

Strongly interacting isotopic Bose-Fermi mixture immersed in a Fermi sea

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(Received 23 March 2011; revised manuscript received 29 April 2011; published 13 July 2011)

We have created a triply quantum-degenerate mixture of bosonic ^{41}K and two fermionic species ^{40}K and ^6Li . The boson is shown to be an efficient coolant for the two fermions, spurring hopes for the observation of fermionic superfluids with imbalanced masses. We observe multiple heteronuclear Feshbach resonances, in particular a wide s -wave resonance for the combination ^{41}K - ^{40}K , opening up studies of strongly interacting *isotopic* Bose-Fermi mixtures. For large imbalance in the local densities of different species, we enter the polaronic regime of dressed impurities immersed in a bosonic or fermionic bath.

DOI: [10.1103/PhysRevA.84.011601](https://doi.org/10.1103/PhysRevA.84.011601)

PACS number(s): 03.75.Ss, 34.50.–s, 37.10.De, 67.60.Fp

Strongly interacting quantum mixtures of ultracold atoms provide an extremely rich platform for the study of many-body physics. They offer control over macroscopic quantum phenomena in and out of equilibrium, enabling a direct quantitative comparison to theoretical models [1]. Two-state mixtures of fermionic atoms near Feshbach resonances allow the creation of fermionic superfluids in the crossover between Bose-Einstein condensation and BCS superfluidity [2,3]. Combining different atomic species gives access to Bose-Bose [4,5], Bose-Fermi [6–12], Fermi-Fermi [13–15], and even triply degenerate Bose-Fermi-Fermi mixtures [13] that each connect to many different areas in condensed-matter, high-energy, or nuclear physics. Bose-Fermi mixtures may provide insight into, for example, boson-mediated Cooper pairing [16], QCD matter [17], and theoretical models of High- T_c superconductivity [18]. A mixture of two different fermions might allow access to a superfluid of unlike fermions. In contrast to superconductors or neutron stars, superfluid pairing will occur between particles that are not related via time-reversal symmetry. Very recently, Fermi-Fermi mixtures of unlike fermionic species have been brought into the strongly interacting regime [19], offering prospects to observe universal physics in imbalanced mixtures, such as universal transport [20,21].

An important class of many-body problems involves the interaction of impurities with a Fermi sea or a bosonic bath, dressing them into quasiparticles known as polarons. For the Fermi polaron, an impurity interacting with a fermionic environment, the resulting energy shift has been experimentally measured [22] and calculated [23–25]. Due to the fermionic nature of the environment, the effective mass is only weakly enhanced [25–27] even for resonant interactions. However, if the impurity swims in a bosonic bath, there is no limit to the number of bosons that interact at close distance with the impurity, and the mass enhancement can be enormous [28].

In this work we present a rather ideal system to study strongly interacting quantum mixtures of different atomic species: a heavy, isotopic Bose-Fermi mixture of ^{40}K - ^{41}K with widely tunable interactions coexisting with a light Fermi sea of ^6Li . We show that ^{41}K is an efficient sympathetic coolant for both ^6Li and ^{40}K , allowing us to reach a triply quantum degenerate mixture. In comparison to experiments employing ^{87}Rb to cool the same fermionic species to triple degeneracy [13], we reach a significantly higher degree of degeneracy in ^6Li . In the quest for optimized cooling schemes

of fermionic atoms, we thus establish ^{41}K as a superior coolant that is available in all current experiments on fermionic ^{40}K . For the potassium isotopes, we identify a strong p -wave Feshbach resonance, as well as a wide s -wave Feshbach resonance. There, at our lowest temperatures, the mixture is in a regime where theory predicts both Bose and Fermi polarons to exist [29,30]. The mass-imbalanced Bose-Fermi mixture ^6Li - ^{41}K also allows for tunable interactions at several Feshbach resonances.

Predating our work, Feshbach resonances in nonisotopic Bose-Fermi mixtures were found in ^{23}Na - ^6Li , ^{87}Rb - ^{40}K , ^{87}Rb - ^6Li [12], and ^{85}Rb - ^6Li [31]. These systems are plagued by typically unequal trapping potentials and the large mass difference between unlike atoms, causing gravitational sag that has to be compensated. Predictions for Feshbach resonances in isotopic Bose-Fermi mixtures are available for ^3He - ^4He [32] and for ^6Li - ^7Li [33], with preliminary experimental findings reported in [34]. An atom-molecule mixture of ^6Li - $^6\text{Li}_2$ allowed access to a part of the phase diagram of strongly interacting bosons and fermions [35]. However, for too strong an interaction the composite nature of the bosonic molecules becomes apparent. With ^{40}K - ^{41}K , we have a Bose-Fermi mixture at our disposal with identical external potentials and essentially equal mass for bosons and fermions, so that the only relevant difference lies in quantum statistics.

The experimental setup, shown in Fig. 1, consists of two independent Zeeman slowers for lithium and potassium, allowing us to simultaneously load large samples of each of the three atomic species directly into a UHV chamber. We trap 3×10^9 ^{41}K atoms in 2 s and 10^9 ^6Li atoms in 1 s. Although the natural abundance of ^{40}K is only 0.01%, the Zeeman slower with a typical flux of 10^{11} atoms/s for abundant species still yields 5×10^7 ^{40}K atoms loaded within 2 s into the magneto-optical trap.

To increase the initial atom density, a 40-ms compressed MOT phase and a 6-ms optical molasses stage compress and cool each gas before loading into the magnetic trap. For ^{41}K , we follow closely the procedure laid out in [36]. ^{40}K and ^6Li require less care, as we deliberately co-trap only a few 10^5 fermionic atoms with the coolant. The maximum number of fermions that can be brought into degeneracy by a given bosonic coolant is roughly given by the number of degenerate bosons the apparatus can provide. For ^{41}K , this limits the

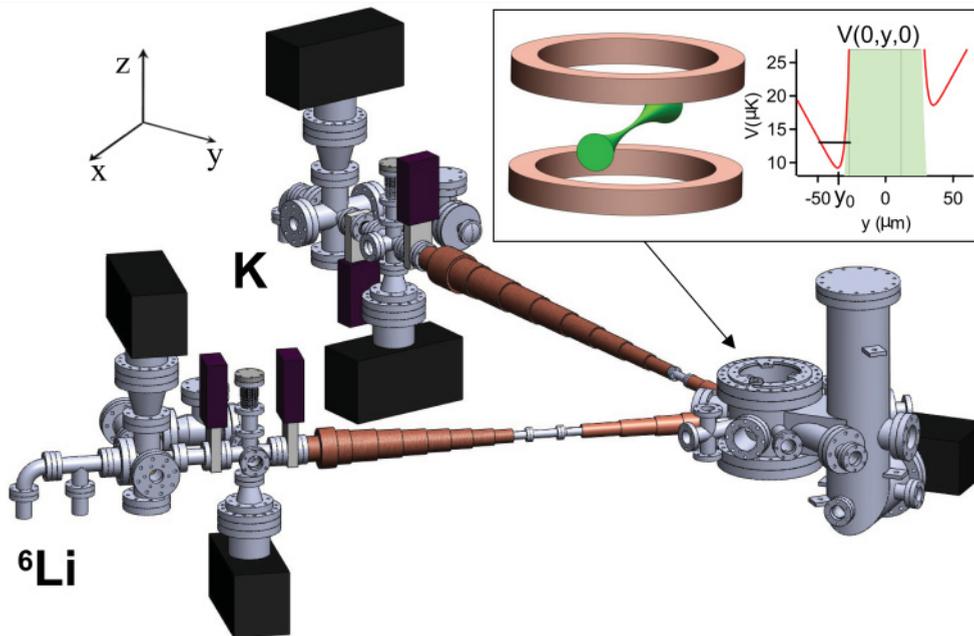


FIG. 1. (Color online) Schematic of the experimental setup. Two Zeeman slowers yield optimized atom flux for ${}^6\text{Li}$ and K, allowing a no-compromise approach to simultaneous magneto-optical trapping of ${}^{41}\text{K}$, ${}^{40}\text{K}$, and ${}^6\text{Li}$ in the main chamber. All species are subsequently loaded into an optically plugged magnetic trap (inset). rf evaporation of ${}^{41}\text{K}$ sympathetically cools the fermionic species. The inset shows the trapping potential, essentially identical for all species, along the horizontal y axis perpendicular to the plug beam.

fermion number to about 2×10^5 , while for ${}^{23}\text{Na}$, the number can be as large as 7×10^7 [37].

After the molasses stage, atoms are prepared in the stretched hyperfine states of $|F, m_F\rangle = |2, 2\rangle$ for ${}^{41}\text{K}$, $|9/2, 9/2\rangle$ for ${}^{40}\text{K}$, and $|3/2, 3/2\rangle$ for ${}^6\text{Li}$ via optical pumping. Evaporative cooling of ${}^{41}\text{K}$ is performed in a quadrupole magnetic trap with a $B'_z = 220$ G/cm ($B'_\perp = 110$ G/cm) magnetic field gradient along the vertical (horizontal) direction. To avoid Majorana spin flips, the magnetic field zero is “plugged” by a repulsive laser beam (power 15 W, wavelength 532 nm) focused to a waist of $20 \mu\text{m}$ [38]. Unwanted hyperfine states from imperfect optical pumping are removed by reducing B'_z for 200 ms to 15 G/cm, only supporting stretched states sufficiently against gravity. Without this cleaning procedure, spin-changing collisions would strongly reduce the atom number during evaporation. Evaporation is performed on ${}^{41}\text{K}$ by driving $|2, 2\rangle \rightarrow |1, 1\rangle$ rf transitions above the hyperfine transition of 254.0 MHz. For the last 2 s of evaporation, the trap is decompressed to $B'_z = 110$ G/cm to suppress three-body losses. A well-centered plugged trap allows for two trap minima on each side of the plug laser (see Fig. 1). To obtain only a single trap minimum, in the final 2 s of evaporation a horizontal bias field is applied in the y direction, perpendicular to the plug beam, thus displacing the center of the magnetic trap by $10 \mu\text{m}$. The resulting trapping potential, shown in the inset of Fig. 1, is approximately harmonic for atoms at energies of $\lesssim 2 \mu\text{K}$. The effect of anharmonicities is strongest along the y direction, and most important for the light fermion ${}^6\text{Li}$ at a typical Fermi energy of $E_F = k_B \cdot 5 \mu\text{K}$ (${}^{40}\text{K}$ only has $E_F \approx k_B \cdot 1.5 \mu\text{K}$).

Even for anharmonic traps, long time-of-flight expansion reveals the momentum distribution of the gas [2]. Time-of-flight images of triply quantum degenerate mixtures are shown

in Fig. 2. Condensation of ${}^{41}\text{K}$ is observed at $T_c = 1.2 \mu\text{K}$ with 3×10^5 atoms. In the harmonic approximation, this translates into a geometric mean of the trapping frequencies of $\bar{\omega}_{41\text{K}} = 2\pi \cdot 380$ Hz. Observing a ${}^{41}\text{K}$ Bose condensate in thermal contact with a cloud of ${}^{40}\text{K}$ and ${}^6\text{Li}$ fermions, each of

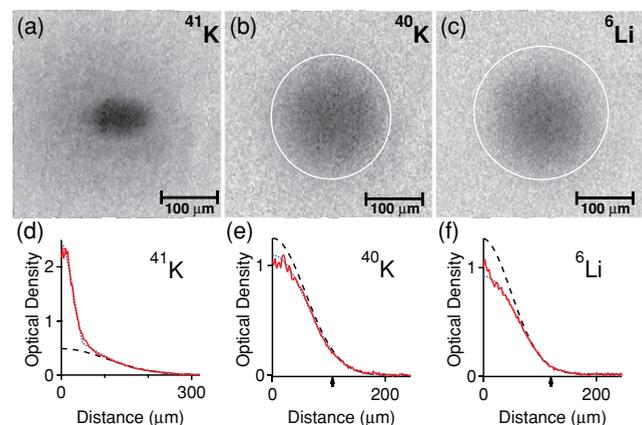


FIG. 2. (Color online) (a)–(c) Absorption images of triply degenerate quantum gases of ${}^{41}\text{K}$, ${}^{40}\text{K}$, and ${}^6\text{Li}$, imaged after 8.12 ms, 4.06 ms, and 1 ms time-of-flight from the magnetic trap, respectively. The final rf-knife frequency was 500 kHz above the 254.0 MHz hyperfine transition of ${}^{41}\text{K}$. The white circles indicate the Fermi radius in time-of-flight t , $R_F = \sqrt{2E_F/m} t$. (d)–(f) Azimuthally averaged column density. Solid dots: Gaussian fit to the wings of the column density. Solid black and blue lines are Gaussian and Fermi-Dirac fits to the entire profile. The deviation of the Gaussian fit from the data is more pronounced for the more deeply degenerate ${}^6\text{Li}$ at $T/T_F = 0.16$ than for ${}^{40}\text{K}$ at $T/T_F = 0.51$. The arrows indicate the Fermi radii. The atom numbers for ${}^6\text{Li}$, ${}^{41}\text{K}$, and ${}^{40}\text{K}$ are 1.6×10^5 , 1.1×10^5 , and 2.0×10^5 , respectively.

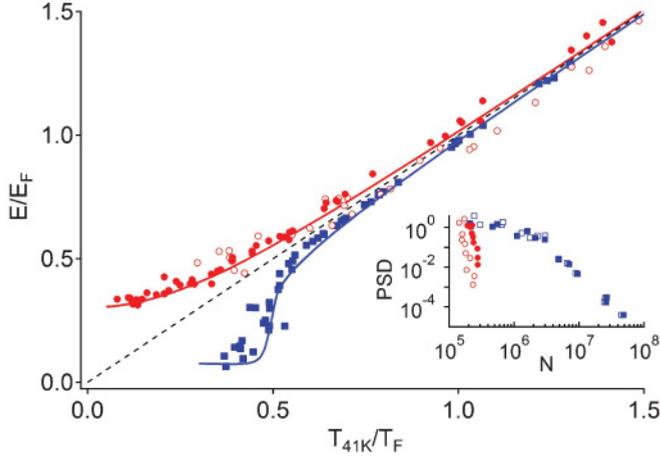


FIG. 3. (Color online) Observation of Pauli pressure and Bose condensation in a triply quantum degenerate mixture. Shown is the normalized release energy E/E_F of each cloud vs the normalized temperature T/T_F . Bose condensation of ^{41}K occurs at $T_c/T_F = 0.52$, causing a sudden reduction in release energy below T_c . For fermions, in contrast, the release energy saturates due to Pauli pressure. Solid circles: ^6Li ; open circles: ^{40}K ; solid squares: ^{41}K . Solid lines: theory for an interacting Bose gas and a noninteracting Fermi gas. Dashed line: Boltzmann gas. The inset shows the evolution of the phase space density (PSD) with atom number (N) during evaporation of ^{41}K . Open squares: Evaporation of ^{41}K without ^6Li and ^{40}K .

roughly the same atom number, already implies degeneracy of the fermionic species. If $T = T_{c,^{41}\text{K}}$, then $T/T_{F,^{40}\text{K}} = \frac{\bar{\omega}_{41\text{K}}}{\bar{\omega}_{40\text{K}}} \frac{1}{(6\zeta(3))^{1/3}} \approx 0.51$ and analogously, $T/T_{F,^6\text{Li}} = 0.2$. Taking into account anharmonicities along the y direction for 10^5 ^6Li atoms gives a small correction to the Fermi energy of -3.5% . Consistent with this expectation, Thomas-Fermi fits to the time-of-flight distributions in Fig. 2 reveal $T/T_{F,^6\text{Li}} = 0.16$ ($N_{^6\text{Li}} = 2.0 \times 10^5$) and $T/T_{F,^{40}\text{K}} = 0.51$ ($N_{^{40}\text{K}} = 1.1 \times 10^4$), while $T/T_{c,^{41}\text{K}} = 0.9$. Evaporating further to obtain essentially pure condensates, we achieve $T/T_{F,^6\text{Li}} = 0.08$ for ^6Li and $T/T_{F,^{40}\text{K}} = 0.35$ for ^{40}K . For ^6Li , the degree of degeneracy is about four times higher than what has been achieved in [13] with ^{87}Rb as the coolant. For ^{40}K , the performance is similar.

We directly observe Pauli pressure and Bose condensation in the triply quantum degenerate mixture. For this, we determine the $1/e$ width R of a Gaussian fitted to the fermionic and bosonic distributions, and compare the release energy $E \equiv \frac{1}{2} m R^2 / t^2$ measured after time-of-flight t to the Fermi energy, defined for each species as $E_F = k_B T_F = \hbar \bar{\omega} (6N)^{1/3}$. In Fig. 3 we show E/E_F as a function of the reduced temperature T/T_F . Thermometry is provided by fitting Bose functions to the wings of the ^{41}K distribution. At high temperatures, E/k_B simply equals the temperature of each gas. At low temperatures, the release energy of a trapped Fermi gas saturates due to Pauli pressure [6,7], while for a Bose cloud E is suddenly reduced as a condensate forms [38].

The inset in Fig. 3 shows the phase space density (PSD) of each atom cloud vs atom number N during sympathetic cooling. The efficiency of evaporation is measured by $\Gamma \equiv -d \ln(\text{PSD})/d \ln(N)$. Thanks to the small fermion number, the evaporation efficiency for ^{41}K is similar with and without load, $\Gamma \approx 3$ [36]. The near-vertical slope of PSD vs N for the

TABLE I. Observed interspecies Feshbach resonances between ^6Li - ^{41}K and ^{40}K - ^{41}K atoms. The width of the resonance, ΔB_{exp} , is determined by a phenomenological Gaussian fit to the observed loss feature (see, e.g., Fig. 4). For the p -wave resonance, the width was measured at $T = 8 \mu\text{K}$.

Mixture	B_0 (G)	ΔB_{exp} (G)	Resonance Type
$^6\text{Li} 1/2, 1/2 \rangle ^{41}\text{K} 1, 1 \rangle$	31.9	0.2	s wave [39]
$^6\text{Li} 1/2, 1/2 \rangle ^{41}\text{K} 1, 1 \rangle$	335.8	1.1	s wave [39]
$^{40}\text{K} 9/2, 9/2 \rangle ^{41}\text{K} 1, 1 \rangle$	472.6	0.2	s wave [40]
$^{40}\text{K} 9/2, 9/2 \rangle ^{41}\text{K} 1, 1 \rangle$	432.9	2.5	p wave [40]
$^{40}\text{K} 9/2, 9/2 \rangle ^{41}\text{K} 1, 1 \rangle$	542.7	12	s wave [40]

fermionic species demonstrates efficient sympathetic cooling by ^{41}K with $\Gamma = 12$ (15) for ^6Li (^{40}K).

We now turn to the creation of strongly interacting quantum mixtures. For this, atoms are loaded after evaporation into an optical dipole trap formed by two crossed laser beams of wavelength 1064 nm, each focused to a waist of $100 \mu\text{m}$ at 7 W of power. For the study of ^6Li - ^{41}K Feshbach resonances, atoms of both species are transferred into the hyperfine ground state via a Landau-Zener sweep of the bias magnetic field in the presence of 261.3 MHz and 234.2 MHz rf radiation. For ^{40}K - ^{41}K , only ^{41}K is transferred into the ground state. This mixture is stable against spin-changing collisions due to the inverted hyperfine structure and the large nuclear spin of ^{40}K . Feshbach resonances are detected via atom loss from three-body collisions, after a fixed wait time, as a function of magnetic field. A list of observed resonances is given in Table I.

We observe a wide Feshbach resonance in collisions of ^{40}K in state $|9/2, 9/2\rangle$ with ^{41}K in state $|1, 1\rangle$ at 543 G

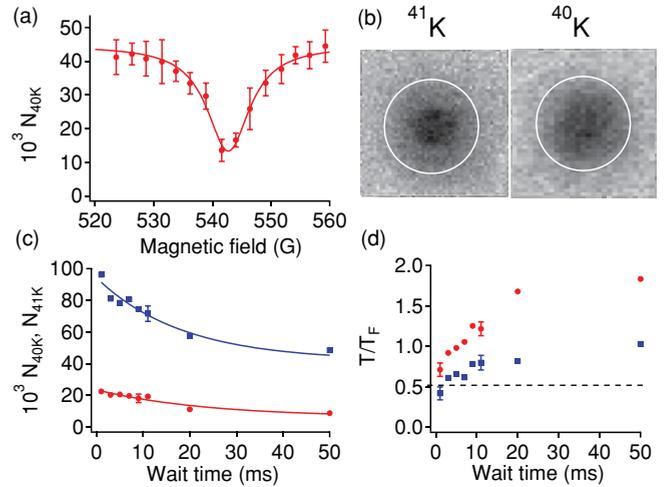


FIG. 4. (Color online) Observation of a wide Feshbach resonance in the isotopic Bose-Fermi mixture of ^{41}K - ^{40}K . (a) The atom loss feature vs magnetic field is centered at $B_0 = 542.7 \pm 0.5$ G. (b) Absorption images of the Bose and Fermi clouds after time of flight. The ^{40}K image was scaled by the ratio of expansion factors of the Bose and Fermi cloud, the images thus approximately illustrate the in-trap density distribution. The white rim indicates the Fermi radius. (c) and (d) Atom number and reduced temperature T/T_F vs wait time at the Feshbach resonance. Circles: ^{40}K ; squares: ^{41}K . Dashed line: Bose-Einstein Condensate threshold $T_c/T_F = 0.52$.

[Fig. 4(a)]. This resonance is theoretically predicted [40] to occur at $B_0 = 541.5$ G with a width of $\Delta B = 52$ G, defined via the scattering length $a = a_{\text{bg}}[1 - \Delta B/(B - B_0)]$, where $a_{\text{bg}} = 65a_0$ is the background scattering length in the vicinity of the resonance. This isotopic Bose-Fermi mixture with essentially no gravitational sag and wide tunability of its interaction strength is very promising for controlled many-body experiments, where the only relevant difference between the two atoms is that of quantum statistics. Figure 4(b) shows the immersion of a Bose-Einstein condensate of ^{41}K into a Fermi sea of ^{40}K with resonant interactions. The condensate survives for about 5 ms, and the remaining thermal atoms decay with a $1/e$ lifetime of 25 ms at initial densities $1(3) \times 10^{12} \text{ cm}^{-3}$ for ^{40}K (^{41}K). Our initial temperatures are low enough, and the condensate lifetime long enough so that polarons should form [29,30]. At the rim of the condensate, where bosons are the minority, bosons are dressed into Fermi polarons, possibly yielding a Fermi polaron condensate [30]. The formation time of such a dressed quasiparticle state should be on the order of $\hbar/E_B \sim 1$ ms, where $E_B = 0.6E_{F,^{40}\text{K}}$ is the polaron energy [22]. In the center of the gas, where fermions are the minority, they can be expected to be dressed by the Bose condensate. It will be intriguing to perform local rf spectroscopy on this unconventional state of polaronic matter and to directly demonstrate dressing of fermionic and bosonic impurities [22].

In conclusion, we have observed triply degenerate quantum gases of ^{41}K , ^{40}K , and ^6Li , through sympathetic cooling of the fermionic species by the boson ^{41}K . In the Bose-Fermi mixtures of ^6Li - ^{41}K and ^{41}K - ^{40}K , five interspecies Feshbach resonances are detected, with s - and p -wave character. The isotopic potassium gas could become a pristine model system for strongly interacting Bose-Fermi mixtures, for example, for the study of polarons [22,27], observation of polaron condensation, and universal transport of mixtures with unlike statistics [20]. The doubly degenerate ^{40}K - ^6Li Fermi-Fermi mixture holds promise for the observation of fermionic superfluidity and Cooper pairing between unlike fermions. Imposing species-dependent optical potentials on mixtures will allow the study of systems with mixed dimensionality [41] and impurity physics such as Anderson localization [42] and the interaction of localized impurities with fermionic superfluids [43].

We would like to acknowledge A. Simoni, T. Hanna, and E. Tiesinga for theoretical determinations of Feshbach resonances. We also thank S. Campbell, C. Clausen, C. Figgatt, K. Fisher, V. Ramasesh, and J. Sharpe for experimental assistance. This work was supported by NSF, AFOSR-MURI and YIP, ARO-MURI, ARO with funding from the DARPA OLE program, the Packard Foundation, and the Alfred P. Sloan Foundation.

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