State protection under collective damping and diffusion

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In this paper we provide a recipe for state protection in a network of oscillators under collective damping and diffusion. Our strategy is to manipulate the network topology, i.e., the way the oscillators are coupled together, the strength of their couplings, and their natural frequencies, in order to create a relaxation-diffusion-free channel. This protected channel defines a decoherence-free subspace (DFS) for nonzero-temperature reservoirs. Our development also furnishes an alternative approach to build up DFSs that offers two advantages over the conventional method: it enables the derivation of all the network-protected states at once, and also reveals, through the network normal modes, the mechanism behind the emergence of these protected domains.

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I. INTRODUCTION

The effects of temperature on the dynamics of quantum states in dissipative networks is a less explored subject that deserves attention due to its prominent role in the process of decoherence of quantum states and in devising strategies to overcome this undesirable phenomenon. Most developments on this topic focus on the case where the network is coupled to reservoir(s) at 0 K, where diffusion is absent [1–4]. However, exploiting the case of finite-temperature reservoir(s), we have recently studied a chain of harmonic oscillators when either each one is coupled to its own reservoir or all of them are coupled to a common reservoir [5]. Based on the development presented in [5], here we demonstrate that both decoherence sources, the collective damping and diffusion, can be circumvented through the manipulation of the network topology, i.e.,the way the oscillators are coupled together, the strength of their couplings, and their natural frequencies. We show how to build a decoherence-free subspace (DFS) for a nonzero-temperature reservoir(s), thus extending DFS for a 0 K reservoir(s) [6] in order to include the diffusion mechanism arising from temperature effects. We provide a technique to build DFSs, adapted in a dissipative bosonic network, that is an alternative to those already presented in literature where temperature effects are taken into account [7,8]. We mention that in Ref. [7] a generalized theory of DFSs is developed, while in Ref. [8] the authors restrict their analysis to a system of two qubits in a common non-Markovian squeezed reservoir.

Despite the expected more restrictive character of the DFS for a nonzero-temperature reservoir(s), due to diffusion, we demonstrate that its protected states turns out to be essentially those belonging to the associated 0 K DFS, with the conditions for building the latter automatically accounting for the former. Therefore, our development also furnishes an alternative approach to build DFSs that offers two advantages over the conventional method [6]: it enables the derivation of all the network-protected states at once, and also reveals, through the network normal modes, the mechanism behind the emergence of such protected domains. More precisely, we construct a relaxation-diffusion-free channel composed of a particular set of noiseless network normal modes by transferring these decoherence mechanisms to the remaining normal modes. A main result of the present work, which contributes to the analysis in Ref. [7], is the elucidation of the mechanism leading to the DFSs for nonzero temperature(s), whose protected states are expanded by a set of normal-modes states that are effectively decoupled from the reservoir(s).

Displaying connections with other fault-tolerant techniques for information processing, such as quantum error correction [9], the quantum Zeno effect [10], and dynamical decoupling [11], the DFSs emerge from symmetries in the system-reservoir(s) coupling that shield a subset of the Hilbert space against fluctuations. A recent contribution [12] has deepened our understanding of the emergence of DFSs, showing that a correlation between the reservoir modes, induced by the system itself, is the mechanism that supports such protected domains. In the present analysis we advance further in the understanding of this mechanism by exploring the symmetries occurring in the matrix weighting the rules of dissipation and diffusion in the incoherent Lindblad dynamics. We finally stress that the experimental implementation of DFSs in linear optics [13], trapped ions [14], and nuclear magnetic resonance [15] indicates that the DFSs is a promising resource for information processing. Quantum computational operations within DFSs have been already accomplished in optics [16] and NMR [17]. The possibility of circumventing nonzero-temperature effects as well, as we demonstrate here, is certainly an additional useful mechanism enlarging perspectives on realistic protocols for information processing.

II. DENSITY OPERATOR AND WIGNER FUNCTION

We start from the Hamiltonian of a network where all N dissipative oscillators are coupled together in addition to interacting with a common reservoir. From this general scenario we derive any other topology, including the case where each oscillator is coupled to its own reservoir [5]. Assuming, from here on, that the subscripts m,n,m',n' run over all the oscillators, from 1 to N, the Hamiltonian reads

$$\mathcal{H} = \frac{\hbar}{2}\sum_m \Bigg[\omega_m a_m^\dagger a_m + \sum_{n(\neq m)} \lambda_{mn} a_m^\dagger a_n + \sum_k (\omega_k b_k^\dagger b_k + 2V_{mk} b_k^\dagger a_m) + \text{H.c.}\Bigg], \tag{1}$$

where $a_m^\dagger$ (a_m) is the creation (annihilation) operator for the mth network oscillator ω_m, which is coupled to the nth one with strengths λ_{mn}, in addition to interacting with the reservoir modes, whose creation (annihilation) operator reads $b_k^\dagger$ (b_k), with strengths V_{mk}. The reduced master equation deduced in Ref. [5] for the bosonic network modeled by Eq. (1) is given by

$$\frac{d}{dt}\rho(t) = \sum_{m,n}\{i[\rho(t), a_m^\dagger H_{mn} a_n] + \mathcal{L}_{mn}\rho(t)\}, \tag{2a}$$

$$\mathcal{L}_{mn}\circ = \frac{\Gamma_{mn} + \Upsilon_{mn}}{2}[a_n \circ, a_m^\dagger] + \frac{\Upsilon_{mn}}{2}[a_n^\dagger \circ, a_m] + \text{H.c.}, \tag{2b}$$

where $H_{mn} = \omega_m \delta_{mn} + \lambda_{mn}(1-\delta_{mn})$ stands for the elements of matrix $\mathbf{H}$ associated with the ideal network: the first two terms on the right-hand-side of Eq. (1). The dynamical damping and the diffusion mechanisms are weighted, respectively, by matrices

$$\mathbf{\Gamma}(t) = \int_0^t d\tau \int_0^\infty \frac{d\nu}{\pi} e^{-i\nu\tau}\gamma(\nu)e^{i\mathbf{H}\tau}, \tag{3a}$$

$$\mathbf{\Upsilon}(t) = \int_0^t d\tau \int_0^\infty \frac{d\nu}{\pi} \bar{n}(\nu) e^{-i\nu\tau}\gamma(\nu)e^{i\mathbf{H}\tau}, \tag{3b}$$

expressed in the continuum limit of the reservoir frequencies. We have also defined, through the correlation $\langle b(\nu)b^\dagger(\nu')\rangle = 1 + 2\pi\bar{n}(\nu)\delta(\nu-\nu')$, the average excitation of the reservoir $\bar{n}(\nu)$ in addition to the decay factors $\gamma_{mn}(\nu) = \sigma(\nu)V_m(\nu)V_n(\nu)$ which give the elements of matrix $\gamma(\nu)$, with $\sigma(\nu)$ being the density of states of the reservoir.

Instead of a common reservoir, we can automatically assign a particular reservoir to each network oscillator by considering diagonal decay factors $\gamma_{mn}(\nu) \to \gamma_{mm}(\nu)\delta_{mn}$ and average excitations $\bar{n}(\nu) \to \bar{n}_m(\nu)$, which imply that there are no indirect ($m \neq n$) decay factors. However, as we conclude from the integrals in Eqs. (3), even when the oscillators are coupled to their own reservoirs, we can recover the indirect decay and diffusion channels (nondiagonal elements Γ_{mn} and Υ_{mn}) when non-Markovian reservoirs are adopted together with strong interoscillator coupling strengths, i.e., $N\lambda_{mn} \approx \omega_m$ [2,5]. As will be demonstrated below, the indirect decay and diffusion channels play a central role in the emergence of DFSs for a nonzero-temperature reservoir(s).

For a network initially in the superposition of the product of coherent states $|\Psi(0)\rangle = \int dr\,\Lambda(r)|\boldsymbol{\beta}(r;0)\rangle$, where $|\boldsymbol{\beta}(r;0)\rangle = \bigotimes_m |\beta_m(r;0)\rangle$, the Wigner-function representation of Eq. (2) is written as

$$W_S(\{\xi_m\},t) = \int dr \int ds\; \Lambda(r)\Lambda^*(s)\mathcal{F}_{rs}(t)\langle\{\beta_m(s;t)\}. \times |\{\beta_m(r;t)\}\rangle \frac{(2/\pi)^N}{\det \mathbf{J}} \exp\{-2[\boldsymbol{\beta}^\dagger(s;t) - \xi^\dagger]\mathbf{J}^{-1} \times [\boldsymbol{\beta}(r;t) - \xi]\}, \tag{4}$$

where the decay function associated with the interference terms ($r \neq s$) is given by

$$\mathcal{F}_{rs}(t) = \frac{\langle\{\beta_m(s)\}|\{\beta_m(r)\}\rangle}{\langle\{\beta_m(s;t)\}|\{\beta_m(r;t)\}\rangle}. \tag{5}$$

Moreover, the evolution $\boldsymbol{\beta}(r;t) = \mathbf{U}(t)\boldsymbol{\beta}(r;0)$ of the excitations of the network oscillators follows from the nonunitary evolution $\mathbf{U}(t) = \exp(-i\mathbb{H}t)$ defined by the non-Hermitian matrix $\mathbb{H} = \mathbf{H} - i\mathbf{\Gamma}/2$. Note that the coherent states form a privileged basis for treating our dissipative bosonic network. The matrix

$$\mathbf{J}(t) = \mathbb{I} + \int_0^t \mathbf{U}(\tau)(\mathbf{\Upsilon} + \mathbf{\Upsilon}^\dagger)\mathbf{U}^\dagger(\tau)\,d\tau \tag{6}$$

in Eq. (4) accounts for the diffusion dynamics associated with temperature effects, with $\mathbb{I}$ being the identity.

III. DFS

In order to analyze the conditions leading to the emergence of a DFS for a nonzero-temperature reservoir, we observe that, in the ideal case where the reservoir does not interact with the oscillators ($\Gamma_{mn} = \Upsilon_{mn} = 0$), the decay of the interference terms and the diffusion process are absent, such that (i) $\mathcal{F}_{rs}(t) = 1$ and (ii) $\mathbf{J} = \mathbb{I}$. In what follows we verify the possibility of attaining both these conditions for the realistic (nonideal) case through the manipulation of the network topology. To this end, assuming $\mathbf{J} = \mathbb{I}$, we verify that condition (i) is satisfied for two distinct situations: The first is the well-known trivial case ($r = s$), where the initial state is a direct product of coherent states $|\Psi(0)\rangle = |\boldsymbol{\eta}\rangle$, ensuring a noise-free dissipative dynamics; we use $\boldsymbol{\eta}$ instead of $\boldsymbol{\beta}$ to label the initial protected states. The second is the nontrivial case ($r \neq s$) following from the eigenvalue equation

$$\mathbf{U}^\dagger(t)\mathbf{U}(t)\boldsymbol{\eta} = \boldsymbol{\eta} \tag{7}$$

($\mathbf{U}^\dagger(t)\mathbf{U}(t) \neq \mathbb{I}$), which prevents the system relaxation since it can be demonstrated that $\sum_m \langle a_m^\dagger a_m\rangle_t = \sum_m \langle a_m^\dagger a_m\rangle_{t=0}$, thus ensuring the more restrictive dissipative-noise-free dynamics of the initial state $|\boldsymbol{\eta}\rangle$. Next, we observe that condition (7) may be derived when imposing the restrictions

$$[\mathbf{H},\mathbf{\Gamma}] = 0, \quad \mathbf{\Gamma}\boldsymbol{\eta} = 0, \tag{8}$$

which constrain the vector $\boldsymbol{\eta}$ to undergo, as if in the absence of the reservoir, the unitary evolution

$$\boldsymbol{\beta}(r;t) = \mathbf{U}(t)\boldsymbol{\eta} = \exp(-i\mathbf{H}t)\boldsymbol{\eta}, \tag{9}$$

which entails, for an absolute-zero reservoir, the emergence of a DFS. Therefore, a protected subspace is spanned by the $\mathbf{\Gamma}$ eigenvectors $\boldsymbol{\eta}^{(\ell)}$ (with null eigenvalues), which follows from the indirect decay and diffusion channels. The remaining $\mathbf{\Gamma}$ eigenvectors (with non-null eigenvalues), let's say $\boldsymbol{\zeta}$, thus span the unprotected states. Assuming a set (labeled by ℓ) of L protected eigenvectors $\boldsymbol{\eta}$, except for the $N - L$ unprotected $\boldsymbol{\zeta}$

(labeled ℓ'), a general initial state reads

$$|\Psi(0)\rangle = \int dr\,\Lambda(r)\left|\boldsymbol{\beta}(r;0) = \sum_{\ell=1}^{L}\alpha_\ell(r)\boldsymbol{\eta}^{(\ell)}\sum_{\ell'=L+1}^{N}\alpha_{\ell'}\boldsymbol{\zeta}^{(\ell')}\right\rangle,$$

where the equality $\mathcal{F}_{rs}(t) = 1$ forbids $\alpha_{\ell'}$ from depending on r, which will be clear below when introducing the network normal-mode representation. In short, whereas the eigenvalue equation $\boldsymbol{\Gamma\eta} = 0$ constrains the initial state $\boldsymbol{\beta}(r;0)$ to be prepared with a portion of its components (labeled ℓ) within a DFS, the condition $[\mathbf{H},\boldsymbol{\Gamma}] = 0$ makes certain that these states will be always within such a DFS despite the evolution governed by $\mathbf{H}$.

A. DFS for a nonzero-temperature reservoir

Next, under the restrictions in Eq. (8) ensuring a DFS at 0 K, we analyze condition (ii), $\mathbf{J} = \mathbb{I}$, aiming to construct a DFS for a nonzero-temperature reservoir. First, we note from Eq. (6) that the equality $\mathbf{J} = \mathbb{I}$ is only established for a 0 K reservoir when $\bar{n}(\nu) = 0$; for the case at hand, a finite-temperature reservoir, we cannot prevent the diffusion of the *whole* network state since

$$\det\mathbf{J}(t) > \det\mathbf{J}(0) = 1. \tag{10}$$

To demonstrate this inequality, we observe that the diffusion and the damping matrices only differ from each other by the average excitation of the reservoir $\bar{n}(\nu)$, such that $[\boldsymbol{\Gamma},\boldsymbol{\Upsilon}] = 0$, enabling us to derive from Eq. (8) the additional condition $[\mathbf{H},\boldsymbol{\Upsilon}] = 0$. Using all these commutations relations, it is straightforward to verify that Eq. (6) simplifies to the expression

$$\mathbf{J}(t) = \mathbb{I} + 2\bar{n}[\mathbb{I} - \exp(-\boldsymbol{\Gamma}t)], \tag{11}$$

showing that the eigenvalues of $\mathbf{J}(t)$ are always larger than or equal to unity and, since we must have at least one nonzero eigenvalue of $\boldsymbol{\Gamma}$, the determinant of $\mathbf{J}(t)$, i.e., the product of its eigenvalues, must satisfy Eq. (10). Moreover, Eq. (11) also shows that the eigenstates of $\boldsymbol{\Gamma}$ with null eigenvalues, those undergoing the dissipative-noise-free dynamics, are also eigenstates of $\mathbf{J}$ with unity eigenvalues. We conclude that the same conditions given by Eq. (8) for the emergence of a DFS automatically apply for the case of a nonzero-temperature reservoir, i.e., a diffusion-relaxation-free channel. This conclusion follows always when the approximations leading to the master equation approach holds for either a Markovian or a non-Markovian reservoir, whose damping and diffusion matrices satisfy Eq. (3).

IV. THE MECHANISM BEHIND THE EMERGENCE OF A DFS

After defining the conditions for building a DFS for a nonzero-temperature reservoir, we next discuss the physical basis underlying the protected subspace. We start by looking closely at the arbitrary initial state $|\Psi(0)\rangle$, under the assumption made above that it encloses a set of L protected eigenvectors. The elements of the vector $\boldsymbol{\beta}(r;0) = \mathbf{T}\boldsymbol{\alpha}(r)$, associated with the excitations of the network oscillators, are thus given by

$$\beta_m(r;0) = \sum_{\ell=1}^{L} T_{m\ell}\,\alpha_\ell(r) + \sum_{\ell'=L+1}^{N} T_{m\ell'}\,\alpha_{\ell'},$$

where the transformation matrix $\mathbf{T}$, whose columns are composed by the $\boldsymbol{\eta}(r;0)$ and $\boldsymbol{\zeta}(0)$ eigenvectors of $\boldsymbol{\Gamma}$, simultaneously diagonalizes $\mathbf{H}$, $\boldsymbol{\Gamma}$, and $\mathbf{J}$. By mapping the protected state $|\Psi(0)\rangle$ into the normal-mode oscillators we obtain $|\tilde{\Psi}(0)\rangle = \int dr\,\Lambda(r)|\tilde{\boldsymbol{\beta}}(r;0)\rangle$, where the elements of the transformed vector $\tilde{\boldsymbol{\beta}}(r;0) = \mathbf{T}^\dagger\boldsymbol{\beta}(r;0) = \mathbf{T}^\dagger\mathbf{T}\boldsymbol{\alpha}(r) = \boldsymbol{\alpha}(r)$ are $\tilde{\beta}_m(r;0) = \alpha_m(r)\delta_{m\ell} + \alpha_m\delta_{m\ell'}$. Therefore, the initial state spanned by the normal mode becomes $|\tilde{\Psi}(0)\rangle = \int dr\,\Lambda(r)|\{\alpha_\ell(r)\}\rangle \otimes |\{\alpha_{\ell'}\}\rangle$, which factorizes the protected subspace from the noisy one. Under the conditions leading to the DFS for a nonzero-temperature reservoir,enclosing this factorized state, we verify that the evolution of the Wigner function (4), rewritten through the normal-mode coordinates $\tilde{\boldsymbol{\xi}}(t) = \mathbf{T}^\dagger\boldsymbol{\xi}(t)$, remains factorized into both the protected and the noise channels: $W_S(\{\tilde{\xi}_m\},t) = W_{\text{DFS}}(\{\tilde{\xi}_\ell\},t)W_{\text{noise}}(\{\tilde{\xi}_{\ell'}\},t)$, with

$$W_{\text{DFS}}(\{\tilde{\xi}_\ell\},t) = (2/\pi)^L \int dr \int ds\ \Lambda(r)\Lambda^*(s)\langle\{\tilde{\beta}_\ell(s;t)\}|\{\tilde{\beta}_\ell(r;t)\}\rangle,$$

accounting for the relaxation-diffusion-free channel, associated with the evolved pure state $\int dr\,\Lambda(r)|\{\tilde{\beta}_\ell(r;t)\}\rangle$, while the remaining noisy normal modes are described by the mixture

$$W_{\text{noise}}(\{\xi_{\ell'}\},t) = \prod_{\ell'}\frac{2}{\pi D_{\ell'}(t)}\exp\left(-\frac{2}{D_{\ell'}}|\tilde{\xi}_{\ell'} - \alpha_{\ell'}|^2\right).$$

The transformation to the normal-mode coordinates thus reveals that the protected states are confined to a set of normal-mode oscillators completely decoupled from the reservoir. All the noisy effects are transferred to the unprotected normal modes. To illustrate this transfer mechanism we sketch in Fig. 1(a) a network of N oscillators (of frequencies ω_m), one coupled to the other apart from interacting with a common reservoir R. The network normal-mode scheme is given in Fig. 1(b), where the noninteracting modes (of frequencies ϖ_m) are only coupled to the reservoir. A DFS for a nonzero-temperature reservoir is represented in Fig. 1(c), where a set of protected normal-mode oscillators (from ϖ_{L+1} to ϖ_N, for example) is decoupled from the reservoir. Since the diffusion necessarily increases with time, as demonstrated above, the noiseless DFS for a nonzero-temperature reservoiris built on at the expense of enhancing the relaxation and diffusion rates of the remaining ϖ_1 and ϖ_L noisy normal modes.

We finally note that in the case where $\alpha_{\ell'}$ (the excitations of the noisy normal modes) equals the average excitation $\bar{n}$ of each reservoir mode, there is no energy drained from the system. However, the injection of noise into the network cannot be stopped since the inequality $\text{Tr}\,\boldsymbol{\Gamma} > 0$ imposes that there must be at least a single unprotected normal mode $(N - L > 1)$ even for a 0 K reservoir. In the case of a nonzero-temperature reservoir the incoherent dynamics of the noisy normal modes is even enhanced by the diffusion process according to Eq. (10). Therefore, the set of noisy normal modes

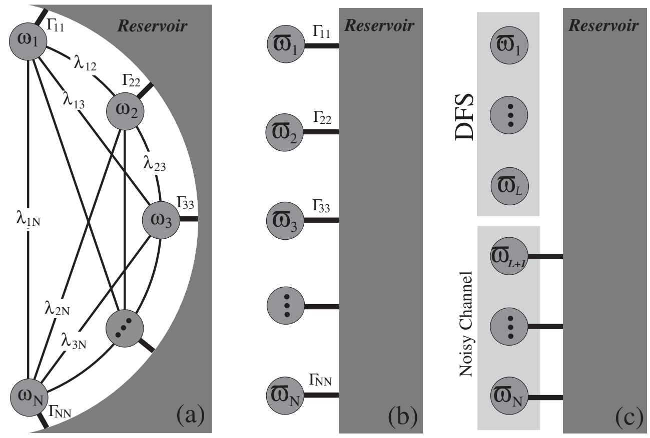

FIG. 1. (a) Sketch of a dissipative network of N oscillators, each interacting with each other separately from being coupled to a common reservoir. (b) As an alternative picture, the noninteracting normal modes (of frequencies ϖ_m) are only coupled to the reservoir. (c) Finally, the DFS is represented by a set of protected normal-mode oscillators (from ϖ_{L+1} to ϖ_N, for example) decoupled from the reservoir.

becomes an absolutely necessary ingredient for the emergence of DFSs.

V. TRANSFERENCE OF NOISE TO THE UNPROTECTED NORMAL MODES

We next pursue, under the conditions underlying the DFS for a nonzero-temperature reservoir, a better understanding of the transference of the noise mechanism to the unprotected normal modes. We first observe that the net energy drained by the reservoir and the amount of noise introduced into the network (from relaxation and diffusion) are obviously independent of the schemes in Figs. 1(a) and 1(c). Consequently, the emergence of a set of protected normal modes in Fig. 1(c), with null relaxation and diffusion rates, must be balanced by the increase of the relaxation and diffusion rates related to the remaining unprotected modes. A crude analysis of the dissipative process of both states $|\Psi(0)\rangle$ and $|\tilde{\Psi}(0)\rangle$ in the schemes in Figs. 1(a) and 1(c) provides a qualitative means to visualize this balancing mechanism. Regarding the state $|\Psi(0)\rangle$, there is an amount of excitation in each natural oscillator, given by $\sum_{\ell'=L+1}^{N} T_{m\ell'}\,\alpha_{\ell'}$, which is out of the DFS. These excitations are drained out of the oscillators by their couplings to the reservoir, as sketched in Fig. 1(a). From the perspective of the normal-mode representation, however, where the state is described by $|\tilde{\Psi}(0)\rangle$, the whole amount of excitation out of the DFS is entirely confined to $N-L$ noisy normal modes. Consequently, in order to attain, with this reduced number of decay channels, the same amount of energy drained from the natural oscillators, the decay rates associated with the noisy normal modes must, on average, be larger than their corresponding values when the conditions in Eq. (8) are not fulfilled. The same analysis applies to the diffusion process.

Quantitatively, the unitary transformation leading from Figs. 1(a) to 1(c) ensures the invariance of $\mathrm{Tr}\,\mathbf{\Gamma}$ and $\mathrm{Tr}\,\mathbf{J}$; the elements whose sums lead to these invariant traces come from the direct coupling of the natural [for Fig. 1(a)] and the normal-modes oscillators [for Fig. 1(c)] with the reservoir, exactly those couplings sketched in Fig. 1. For the particular case where we assign the same decay rate Γ to all the natural oscillators, we obtain $\mathrm{Tr}\,\mathbf{\Gamma} = N\Gamma$ and $\mathrm{Tr}\,\mathbf{J} = N + 2\bar{n}(1-e^{-N\gamma t})$, and assuming the maximum enabled number $L = N-1$ of protected normal modes, we verify that the single remaining unprotected mode presents the highly enhanced relaxation and diffusion strengths given by $\mathrm{Tr}\,\Gamma$ and $\mathrm{Tr}\,\mathbf{J}$, respectively.

VI. ILLUSTRATIVE EXAMPLES AND CONCLUSION

To illustrate our protocol for the construction of a DFS for a nonzero-temperature reservoir, we consider a degenerate network ($\omega_m = \omega$), where all N oscillators interact with each other ($\lambda_{mn} = \lambda$) in addition to being coupled to a common reservoir. We also assume a Markovian white-noise reservoir whose spectral density is invariant over translation in frequency space, such that $\gamma_{mn}(\nu) = \gamma_{mn}$. Under these conditions, the elements of both matrices in Eq. (3) become $\Gamma_{mn} = \Gamma$ and $\Upsilon_{mn} = \bar{n}\Gamma$ [5], thus resulting in the eigenvalues $N\Gamma\delta_{mN}$ and $1 + 2\bar{n}(1-e^{-N\Gamma t})\delta_{mN}$ of matrices $\mathbf{\Gamma}$ and $\mathbf{J}$, respectively. Therefore, the specific network parameters considered above turn out to be particularly convenient since only one of the normal-mode oscillators is under relaxation and diffusion, thus giving the protected state $|\tilde{\Psi}(0)\rangle = \int dr\,\Lambda(r)|\boldsymbol{\alpha}(r)\rangle = \int dr\,\Lambda(r)|\{\alpha_\ell(r)\}\rangle \otimes |\alpha_N\rangle$, where the components of the vector $\boldsymbol{\alpha}(r) = (\alpha_1(r),\alpha_2(r),\ldots,\alpha_{N-1}(r),\alpha_N)^\top$ are the excitations of the normal-mode oscillators. Alternatively, within the natural oscillators we have $|\Psi(0)\rangle = \int dr\,\Lambda(r)|\boldsymbol{\beta}(r;0)\rangle$, with $\boldsymbol{\beta}(r) = \mathbf{T}\boldsymbol{\alpha}(r)$ and the elements of the transformation matrix satisfying the relations $T_{mN} = 1/\sqrt{N}$, $T_{mm} = -\sqrt{(N-m)/(N-m+1)}$ for $m < N$, $T_{mn} = 0$ for $m < n < N$, and $T_{mn} = 1/\sqrt{(N-m)(N-m+1)}$ for $n < m < N$. We observe that other DFSs, following from topologies with a small number of protected normal modes, necessarily fall into a subclass of the above-obtained larger DFS.

As another example we consider the same degenerate network defined above ($\omega_m = \omega$), where all N oscillators interact with each other ($\lambda_{mn} = \lambda$). However, instead of being coupled to a common reservoir, we now assume that each oscillator is coupled to its own Markovian white-noise reservoir. In this case we have only diagonal elements in both matrices in Eq. (3), i.e., $\Gamma_{mn} = \Gamma\delta_{mn}$ and $\Upsilon_{mn} = \bar{n}\Gamma\delta_{mn}$. Since none of these diagonal elements is zero, we have no DFSs in this case. However, by assuming structured [1,20] instead of white-noise reservoirs, we may generate DFSs by manipulating the network topology, the natural frequencies $\{\omega_m\}$ and coupling strengths $\{\lambda_{mn}\}$, so as to shift the network normal modes to regions where the spectral density of the reservoir is negligible. In this way we minimize the effects of the reservoirs on the coherent evolution of the network states, and the greater the number of normal modes displaced to these regions is, the greater the dimension of the DFS generated is.

We have thus presented a protocol to construct a relaxation-diffusion-free channel, a DFS where finite-temperature reservoirs are considered, by manipulating the network topology, particularly through the parameters giving rise to indirect decay and diffusion rates. We explored the issue of protected states, deepening our understanding of the mechanisms underlying the emergence of protected subspaces by focusing on the relaxation and diffusion rates associated with the network normal-mode scheme. We believe that our approach, extending the DFSs to encompass diffusion effects, contributes to the efforts toward realistic protocols for information processing. Finally, we observe that recent networks, such as circuit QED [18] and arrays of microcavities coupled by optical fibers [19], have attracted considerable attention as candidates for information processing. The efficiency in handling the strength of the parameters involved in these promising networks is an advantageous characteristic that has motivated our efforts.

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[1] M. A. de Ponte, M. C. de Oliveria, and M. H. Y. Moussa, Phys. Rev. A **70**, 022324 (2004); **70**, 022325 (2004).

[2] M. A. de Ponte, S. S. Mizrahi, and M. H. Y. Moussa, Phys. Rev. A **76**, 032101 (2007).

[3] Z. Ficek and R. Tanas, Phys. Rep. **372**, 369 (2002).

[4] G. Burkard and F. Brito, Phys. Rev. B **72**, 054528 (2005).

[5] M. de Ponte *et al.*, J. Phys. A **42**, 365304 (2009).

[6] G. M. Palma *et al.*, Proc. R. Soc. London, Ser. A **452**, 567 (1996); P. Zanardi and M. Rasetti, Phys. Rev. Lett. **79**, 3306 (1997); D. A. Lidar, I. L. Chuang, and K. B. Whaley, *ibid.* **81**, 2594 (1998); E. Knill,, R. Laflamme, and L. Viola, *ibid.* **84**, 2525 (2000); A. Beige,, D. Braun, B. Tregenna, and P. L. Knight, *ibid.* **85**, 1762 (2000).

[7] A. Shabani and D. A. Lidar, Phys. Rev. A **72**, 042303 (2005).

[8] Md. Manirul Ali, P.-W. Chen, and H.-S. Goan, Phys. Rev. A **82**, 022103 (2010).

[9] P. W. Shor, Phys. Rev. A **52**, R2493 (1995); J. Chiaverini *et al.*, Nature (London) **432**, 602 (2004).

[10] A. G. Kofman and G. Kurizki, Nature (London) **405**, 546 (2000); P. Facchi, H. Nakazato, and S. Pascazio, Phys. Rev. Lett. **86**, 2699 (2001); M. C. Fischer, B. Gutierrez-Medina, and M. G. Raizen, *ibid.* **87**, 040402 (2001).

[11] L. Viola, E. Knill, and S. Lloyd, Phys. Rev. Lett. **82**, 2417 (1999).

[12] M. A. de Ponte, S. S. Mizrahi, and M. H. Y. Moussa, Ann. Phys. (NY) **322**, 2077 (2007).

[13] P. G. Kwiat *et al.*, Science **290**, 498 (2000); M. Bourennane, M. Eibl, S. Gaertner, C. Kurtsiefer, A. Cabello, and H. Weinfurter, Phys. Rev. Lett. **92**, 107901 (2004).

[14] C. F. Roos, G. P. T. Lancaster, M. Riebe, H. Haffner, W. Hansel, S. Gulde, C. Becher, J. Eschner, F. Schmidt-Kaler, and R. Blatt, Phys. Rev. Lett. **92**, 220402 (2004); H. Häffner *et al.*, Appl. Phys. B **81**, 151 (2005); D. Kielpinsky *et al.*, Science **291**, 1013 (2001).

[15] L. Viola *et al.*, Science **293**, 2059 (2001).

[16] M. Mohseni, J. S. Lundeen, K. J. Resch, and A. M. Steinberg, Phys. Rev. Lett. **91**, 187903 (2003); T. Monz *et al.*, *ibid.* **103**, 200503 (2009).

[17] J. E. Ollerenshaw, D. A. Lidar, and L. E. Kay, Phys. Rev. Lett. **91**, 217904 (2003).

[18] J. Clarke and F. K. Wilhelm, Nature (London) **453**, 1031 (2008); Y. Makhlin *et al.*, Rev. Mod. Phys. **73**, 357 (2001); M. A. Sillanpää *et al.*, Nature (London) **449**, 438 (2007); J. Majer *et al.*, *ibid.* **449**, 443 (2007); M. Neeley *et al.*, Nat. Phys. **4**, 523 (2008); M. Hofheinz *et al.*, Nature (London) **459**, 546 (2009); L. Tian and R. W. Simmonds, Phys. Rev. Lett. **99**, 137002 (2007).

[19] K. J. Vahala, Nature (London) **424**, 839 (2003); G. Khitrova *et al.*, Nat. Phys. **2**, 81 (2006).

[20] G. S. Agarwal, M. O. Scully, and H. Walther, Phys. Rev. Lett. **86**, 4271 (2001); G. S. Agarwal and S. Das, New J. Phys. **10**, 013014 (2008); M. A. de Ponte, M. C. de Oliveira, and M. H. Y. Moussa, Ann. Phys. (NY) **317**, 72 (2005).