PHYSICAL REVIEW A **84**, 020302(R) (2011)

Entanglement-enhanced information transfer through strongly correlated systems and its application to optical lattices

Song Yang,[1,2] Abolfazl Bayat,[1] and Sougato Bose[1]

[1] *Department of Physics and Astronomy, University College London, London WC1E 6BT, United Kingdom*

[2] *Key Laboratory of Quantum Information, University of Science and Technology of China, Hefei, 230026, People's Republic of China*

(Received 14 January 2011; revised manuscript received 30 March 2011; published 4 August 2011)

We show that the inherent entanglement of the ground state of strongly correlated systems can be exploited for both classical and quantum communications. Our strategy is based on a single-qubit rotation that encodes information in the entangled nature of the ground state. In classical communication, our mechanism conveys more than one bit of information in each shot, just as dense coding does, without demanding long-range entanglement. In our scheme for quantum communication, the quality is higher than the widely studied attaching scenarios. Moreover, we propose to implement this way of communication in optical lattices.

DOI: 10.1103/PhysRevA.84.020302 PACS number(s): 03.67.Hk, 03.65.Ud, 37.10.Jk, 42.79.Sz

I. INTRODUCTION

Strongly correlated systems often have nontrivial entangled ground states. When one wants to use the dynamics, as opposed to measurements [1], of such a system for propagating information [2], the inherent entanglement plays a little role and only the symmetries of the state and the Hamiltonian seem to be important [3]. The only mode of transmission studied so far is to attach a qubit encoding an unknown quantum state to the system [2,3], which does not seek to harness the entanglement of many-body systems. Note that the entanglement in strongly correlated system is notoriously short ranged [4] (with rare exceptions [5]), so it cannot be used directly to teleport [6] an unknown state, accomplish remote state preparation [7] of a known state, or double the rate of classical communication by dense coding [8]. Thus, a natural question is whether the entanglement in many-body systems can practically benefit some mode of information transfer. In particular, it is highly desirable to design alternative protocols for dynamic communication through strongly correlated systems, which are dense-coding-like or remote-state-preparation-like despite the absence of long-range entanglement.

Cold atoms in an optical lattice are now an established field for many-body experiments. Both bosons [9] and fermions [10] have been realized in the Mott-insulator phase, where there is exactly one atom per site, and by properly controlling the intensity of laser beams one can get an effective spin Hamiltonian [11] between atoms. Superlattices have been used to take singlet-triplet measurements of simulated spins in such systems [12–14]. Striking recent developments [15–19] make it timely to seriously consider the implementation of communication schemes in optical lattices. Alternative cooling techniques [15] have enabled the attainment of the temperatures required for observing quantum magnetic phases. Moreover, single-atom detection [16] with single-site resolution as well as single-qubit operation and measurement [17–19] has been achieved. Hence, near-future experiments are likely to look at the effects of local actions, e.g., spin flips [17,18], in optical lattice simulated spin systems. A good question then is whether the same experiments can also provide interesting quantum information protocols.

In this Rapid Communication we introduce a mechanism for both quantum and classical communications in spin chains that uses a local rotation by a sender, followed by nonequilibrium dynamics and subsequent reception and measurements by a receiver. Surprisingly, despite the absence of long-range entanglement, we find that even a single-qubit rotation can convey more than one bit of information in the same spirit used by the dense-coding protocol [8]. We also find that the fidelity of quantum state transfer is enhanced compared to the fidelity achieved by attaching a qubit to the system.

II. SETUP

We consider a chain of N spin-1/2 particles, where N is even, interacting through a dimerized Hamiltonian

$$H = J\sum_{k=1}^{N-1}[1+(-1)^{k+1}\delta]\hat{\sigma}_k\cdot\hat{\sigma}_{k+1}, \tag{1}$$

where $J>0$ is the coupling, $\hat{\sigma}_k=(\sigma_k^x,\sigma_k^y,\sigma_k^z)$ denotes the Pauli operators at site k, and $0<\delta<1$ determines the dimerization of the chain. A schematic of this system is shown in Fig . 1(a). We assume that the system is initially in its ground state $|\text{g.s.}\rangle$. Due to the SU(2) symmetry of the Hamiltonian, the reduced density matrix of the first two qubits is a Werner state $\rho_{1,2}=p|\psi^-\rangle\langle\psi^-|+(1-p)\hat{I}_4/4$, where $\hat{I}_n$ represents an $n\times n$ identity matrix, $|\psi^-\rangle=(|01\rangle-|10\rangle)/\sqrt{2}$ is the singlet state, and $0\leqslant p\leqslant 1$ is controlled by δ (if $\delta\to 1$ then $p\to 1$). We assume that the sender (Alice) controls qubit 1, as shown in Fig. 1(a), while the receiver (Bob) controls the qubits $N-1$ and N, as shown in Fig. 1(b).

III. CLASSICAL COMMUNICATION

For classical communication, Alice encodes two bits of classical information (i.e., 00, 01, 10, and 11) in the state of the chain by a single operation σ_1^α on qubit 1, where $\alpha\in\{I,x,y,z\}$ and $\sigma_1^I=\hat{I}_2$ such that σ_1^I represents 00, σ_1^x represents 01, etc. Unlike the dense-coding proposal put forth in Ref. [8], this encoding is local since there is no long-range entanglement between Alice and Bob. Accordingly, the quantum state of the whole system changes to $|\psi^\alpha(0)\rangle=\sigma_1^\alpha|\text{g.s.}\rangle$. The reduced density matrix of the first two qubits after this local action

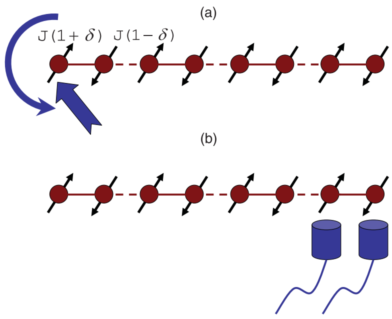

FIG. 1. (Color online) (a) Dimerized chain where solid (dashed) lines represent strong (weak) bonds and encoding is achieved through a local rotation on qubit 1. (b) Decoding is done through measurements on qubits $N-1$ and N.

becomes $\rho^{\alpha}_{1,2}(0) = \sigma^{\alpha}_1 \rho_{1,2} \sigma^{\alpha}_1 = p|b^{\alpha}\rangle\langle b^{\alpha}| + (1-p)\hat{I}_4/4$, with Bell states $|b^{\alpha}\rangle = \sigma^{\alpha}_1|\psi^-\rangle$. Notice that, for $p<1$ the encoded states are not fully distinguishable.

After encoding, the system evolves as $|\psi^{\alpha}(t)\rangle = e^{-iHt}|\psi^{\alpha}(0)\rangle$ and the time evolution of the system transfers $\rho^{\alpha}_{1,2}(0)$ dispersively along the chain. At time t the density matrix of qubits $N-1$ and N is $\rho^{\alpha}_{N-1,N}(t)$, for which Bob can define a superoperator $\mathcal{E}$ as $\rho^{\alpha}_{N-1,N}(t) = \mathcal{E}(\rho^{\alpha}_{1,2}(0))$. At an optimal time $t=t^*$, the density matrix $\rho^{\alpha}_{N-1,N}(t^*)$ has its maximal fidelity with $\rho^{\alpha}_{1,2}(0)$ and by performing Bell measurement on qubits $N-1$ and N Bob can identify the operator σ^{α}_1 (and accordingly two classical bits encoded by Alice) through his measurement outcome $|b^{\alpha}\rangle$. However, Bob may have some errors in his decoding as (i) the initial encoding may not be perfect ($p<1$) and (ii) the dynamics is dispersive and the received state $\rho^{\alpha}_{N-1,N}(t^*)$ is not exactly equal to $\rho^{\alpha}_{1,2}(0)$. To quantify the quality of communication for each σ^{α}_1 one can numerically compute the fidelity $F^{\alpha}(t) = \langle b^{\alpha}|\rho^{\alpha}_{N-1,N}(t)|b^{\alpha}\rangle$. In Fig. 2(a) we plot $F^{\alpha}(t)$ versus time for a chain of $N=20$ in which the fidelity peaks at the time $t=t^*$.

In classical communication, Holevo information is usually employed to quantify the information that is sent. For our proposed mechanism, the Holevo information is $C(t) = S(\mathcal{E}(\sum_{\alpha} q_{\alpha}\rho^{\alpha}_{1,2})) - \sum_{\alpha} q_{\alpha} S(\mathcal{E}(\rho^{\alpha}_{1,2}))$, where $S(\rho) = -\mathrm{tr}(\rho\log\rho)$ is the von Neumann entropy and q_{α} is the probability of applying σ^{α}_1. We assume $q_{\alpha}=1/4$, which also maximizes $C(t)$. Holevo information also peaks at $t=t^*$ and with time-dependent density-matrix renormalization-group techniques we numerically simulate systems up to 30 sites. In Fig. 2(b) we plot $C(t^*)$ as a function of N. It shows that $C(t^*)$ is still above 1 bit for a chain of length $N=30$ (for $\delta=0.7$). In the inset of Fig. 2(b) we plot the optimal time t^* versus N, which linearly increases with length.

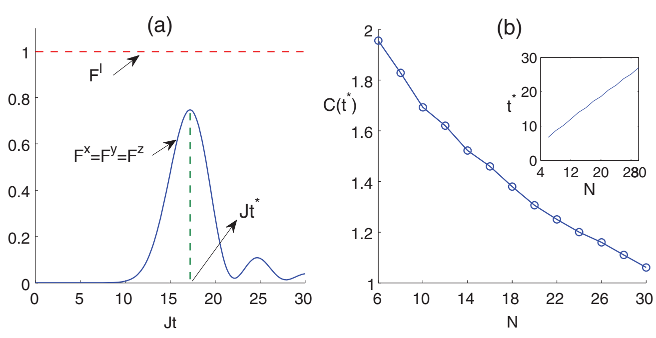

FIG. 2. (Color online) (a) F^{α} as a function of time for a chain of $N=20$ and $\delta=0.7$. (b) Scaling of $C(t^*)$ in terms of N. The inset shows the scaling of the optimal time t^* versus N ($\delta=0.7$).

IV. QUANTUM COMMUNICATION

We can use the same recipe for quantum communication. To encode one qubit in a pure state Alice applies

$$R_1(\theta,\phi) = \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2}e^{-i\phi} \\ \sin\frac{\theta}{2}e^{i\phi} & \cos\frac{\theta}{2} \end{pmatrix} \tag{2}$$

on the first qubit of the chain. The application of the operator R_1 on state $|0\rangle$ (or $|1\rangle$) gives the most general pure state of a qubit on the surface of the Bloch sphere determined by two angles θ and ϕ. After the operation of R_1, the state of the system changes to $|\psi^{\theta,\phi}(0)\rangle = R_1(\theta,\phi)|\text{g.s.}\rangle$ and the reduced density matrix of the first two qubits becomes $\rho^{\theta,\phi}_{1,2}(0) = pR_1|\psi^-\rangle\langle\psi^-|R_1^{\dagger} + (1-p)\hat{I}_4/4$. After encoding, the system evolves as $|\psi^{\theta,\phi}(t)\rangle = e^{-iHt}|\psi^{\theta,\phi}(0)\rangle$ and the density matrix of Bob's qubits, i.e., $\rho^{\theta,\phi}_{N-1,N}(t)$, accordingly changes with time. At time $t=t^*$, the two parameters of R_1 (i.e., θ and ϕ) are encoded in the density matrix of Bob's qubits, i.e., $\rho^{\theta,\phi}_{N-1,N}(t^*)$, and he can localize this information in a single qubit by performing a single-qubit measurement in the computational basis on site $N-1$. In an ideal case, where both encoding and transmission are perfect, Bob receives $R_1|\psi^-\rangle\langle\psi^-|R_1^{\dagger}$. One can easily show that in the state $R_1|\psi^-\rangle$ when qubit $N-1$ is projected on $|0\rangle$ or $|1\rangle$ the state of qubit N is collapsed according to

$$\begin{aligned} |0\rangle_{N-1} &\to |\psi_0\rangle_N = \cos\frac{\theta}{2}|1\rangle + \sin\frac{\theta}{2}e^{-i\phi}|0\rangle, \\ |1\rangle_{N-1} &\to |\psi_1\rangle_N = \cos\frac{\theta}{2}|0\rangle - \sin\frac{\theta}{2}e^{+i\phi}|1\rangle, \end{aligned} \tag{3}$$

where $|\psi_k\rangle_N$ $(k=0,1)$ is the state of site N when site $N-1$ is projected on state $|k\rangle$. However, in a realistic situation qubit N remains mixed even after measuring qubit $N-1$. So the measurement fidelity is defined as $F^M(\theta,\phi,t) = p_0\langle 0,\psi_0|\rho^{\theta,\phi}_{N-1,N}(t)|0,\psi_0\rangle + p_1\langle 1,\psi_1|\rho^{\theta,\phi}_{N-1,N}(t)|1,\psi_1\rangle$, where p_k denotes the probability of projecting qubit $N-1$ on state $|k\rangle$. We compute the average measurement fidelity by integrating $F^M(\theta,\phi,t)$ over the surface of the Bloch sphere as $F^M_{\text{av}}(t) = \int F^M(\theta,\phi,t)d\Omega$. One can show that

$$F^M_{\text{av}}(t) = \tfrac{1}{2} + \tfrac{1}{12}(F_2 - F_1) + \tfrac{2}{3}F_3, \tag{4}$$

where

$$\begin{aligned} F_1 &= \langle \text{g.s.}|\sigma^z_{N-1}(0)\sigma^z_N(0)|\text{g.s.}\rangle, \\ F_2 &= 2\langle \text{g.s.}|\sigma^+_1(0)\sigma^z_{N-1}(t)\sigma^z_N(t)\sigma^-_1(0)|\text{g.s.}\rangle, \\ F_3 &= \mathrm{Re}[\langle \text{g.s.}|\sigma^+_1(0)\sigma^z_{N-1}(t)\sigma^-_N(t)|\text{g.s.}\rangle]. \end{aligned} \tag{5}$$

These components can be computed numerically by means of exact diagonalization. In Fig. 3(a) we plot $F^M_{\text{av}}(t^*)$ as a function of N for $\delta=0.7$. According to Fig. 3(a), the average fidelity decays very slowly and fits by the line $F^M_{\text{av}}(t^*) = -0.0062N + 1.03$. Remarkably, extrapolation shows that the average fidelity is above the classical threshold $2/3$ for chains up to $N=58$.

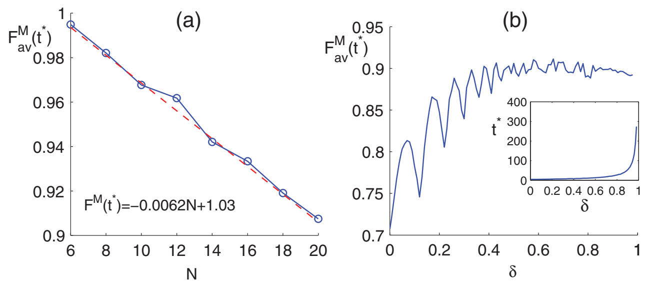

FIG. 3. (Color online) $F_{\rm av}^M(t^*)$ as a function of N (blue circles) and its linear fit (red dashed line) for a chain with $\delta = 0.7$. (b) $F_{\rm av}^M(t^*)$ as a function of dimerization δ for a chain of $N = 20$. The inset shows the optimal time t^* as a function of δ.

Our protocol is fundamentally different from the usual attaching schemes because of the direct role of the inherent short-range entanglement in strongly correlated systems. We compare the average fidelities achieved in our proposal and ferromagnetic-antiferromagnetic attaching scenarios in Table I. As the numbers evidently show, $F_{\rm av}^M(t^*)$ is always higher than both attaching schemes.

V. MECHANISM

In almost all works in the context of quantum communication through many-body systems (see, for instance, Ref. [20]) people first establish long-range entanglement between the sender and receiver through entanglement propagation and then use that entanglement for teleportation. Our mechanism is fundamentally different as distributing entanglement is not our aim. We instead exploit the inherent local entanglement (i.e., between proximal spins) in the initial state of the system for communication. In the absence of local entanglement, for instance, in a ferromagnetic initial state, the actions of σ_1^x and σ_1^y are identical (spin flip) and hence cannot be used for encoding different states. The capability of using localized entanglement for some tasks that in general need long-range entanglement is the unique feature of strongly correlated systems that we point out. To have a purer local entanglement, and thus a better encoding, one has to use a proper δ. In a chain of length $N = 20$ for $\delta > 0.5$ we have $p > 0.99$. In contrast, by increasing δ the propagation becomes slower due to the emergence of small couplings [i.e., $J(1-\delta)$], which favors

TABLE I. Comparison between different strategies of quantum communication, namely, antiferromagnetic (AFM) and ferromagnetic (FM) chains with attaching an extra spin as well as $F_{\rm av}^M(t^*)$ achieved in our scheme.

N	FM	AFM	$F_{\rm av}^M(t^*)$
6	0.820	0.954	0.993
8	0.787	0.935	0.980
10	0.763	0.919	0.967
12	0.745	0.906	0.961
14	0.731	0.895	0.941
16	0.719	0.885	0.932
18	0.708	0.877	0.918
20	0.699	0.871	0.906

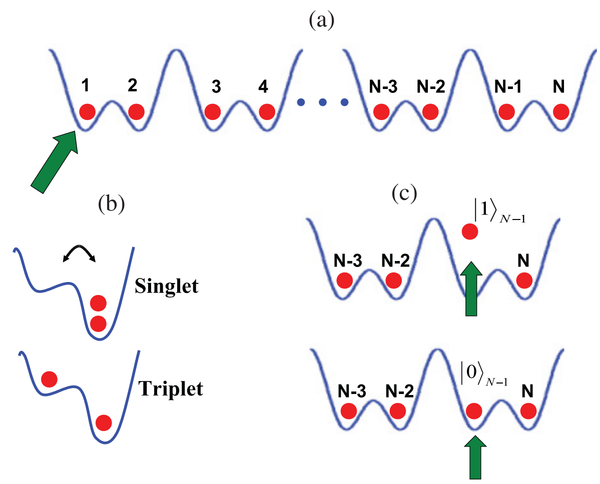

FIG. 4. (Color online) Schematic for optical lattice realization. (a) Encoding through a local rotation on the first qubit. (b) Bell measurement in which two atoms hop to a single site when their internal state is singlet and remain apart otherwise. (c) Single-site measurement by an intense laser beam that pushes the atom of the lattice if it is in state $|1\rangle$ and leaves it there if it is in $|0\rangle$.

intermediate values of δ. In Fig. 3(b) we plot $F_{\rm av}^M(t^*)$ as a function of δ for a chain of length $N = 20$, which oscillates upward to take its maximum value around $\delta = 0.7$. In the inset of Fig. 3(b) we plot t^* as a function of δ, which increases exponentially for $\delta > 0.8$. For relatively large δ, the ground state of the system is almost a series of singlets; thus, instead of the $|\text{g.s.}\rangle$, a series of singlets can be prepared, which may be possible in an independent process that avoids sophisticated cooling.

VI. APPLICATION

As an application, we propose an array of cold atoms in their Mott-insulator phase sitting in the minima of a superlattice potential [12–14] formed by counterpropagating laser beams with two frequencies, with one being twice the other [Fig. 4(a)]. In the limit of high on-site energy the interaction between atoms is effectively modeled by a spin Hamiltonian [11]. The alternating barriers between the atoms allows for realizing the dimerized Hamiltonian of Eq. (1). For encoding information, classical or quantum, we need a unitary operation acting on site 1, which is achieved by shining a laser on qubit 1, as shown in Fig. 4(a). To have a local gate operation without affecting the neighboring qubits one may apply a weak magnetic field gradient [17] or use a tightly focused laser beam [18] to split the hyperfine levels of the target atom. Then a microwave pulse, tuned only for the target qubit, operates the gate locally, as has been realized in Refs. [17,18].

When encoding finishes, Bob should wait for a period of t^* and then decodes information. As discussed before, in classical communication decoding is a Bell measurement on sites $N-1$ and N while in quantum communication it is a single-site measurement on site $N-1$. Since the measurement itself takes time and in particular a Bell measurement may not be very fast, we may stop the dynamics at $t = t^*$ by raising the barriers quickly such that $J \to 0$. Notice that in a superlattice one can control the even (odd) couplings independently by tuning the intensity of the low- (high-) frequency trapping laser beam [13,14]. Freezing the dynamics allows for slow measurements to be accomplished. For a Bell measurement, we can measure the singlet fraction through the spin triplet blockade technique, which has been recently proposed [12] and subsequently realized [13,14]. In that method a spin-dependent offset is applied on both sites to deform the superlattice such that one atom hops to the other site to have both atoms together only if their internal state is a singlet [12] [Fig. 4(b)]. Subsequent fluorescent photography [16], which can be done without disturbing the internal states [19], reveals the position of the atoms and therefore determines the singlet or triplet state of the atoms; however, it does not discriminate the nonsinglet Bell states. To distinguish between nonsinglet Bell states Bob has to apply a local Pauli rotation on qubit $N-1$ to convert one of the nonsinglet Bell states to singlet and then a subsequent singlet or triplet measurement determines whether or not the new state is singlet. In the worst case with two local operations followed by singlet or triplet measurements, Bob accomplishes his Bell measurement on qubits $N-1$ and N. In contrast, in quantum communication in which a single-site measurement is expected on site $N-1$, we can use the technique of Ref. [17]. In that methodology state $|1\rangle$ is coupled to an excited state through an intense perpendicular laser beam whose radiation pressure pushes the atom out of the lattice. This leaves the site empty if its atom is in state $|1\rangle$ and full if the atom is in state $|0\rangle$, as shown schematically in Fig. 4(c). This can thereby be read by fluorescent imaging.

VII. INITIALIZATION

Interestingly, one does not need sophisticated cooling methods to create the initial state of our protocol. An ideal initialization is a series of singlets ($\delta = 1$) as long as the subsequent nonequilibrium dynamics happens at $\delta \neq 1$. This initial state has already been prepared [13]. Independently, by starting from a band insulator and adiabatically changing the lattice potential as proposed in Ref. [21], ground states with other δ can be realized as initial states. Thus the temperature only needs to be less than the band gap of the insulator ($\gg J$). A more direct strategy of cooling to the ground state requires the temperature of the system to be less than the energy gap [$\sim 4J(1+\delta)$]. For a typical value of $J = 360$ Hz [14], one finds $K_B T < 100$ nK. Current experiments are on the verge of reaching this range of temperatures [15,22].

VIII. CONCLUSION

We introduced a methodology for truly exploiting the inherent entanglement in the ground state of strongly correlated systems for both classical and quantum communications. In our proposed scheme a local rotation on a single qubit encodes information in the entangled ground state of the many-body system. In classical communication, this encoding enables conveying more than one bit of information, just as in dense coding, without any prior shared entanglement. We also showed that the same recipe can be used for quantum communication, which gives better quality in comparison to the usual attaching scenarios. Moreover, this proposal is especially timely in the context of optical lattice implementations where all the requirements of our proposal have been achieved in recent experiments [17,18].

ACKNOWLEDGMENTS

Discussion with A. Beige, K. Bongs, D. Jaksch, D. Lucas, F. Renzoni, and M. Lubasch are gratefully acknowledged. S.Y. thanks CSC for financial support. A.B. and S.B. acknowledge the Engineering and Physical Sciences Research Council. S.B. also thanks the Royal Society and the Wolfson Foundation.

[1] H. J. Briegel, *et al.* Nature Phys. **5**, 19 (2009); G. K. Brennen and A. Miyake, Phys. Rev. Lett. **101**, 010502 (2008).
[2] S. Bose, Contemp. Phys. **48**, 13 (2007).
[3] A. Bayat and S. Bose, Phys. Rev. A **81**, 012304 (2010).
[4] A. Osterloh *et al.*, Nature (London) **416**, 608 (2002); T. J. Osborne and M. A. Nielsen, Phys. Rev. A **66**, 032110 (2002); L. Amico *et al.*, Rev. Mod. Phys. **80**, 517 (2008).
[5] L. Campos Venuti, C. Degli Esposti Boschi, and M. Roncaglia, Phys. Rev. Lett. **96**, 247206 (2006); S. Oh, M. Friesen, and X. Hu, Phys. Rev. B **82**, 140403(R) (2010).
[6] C. H. Bennett *et al.*, Phys. Rev. Lett. **70**, 1895 (1993).
[7] C. H. Bennett *et al.*, Phys. Rev. Lett. **87**, 077902 (2001).
[8] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. **69**, 2881 (1992).
[9] W. S. Bakr *et al.*, Nature (London) **462**, 74 (2009); M. Greiner *et al.*, *ibid.* **415**, 39 (2002); W. S. Bakr *et al.*, Science **329**, 547 (2010).
[10] R. Jördens *et al.*, Nature (London) **455**, 204 (2008); U. Schneider *et al.*, Science **322**, 1520 (2008).
[11] L. M. Duan, E. Demler, and M. D. Lukin, Phys. Rev. Lett. **91**, 090402 (2003).
[12] A. M. Rey, V. Gritsev, I. Bloch, E. Demler, and M. D. Lukin, Phys. Rev. Lett. **99**, 140601 (2007).
[13] S. Trotzky *et al.*, Science **319**, 295 (2008).
[14] S. Trotzky, Y. A. Chen, U. Schnorrberger, P. Cheinet, and I. Bloch, Phys. Rev. Lett. **105**, 265303 (2010).
[15] P. Medley, D. M. Weld, H. Miyake, D. E. Pritchard, and W. Ketterle, Phys. Rev. Lett. **106**, 195301 (2011).
[16] J. F. Sherson *et al.*, Nature (London) **467**, 68 (2010); C. Weitenberg *et al.*, *ibid.* **471**, 319 (2011).
[17] M. Karski *et al.*, New J. Phys. **12**, 065027 (2010).
[18] C. Weitenberg, M. Endres, J. F. Sherson, M. Cheneau, P. Schauss, T. Fukuhara, I. Bloch, and S. Kuhr, Nature (London) **471**, 319 (2011).
[19] M. J. Gibbons, C. D. Hamley, C. Y. Shih, and M. S. Chapman, e-print arXiv:1012.1682.
[20] S. Hengl, J. Aberg, and R. Renner, e-print arXiv:0910.1745.
[21] M. Lubasch *et al.*, e-print arXiv:1106.1628.
[22] S. Trotzky *et al.*, Nature Phys. **6**, 998 (2010).