In the latter case, the theoretical predictions describe the experimental cross section even on a quantitative level quite well, i.e., these two-dimensional calculations are able to describe the essential features of the full 3D problem.

Although Ericson fluctuations were absent in the photoionization cross section of doubly excited He below I_{13}, we studied the autocorrelation function, with the result that its behavior was quite similar to that observed experimentally and theoretically for ^{85}Rb [6,7] and similar to the results of the previous theoretical study of He using the s^2 model [8]. Therefore, we have to conclude that the observation of an autocorrelation function with a Lorentzian shape is a necessary, but is not a sufficient prerequisite for Ericson fluctuations to occur.

II. EXPERIMENTAL SETUP AND CALCULATIONS

The experiments were performed at the high-resolution undulator beamline UE56-2/PGM2 of the Berliner Elektronenspeicherring für Synchrotronstrahlung (BESSY II), with a photon-energy resolution of $\Omega \cong 1.7$ meV [full width at half maximum (FWHM)]. The spectra were measured with an ionization cell, filled with $\cong 1$ mbar of He and operated with an electric field of 100 V/cm between the two plates in the ionization cell. A step width of 250 μeV was used, and the absolute photon-energy scale was calibrated on the basis of the theoretical results of this paper. It should be noted that it is rather challenging to record spectra in this energy region of doubly excited helium since the amplitudes of the resonances amount to only 0.04–0.2% of the signal background caused by direct photoionization. Small variations in the gas pressure or insufficient normalization to the photon flux can easily make the observation of these resonances impossible.

The photoionization cross section of He was calculated by diagonalization of the nonrelativistic Hamiltonian in a perimetric basis set that was chosen in a way to correctly reproduce the long-range behavior of the electronic wave function as well as its behavior close to the nucleus [9]. Above I_1, only autoionizing resonances are found, but no bound states are found. Close to the double-ionization threshold, the number of open channels increases dramatically, and it is essential to correctly calculate both the positions and widths of the various resonances. Complex rotation renders it possible to obtain these quantities accurately and to express the photoionization cross section as a simple sum of Fano profiles of the various resonances. In the region of overlapping resonances with $\bar{\Gamma} \simeq \bar{S}$, the direct extraction of positions and widths is a major advantage of the present approach as compared to other

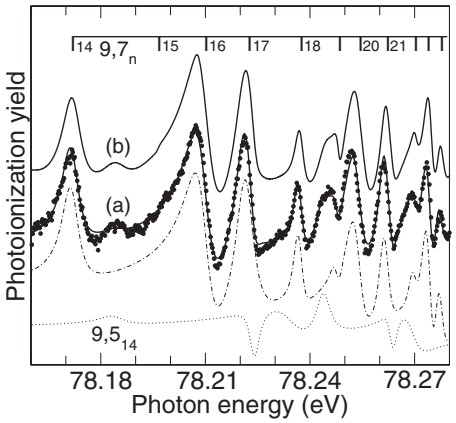

FIG. 1. Total photoionization cross section below the SIT I_9: (a) The experimental data are represented by the data points, with the fit result represented by the solid curve through the data points. In the lower part of the figure, the contributions of the principal $F = N - K = 2$ and the secondary $F = 4$ Rydberg series are represented by the dashed-dotted and dotted lines, respectively. (b) The results of complex-rotation calculations are represented by the solid line. The vertical-bar diagram in the upper part of the figure indicates the energy positions of the resonances belonging to the Rydberg series $9,7_n$.

methods [20], since it ensures that the photoionization spectra as well as the properties of individual resonances are fully converged.

III. PROOF OF ABSENCE OF ERICSON FLUCTUATIONS AND AUTOCORRELATION FUNCTION

As stated above, in a combined experimental and theoretical study up to the SIT I_{15}, we proved that Ericson fluctuations were absent [11,12]. First, we want to describe the data analysis used in Ref. [11] to exclude Ericson fluctuations in more detail. For reasons of simplicity, in the present paper, we will focus on the energy region below the ionization threshold I_9 as an example.

To this end, the high-resolution photoionization spectrum below the SIT I_9 is presented in Fig. 1, together with the results of complex-rotation calculations (upper solid line). To simulate the experimental resolution, the theoretical data were convoluted by a Gaussian function with a FWHM of 1.7 meV. A comparison of the experimental and theoretical results shows impressively good agreement for the entire region below I_9.

In the fit procedure, the resonance positions, linewidths, and Fano parameters q are fixed to the values obtained from the calculation. The intensities represent the only free parameters, with the result that the relative intensities obtained for most of the resonances agree with the calculated ones within a factor of 2. This data-analysis procedure was first applied in Ref. [21], where minor discrepancies between experiment and theory were discussed, with the conclusion that they were not devaluating the theoretical results. The main goal of the fit procedure was to deconvolute the spectrum and to determine the lowest possible number of resonances that were necessary for a reasonably good description. Such a fit can be called a describing fit and is directly related to the question whether Ericson fluctuations are present in this region of the spectrum or not. In the energy interval between 78.16 and 78.28 eV, presented in Fig. 1, a total of 139 resonances can be identified. However, from our fit represented by the solid line through the data points, it becomes obvious that only about 15 resonances significantly contribute to the spectrum. Most of these resonances can be assigned to the principal series $9,7_n$, with $F = N - K = 2$; these resonances are represented by the dashed-dotted subspectrum, and their energy positions are indicated by the vertical bars above the spectrum. A much smaller fraction of resonances belongs to the secondary series $9,5_n$, with $F = N - K = 4$; they are represented by the dotted subspectrum. The describing fits performed for the energy regions below the SITs I_{10}–I_{15} essentially show the same behavior, clearly indicating that the spectra are still dominated by only one nonoverlapping or weakly overlapping Rydberg series, namely, the principal series with $F = 2$. Consequently, Ericson fluctuations caused by a large number of overlapping resonances are absent in the spectra up to the ionization threshold I_{15}. These results are in full accordance with the theoretical description that predicts all other Rydberg series intensities that are by one to several orders of magnitude smaller than those of the principal series.

Although we have shown that Ericson fluctuations are absent in the spectrum, in the following, we discuss its autocorrelation function, since this function was used, e.g., by Stania and Walther [6] as the sole criterion for the existence of Ericson fluctuations in the photoionization spectra of ^{85}Rb in crossed magnetic and electric fields. For comparison with our helium results, a simulated spectrum that displays Ericson fluctuations is shown in Fig. 2. It was generated by 1000 Fano resonances randomly distributed in an energy region of 200 meV, with an average width of $\bar{\Gamma} = 5$ meV, corresponding to an Ericson parameter of $\Lambda \cong 25$. In detail, Γ varies randomly between 0 and 10 meV, the Fano parameter q varies randomly between −1 and 1, and the intensity varies randomly between 0 and a maximum value, i.e., we used a uniform distribution of all of these parameters. For a better comparison with the total photoionization cross section given in Fig. 1, the spectrum was convoluted with a Gaussian function of $\sigma_{\mathrm{FWHM}} = 1.5$ meV (FWHM). Note that the simulated spectrum in Fig. 2 exhibits spectral features similar to those of

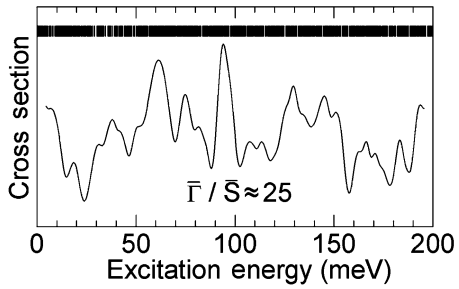

FIG. 2. Simulated spectrum with Ericson fluctuations characterized by $\bar{\Gamma} = 5$ meV and $\Lambda = \bar{\Gamma}/\bar{S} \cong 25$. The spectrum is convoluted with a Gaussian function of 1.5 meV (FWHM) and is plotted as a function of relative excitation energies. The energy positions of the 1000 individual resonances (see text) are indicated by black vertical narrow bars in the upper part of the spectrum that overlap strongly; the resulting white bars define regions without resonances.

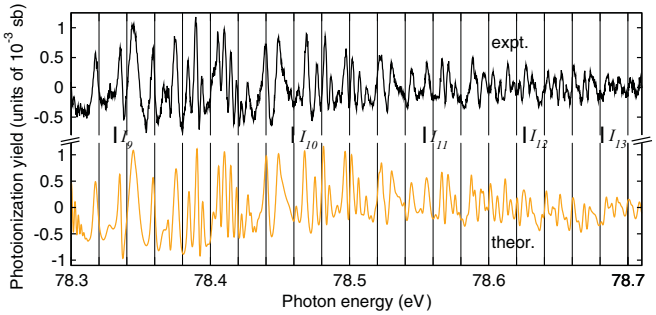

FIG. 3. (Color online) Total cross section of doubly excited He between 78.3 and 78.71 eV: experimental data (upper black curve) and results of complex-rotation calculations (lower orange curve). The photoionization yield is given in units of the single ionization background (sb), and the vertical bars indicate the positions of the SITs from I_9 to I_{13}.

the spectrum of doubly excited helium given in Fig. 1. In both spectra, the Fano line shapes of the individual resonances cannot be recognized.

The autocorrelation functions defined by Eq. (1) of the simulated spectrum, shown in Fig. 2, and of the spectra of helium below the ionization thresholds I_9–I_{13}, shown in Figs. 1 and 3, are presented in Fig. 4. Since all spectra are broadened by the experimental resolution, we want to point out that a convolution of the cross section with a Gaussian function of the width σ_{expt} leads to a convolution of the corresponding autocorrelation function with a Gaussian function of the width $\sigma_{\text{autocor}} = \sigma_{\text{expt}}\sqrt{2}$, see the Appendix. This leads in the present case of $\sigma_{\text{expt}} = 1.7$ meV to $\sigma_{\text{autocor}} = 2.4$ meV. For small displacements ε, the best agreement between the autocorrelation functions and the convoluted Lorentzian functions that is represented by the dashed lines in Fig. 4 was obtained using $\sigma_{\text{autocor}} = 2.1$ meV, which is in good agreement with the expected value. For small values of ε, the Lorentzian functions obviously agree very well with the autocorrelation functions, i.e., they fulfill the prediction expected for the presence of Ericson fluctuations. However, the photoionization cross section of helium, where Ericson fluctuations are absent, shows the same behavior as the simulated spectrum, where Ericson fluctuations are present! For larger displacements ε, oscillations around zero are observed. The average widths of $\cong 2.3$ and $\cong 5$ meV for the resonances in the spectra in Figs. 1 and 2, respectively, are in good agreement with the values of $\cong 3$ and $\cong 4$ meV obtained from the Lorentzian fits. The observed features of the autocorrelation functions are quite similar to those of Stania and Walther [6], of Madroñero and Buchleitner [7], and of Xu *et al.* [8].

Interestingly, the autocorrelation function of the spectrum below I_9 starts to deviate strongly from a Lorentzian shape already at $\epsilon \cong \Gamma_{\text{fit}}$, while the autocorrelation function of the simulated spectrum with Ericson fluctuations exhibit such strong deviations only above $\epsilon \cong 3\Gamma_{\text{fit}}$. To see if there was any systematics in this observation, we also analyzed the autocorrelation functions of the photoionization cross sections below the ionization thresholds I_{10}–I_{13} that were shown in Figs. 3(c)–3(f). For a better comparison, all these functions are plotted for displacements up to $15\Gamma_{\text{av}}$, with $\Gamma_{\text{av}} = \Gamma_{\text{fit}}$ for the autocorrelation functions of the simulated spectrum

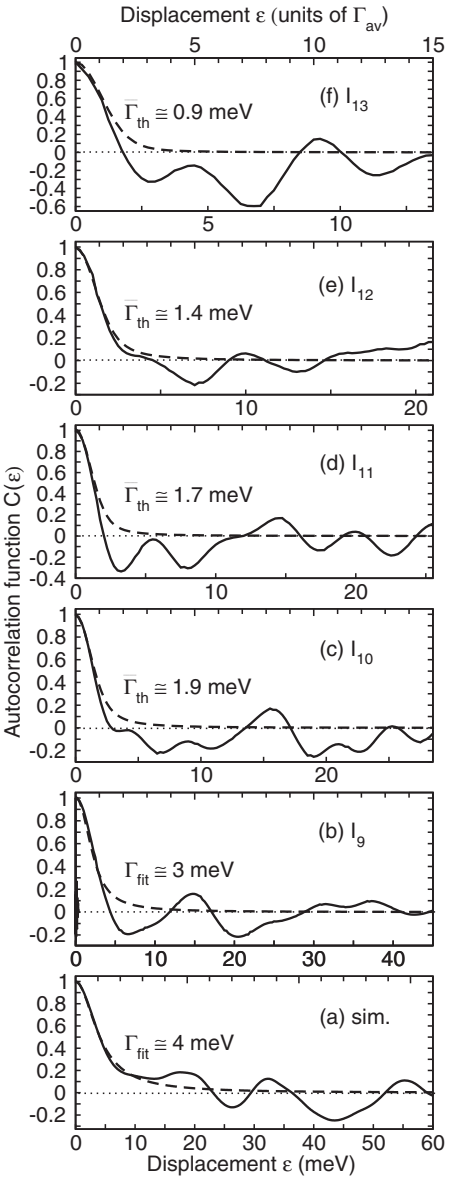

FIG. 4. Autocorrelation functions (a) of the simulated spectrum given in Fig. 2 and (b)–(f) of the total ionization cross sections of doubly excited helium below I_9–I_{13}. The displacements are given in meV (bottom abscissa of each panel) and in units of Γ_{av} (top abscissa) up to $\varepsilon = 15\Gamma_{\text{av}}$; for Γ_{av}, see text. The dashed lines in (a) and (b) represent the fit results of Lorentzian functions to the autocorrelation functions and those in (c)–(e) show simulated Lorentzian functions using the average values of the theoretical linewidths $\bar{\Gamma}_{\text{th}}$ as widths below the corresponding ionization threshold; all Lorentzians were convoluted to simulate the experimental resolution (for details, see text). The dotted horizontal lines mark zero.

and of the photoionization cross section below I_9. Below all higher ionization thresholds, the average widths are smaller than σ_{autocor} so that no reliable fit value could be obtained from the autocorrelation function. We, therefore, used the average value of the calculated lifetime widths of the resonances below a given threshold $\bar{\Gamma}_{\text{th}}$ as widths to simulate the Lorentzian function for comparison. These functions were convoluted with a Gaussian of the width $\sigma_{\text{autocor}} = 2.1$ meV (see the Appendix) and are displayed in Figs. 4(c)–4(f) as dashed lines;

it is obvious that—for small displacements $\varepsilon < \Gamma_{av}$—there is good agreement between the convoluted Lorentzians and the corresponding autocorrelation functions.

For displacements ϵ larger than Γ_{av}, a different behavior of the various autocorrelation functions is observed. The autocorrelation functions obtained below I_9, I_{11}, and I_{13} exhibited significant deviations from a Lorentzian shape already at $\epsilon \cong \Gamma_{av}$, while the one below I_{10}—and, in particular, the one below I_{12}—followed a Lorentzian shape quite well up to $\epsilon \cong 2\Gamma_{av}$ and $3\Gamma_{av}$, respectively. We, therefore, conclude that there is no systematics in the behavior of the autocorrelation function for displacements between Γ_{av} and $3\Gamma_{av}$. In contrast to this, there is a continuous transition in the nearest-neighbor spacing distribution from regular to chaotic between I_9 and I_{17} [11]. In addition, our calculations show that the Ericson parameter Λ increases with the ionization threshold. From these considerations, we conclude that the shape of the autocorrelation function for $\epsilon \geqslant \Gamma_{av}$ does not provide information on the degree of chaos inherent in the system. Instead, it depends on the details of the cross section.

We also checked the influence of the finite resolution used to convolute the helium spectra on our observations with the result that there is no effect as long as it is smaller than the average width in agreement with Ref. [8]. These results clearly demonstrate that the occurrence of Ericson fluctuations in a spectrum can be identified neither by the shape of the spectral variations nor by the autocorrelation function alone. Therefore, it is essential to independently ensure that the intensities are quasiuniformly distributed, since only in this case, will a large number of overlapping resonances contribute to the observed cross section. This, however, can hardly be done by experimental studies alone; instead, complementary calculations are required. In addition, we proved that the average width of the resonances contributing to the spectra can be obtained by fitting a Lorentzian to the autocorrelation function in the region of small displacements ε; however, this was once again independent of the presence of Ericson fluctuations.

IV. SUMMARY AND CONCLUSION

To summarize, the total photoionization cross section of doubly excited helium was studied experimentally up to the SIT I_{13} of the He^+ ion and theoretically up to I_{16}. Based on the excellent agreement of the experimental data with the results of complex-rotation calculations, a describing fit was performed. As shown for the spectrum below I_9, as an example, the fit analysis revealed that the spectra were dominated by the principal Rydberg series, implying that the intensities of the various resonances were far from uniformly distributed. Consequently, Ericson *fluctuations* generated by strongly overlapping resonances were absent, although the Ericson *regime* with $\Lambda = \bar{\Gamma}/\bar{S} \gg 1$ had been reached. Nevertheless, the autocorrelation function of the spectrum displays a Lorentzian form for small displacements. Therefore, we have to conclude that a Lorentzian form of the autocorrelation function for small displacements, although it can help in the analysis of spectral properties, such as average linewidths, is a necessary but is not a sufficient prerequisite for the existence of Ericson fluctuations—contrary to a widespread assumption. Additionally, for a clear proof of Ericson fluctuations, one has to show that the intensities of the overlapping resonances are comparable. The present case of helium shows that this omission may lead to the wrong conclusions about the presence of Ericson fluctuations in a spectrum if there is a strong hierarchy in the intensities of the various resonance series. In some sense, a Lorentzian autocorrelation function is nothing else but a rough indication that there is no simple structure in the spectrum, i.e., it is a consequence of the level repulsion leading to a Wigner distribution as well as of fluctuations in the Fano parameter q and the intensities within the dominant $F = 2$ series [12,21].

An interesting and partly unsolved question concerns the specific excitation energy above which true Ericson fluctuations in the double-excitation cross section of He occur. This requires strongly overlapping resonances within the dominant $F = 2$ series, i.e., at significantly higher N values than experimentally studied up to now. On the other hand, Xu *et al.* claimed that the transition to Ericson fluctuations occurred above I_{16} [8]. In contrast, calculations for the one-dimensional helium atom—where a single Rydberg series converges to each SIT—suggest that this does not happen below $N \cong 34$ [22]. In this context, recent studies by Eigelsperger and Madroñero [18,19] found, for doubly excited P states in planar helium, that the intensity ratio between the principal series and the first secondary series decreased when approaching the double-ionization threshold. If this would also hold for real 3D helium, Ericson fluctuations might already occur for N values considerably smaller than $N \cong 34$, but not below $N = 20$ [19]. However, for a definite conclusion, further experimental results and numerical simulations at higher energies are needed.

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APPENDIX: EFFECT OF FINITE RESOLUTION

Here, we show that the finite resolution of the spectra (either experimental or calculated) has a well-controlled effect on the analysis, namely, a simple smoothing of the autocorrelation function $C(\varepsilon)$, Eq. (1). With a subscript "f," let us denote the fluctuating part of the cross section,

$$\sigma_f(E) = \sigma(E) - \bar{\sigma}. \tag{A1}$$

Then, the autocorrelation function $C(\varepsilon)$, Eq. (1), simply writes as

$$C(\varepsilon) = \frac{1}{\Delta\sigma^2(E_2 - E_1)} \int_{E_1}^{E_2} \sigma_f(E+\varepsilon)\sigma_f(E)\, dE. \tag{A2}$$

Let us now consider a finite-resolution cross section, modeled by the convolution of the cross section with a smoothing function, for example, a Gaussian with variance γ,

$$\Sigma(E) = \int_{\infty}^{+\infty} \sigma(E-x) f(x)\, dx, \tag{A3}$$

with

$$f(x) = \frac{1}{\sqrt{2\pi}\gamma} \exp\left(-\frac{x^2}{2\gamma^2}\right). \quad \text{(A4)}$$

Provided the width of this Gaussian is much smaller than the energy interval $E_2 - E_1$ (which is assumed in the following), the finite-resolution cross section has the same average than the raw one, and the finite-resolution autocorrelation function can be written as

$$\mathcal{C}(\varepsilon) = \frac{1}{\Delta\sigma^2(E_2 - E_1)} \int_{E_1}^{E_2} \Sigma_f(E + \varepsilon)\, \Sigma_f(E)\, dE, \quad \text{(A5)}$$

with $\Sigma_f(E) = \Sigma(E) - \bar{\sigma}$. Note that this finite-resolution autocorrelation is not properly normalized here because we use the variance $\Delta\sigma$ of the raw cross section in the denominator, which is larger than the variance of the finite-resolution cross section. This is unimportant as all results displayed in the figures are *a posteriori* normalized. Inserting Eq. (A3) in Eq. (A5), one obtains

$$\mathcal{C}(\varepsilon) = \frac{1}{\Delta\sigma^2(E_2 - E_1)} \int \sigma_f(E + \varepsilon - x)\, \sigma_f(E - y) \times f(x) f(y) dx\, dy\, dE. \quad \text{(A6)}$$

The following change of variables:

$$u = E - y, \quad \text{(A7)}$$

$$v = -x + y + \varepsilon, \quad \text{(A8)}$$

$$w = \frac{x + y}{2} \quad \text{(A9)}$$

leads to the triple integral,

$$\mathcal{C}(\varepsilon) = \frac{1}{\Delta\sigma^2(E_2 - E_1)} \int \sigma_f(u)\sigma_f(u + v) f\left(w + \frac{\varepsilon - v}{2}\right) \times f\left(w - \frac{\varepsilon - v}{2}\right) du\, dv\, dw. \quad \text{(A10)}$$

The integral over w is a Gaussian integral easily performed, while the integral over u simply reduces to the raw autocorrelation function $C(v)$, finally giving

$$\mathcal{C}(\varepsilon) = \int_{\infty}^{+\infty} \frac{1}{\sqrt{4\pi}\gamma} C(v) \exp\left(-\frac{(v - \varepsilon)^2}{4\gamma^2}\right) dv. \quad \text{(A11)}$$

This proves that the autocorrelation function for finite-experimental resolution γ is simply obtained from the raw one by a convolution with a Gaussian of variance $\sqrt{2}\gamma$.

Note that a similar property is valid if the smoothing is not a Gaussian function: The only difference is that the smoothing function is then related to the autocorrelation of the function f itself and, in general, is not a Gaussian function.

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