PHYSICAL REVIEW A **84**, 010304(R) (2011)

Heralded-qubit amplifiers for practical device-independent quantum key distribution

Marcos Curty[1] and Tobias Moroder[2]

[1]*ETSI Telecomunicación, Department of Signal Theory and Communications, University of Vigo, Campus Universitario, E-36310 Vigo, Pontevedra, Spain*

[2]*Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften, Technikerstraße 21A, A-6020 Innsbruck, Austria*

(Received 29 March 2011; revised manuscript received 15 June 2011; published 27 July 2011)

Device-independent quantum key distribution does not need a precise quantum mechanical model of employed devices to guarantee security. Despite its beauty, it is still a very challenging experimental task. We compare a recent proposal by Gisin *et al*. [Phys. Rev. Lett. **105**, 070501 (2010)] to close the detection loophole problem with that of a simpler quantum relay based on entanglement swapping with linear optics. Our full-mode analysis for both schemes confirms that, in contrast to recent beliefs, the second scheme can indeed provide a positive key rate which is even considerably higher than that of the first alternative. The resulting key rates and required detection efficiencies of approximately 95% for both schemes, however, strongly depend on the underlying security proof.

DOI: 10.1103/PhysRevA.84.010304 PACS number(s): 03.67.Dd, 03.65.Ud, 03.67.Hk, 42.50.−p

Despite its often praised unconditional security, quantum cryptography also relies on some assumptions. Some of them are quite natural, such as the validity of quantum mechanics, the existence of true random number generators, or the assumption that the legitimate users are well shielded from the eavesdropper. Other assumptions, such as considering that the honest parties have an accurate and complete description of their physical devices, are more severe. Obviously, if the functioning of the real setup differs from that considered in the mathematical model, this may become completely vulnerable to new types of attacks not covered by the security proof [1].

In principle, this presumably hard-verifiable requirement of characterizing real devices can be circumvented using device-independent quantum key distribution (diQKD) [2–4]. Here, the legitimate users only need to specify a certain number of possible inputs and outputs for each "black box," and they can prove the security of the protocol based on the violation of an appropriate Bell inequality, which certifies the presence of quantum correlations. In practice, however, diQKD is a very challenging experimental problem. Specially, it is necessary to close the detection loophole which is present in all optical tests of Bell's inequalities realized so far, even at short distances [5]. Current experimental nonlocality tests use the so-called fair-sampling assumption to cope with the low efficiencies of both the quantum channel and detectors, but unfortunately this premise cannot be justified in a complete device-independent scenario.

In this Rapid Communication we investigate a potential solution to bypass this detection loophole problem due to channel losses in diQKD in order to cover long distances. In particular, we compare a recent proposal by Gisin *et al*. [6] based on so-called qubit amplification, with that of a standard quantum relay which employs entanglement swapping. Contrary to recent arguments [6,7], our full-mode simulation for both schemes demonstrates that the second alternative can indeed provide a positive key rate using only linear optical components. This key rate is also considerably higher than that of the first alternative. Let us stress that our main motivation lies in experimental realizations of diQKD over long distances, rather than presenting a rigorous full security proof for such schemes in the presence of losses. For that, we employ the security analysis provided in Ref. [6], which holds for specific kinds of eavesdropping attacks that are assumed, but unproven, to be optimal. For comparison reasons, we also evaluate a conservative lower bound on the secret key generation rate that can be obtained by deterministic or random assignment of inconclusive to conclusive events [4]. In this last scenario, however, the employed detectors must have almost perfect efficiency in order to distribute a secret key with practical signals. These differences should further emphasize the strong performance and requirements dependence of these schemes with respect to the underlying security analysis.

As a starting point of our considerations let us recall the heralded-qubit amplifier introduced by Gisin *et al*. in Ref. [6], extending an earlier work by Ralph and Lund [8]. The goal is to determine if an arriving light pulse contains precisely one photon or not, without disturbing its state of polarization. Such a scheme can be seen as a quantum-nondemolition measurement that distinguishes single-photon signals from vacuum or multiphoton pulses. The basic setup is illustrated in Fig. 1. The amplifier consists of a linear optics network, together with two single-photon sources, which are denoted in the figure as ρ^{h}_{single} and ρ^{v}_{single}, emitting horizontally (h) and vertically (v) polarized photons, respectively. Whenever a single-photon pulse from the channel enters the amplifier, its state of polarization is teleported to a photon situated at its output port. If the incoming light pulse is empty or contains more than one photon, however, the teleportation process fails with high probability. By looking at the detection pattern observed in the photodetectors D_i, with $i \in \{h,v\}$, Bob can verify which of these two possible events occurred.

Let us consider first for simplicity the scenario where all optical elements within the amplifier are lossless, and all detectors D_i are noiseless, have photon number resolution, and have perfect detection efficiency. Moreover, let us assume that ρ^{h}_{single} and ρ^{v}_{single} emit exactly one photon each in the correct polarization, and

$$\rho_{AB} = (1-p)|0\rangle\langle 0| + p\left|\frac{a_h^\dagger b_h^\dagger + a_v^\dagger b_v^\dagger}{\sqrt{2}}\right\rangle\left\langle\frac{a_h^\dagger b_h^\dagger + a_v^\dagger b_v^\dagger}{\sqrt{2}}\right|, \quad (1)$$

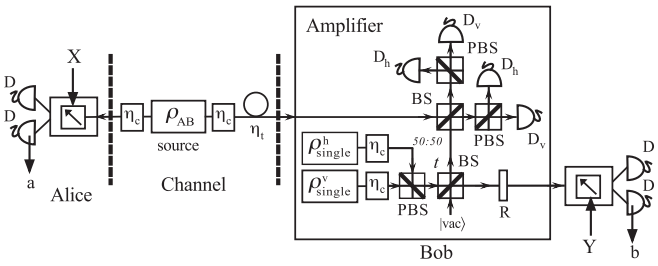

FIG. 1. Basic setup of a diQKD scheme with a quantum teleportation-based heralded-qubit amplifier located on Bob's side [6]. The entanglement source ρ_{AB} is near Alice's transmitter. The parameter η_c denotes the efficiency of optical couplers, t is the transmittance of a beamsplitter (BS), PBS stands for a polarizing BS, ρ^h_{single} and ρ^v_{single} represent two single-photon sources generating horizontally (h) and vertically (v) polarized photons respectively, R is a polarization rotator, and D and D_i, with $i \in \{h,v\}$, denote photodetectors. The single-photon sources ρ_{single} can be realized, for instance, with heralded entanglement sources like spontaneous parametric down-conversion sources.

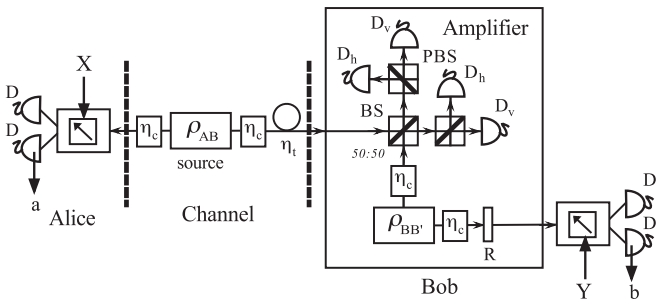

FIG. 2. Basic setup of a diQKD system with a standard quantum relay using linear optics. When compared to Fig. 1, now the two single-photon sources ρ^h_{single} and ρ^v_{single}, together with the BS of transmittance t, have been replaced by just one entanglement source $\rho_{BB'}$.

where $|0\rangle$ denotes the vacuum state, $a_h^\dagger$ and $b_h^\dagger$ ($a_v^\dagger$ and $b_v^\dagger$) represent the creation operators for the horizontal (vertical) polarization modes, and $0 < p \leqslant 1$. In this situation, it turns out that whenever Bob's detectors D_i observe two photons prepared in orthogonal polarizations, the unnormalized conditional state at the input ports of Alice's and Bob's measurement devices X and Y (see Fig. 1) has the form (after an appropriate one-photon rotation R)

$$\sigma_{AB} = (1-p)\frac{(1-t)^2}{4}|0\rangle\langle 0| + p\frac{(1-\eta_t)(1-t)^2}{8}[|a_h^\dagger\rangle\langle a_h^\dagger| + |a_v^\dagger\rangle\langle a_v^\dagger|] + p\frac{\eta_t t(1-t)}{4}\left|\frac{a_h^\dagger b_h^\dagger + a_v^\dagger b_v^\dagger}{\sqrt{2}}\right\rangle \times \left\langle\frac{a_h^\dagger b_h^\dagger + a_v^\dagger b_v^\dagger}{\sqrt{2}}\right|. \quad (2)$$

Here, t is the transmittance of a beamsplitter (BS) within the amplifier, and η_t denotes the transmission efficiency of the quantum channel.

By selecting a sufficiently high value for the parameter t, Alice and Bob can always amplify the maximally entangled component of σ_{AB} for any transmission efficiency of the quantum channel. This technique provides them with a powerful tool to overcome the problem of transmission losses in diQKD. Any successful amplifier event acts as a kind of fair-sampling device. Since the real measurement input is only chosen afterward, there should be no correlations between the trigger and the input choice [9]. As a result, it turns out that the overall detection efficiency which is needed to close the detection loophole in diQKD can be reduced to basically that of Alice's and Bob's devices, but it no longer depends on the loss of the quantum channel. A drawback of this technique is, however, the small success probability, $P_{\text{succ}} = (1-t)[1-t-p\eta_t(1-2t)]$, of having a successful heralding signal from the amplifier for large t, which might strongly reduce the achievable secret key rate of the protocols. This imposes a trade-off on the value of the transmittance t. Guaranteeing that Alice and Bob share a high entangled state σ_{AB} suitable for diQKD favors $t \approx 1$, but for small transmission efficiencies this implies an almost zero success probability.

A more direct approach to implement a heralded-qubit amplifier is to use a standard quantum relay with linear optics. The basic setup is illustrated in Fig. 2. The working principle of this scheme is essentially the same as that of a teleportation-based amplifier. The only difference between the two solutions relies on Bob's mechanism of generating an entangled state in the amplifier for teleportation. While in Ref. [6] Bob mixes two-photon pulses with a vacuum signal at a BS whose transmittance is optimized, in a quantum relay architecture he directly uses an entanglement photon source, which might be easier to realize experimentally. Intuitively speaking, one could expect that a linear optics quantum relay might be valuable for long-distance diQKD *only* when Alice and Bob have a high-quality entanglement source at their disposal. Otherwise, the conditional signals σ_{AB} shared by the legitimate users (after a successful amplifier event) might be poorly entangled. That is the case, for instance, when ρ_{AB} and $\rho_{BB'}$ (see Fig. 2) are generated with spontaneous parametric down-conversion (SPDC) sources. We will show that this intuition is wrong, and a quantum relay can indeed be used to achieve considerable higher secret key rates than those obtained with the teleportation-based amplifier of Fig. 1, even with practical signals.

Let us begin again by considering a simplified scenario where all detectors D_i are noiseless, photon number resolving, and perfectly efficient. Moreover, we assume only for the moment that $\eta_t = 1$ and the states $\rho_{AB} = \rho_{BB'}$ have the form

$$\rho_{AB} = p_0|0\rangle\langle 0| + p_1\left|\frac{a_h^\dagger b_h^\dagger + a_v^\dagger b_v^\dagger}{\sqrt{2}}\right\rangle\left\langle\frac{a_h^\dagger b_h^\dagger + a_v^\dagger b_v^\dagger}{\sqrt{2}}\right| + p_2\left|\frac{(a_h^\dagger b_h^\dagger + a_v^\dagger b_v^\dagger)^2}{2\sqrt{3}}\right\rangle\left\langle\frac{(a_h^\dagger b_h^\dagger + a_v^\dagger b_v^\dagger)^2}{2\sqrt{3}}\right|, \quad (3)$$

with $p_0 + p_1 + p_2 = 1$. In this situation, it can be shown that whenever Bob's detectors D_i observe precisely two photons

prepared in orthogonal polarizations, the unnormalized conditional quantum state shared with Alice is given by

$$\sigma_{AB} = \frac{p_0 p_2}{3}[|a_h^\dagger a_v^\dagger\rangle\langle a_h^\dagger a_v^\dagger| + |b_h^\dagger b_v^\dagger\rangle\langle b_h^\dagger b_v^\dagger|] + \frac{p_1^2}{2}\left|\frac{a_h^\dagger b_h^\dagger + a_v^\dagger b_v^\dagger}{\sqrt{2}}\right\rangle\left\langle\frac{a_h^\dagger b_h^\dagger + a_v^\dagger b_v^\dagger}{\sqrt{2}}\right|. \quad (4)$$

In contrast to Eq. (2), there is no parameter now that Alice and Bob could tune to amplify further the maximally entangled component of σ_{AB}. Actually, depending on the probability distribution p_n of the entanglement sources ρ_{AB} and $\rho_{BB'}$, the fidelity of the outgoing state σ_{AB} with respect to a maximally entangled state could be very low. This occurs, for example, when Bob uses SPDC sources with a photon number distribution given by $p_n = (n+1)\lambda^n/(1+\lambda)^{n+2}$, where λ denotes a parameter related to the pump amplitude of the laser. For small values of λ, this fidelity is roughly equal to $1/2$. The scenario changes if Alice and Bob post-select only those detection events where both of them see precisely one photon in their measurement devices X and Y. However, such a strategy seems to open the detection loophole. Note that the probability μ_{cc} that Alice and Bob obtain a conclusive result (i.e., both of them observe exactly one photon each in their measurement apparatuses) is also about $1/2$ for small values of λ. This result is far below the typical detection efficiency of 82.8% that is required to violate the Clauser-Horne-Shimony-Holt (CHSH) inequality [10] that prevents eavesdropping exploiting the detection loophole. This argumentation seems to render the conditional signal states σ_{AB} given by Eq. (4) unsuitable for diQKD [6,7].

The key point here, however, is simple, but counterintuitive: the detection efficiency limit of 82.8% does not apply to the correlations observed when measuring the signals σ_{AB}. This can be seen with a simple example. Suppose, for instance, that Alice and Bob employ a post-processing strategy where inconclusive outcomes are assigned to conclusive "+1" outcomes in a deterministic fashion [4]. In this situation, the CHSH quantity becomes

$$S = \mu_{cc} S_{cc} + 2(1 - \mu_{cc}), \quad (5)$$

where the parameter S_{cc} denotes the CHSH value computed only on the set of conclusive results obtained before applying the post-processing step. For small λ, we find that S is roughly given by $S = 1 + \sqrt{2} > 2$. That is, Alice and Bob can indeed detect the presence of nonlocal correlations in the signals σ_{AB}. This result mainly arises because σ_{AB} provides Alice and Bob with an atypical detection pattern: both of them obtain either a conclusive or an inconclusive outcome. But no conclusive-inconclusive or inconclusive-conclusive outcomes are observed, as typically present for "local detection losses." This argument actually holds for any value of $p_1 > 0$ in Eq. (3). When the detection efficiency of Alice's and Bob's detectors is not perfect (but high enough), it turns out that the probability to observe conclusive-inconclusive or inconclusive-conclusive results is very low, and they can still violate the CHSH inequality and distribute a secret key.

To evaluate the performance of both setups in a more realistic situation, we consider a full-mode description of the sources for the case where Alice and Bob use both

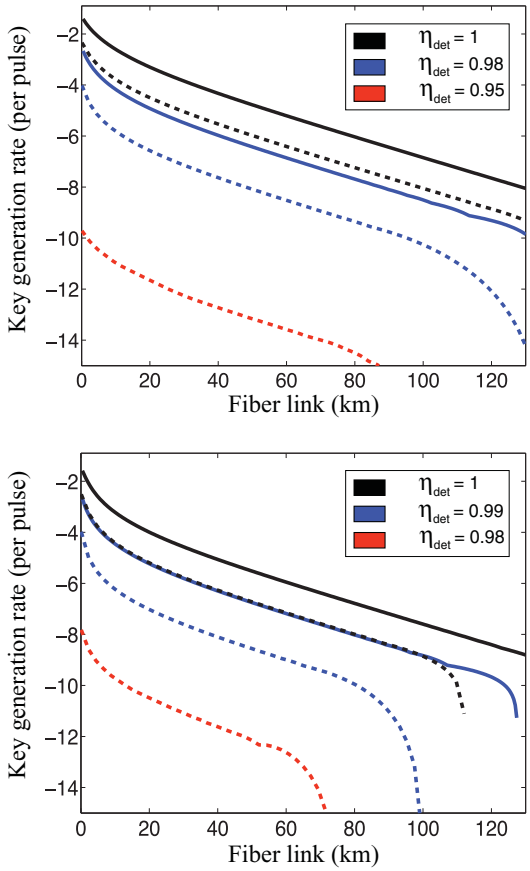

FIG. 3. (Color online) Lower bound on the secret key rate on a logarithmic scale (base 10) vs distance for a diQKD setup using the amplifier illustrated in Fig. 1 (dashed line) and the quantum relay with linear optics shown in Fig. 2 (solid line). The upper figure corresponds to the security analysis provided in Refs. [6,12]. The lower figure represents the situation where the legitimate users assign inconclusive to conclusive results deterministically [4]. In this last case, the minimum detection efficiency of Alice's and Bob's detectors is around 98% (for the teleportation-based amplifier) and 99% (for the quantum relay). In the simulations we assume $\eta_c = \eta_{det}$ and a loss coefficient of the optical fiber of $\alpha = 0.2$ dB/km.

entangled and heralded single-photon sources based on SPDC, together with inefficient photon-number-resolving detectors. For simplicity, however, we do not consider any misalignment effect in the quantum channel or in Alice's and Bob's detection apparatuses (in the view that photon loss is the dominant error mechanism), and we also neglect the effect of dark counts in the photodetectors (which typically results in a cutoff distance where the dark-count free key generation rate and the overall dark count probability are roughly equal). To compute a lower bound on the secret key rate we employ the diQKD protocol based on the violation of the CHSH inequality analyzed in Refs. [3,4,11], and we evaluate two different secret key rate formulas. The first follows the security analysis presented in Ref. [6] and holds for particular eavesdropping attacks that are assumed to be optimal [12]. The second one corresponds to the conservative situation where the legitimate users assign inconclusive to conclusive events in a deterministic fashion [4]. For the cases studied, this last strategy seems to perform better than that based on a purely random assignment of inconclusive

to conclusive outcomes [13]. The results are illustrated in Fig. 3 for a few values of the coupling efficiency η_c and the detection efficiency η_{det} of Alice's and Bob's detectors. For a given distance, we optimize the transmittance t and the intensity of each laser numerically to maximize the resulting key rate. When η_c and η_{det} are high enough, Fig. 3 shows that the use of a quantum relay can provide significantly higher key rates than a teleportation-based amplifier, while this last alternative can tolerate slightly lower detection efficiencies (around 95%) than the former one (around 96%), though the achievable key rates in this regime are already quite low. The improved performance of the quantum relay in comparison with the amplifier scheme seems to rest mainly on the quality of the single-photon sources ρ^h_{single} and ρ^v_{single}. Only if one considers the ideal scenario where these sources are perfect and on-demand can the second scheme deliver key rates similar to those of a quantum relay with practical SPDC sources. However, when Bob uses heralded single-photon sources based on SPDC instead (see Fig. 3), the small probability of finding one photon in the idler mode of both sources at the same time strongly reduces the resulting key rate of a teleportation-based amplifier. Although less efficient, the use of threshold detectors might also be an alternative to photon-number-resolving detectors. However, the analysis of this scenario is more involved since the probability that Alice and Bob obtain conclusive or inconclusive events depends on the basis choice. Details of this analysis will be presented somewhere else.

To conclude, we have performed a full-mode analysis of two potential solutions to circumvent the problem of transmission losses in diQKD using only linear optical components. Contrary to recent findings, we have demonstrated that a standard quantum relay is indeed an alternative and can outperform a teleportation-based amplifier. Still, a main technological challenge here is to develop photodetectors with nearly perfect detection efficiency and negligible noise. Recent results in this field give reasons to be optimistic [14].

The authors specially thank H.-K. Lo for bringing the subject of heralded-qubit amplification to our attention. We also thank S. Pironio for explaining to us the security analysis presented in Ref. [6] and for stimulating discussions on this topic, together with N. Sangouard and N. Lütkenhaus. We are indebt to J. M. Taboada and F. Obelleiro for their support in the use of a cluster to run the simulations. This work was supported by Xunta de Galicia, Spain (Grant No. INCITE08PXIB322257PR) and by the FWF (START Prize and SFB FOQUS).

[1] Y. Zhao, C.-H. F. Fung, B. Qi, C. Chen, and H. K. Lo, Phys. Rev. A **78**, 042333 (2008); L. Lydersen *et al.*, Nature Photonics **4**, 686 (2010); F. Xu, B. Qi, and H.-K. Lo, New J. Phys. **12**, 113026 (2010); I. Gerhardt *et al.*, Nature Comm. **2**, 349 (2011).

[2] D. Mayers and A. C.-C. Yao, in *Proceedings of the 39th Annual Symposium on Foundations of Computer Science (FOCS'98)* (IEEE Computer Society, Washington, DC, 1998), p. 503; A. Acín, N. Gisin, and L. Masanes, Phys. Rev. Lett. **97**, 120405 (2006).

[3] A. Acín *et al.*, Phys. Rev. Lett. **98**, 230501 (2007).

[4] S. Pironio *et al.*, New J. Phys. **11**, 045021 (2009).

[5] P. Pearle, Phys. Rev. D **2**, 1418 (1970).

[6] N. Gisin, S. Pironio, and N. Sangouard, Phys. Rev. Lett. **105**, 070501 (2010).

[7] N. Sangouard *et al.*, Phys. Rev. Lett. **106**, 120403 (2011).

[8] T. C. Ralph and A. P. Lund, in *Proceedings of the 9th International Conference of Quantum Communication, Measurement and Computing*, edited by A. Lvovsky (American Institute of Physics, New York, 2009), p. 155.

[9] If the measurement setting is chosen in advance, the trigger outcome could again depend on the input choice. That is, a real experiment should not post-select those events where the trigger has succeeded.

[10] J. F. Clauser *et al.*, Phys. Rev. Lett. **23**, 880 (1969).

[11] M. McKague, New J. Phys. **11**, 103037 (2009).

[12] We like to point out that we do not exactly employ the key rate formula provided in Ref. [6], although this would not change the main results of this Rapid Communication; only the required detection efficiencies become lower but still the quantum relay clearly outperforms the amplifier scheme. In the simulation of Fig. 3 we use Eq. (10) from the supplementary material of Ref. [6] but with a parameter μ given by $\mu = (\mu_{ci} + 2\mu_{ic})/\mu_{cc}$. This accounts for the case that the only quantum strategy, following the terminology of Ref. [6], for which one can directly employ the key rate formula of the lossless case, must give conclusive outcomes *for all* different basis settings and not just for the settings used to evaluate the CHSH parameter. Otherwise the eavesdropper could pretend a large value of this parameter by a strategy that always gives conclusive outcomes (and a large violation) for the CHSH settings but if Alice measures in her key setting then it gives an inconclusive result and hence is discarded in the post-processing phase.

[13] X Ma, T. Moroder and N. Lütkenhaus, e-print arXiv:0812.4301.

[14] A. E. Lita, A. J. Miller and S. W. Nam, Opt. Express **16**, 3032 (2008).