Equivalence between Redfield- and master-equation approaches for a time-dependent quantum system and coherence control

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We present a derivation of the Redfield formalism for treating the dissipative dynamics of a time-dependent quantum system coupled to a classical environment. We compare such a formalism with the master equation approach where the environments are treated quantum mechanically. Focusing on a time-dependent spin-1/2 system we demonstrate the equivalence between both approaches by showing that they lead to the same Bloch equations and, as a consequence, to the same characteristic times T_1 and T_2 (associated with the longitudinal and transverse relaxations, respectively). These characteristic times are shown to be related to the operator-sum representation and the equivalent phenomenological-operator approach. Finally, we present a protocol to circumvent the decoherence processes due to the loss of energy (and thus, associated with T_1). To this end, we simply associate the time dependence of the quantum system to an easily achieved modulated frequency. A possible implementation of the protocol is also proposed in the context of nuclear magnetic resonance.

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I. INTRODUCTION

The rapid development of quantum-information science has brought together several areas of theoretical and experimental physics [1]. Much effort has been concentrated in the search for solutions to sensitive problems that prevent the efficient realization of quantum-information processing [2]. We first mention the system-environment coupling which induces the decoherence of quantum states [3], apart from other barriers such as scalability [4] and optimal control of individual systems [5]. These challenges motivate both fundamental physical phenomena and outstanding technological issues such as individually addressing quantum systems, separated by only few nm, with small errors [6].

Potential platforms for the implementation of quantum logic operations have appeared in many fields such as condensed matter, quantum optics, and atomic physics [1]. However, the problems mentioned above are faced by all the different communities when employing their particular techniques. In the particular case of the dissipation and decoherence phenomena—in which we focus in the present work—the Redfield formalism [7,8] and the master equation [9] have been the most applied approaches to address the environment effects on the proposed protocols for quantum-information processing. Whereas the semiclassical Redfield formalism relies on a classical noise source, a quantum environment is assumed in the master equation approach. In this article, considering the general case of a time-dependent system, we discuss general similarities and differences between both approaches and show that they are equivalent, in the sense that they lead to the same phenomenological Bloch equations [10]. Consequently, both of them result in the same characteristic relaxation times T_1 and T_2 associated with the longitudinal and transverse relaxations, respectively [7,8]. From this identification we show how these characteristic times are related to the operator-sum representation [2] and the phenomenological-operator approach [11].

The Redfield formalism was intended to offer a microscopic description of the relaxation phenomenon, thus providing a deeper understanding of the parameters T_1 and T_2. Whereas the classical noise source employed in the Redfield theory suffices to derive both relaxation times, two distinct quantum environments must be adopted to derive these time scales from the master equation formalism. On this regard, an amplitude and a phase damping environment are assumed to define the longitudinal and the transverse relaxation times, respectively. These quantum environments represent an energy-draining and a phase-shuffle channel by which the system loses excitations and phase relations.

After presenting a detailed derivation of the Redfield theory and comparing the derived characteristics times with those obtained from the master equation, we finally apply these equivalent formulations to the problem of state protection. We note that several distinct techniques have been proposed to control the effects of decoherence on quantum states, aiming to enlarge the fidelity of quantum-information protocols. Among others, we mention the quantum-error correction codes [12], environments engineering [13], decoherence-free subspaces [14], and dynamical decoupling [15]. We finally mention that in a previous work [16], addressing the energy draining and decoherence of a harmonic oscillator, it was demonstrated that the inevitable action of the environment can be substantially weakened when considering appropriate nonstationary quantum systems. Reasoning by analogy with the technique presented in Ref. [16], we show how to enlarge the longitudinal relaxation time associated with the amplitude-damping channel focusing a spin-1/2 system. The ideas presented here for decoherence control can be easily implemented in the nuclear magnetic resonance (NMR) context.

This article is organized as follows: In Sec. II we present the derivation of the Redfield equation for the general scenario of a time-dependent system. In Sec. III we apply the master equation approach to the same case. In Sec. IV we show the equivalence between the Redfield and the master equation formalisms by deriving the Bloch equations from both approaches. Focusing on a time-dependent spin-1/2 system, in

Secs. V and VI we present the operator-sum representation and the phenomenological operator approaches and their relation with the previous techniques. As an application of the theory, in Sec. VII we address the state protection of a nonstationary spin-1/2 system. Finally, Sec. VIII is dedicated to our final remarks where we discuss the generalization of the methods presented in this article for larger systems. In general, we will adopt the language of the NMR quantum-information processing [17], although the theory presented here is valid for several other platforms, as quantum dots [18], superconducting artificial atoms [19], and so on. Throughout the article we will use natural units such that $\hbar = k_B = 1$.

II. REDFIELD FORMALISM FOR A TIME-DEPENDENT SPIN SYSTEM

Considering the interaction of a time-dependent spin system, described by the Hamiltonian $H_S(t)$, with a spin lattice modeling the environment and represented by the Hamiltonian H_L, the total density operator $\rho^{\mathrm{sch}}(t)$ in the Schrödinger picture, evolves as

$$\frac{d\rho^{\mathrm{sch}}(t)}{dt} = -i[H_S(t) + H_L + H_{\mathrm{SL}}(t), \rho^{\mathrm{sch}}(t)], \quad (1)$$

with $H_{\mathrm{SL}}(t)$ being the time-dependent spin-lattice interaction. In the interaction picture, defined by the unitary transformation $U(t) = \exp[-i(\mathcal{H}_S(t) + H_L t)]$, where $\mathcal{H}_S(t) = \int_0^t dt' H_S(t')$, we simplify the above evolution equation to the form

$$\frac{d\rho(t)}{dt} = -i[V_{\mathrm{SL}}(t), \rho(t)], \quad (2)$$

where $V_{\mathrm{SL}}(t) = e^{i\mathcal{H}_S(t)}\mathcal{H}_{\mathrm{SL}}(t)e^{-i\mathcal{H}_S(t)}$, with $\mathcal{H}_{\mathrm{SL}}(t) = e^{iH_L t}H_{\mathrm{SL}}(t)e^{-iH_L t}$. We have assumed the condition $[H_S(t), H_S(t')] = 0$ which is always fulfilled whenever the free Hamiltonian of the system can be written as a diagonal time-independent operator, with a time-dependent coefficient $H_S(t) = f(t)O_S$. Moreover, we observe that in NMR relaxation experiments all the required pulses to perform the necessary rotations are applied either at the beginning, to prepare the initial state $\rho^{\mathrm{sch}}(0)$, or at the end of the experiment, to implement the tomography of the evolved state. Between the applications of these pulses, the prepared state $\rho^{\mathrm{sch}}(0)$ of the system, described by the diagonal Hamiltonian $H_S(t)$, evolves only under the action of the environment.

By its turn, the density operator in the rotating frame is given by $\rho(t) = e^{i\mathcal{H}_S(t)}\widetilde{\rho}^{\mathrm{sch}}(t)e^{-i\mathcal{H}_S(t)}$, where $\widetilde{\rho}^{\mathrm{sch}}(t) = e^{iH_L t}\rho^{\mathrm{sch}}(t)e^{-iH_L t}$. By assuming a weak system-environment coupling and getting rid of the degrees of freedom of the spin lattice, we solve Eq. (2) up to second order of perturbation theory, obtaining

$$\frac{d\sigma(t)}{dt} = -\mathrm{Tr}_L \int_0^t dt'[V_{\mathrm{SL}}(t), [V_{\mathrm{SL}}(t'), \rho(t')]].$$

We considered that the interaction $V_{\mathrm{SL}}(t)$ is a stochastic operator with null mean value [7,8], which results in a zero first-order term. Next, let us use the Markov approximation $\rho(t') \to \rho(t) \simeq \sigma(t) \otimes \rho_L(0)$, with $\sigma(t)$ being the reduced density operator of the spin system, $\sigma(t) = \mathrm{Tr}_L \rho(t)$, and ρ_L the reduced density operator of the environmental spin lattice.

This approximation means that the state of the lattice is not affected by the interaction with the system. In other words, it means that the lattice presents a sufficiently large heat capacity in order to remain in the thermal equilibrium state $\rho_L(0) = e^{-\beta H_L}/\mathrm{Tr}[e^{-\beta H_L}]$, with $\beta = 1/T$, T being the environment temperature. Finally, inspired in NMR systems [8,17], we are going to apply the high-temperature approximation, which takes into consideration systems were the energy gap between the spin levels, $\hbar\omega$ (where ω is the characteristic transition frequency among levels), is much smaller than the thermal energy, $k_B T$, of the system, i.e., $\hbar\omega/k_B T \ll 1$. In this sense, the density operator of the system can be written as $\sigma(t) = e^{-\beta H_S(t)}/Z \simeq 1 - \beta H_S(t)$. We stress that there is a crucial difference between high and infinite temperature limits; differently from the latter case, in the former there is still a population difference between the spin levels which accounts for the reminiscent equilibrium magnetization. Thus, applying the high-temperature approximation we obtain $\sigma(t) \otimes \rho_L(0) \simeq \sigma(t) - \beta H_L$ and, consequently,

$$\frac{d\sigma(t)}{dt} = \mathrm{Tr}_L \int_0^t dt'[V_{\mathrm{SL}}(t), [V_{\mathrm{SL}}(t'), \beta H_L - \sigma(t)]]. \quad (3)$$

From the Heisenberg equation of motion for $V_{\mathrm{SL}}(t)$, we obtain

$$[V_{\mathrm{SL}}(t), \sigma(t) - \beta H_L] = [V_{\mathrm{SL}}(t), \sigma(t) + \beta H_S(t)] i\beta \frac{dV_{\mathrm{SL}}(t)}{dt}$$
$$+ i\beta U^\dagger(t)\frac{dH_{\mathrm{SL}}(t)}{dt}U(t),$$

and, consequently, the evolution equation reads

$$\frac{d\sigma(t)}{dt} = i\beta \mathrm{Tr}_L\{[V_{\mathrm{SL}}(t), V_{\mathrm{SL}}(0)]\}$$
$$- \mathrm{Tr}_L \int_0^t dt'[V_{\mathrm{SL}}(t), [V_{\mathrm{SL}}(t'), \sigma(t) + \beta H_S(t')]]$$
$$- i\beta \mathrm{Tr}_L \int_0^t dt'[V_{\mathrm{SL}}(t), U^\dagger(t')\frac{dH_{\mathrm{SL}}(t')}{dt'}U(t')]. \quad (4)$$

By rewriting the spin-lattice interaction as $V_{\mathrm{SL}}(t) \propto \lambda(t)O$, where $\lambda(t)$ models the lattice stochastic fluctuation and O stems for an operator acting on the spin system space, we verify straightforwardly that the first term of the right-hand side of Eq. (4) is null, in accordance with the assumption $\langle\lambda(t)\rangle = 0$. Moreover, with the above definition for the spin-lattice interaction, we verify that $\mathcal{H}_{\mathrm{SL}}(t) = H_{\mathrm{SL}}(t)$ and, consequently, $V_{\mathrm{SL}}(t) = e^{i\mathcal{H}_S(t)}H_{\mathrm{SL}}(t)e^{-i\mathcal{H}_S(t)}$. Integrating by parts the third term of the right-hand side of Eq. (4) and considering, as usual, that the time oscillations of the operator $U^\dagger(t')OU(t')$ is much faster than that of $V_{\mathrm{SL}}(t)$, we apply the rotating-wave approximation to conclude that this term is also null. The fact that this is indeed the case can be seen as follows: the operator $V_{\mathrm{SL}}(t)$ oscillates with the spin-lattice coupling frequency, while the operator $U^\dagger(t')OU(t')$ oscillates with the bare spin frequency which (in the assumed system-environment weak coupling regime) is much higher than the interaction frequency. Putting all this together, we finally obtain the simplified equation of motion for the spin system

$$\frac{d\sigma(t)}{dt} = \mathrm{Tr}_L \int_0^t dt'[V_{\mathrm{SL}}(t), [V_{\mathrm{SL}}(t'), \beta H_S(t') - \sigma(t)]],$$

which, in accordance with the high-temperature approximation, where $\sigma_{\rm eq}(0) \simeq 1 - \beta H_S(0)$ and $\beta H_S(t') \simeq 1 - \sigma_{\rm eq}(0)$, becomes

$$\frac{d\Sigma(t)}{dt} = -\int_0^t dt' \overline{[V_{\rm SL}(t),[V_{\rm SL}(t'),\Sigma(t)]]}, \qquad (5)$$

where we have defined the operator $\Sigma(t) = \sigma(t) - \sigma_{\rm eq}(0)$ and substituted the trace over the lattice degrees of freedom by the ensemble average over stochastic realizations, represented by the overbar. We have thus obtained the Redfield equation for a time-dependent spin system and we note, in spite of the c-number character of the environment degrees of freedom, its resemblance with the master equation to be presented below.

It is important to stress that the high-temperature approximation, allowing us to define the latter operator $\Sigma(t)$, indicates a relaxation to the highly mixed thermal Gibbs state. However, we mention that in the whole calculation to obtain the Redfield equation (5) it is not necessary to impose such an approximation. It was only done because it is characterisitic of NMR systems, on which we focus in the present work.

Toward the definition of the lattice spectral density we next introduce, through the eigenvalue equation $H_S(t)|k\rangle = \epsilon_k(t)|k\rangle$, the spin basis $\{|k\rangle\}$. Taking the matrix element kk' of Eq. (5) and back to the Schrödinger picture where $\Sigma^{\rm sch}(t) = e^{-i\mathcal{H}_S(t)}\Sigma(t)e^{i\mathcal{H}_S(t)}$, we obtain the Redfield equations for the evolution of the density matrix elements

$$\frac{d\Sigma_{kk'}^{\rm sch}(t)}{dt} = -i\langle k|[H_S,\Sigma^{\rm sch}]|k'\rangle \\ + \sum_{n,n'} e^{-i(\Omega_{kk'}+\Omega_{n'n})} R_{kn,n'k'} \Sigma_{nn'}^{\rm sch}, \qquad (6)$$

where we have used the short-hand notation $\Omega_{kn}(t) = E_k(t) - E_n(t)$, $E_k(t) = \int_0^t dt'\epsilon_k(t')$, $\Sigma_{nn'}^{\rm sch}(t) = \langle n|\Sigma^{\rm sch}(t)|n'\rangle$, and defined the relaxation matrix elements

$$R_{kn,n'k'}(t) = J_{kn,n'k'}(t,\Omega_{n'k'})e^{i\Omega_{kn}(t)} + J_{n'k',kn}(t,\Omega_{kn})e^{i\Omega_{n'k'}(t)} \\ - \delta_{k'n'} \sum_j J_{kj,jn}(t,\Omega_{jn})e^{i\Omega_{kj}(t)} \\ - \delta_{kn} \sum_j J_{jk',n'j}(t,\Omega_{n'j})e^{i\Omega_{jk'}(t)}, \qquad (7)$$

with the environment spectral densities given by

$$J_{kn,n'k'}(t,\Omega_{n'k'}) = \int_0^t dt' G_{kn,n'k'}(t,t')e^{i\Omega_{n'k'}(t')}, \qquad (8a)$$

$$G_{kn,n'k'}(t,t') = \overline{\langle k| H_{\rm SL}(t) |n\rangle \langle n'| H_{\rm SL}(t') |k'\rangle}. \qquad (8b)$$

To simplify the notation we have omitted the explicit time dependence of all functions in Eq. (6).

For the particular case of a time-independent system, we obtain $E_m(t) = \epsilon_m t$, $\Omega_{kn}(t) = (\epsilon_k - \epsilon_n)t \equiv \omega_{kn}t$ and the above Redfield equations reduce to the well-known textbook result [7,8]

$$\frac{d\Sigma_{kk'}^{\rm sch}(t)}{dt} = -i\langle k|[H_S,\Sigma^{\rm sch}(t)]|k'\rangle \\ + \sum_{n,n'} e^{-i(\omega_{kk'}-\omega_{n'n})t} R_{kn,n'k'}(t)\Sigma_{nn'}^{\rm sch}(t),$$

with

$$R_{kn,n'k'} = J_{kn,n'k'}(\omega_{n'k'})e^{i\omega_{kn}t} + J_{n'k',kn}(\omega_{kn})e^{i\omega_{n'k'}t} \\ - \delta_{k'n'} \sum_j J_{kj,jn}(\omega_{jn})e^{i\omega_{kj}t} \\ - \delta_{kn} \sum_j J_{jk',n'j}(\omega_{nj})e^{i\omega_{jk'}t},$$

and

$$J_{kn,nn'}(\omega_{nn'}) = \int_0^\infty dt' G_{kn,nn'}(t') \exp\{i\omega_{nn'}t'\}.$$

We observe that, although we have focused on a spin system, the equations obtained here are completely general, being valid for the Hamiltonian $H_S(t)$, provided that the three following conditions are met: (i) $[H_S(t),H_S(t')] = 0$, (ii) system-environment weak coupling regime (Markovian environment), and (iii) high-temperature approximation [specifically, to derive Eqs. (6), (7), and (8)]. The restrictions and the validity of these approximations will be discussed in the conclusions of the article. Let us now turn to the master equation approach.

III. THE MASTER EQUATION APPROACH

In this section, in contrast to the semiclassical approach of the Redfield formalism, we derive the master equation governing the dynamics of the dissipative time-dependent spin system where the environment is assumed to be modeled within the quantum formalism. We start from Eq. (3), such that

$$\frac{d\sigma(t)}{dt} = -{\rm Tr}_L \int_0^t dt' [V_{\rm SL}(t),[V_{\rm SL}(t'),\sigma(t)]]. \qquad (9)$$

Now, instead of assuming a classical environment leading to the above defined spin-lattice interaction as $V_{\rm SL}(t) \propto \lambda(t)O$, we consider two distinct quantum environments, to be defined below as the amplitude- and the phase-damping channels, each one being modeled by an infinite collection of decoupled harmonic oscillators, described by the Hamiltonian $H_L = \sum_{r,\ell} \varpi_{r\ell} a_{r\ell}^\dagger a_{r\ell}$, where $r = 1,2$ labels the environments while ℓ stands for the infinity set of oscillators whose frequencies are denoted by $\varpi_{r\ell}$. $a_{r\ell}^\dagger$ ($a_{r\ell}$) represents the creation (annihilation) operator for the lth mode of the rth environment. The action of these environments on the spin system is modeled by the interaction

$$V_{\rm SL}(t) = \sum_r [O_r^\dagger \Gamma_r(t) + O_r \Gamma_r^\dagger(t)], \qquad (10)$$

where $\Gamma_r(t) = \sum_\ell \gamma_{r\ell}(t)a_{r\ell}$ and $\gamma_{r\ell}(t) = \gamma_{r\ell}^{\rm Schr}(t)\exp[i\,\Delta_k(t)]$ with $\Delta_k(t)$ being the phase factor coming from the transformation $U^\dagger(t)O_r^\dagger a_{r\ell}U(t)$ to the interaction picture. It is worth mentioning that the time dependence of the system-environment coupling in the Schrödinger picture, $\gamma_{r\ell}^{\rm Schr}(t)$, comes from the assumption of a time-dependent spin system Hamiltonian, $H_S(t)$. In fact, the coupling strength $\gamma_{r\ell}^{\rm Schr}(t)$ leads to the decay rate of the master equation which plays the role of the time-dependent relaxation matrix in the Redfield equation (6).

By inserting Eq. (10) into Eq. (9) and performing the trace over the environments degrees of freedom, we obtain the master equation in the interaction picture

$$\frac{d\sigma(t)}{dt} = \sum_{r,r'} \{ \mathcal{F}_{rr'}(t)[O_{r'}\sigma(t), O_r^\dagger] $$
$$ + \mathcal{G}_{rr'}(t)[O_{r'}^\dagger \sigma(t), O_r] + \text{H.c.} \}, \quad (11)$$

where we have defined the functions

$$\mathcal{F}_{rr'}(t) = 2 \lim_{\tau \to 0} \left[\frac{1}{\tau} \int_t^{t+\tau} dx \int_t^x dx' \langle \Gamma_r^\dagger(x)\Gamma_{r'}(x') \rangle \right],$$

$$\mathcal{G}_{rr'}(t) = 2 \lim_{\tau \to 0} \left[\frac{1}{\tau} \int_t^{t+\tau} dx \int_t^x dx' \langle \Gamma_r(x)\Gamma_{r'}^\dagger(x') \rangle \right].$$

For the environments considered in this work it follows that $\langle \Gamma_r^\dagger(t)\Gamma_{r'}^\dagger(t') \rangle = \langle \Gamma_r(t)\Gamma_{r'}(t') \rangle = 0$, $\langle \Gamma_r^\dagger(t)\Gamma_{r'}(t') \rangle = \langle n_r \rangle \delta_{rr'}$ and $\langle \Gamma_r(t)\Gamma_{r'}^\dagger(t') \rangle = (\langle n_r \rangle + 1)\delta_{rr'}$, $\langle n_r \rangle$ being the thermal average excitation of the rth environment. These relations, of course, depend on the state of the environment. We observe that this master equation describes the Markovian evolution of a general time-dependent system, provided that the conditions (i) and (ii) of the last section are satisfied.

IV. THE CHARACTERISTIC RELAXATION TIMES

In this section, restricting us to the case of spin-1/2 systems, we aim to derive the Bloch equations for the evolution of the magnetization components of N noninteracting spins. First, we obtain the Bloch equations from the Redfield formalism, relating the characteristic relaxation times with the properties of the associated classical stochastic environment. Next, computing the evolution of the average magnetization from the master equation formalism, we are able to link the characteristic relaxation times with the properties of the quantum environment.

A. From the Redfield to the Bloch equations

Let us consider here a spin-1/2 system placed in a constant magnetic field in the z direction. The frequency gap between the two Zeeman levels defines the Larmor frequency $\omega_L(t) = \omega_1(t) - \omega_0(t)$, with $\omega_1(t)$ and $\omega_0(t)$ representing the frequencies of the excited and the ground state, respectively. The modulation of these frequencies are due to some external influence, like an additional time-dependent magnetic field. The bare Hamiltonian of the spin-1/2 system is then given by $H_S(t) = \omega_L(t)I_z$. The action of the environment over the system is modeled by the spin-lattice Hamiltonian

$$H_{\text{SL}}(t) = -\gamma_n \sum_q \lambda_q(t)I_q, \quad (12)$$

where γ_n is the gyromagnetic factor, q labels the orthogonal Cartesian directions $\{x, y, z\}$, $\lambda_q(t)$ refers to the lattice stochastic fluctuation in q direction, and $I_q = \sigma_q/2$ stands for the spin (Pauli) operator.

For this system, the spectral density given in Eqs. (8) becomes

$$J_{kn,n'k'}(t, \Omega_{n'k'}) = \sum_q I_q^{kn} I_q^{n'k'} \Theta_q(t, \Omega_{n'k'}), \quad (13)$$

where $I_q^{kn} = \langle k| I_q |n \rangle$ and

$$\Theta_q(t, \Omega_{n'k'}) = \gamma_n^2 \lambda_q^2 \int_0^t dt' e^{-|t'|/\tau_0} e^{i\Omega_{n'k'}(t+t')}. \quad (14)$$

To derive Eq. (14) we have assumed isotropic stochastic fluctuations [8], by which

$$\overline{\lambda_q(t)\lambda_{q'}(t+t')} = \delta_{qq'}\lambda_q^2 e^{-|t'|/\tau_0},$$

λ_q^2 being a mean value depending on the specific nature of the spin system and τ_0 the environment correlation time, measuring the rate of flips between the bath spins due to a specific anisotropic spin interaction (e.g., chemical shift and dipolar coupling) [20]. Note that we have assumed that the mean values of the coupling $\lambda_q(t)$ are not affected due to the time dependence of the system. This is quite reasonable since we are modeling the environment as a stochastic noise source. Remembering Sec. II, we have

$$\Omega_{kn}(t) = \int_0^t d\tau \left[\omega_k(\tau) - \omega_n(\tau) \right],$$

which is just the integral of the Larmor frequency with a positive or negative signal, depending on the difference $k - n$ ($k, n = 0, 1$).

Next, by substituting Eq. (13) into Eq. (7), we obtain the elements of the relaxation matrix

$$R_{kn,n'k'}(t) = \sum_q \Big\{ [\Theta_q(t, \Omega_{n'k'})e^{i\Omega_{kn}(t)} $$
$$ + \Theta_q(t, \Omega_{kn})e^{i\Omega_{n'k'}(t)}]I_q^{kn}I_q^{n'k'} $$
$$ - \sum_j [\delta_{k'n'}I_q^{kj}I_q^{jn}\Theta_q(t, \Omega_{jn})e^{i\Omega_{kj}(t)} $$
$$ + \delta_{kn}I_q^{jk'}I_q^{n'j}\Theta_q(t, \Omega_{n'j})e^{i\Omega_{jk'}(t)}] \Big\}, \quad (15)$$

which enables us to compute the evolution of the mean value of the magnetization $\langle I_d \rangle$ in an arbitrary d direction:

$$\frac{d\langle I_d \rangle}{dt} = \frac{d}{dt}\text{Tr}[I_d\Sigma(t)] = \text{Tr}\left[I_d \frac{d\Sigma(t)}{dt} \right].$$

By replacing Eqs. (6) and (15) into the right-hand side of the last equation, we obtain

$$\frac{d\langle I_d \rangle}{dt} = -i \sum_{l,m} I_d^{lm} \langle m|[H_S(t), \Sigma]|l \rangle $$
$$ + \sum_q \sum_{l,m} \Theta_q(t, \Omega_{ml})e^{-i\Omega_{ml}} I_d^{ml} \langle l|[[I_d, I_q], \Sigma]|m \rangle.$$

Since, for spin-1/2 systems, $|\Omega_{ml}(t)| = \Omega(t)(1 - \delta_{ml})$, the above equation can be separated into the longitudinal and transverse field components,

$$\frac{d\langle I_d \rangle}{dt} = \sum_{q=\{x,y\}} \sum_{l,m} \kappa_q(t, \Omega_{ml})I_d^{ml} \langle l|[[I_d, I_q], \Sigma]|m \rangle $$
$$ + \kappa_z \text{Tr}\{I_z[[I_d, I_z], \Sigma]\},$$

where $\kappa_q(t, \Omega_{ml}) = \Theta_q(t, \Omega_{ml})e^{-i\Omega_{ml}(t)}$. We stress that we have neglected the free-evolution term in the above equation because we are interested only in the effect of the environment induced dynamics. We also note that $\langle m|I_{x(y)}|l \rangle \neq 0$ if and only if $m \neq l$ and $\langle m|I_z|l \rangle \neq 0$ when $m = l$, which explains why

the term $\kappa_{x(y)}(t,\Omega_{ml})$ is a time-dependent function while κ_z is a constant. Writing the last equation in terms of the longitudinal and transversal magnetizations, defined as $M_z = \langle I_z \rangle$ and $\mathbf{M}_\perp = \langle I_x \rangle \hat{x} + \langle I_x \rangle \hat{y}$, respectively, we obtain

$$\frac{dM_z(t)}{dt} = -\mathrm{Re}[\kappa_x(t,\Omega) + \kappa_y(t,\Omega)]\{M_z(t) - M_0\},$$

$$\frac{d\mathbf{M}_\perp(t)}{dt} = -\frac{1}{2}\mathrm{Re}[\kappa_x(t,\Omega) + \kappa_y(t,\Omega) + 2\kappa_z]\mathbf{M}_\perp(t),$$

with $M_0 = \langle I_z \rangle_{\mathrm{eq}} = \mathrm{Tr}\{I_z \sigma_{\mathrm{eq}}\}$ being the equilibrium longitudinal magnetization. Now, comparing these results with the phenomenological Bloch equations

$$\frac{dM_z(t)}{dt} = \frac{1}{T_1}\{M_z(t) - M_0\}, \tag{16a}$$

$$\frac{d\mathbf{M}_\perp(t)}{dt} = -\frac{\mathbf{M}_\perp(t)}{T_2}, \tag{16b}$$

the characteristic relaxation times, in terms of the time-dependent decay rates κ_q in the classical stochastic environment, is defined as

$$\frac{1}{T_1} \equiv \mathrm{Re}[\kappa_x(t,\Omega) + \kappa_y(t,\Omega)], \tag{17a}$$

$$\frac{1}{T_2} \equiv \frac{1}{2}\mathrm{Re}[\kappa_x(t,\Omega) + \kappa_y(t,\Omega)] + \kappa_z, \tag{17b}$$

Equations (16) and (17) show that, in contrast to the longitudinal rate T_1, the transverse decay rate T_2 is related to an energy-conserving process, affecting only the quantum coherence of the system. This fact justifies the choice of the amplitude- and phase-damping channels for the quantum description of the spin system. It is worth mentioning that T_1 can be controlled through the time-dependent parameter $\Omega(t)$ while T_2 can only be partially controlled since the decay rate κ_z does not depend on $\Omega(t)$. Finally, from Eqs. (17) we obtain the well-known relation between both characteristic times

$$\frac{1}{T_2} = \frac{1}{2T_1} + \kappa_z. \tag{18}$$

B. From the master equation to the Bloch equations

In this section we consider the same system as before, but instead of a classical noise the spin system interacts with two quantum environments. In order to compute the evolution of the average magnetization from the master equation (11), which takes into account the amplitude- and phase-damping channels, we first address the decay rates $\mathcal{F}_{rr'}(t)$ and $\mathcal{G}_{rr'}(t)$ defined in the end of Sec. III. When considering the amplitude-damping channel ($r = a$) we associate O_a ($O_a^\dagger$) with the lowering (raising) spin operators, whereas in the case of phase-damping ($r = p$) we define O_p as the Hermitian number excitation operator. We then set $O_a = I_-$ and $O_p = I_z$. For both cases we set the mean values for the environment operators $\langle a_{rl}^\dagger a_{rk} \rangle = \langle n_{r,k} \rangle \delta_{lk}$ and, consequently, $\langle a_{rk} a_{rl}^\dagger \rangle = (\langle n_{r,k} \rangle + 1)\delta_{lk}$, $\langle n_{r,k} \rangle$ being the thermal average excitation of the kth mode of the rth environment. Considering that the environment frequencies are very closely spaced to allow a continuum summation, such that $\sum_\ell \to (2\pi)^{-1}\int_{-\infty}^{\infty} d\nu J_r(\nu)$,

$J(\nu)$ being the spectral density of the environment, we obtain, for the amplitude damping case, the effective time-dependent decay rates

$$\mathcal{F}_a(t) = \frac{\langle n_a \rangle}{2\pi}\Theta_a(t), \tag{19a}$$

$$\mathcal{G}_a(t) = \frac{(\langle n_a \rangle + 1)}{2\pi}\Theta_a(t), \tag{19b}$$

where, referring to the time dependence of the system-environment coupling in the Schrödinger picture $\gamma_{r\ell}^{\mathrm{Schr}}(t)$,

$$\Theta_a(t) = \lim_{\tau \to 0}\left\{\frac{1}{\tau}\int_t^{t+\tau} dx \int_t^x dx' \int_{-\infty}^{\infty} d\nu J_a(\nu)\right.$$
$$\left. \times \gamma_a^{\mathrm{Schr}}(\nu,x')\gamma_a^{\mathrm{Schr}}(\nu,x)e^{i[\Omega(x')-\Omega(x)+\nu(x-x')]}\right\}.$$

To obtain the effective decay rates in Eq. (19) it was assumed, as usual, that the thermal average excitation of the environment modes vary slowly around the range of variation of the spin system frequency. This is a good approximation when the environment is in a thermal state, as the present case [9].

For the phase-damping channel, the effective decay rates are given by

$$\mathcal{F}_p(t) = \frac{\langle n_p \rangle}{2\pi}\Theta_p(t), \tag{20a}$$

$$\mathcal{G}_p(t) = \frac{(\langle n_p \rangle + 1)}{2\pi}\Theta_p(t), \tag{20b}$$

where we defined

$$\Theta_p(t) = \lim_{\tau \to 0}\left[\frac{1}{\tau}\int_t^{t+\tau} dx \int_t^x dx' \int_{-\infty}^{\infty} d\nu J_p(\nu)\right.$$
$$\left. \times \gamma_p^{\mathrm{Schr}}(\nu,x')\gamma_p^{\mathrm{Schr}}(\nu,x)e^{i\nu(x'-x)}\right].$$

Due to the diagonal system-environment coupling associated with the phase-damping case, we are able to compute $\Theta_p(t)$ without having the explicit form of $\Omega(t)$. In fact, assuming that the spectral density $J_p(\nu)$ as well as the system-environment coupling $\gamma_p^{\mathrm{Schr}}(\nu,x')$ vary slowly around $\nu = 0$, we obtain the time-independent parameter

$$\Theta_p = \lim_{\tau \to 0}\left\{\frac{J_p(0)}{\tau}\int_t^{t+\tau}\left[\gamma_p^{\mathrm{Schr}}(0,x)\right]^2 dx\right\}$$
$$= J_p\left[\gamma_p^{\mathrm{Schr}}\right]^2.$$

From this result we see that, in contrast to the case of the amplitude-damping channel, the decay rates $\mathcal{F}_p$ and $\mathcal{G}_p$ do not acquire a time dependence due to the modulation of the system frequency, resembling the result obtained in the Redfield formalism. Finally, the master equation (11) becomes

$$\frac{d\sigma(t)}{dt} = \frac{\langle n_a \rangle}{2\pi}\Theta_a(t)[I_-\sigma(t),I_+]$$
$$+ \frac{(\langle n_a \rangle + 1)}{2\pi}\Theta_a(t)[I_+\sigma(t),I_-]$$
$$+ \frac{\Theta_p}{2\pi}(2\langle n_p \rangle + 1)[I_z\sigma(t),I_z] + \mathrm{H.c.} \tag{21}$$

Computing the evolution of the mean value of the magnetization $\langle I_d \rangle$ in an arbitrary d direction we obtain for the longitudinal and transversal magnetizations

$$\frac{dM_z}{dt} = -\frac{\mathrm{Re}\Theta_a(t)}{\pi}(2\langle n_a \rangle + 1)M_z,$$

$$\frac{d\mathbf{M}_\perp}{dt} = -\left[\frac{\mathrm{Re}\Theta_a(t)}{2\pi}(2\langle n_a \rangle + 1) + \frac{\mathrm{Re}\Theta_p}{\pi}(2\langle n_p \rangle + 1) \right]\mathbf{M}_\perp.$$

As in the preceding subsection, we compose these equations with the Bloch equations (16) to obtain

$$\frac{1}{T_1} \equiv \frac{\mathrm{Re}\Theta_a(t)}{\pi}(2\langle n_a \rangle + 1), \tag{22a}$$

$$\frac{1}{T_2} \equiv \frac{\mathrm{Re}\Theta_a(t)}{2\pi}(2\langle n_a \rangle + 1) + \frac{\mathrm{Re}\Theta_p}{\pi}(2\langle n_p \rangle + 1), \tag{22b}$$

and, consequently, the relation

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{\mathrm{Re}\Theta_p}{\pi}(2\langle n_p \rangle + 1),$$

which has the same structure as Eq. (18), obtained by the Redfield formalism. From Eqs. (17) and (22) we can make the identifications

$$\frac{\mathrm{Re}\Theta_a(t)}{\pi}(2\langle n_a \rangle + 1) \equiv \mathrm{Re}[\kappa_x(t,\Omega) + \kappa_y(t,\Omega)], \tag{23a}$$

$$\frac{\mathrm{Re}\Theta_p}{\pi}(2\langle n_p \rangle + 1) \equiv \kappa_z. \tag{23b}$$

These equations show the connections between the semiclassical and quantum approaches to open system dynamics. In the next two sections we will construct the Kraus and the phenomenological operators for the time-dependent system studied in this section.

V. THE OPERATOR-SUM REPRESENTATION

It is well known that every transformation that is given by a completely positive map admits a representation of the form [21]

$$\sigma(t) = \sum_k E_k(t)\sigma(0)E_k^\dagger(t), \tag{24}$$

with the Kraus operators $E_k(t)$ satisfying the following relation

$$\sum_k E_k^\dagger(t)E_k(t) = 1.$$

Our goal in this section is to construct the operators $E_k(t)$ for both channels studied in the last section. To achieve this we will consider the density operator evolution equations, which follows from the Redfield or the master equation formalisms. We then compare these equations with those shown in Eq. (24) to obtain the time dependence of the Kraus operators.

In the next two subsections we will adopt the basis defined in Sec. IV that diagonalizes the spin operator I_z, $\{|0\rangle, |1\rangle\}$. Since both channels studied here are independent, let us then consider each one of them separately.

A. Phase-damping channel

Considering only the phase damping ($\Theta_a = 0$), the master equation (21) leads us to the following set of differential equations satisfied by the elements of the density operator

$$\frac{d\sigma_{11}(t)}{dt} = 0, \tag{25a}$$

$$\frac{d\sigma_{00}(t)}{dt} = 0, \tag{25b}$$

$$\frac{d\sigma_{10}(t)}{dt} = -\Gamma_p \sigma_{10}(t), \tag{25c}$$

$$\frac{d\sigma_{01}(t)}{dt} = -\Gamma_p \sigma_{01}(t), \tag{25d}$$

where we have defined $\Gamma_p = 2\mathrm{Re}[\mathcal{F}_p + \mathcal{G}_p] = \mathrm{Re}\Theta_p(2\langle n_p \rangle + 1)/\pi$. Note that, as expected, the populations are not affected by this noisy channel. We assume that the Kraus operators for this case are given by [2]

$$E_0^p = \sqrt{1 - p(t)}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{26a}$$

$$E_1^p = \sqrt{p(t)}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{26b}$$

with $p(t)$ being the parameter to be determined. Starting from the initial density operator

$$\sigma(0) = \begin{pmatrix} \sigma_{11}^0 & \sigma_{10}^0 \\ \sigma_{01}^0 & \sigma_{00}^0 \end{pmatrix},$$

and substituting Eq. (26) into Eq. (24), we thus obtain

$$\begin{bmatrix} \sigma_{11}(t) & \sigma_{10}(t) \\ \sigma_{01}(t) & \sigma_{00}(t) \end{bmatrix} = \begin{bmatrix} \sigma_{11}^0 & (1-2p)\sigma_{10}^0 \\ (1-2p)\sigma_{01}^0 & \sigma_{00}^0 \end{bmatrix}. \tag{28}$$

By imposing that the time derivative of the elements of the above evolved density operator must be identical to those in Eqs. (25), we derive the following differential equation for the parameter $p(t)$:

$$\frac{dp}{dt} = -\frac{\Gamma_p}{2}(2p - 1),$$

with the initial condition $p(0) = 0$, arising from the fact that the Kraus operators must reduce to the identity at the initial time. The solution for $p(t)$ is thus given by

$$p(t) = \tfrac{1}{2}\{1 - \exp[-\Gamma_p t]\}, \tag{29}$$

which finally defines the Kraus operators in Eqs. (26).

B. Amplitude-damping channel

For the case of amplitude damping we assume that the Kraus operators are given by [2]

$$E_0^a = \sqrt{\gamma_T}\begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1 - a(t)} \end{bmatrix}, \tag{30a}$$

$$E_1^a = \sqrt{\gamma_T}\begin{bmatrix} 0 & \sqrt{a(t)} \\ 0 & 0 \end{bmatrix}, \tag{30b}$$

$$E_2^a = \sqrt{1 - \gamma_T}\begin{bmatrix} \sqrt{1 - a(t)} & 0 \\ 0 & 1 \end{bmatrix}, \tag{30c}$$

$$E_3^a = \sqrt{1 - \gamma_T}\begin{bmatrix} 0 & 0 \\ \sqrt{a(t)} & 0 \end{bmatrix}, \tag{30d}$$

where $\gamma_T = \exp[-\beta E]/Z$ is the Boltzmann factor, E is the energy gap of the spin-1/2 levels, and $Z = 1 + \exp[-\beta E]$ is the partition function. By analogy with the preceding subsection, our aim is to obtain the differential equation obeyed by the parameter $a(t)$. To this end, we compute the time evolution given by Eq. (24), to obtain

$$
\begin{bmatrix} \sigma_{11}(t) & \sigma_{10}(t) \\ \sigma_{01}(t) & \sigma_{00}(t) \end{bmatrix} = \begin{bmatrix} \gamma_T a + q\left(1 - \sigma_{00}^0\right) & \sqrt{q}\sigma_{10}^0 \\ \sqrt{q}\sigma_{01}^0 & (1 - \gamma_T) a + q\sigma_{00}^0 \end{bmatrix},
$$

with $q = 1 - a$. By imposing that the differential equations derived from the above density operator must be identical to those obtained from the master equation (21), with $\Theta_p = 0$, given by

$$
\frac{d\sigma_{11}(t)}{dt} = 2\text{Re}\{\mathcal{F}_a(t)\sigma_{00}(t) - \mathcal{G}_a(t)\sigma_{11}(t)\},
$$
$$
\frac{d\sigma_{10}(t)}{dt} = -\{\mathcal{F}_a^*(t) + \mathcal{G}_a(t)\}\sigma_{10}(t),
$$
$$
\frac{d\sigma_{01}(t)}{dt} = -\{\mathcal{F}_a(t) + \mathcal{G}_a^*(t)\}\sigma_{01}(t),
$$
$$
\frac{d\sigma_{00}(t)}{dt} = -2\text{Re}\{\mathcal{F}_a(t)\sigma_{00}(t) - \mathcal{G}_a(t)\sigma_{11}(t)\},
$$

we finally obtain

$$
[\gamma_T - \sigma_{11}^0]\frac{da}{dt} = -2\text{Re}\{\mathcal{F}_a(t) + \mathcal{G}_a(t)\}\left(\gamma_T a + q\sigma_{11}^0\right)
$$
$$
+ 2\text{Re}\{\mathcal{F}_a(t)\},
$$
$$
\frac{da}{dt}\sigma_{10}^0 = 2\{\mathcal{F}_a(t) + \mathcal{G}_a^*(t)\}q\sigma_{10}^0,
$$
$$
\frac{da}{dt}\sigma_{01}^0 = 2\{\mathcal{F}_a^*(t) + \mathcal{G}_a(t)\}q\sigma_{01}^0.
$$

The fourth equation is identical to the first one. Adding the last two equations and noting that the initial conditions for the elements of $\sigma(0)$ are arbitrary, the following set of differential equations must be satisfied:

$$
\frac{\sigma_{11}^0 - \gamma_T}{2}\frac{da}{dt} = \text{Re}\{\mathcal{F}_a(t) + \mathcal{G}_a(t)\}\left(\gamma_T a + q\sigma_{11}^0\right)
$$
$$
+ \text{Re}\{\mathcal{F}_a(t)\}, \tag{31}
$$
$$
\frac{da}{dt} = 2\text{Re}\{\mathcal{F}_a(t) + \mathcal{G}_a(t)\}q, \tag{32}
$$

together with the condition $a(0) = 0$. The solution of Eq. (32) is readily obtained as

$$
a(t) = 1 - \exp\left[-2\int_0^t \text{Re}\{\mathcal{F}_a(\tau) + \mathcal{G}_a(\tau)\}d\tau\right], \quad (33)
$$

and its substitution into Eq. (31) leads to the relation

$$
(\gamma_T - 1)\,\text{Re}\{\mathcal{F}_a(t)\} + \gamma_T\text{Re}\{\mathcal{G}_a(t)\} = 0,
$$

which, together with Eqs. (19), results in the identity

$$
\langle n_a \rangle = \frac{\gamma_T}{1 - 2\gamma_T} = \frac{1}{e^{\beta E} - 1}.
$$

Equation (33) satisfies both Eqs. (31) and (32) and is, thus, the desired solution.

It is now straightforward to see the connection between both *ab initio* approaches, the Redfield and the master equation, with the operator sum representation. Referring

to the identifications in Eqs. (23) and the definition of the relaxation times in Eqs. (22), we can rewrite the time dependence of the Kraus operators for the phase-damping channel as

$$
p(t) = \frac{1}{2}\left\{1 - \exp\left[-\frac{\kappa_z t}{2}\right]\right\}
$$

and for the amplitude-damping channel as

$$
a(t) = 1 - \exp\left[-\int_0^t \frac{1}{T_1(\tau)}d\tau\right].
$$

It is interesting to point out that within the operator sum formalism the distinction between the decay of the coherences and the T_2 decay in NMR systems becomes evident. While the density matrix coherences decay is completely independent from the T_1 relaxation time, there is an intrinsic dependence of T_2 with T_1 through Eq. (18). Thus, one can conclude that the phenomenological T_2 does not reflect only the decay of the quantum coherences of the system.

Before applying this formalism to a specific situation, let us make some remarks regarding a very closely related technique, the phenomenological-operator approach [11], which is also introduced to simplify the treatment of dissipative quantum systems.

VI. THE PHENOMENOLOGICAL-OPERATOR APPROACH

The phenomenological-operator approach is a technique equivalent to the operator sum representation but taking explicitly into account the state of the environment together with those of the open quantum system. From the phenomenological operators we automatically derive the Kraus operators and vice versa.

A. Phase-damping channel

First, let us consider the coupling of the spin-1/2 states to a surrounding phase-damping environment, which can be described by the map

$$
|0\rangle|\mathcal{E}\rangle \rightarrow |0\rangle\hat{\mathcal{T}}_{00}^{(p)}|\mathcal{E}\rangle \tag{34a}
$$
$$
|1\rangle|\mathcal{E}\rangle \rightarrow |1\rangle\hat{\mathcal{T}}_{11}^{(p)}|\mathcal{E}\rangle + |1\rangle\hat{\mathcal{T}}_{10}^{(p)}|\mathcal{E}\rangle, \tag{34b}
$$

where $|\mathcal{E}\rangle$ denotes the initial state of the environment and the operators $\hat{\mathcal{T}}$, acting on this state, account for the system-environment coupling. Since a phase-damping channel does not exchange energy with the system, we obviously have the identity operator $\hat{\mathcal{T}}_{00}^{(p)} = \mathbf{1}$. Regarding the excited initial state $|1\rangle$, it will remain as such, with or without an additional phase shift relative to the ground state $|0\rangle$, due to the action of the environment. In the latter case, we must impose (after the computed master equation (21) with $\Theta_a = 0$), the decaying probability $\hat{\mathcal{T}}_{11}^{(p)} = e^{-\Gamma_p t}\mathbf{1}$, with the above defined rate $\Gamma_p = 2\text{Re}[\mathcal{F}_p + \mathcal{G}_p]$, remembering that the time-dependent frequency does not lead to a time-dependent relation rate Γ_p. In the former case we have $\hat{\mathcal{T}}_{10}^{(p)} = \sum_j \mathbf{f}_j(t)(a_j^\dagger + a_j)$, with $\mathbf{f}_j(t)$ giving the probability amplitude for environment, described by the creation and annihilation operators $a_j^\dagger$ and a_j, respectively, to induce a phase shift on the excited state of the

system. Assuming that the state of the environment is modified such that $\langle \mathcal{E} | \hat{T}_{10}^{(p)} | \mathcal{E} \rangle = 0$, we obtain, after normalization of the state vector $|1\rangle |\mathcal{E}\rangle$, the relation $\sum_j |\mathbf{f}_j(t)|^2 = 1 - e^{-2\Gamma_p t}$. It is straightforward to verify that the map in Eqs. (34) leads exactly to the density operator (28) derived for the phase-damping process.

B. Amplitude-damping channel

Before addressing the nonzero temperature case, we first consider the coupling of the spin-1/2 states to an amplitude-damping environment at $T = 0$ K, described by the map

$$|0\rangle |\mathcal{E}\rangle \rightarrow |0\rangle \hat{T}_{00}^{(a)} |\mathcal{E}\rangle,$$
$$|1\rangle |\mathcal{E}\rangle \rightarrow |1\rangle \hat{T}_{11}^{(a)} |\mathcal{E}\rangle + |0\rangle \hat{T}_{10}^{(a)} |\mathcal{E}\rangle.$$

With the environment in the vacuum state we obviously have the identity operator $\hat{T}_{00}^{(a)} = \mathbf{1}$, and after the computed master equation (21) with $\Theta_p = 0$, we must impose that $\hat{T}_{11}^{(a)} = \exp[-2\int_0^t \mathrm{Re}\{\mathcal{F}_a(\tau) + \mathcal{G}_a(\tau)\} d\tau] \mathbf{1}$. For a time-independent system we obtain the usual solution $\hat{T}_{11}^{(a)} = e^{-\Gamma_a t} \mathbf{1}$, where $\Gamma_a = 2\mathrm{Re}[\mathcal{F}_a + \mathcal{G}_a]$. For the operator $\hat{T}_{10}^{(a)}$ associated with the excitation of one of the infinite environment modes, we have $\hat{T}_{10}^{(a)} = \sum_j \mathbf{g}_j(t) a_j^\dagger$, with $\mathbf{g}_j(t)$ giving the probability amplitude for the excitation of the jth oscillator mode of the environment. After normalization of the wave vector $|1\rangle |\mathcal{E}\rangle$, we obtain $\sum_j |\mathbf{g}_j(t)|^2 = 1 - e^{-2\Gamma_a t}$.

By turning our attention to the case of a nonzero temperature environment, we can write the extended map

$$|0\rangle |\mathcal{E}\rangle \rightarrow |0\rangle \hat{T}_{00}^{(a)} |\mathcal{E}\rangle + |1\rangle \hat{T}_{01}^{(a)} |\mathcal{E}\rangle,$$
$$|1\rangle |\mathcal{E}\rangle \rightarrow |1\rangle \hat{T}_{11}^{(a)} |\mathcal{E}\rangle + |0\rangle \hat{T}_{10}^{(a)} |\mathcal{E}\rangle,$$

where, now, instead of the identity operator $\hat{T}_{00}^{(a)} = \mathbf{1}$, we must account for the probability of excitation of the system due to the environment background photons. Since the operator $\hat{T}_{00}^{(a)}$ is associated with an event at which the environment is not excited, it must remain proportional to the identity. Moreover, as far as the probability for the system to remain in the ground state must decrease in a rate proportional to the environment temperature, we naturally impose that $\hat{T}_{00}^{(a)} = \sqrt{1 - (1 - e^{-2\Gamma_a t})\gamma_T} \mathbf{1}$. From the above assumption for $\hat{T}_{00}^{(a)}$ we straightforwardly obtain for $\hat{T}_{01}^{(a)} = \sum_j \mathbf{h}_j(t) a_j$, the relation $\sum_j |\mathbf{h}_j(t)|^2 = \sqrt{(1 - e^{-2\Gamma_a t})\gamma_T}$, with $\mathbf{g}_j(t)$ giving the probability amplitude for the system to be excited by the jth oscillator mode of the environment. Regarding the operator $\hat{T}_{11}$, we know that it must be also proportional to the identity since the environment must remain unaffected. However, for the case of nonzero temperature, the probability for the system to remain in the excited state must decrease in a rate smaller than the exponential decay factor $e^{-2\Gamma_a t/2}$ coming from an environment at absolute zero. Moreover, the equilibrium probability must depend on the thermal average photon number $\langle n_a \rangle$, such that we impose $\hat{T}_{11}^{(a)} = \sqrt{e^{-2\Gamma_a t t} + (1 - e^{-2\Gamma_a t t})\gamma_T} \mathbf{1}$. Consequently, for the complementary operator $\hat{T}_{10}^{(a)} = \sum_j \tilde{\mathbf{h}}_j(t) a_j^\dagger$ we obtain $\sum_j |\tilde{\mathbf{h}}_j(t)|^2 = \sqrt{(1 - e^{-\gamma t})(1 - \gamma_T)}$, $\mathbf{h}_j(t)$ being the probability amplitude

for the excitation of the jth oscillator mode of the environment at nonzero temperature.

VII. COHERENCE CONTROL

Reference [16] proposed a method to circumvent the decoherence process of a nonstationary system under an amplitude-damping channel. In that work, the authors focused on the state protection of a cavity mode whose modulation of the frequency $\omega(t)$ was engineered through the atom-field interaction. The master equation approach was used to investigate the dynamics of the cavity mode, assuming its interaction with the environment to be proportional to

$$\frac{\xi^2}{[\omega(t) - \nu]^2 + \xi^2}, \tag{36}$$

ν being the continuous frequency of the environment and the parameter ξ accounting for the Lorentzian sharpness of the coupling around the frequency $\omega(t)$. Thus the Lorentzian coupling, which is justified in the system-environment weak-coupling regime, "follows" the evolution of the frequency of the system, as expected under the sudden coupling approximation.

The modulation of the frequency was engineered to be of the form

$$\omega(t) = \omega_0 + \chi \sin \zeta t, \tag{37}$$

with ω_0 being the static frequency of the cavity mode. The condition $\zeta/\omega_0 \ll 1$, easily achieved within typical experimental conditions as in NMR and cavity quantum electrodynamics, define the adiabatic modulation of the frequency. Out of this regime, when $\zeta \gtrsim \omega_0$ we reach the regime of the Casimir-like effect [22], where the decoherence mechanism of the cavity mode is completely distinct from the one discussed here.

Within the condition $\zeta/\omega_0 \ll 1$, we show that the control of the decoherence process is achieved by means of the two parameters,

$$\eta \equiv \frac{\Gamma_0}{\zeta}, \tag{38}$$

$$\varepsilon \equiv \frac{\xi}{\chi} \sim \frac{\Gamma_0}{\chi}, \tag{39}$$

where Γ_0 is the natural decay rate of the cavity mode and we have assumed, as it is expected, that $\xi \sim \Gamma_0$. It was demonstrated in Ref. [16] that a significant attenuation of the decoherence occurs when both of these parameters are smaller than unit. This is seen from the derived time-dependent decay rate of the cavity mode, which gets weaker as one or both parameters η and ε decreases. The physical reason for this can be seen as follows: The characteristic time interval for an appreciable action of the environment over the stationary system is around Γ_0^{-1}. However, when the frequency of the system changes continuously, its rate of variation (proportional to ζ) plays a crucial role in the effective coupling between the system and the environment. Remembering that this coupling occurs around $\omega(t)$, in a region defined by the Lorentzian sharpness ξ [see Eq. (36)], a rate of variation ζ significantly

larger than Γ_0, such that $\eta \ll 1$, makes an effective action of the environment over the system difficult since their interaction time is reduced proportionally to η. Otherwise, when ζ is smaller than Γ_0, an effective action of the environment takes place, inducing the relaxation of the system before a significant change of its frequency. In its turn, the role of the amplitude of the oscillation χ is to trigger the action of the rate of variation ζ. In fact, when the amplitude χ is smaller than the Lorentzian sharpness ξ, the nonstationary system does not leave the region (in frequency space) of its effective coupling with the environment, thus decaying as a stationary system, whatever the value of ζ. However, when χ is larger than ξ, the effective system-environment coupling moves to different regions of the spectrum, thus triggering the action of the rate of variation ζ as described above [16].

We now apply the same idea to the case of a spin-1/2 system considering the Redfield formalism to treat the decoherence process. The same conclusions of Ref. [16] are obtained here, *but without imposing any specific form for the system-environment coupling*, as in Eq. (36), since a Lorentzian time-dependent effective decay rate for the spin-1/2 system automatically appears from the Redfield formalism applied to NMR systems. As far as we do not have to define the function in Eq. (36), we stress that the parameter ε, which takes place explicitly in the effective time-dependent decay rate of the cavity mode in Ref. [16], does not appear in the present spin-1/2 case. Instead, we must use the ratio χ/ζ, also defined in Ref. [16], weighting the contributions of parameters η and ε.

The spin-1/2 system, interacting with the amplitude-damping environment, is described by the Hamiltonian $H_S(t) = \omega_L(t) I_z$, with $\omega_L(t)$ being modulated as in Eq. (37). We note that such a frequency modulation bears no connection with how the protocol works, considering the solution of the differential equation governing the evolution of the longitudinal magnetization [see Eq. (16a)], which decays with the effective rate given by

$$D(t) = \int_0^t d\tau \frac{1}{T_1(\tau)},$$

$T_1(t)$ defined in Eq. (17a). Therefore, to circumvent decoherence we have to make $T_1(t)$ greater than its static value T_1^0 (equivalent to Γ_0^{-1} in the previous case of a damped cavity mode). This fact can also be directly seen from the operators in Eqs. (30), which reduces to the identity when $a(t) \to 0$, i.e., when $D(t) \to 0$. By considering an isotropic and homogeneous environment, Eqs. (14), (17a), and (37) lead to the following expression for D

$$D(\tau) = 2(\gamma_n \lambda T_1^0)^2 \int_0^\tau d\tau_2 \int_0^{\tau_2} d\tau_1 \exp\left(-\frac{T_1^0 \tau_1}{\tau_0}\right)$$
$$\times \cos\left[T_1^0 \omega_L \tau_1 + 2\frac{\chi}{\zeta} \sin^2\left(2\frac{\tau_1}{\eta}\right)\right], \quad (40)$$

where we have performed the change of variable $\tau = t/T_1^0$ and $\eta = (T_1^0 \zeta)^{-1}$ as in Eq. (38). The gyromagnetic factor γ_n and the lattice stochastic fluctuation in q direction $\lambda \equiv \lambda_q$ is defined in Eq. (12).
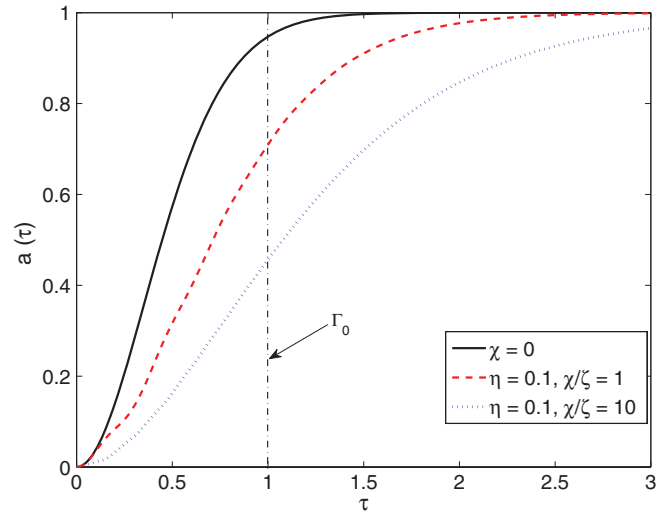

FIG. 1. (Color online) Plot of the decay function $a(t)$ defined in Eq. (33) for some values of the control parameters η and χ/ζ. The black solid line represents the uncontrolled case, where $\chi = 0$. The other two shows the effective control of the relaxation process due to the modulation of the system frequency. The vertical green dash-dotted line marks the relaxation time for the static (uncontrolled) case.

In Fig. 1 we plot the function $a(\tau) = 1 - e^{-D(\tau)}$ against the dimensionless time τ, for some values of η and the ratio χ/ζ. As can be seen, the relaxation decay rate for the stationary case $\chi = 0$, the black solid curve, is significantly attenuated when the control parameter η decreases and/or the ratio χ/ζ increases. The red-dashed and the blue-dotted curves correspond to the pairs of values $(\eta = 0.1, \chi/\zeta = 1)$ and $(\eta = 0.1, \chi/\zeta = 10)$, respectively. The curves corresponding to the pairs $(\eta = 0.01, \chi/\zeta = 1)$ and $(\eta = 0.01, \chi/\zeta = 10)$ are very close to the red dashed and blue dotted lines, respectively, and, therefore, they are not shown in the figure. From this fact we conclude that the role of parameter χ (the amplitude of the modulation) in the decoherence control is more effective than the role played by ζ (the modulation frequency). This result is in perfect agreement with the one obtained in Ref. [16]. An important observation is that this protocol is completely independent of the temperature of the environment, as well as of the initial state of the system.

A possible implementation of the proposed scheme could be realized in an NMR experiment as discussed below. In addition to the static Zeeman field $\mathbf{B}_0$ which defined the Larmor frequency $\omega_L = \gamma_n B_0$, another parallel time-dependent field $\mathbf{B}(t)$ is applied to modulate the frequency of the spin-1/2 system. This additional field may be furnished by a Helmholtz like coil surrounding the probe, traversed by a tailored current which provides the time-dependent component $\gamma_n B(t)$ of the frequency. By imposing the time dependence of the auxiliary field to be of the form

$$\gamma_n B(t) = \chi \sin(\zeta t),$$

we obtain the frequency modulation given by Eq. (37). To circumvent decoherence we must have $\eta \ll 1$ [see Eq. (38)

and Fig. 1], which implies that the modulation frequency must obeys

$$\zeta \gg \left(T_1^0\right)^{-1}. \tag{41}$$

This relation is in agreement with the adiabatic condition, since we have, in general, $T_1 \gg \omega_L^{-1}$. The other condition for the protocol to work is that the amplitude χ of the oscillation of the system frequency must be greater than the spectral sharpness of the coupling which, as said before, is expect to be of the order of $(T_1^0)^{-1}$, such that

$$\chi \gg \left(T_1^0\right)^{-1}. \tag{42}$$

Besides the conditions (41) and (42), the magnetic field $B(t)$ must also obey the adiabatic condition [see the equation below Eq. (37)], which leads to $|B(t)| \ll |B_0|$. This condition can be easily attained in a typical NMR experiment. Considering a hydrogen nuclei at $B_0 \sim 10$ T ($\omega_L \simeq 400$ MHz) and $T_1 \sim 1$ s, we obtain, together with condition (41), the following range for the magnitude of the control field $B(t)$ for the protocol to work, i.e., $(\gamma_n T_1^0)^{-1} \ll |B(t)| \ll |B_0|$:

$$10^{-6} \text{ T} \ll |B(t)| \ll 10 \text{ T}.$$

Note that the NMR setup is appropriate for this kind of experiment due to the fact that the relaxation time is relatively large compared with other platforms. This fact permits us to realize the experiment in a high-temperature environment. It is also important to observe that, despite the fact that in the NMR experiments both amplitude and phase damping are present, there exist experiments where only the amplitude damping is probed, so they can be used to verify our proposals. Therefore, we can control only the relaxation time T_1 but not T_2.

VIII. FINAL DISCUSSIONS

We have presented here a unified view of the semiclassical Redfield formalism and the quantum master equation approach for a time-dependent spin system. Focusing on a spin-1/2 system, we showed the equivalence between both approaches through the fact that they lead to the same Bloch equations and, consequently, to the same characteristic longitudinal T_1 and transversal T_2 relaxations times. We verified that only T_1 is affected by the time dependency of the system frequency and built the Kraus and the phenomenological operators for the spin-1/2 system under both the amplitude- and the phase-damping channels assumed within the master equation approach.

As an application, we revisited a protocol to circumvent relaxation and, consequently, the coherence control of a nonstationary system [16]. In contrast to the protocol in Ref. [16], in the present case, we do not imposed a functional form for the system-environment coupling, which emerges naturally from the Redfield semiclassical formalism. The coherence control of the spin-1/2 system was demonstrated by enlarging the longitudinal relaxation time through the modulation of the system frequency. We stress that the protocol in Ref. [16], translated here for the spin-1/2 system, differs from the dynamical decoupling methods presented in the literature [15], where one must interfere in the system in time scales less than the bath correlation time. The frequency modulation technique takes advantage of the pre-existing natural frequency of the system, adding to it a small amplitude to achieve such time scales. We also discussed the implementation of this protocol to control the longitudinal relaxation time in the NMR context.

It is worth stressing that the development present here applies for Markovian environments and adiabatic modulation of the required time-dependent frequency. Therefore, an extension of the present development for non-Markovian environments as well as for interacting time-dependent systems would be desirable, since non-Markovian environments is present in many promising platforms for quantum-information processing such as photonic crystals [23]. Moreover, time-dependent interacting system would considerably enlarge the perspective of the present work for quantum-information purposes.

Although the entanglement is not usually present in the NMR system [24], there exists classical and quantum correlations that could be useful for some quantum-information protocols [25]. The decoherence control exposed here can then be directly applied to protect these correlations from the action of the amplitude-damping channel. Finally, we stress that the theory presented here is completely general and can be applied to different platforms for quantum-information processing such as cavity quantum electrodynamics, trapped ions, quantum dots, and so on. However, for this statement to be valid we must assume that all the rotations needed to implement quantum-information processing occur at the beginning or at the end of the experiment; in between them, the system evolves only under the action of the environments.

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